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CP VIOLATION, FLAVOUR VIOLATION AND FERMION MASS RELATIONS IN
SOME HORIZONTAL GAUGE THEORIES

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Abstract

Six quark horizontal gauge models incorporating a natural suppression mechanism for diagonal flavour-changing currents are considered. Some interesting possibilities for CP violation, flavour violation, fermion mass and mixing angle relations in these models are studied.

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A previous work [1] dealt with a natural mechanism for suppressing flavour-changing currents in horizontal gauge models. The existence of this mechanism was pointed out to the author by Lincoln Wolfenstein.

Ref. [1] did not present a fully developed model with six quarks. In this paper it is shown that models incorporating the flavour-change suppression mechanism can be constructed. An interesting aspect is that in some of the models CP violation in the theory arises only due to the horizontal gauge boson (S boson) interactions, and the CP-violating phase in the usual weak interactions is zero. In other cases CP violation in the $K^0-\bar{K}^0$ transition is due to the weak interactions only, and the S-boson interactions are CP conserving in this process. The model uses a $SU(2)_H$ group and puts the fermions in triplet representations of this horizontal group. A similar model was considered by F. Wilczek and A. Zee [2] to predict relations between quark masses and mixing angles. A four-quark $SU(2)_H$ model was considered by T. Maehara and T. Yanagida [3] to get CP violation with only four quarks. R.N. Cahn and H. Harari [4] also considered the four-quark $SU(2)_H$ model.

The model presented here uses a slightly more general version of the suppression mechanism of ref. [1]. It was shown there that if a $SU(n)_H$ gauge group is introduced and fermions are put into fundamental multiplets, the horizontal gauge bosons conserve flavour in diagonal processes (processes involving only two flavours) under certain conditions. The condition that fermions occur in fundamental multiplets can be relaxed in the following way: If the fermions occur in a (ntm) dimensional representation with $m \neq 0$, the (ntm) dimensional unitary matrix U which diagonalizes the fermion mass matrix must be generated by the (ntm) dimensional $SU(n)$ generators τ^a which occur in the fermion

covariant derivatives, i.e., $U = e^{ie\tau^a}$. [Repeated indices are summed from 1 to (n^2-1) .] This condition can be rephrased as follows: the unitary matrix which diagonalizes the fermion mass matrix must belong to the $SU(n)$ subgroup of $U(n+m)$ generated by the τ^a . In ref. [1] the fermions were put into fundamental representations because in that case the traceless hermitian matrix $U^\dagger \tau^a U$ could be expressed as a linear combination of the τ^a . If this expansion is possible even under the relaxed condition stated here, then the rest of the proof is the same as in ref. [1]. The possibility of the expansion follows immediately from the identity [5]:

$$e^{-ie\tau^a \tau^b} = \tau^a + (-i) \left[\epsilon_{\tau^a \tau^b} \right] + \dots + \frac{(-i)^n}{n!} \left[\epsilon_{\tau^a \tau^b}, \left[\epsilon_{\tau^c \tau^d}, \left[\epsilon_{\tau^e \tau^f}, \dots \left[\epsilon_{\tau^g \tau^h}, \tau^a \right] \dots \right] \right] \right] + \dots \quad (1)$$

and from the $SU(n)$ commutation relations $[\tau^a, \tau^b] = C_{ab}^c \tau^c$. The rest of the proof is the same as in ref. [1]. The proof essentially states that if the S bosons are degenerate the effect of fermion mixing can be cancelled by suitably mixing the S bosons, and processes forbidden before fermion mixing remain forbidden after the mixing.

The key idea behind the model is that if the mass matrix of the charge $-1/3$ quarks is a real symmetric matrix it can be diagonalized by a real orthogonal transformation. Since the group of real orthogonal three-dimensional matrices is generated by the $SU(2)$ generators τ^a occurring in the fermion covariant derivatives, the suppression mechanism of ref. [1] operates and the strangeness-changing neutral current is suppressed. The model consists of three generations of quarks and leptons with the usual $SU(2)_W \times U(1)$ multiplets. A local $SU(2)$ gauge symmetry is introduced and the fermions transform as triplets under $SU(2)_H$. The theory has five Higgs multiplets ξ_a , ξ_b , ω_a , ω_b and χ . The first four transform as doublets under $SU(2)_W$.

ξ_a and ξ_b transform as quintuplets (Isospin 2) under $SU(2)_H$. ω_a and ω_b are singlets under $SU(2)_H$. χ is a doublet under $SU(2)_H$ and a singlet under $SU(2)_W$. It gives a heavy mass to the horizontal gauge bosons. An additional discrete symmetry D_1 is introduced. Different choices of D_1 lead to different predictions. The choices considered here are given below (note that in some of the cases additional Higgs fields are introduced):

case (i) : $\{u_R\} \rightarrow -\{u_R\}$, $\xi_b \rightarrow -\xi_b$, $\omega_b \rightarrow -\omega_b$, every other multiplet invariant.

case (ii) : Same as above except $\{e_R\} \rightarrow -\{e_R\}$.

case (iii) : Same as case (ii) except no ω_b is introduced.

case (iv) : Same as case (ii). (Assumptions for vacuum expectation values differ.)

case (v) : Same as case (i) except that no ω_b is introduced.

case (vi) : Same as case (v) except that a Higgs field ξ_c is introduced transforming like ξ_a .

The choice of discrete symmetry determines which Higgs and fermion fields are allowed to couple. For example, the up-type quarks can only get their masses from the Higgs labelled by "b". The down-type quarks never couple to these Higgs fields. In cases (i) and (ii) it is assumed that the vacuum expectation values (VEVs) of all Higgs fields are real except for that of ω_b . In cases (iii)-(vi) it is assumed that all VEVs are real except for the VEVs of the ξ_b field. Since such patterns of symmetry-breaking can conceivably occur for a continuous range of the Higgs potential parameters, the assumptions are not unnatural (in the technical sense of the term). The possibility of writing down a renormalizable, gauge-invariant Higgs potential leading to the assumed pattern of symmetry-breaking has, however, to be studied. In this paper we will not study

the Higgs sector in any great detail. We will also neglect the possibility of mixing of the Higgs fields. Table 1 summarizes the transformation properties of the Higgs.

We will now consider the phenomenological implications of the different cases. The $SU(2)_H$ quintuplets lead to symmetric mass matrices for the fermions because they couple to the fermions as $f_{\xi q} \bar{\psi}_L^i \xi^j \psi_R^j$ where ψ_L and q_R are $SU(2)_W$ doublet and singlet fermion fields, respectively, and i, j are $SU(2)_H$ indices. $f_{\xi q}$ is a Higgs-fermion Yukawa coupling, and ξ^j is the symmetric traceless representation of the ξ Higgs multiplet. Because the $\{d_R\}$ multiplet is left unchanged by D_1 it cannot couple to the Higgs fields ξ_b or ω_b . Since these are the only fields which get complex VEVs in all cases, the mass matrix of the d-type quarks is a real symmetric matrix. It is not traceless because $\{d\}$ also couples to the $SU(2)_H$ singlet ω_a . The mass matrix of the d-type quarks can be diagonalized by a real orthogonal transformation, and all the conditions for natural flavour-change suppression are satisfied. Thus, diagonal flavour-changing processes involving the charge $-1/3$ quarks, in particular the $K^0-\bar{K}^0$ transition, are suppressed in all cases.

When we consider cases (i) and (ii) we get very interesting consequences for CP violation. In these cases the only complex VEV is due to ω_b , which contributes a multiple of the unit matrix to the mass matrix. This means that all fermion mass matrices can be diagonalized by real orthogonal transformations, except that fermions coupling to ω_b pick up complex phase factors which have to be absorbed by either the left or right helicity fermion fields. For example, the phase terms which remain in the charge $2/3$ mass matrix after it is diagonalized by a real orthogonal transformation can be absorbed by the right-handed fields, which

implies that all mixing angles in the left-hand sector are real and the Kobayashi-Maskawa (KM) matrix cannot have a complex CP-violating phase. Thus, in cases (i) and (ii) the weak interactions are CP conserving.

This situation regarding the mass matrix is very similar to the situation which occurs in a discussion by G. Branco [6] of CP violation due to Higgs particles in theories where the Higgs particles conserve flavour naturally. In his case the resulting model was purely millitweak while in our case the model is superweak [7] CP violating*. Since the phase angles in the charge $2/3$ sector can be absorbed in the right-hand fields or in the left-hand fields, interactions involving fields of only one helicity will not violate CP. This is why the weak interactions are CP conserving in this case. This helicity suppression will also manifest itself in CP violation in the $K^0-\bar{K}^0$ transition. Let us consider this process. The S boson interactions in the d-type quark sector are CP conserving, and CP violation in $K^0-\bar{K}^0$ must occur due to both weak and horizontal boson interactions occurring together in the Feynman diagrams. A diagram which might contribute to CP violation is shown in fig. 1. An estimate of the order of magnitude for this diagram would be $g_W^2 \frac{m_c m_c}{M_W^2 M_S^2} \frac{g_S^2}{M_S^2}$ times mixing angle factors and possible logarithmic factors. m_t and m_c are the masses of the top and charmed quarks, g_W and g_S are the weak and horizontal gauge coupling constants, and M_S is the mass of the S boson. For reasonable values of parameters like m_t , mixing angles, etc. the value of M_S needed in the above estimate to give the observed value of the ϵ is $\sim 10^4$ GeV.

*In such cases where the Kobayashi-Maskawa matrix is real, higher-order corrections will generally contribute small complex terms to the mass matrix. Thus, the KM matrix may ultimately pick up a complex phase which presumably will be very small.

imaginary part of the K_L - K_S mass difference is consistent with the limits set by flavour-changing processes [1]. CP violation for D^0 - $\bar{D}^0$ is suppressed by the S boson mass differences. Fig. 2 shows a diagram contributing to the neutron dipole moment. CP violation from this diagram is suppressed when left and right mixing angles are equal [3]. The reason is the same as for the suppression of diagonal flavour-changing processes: one can rotate the S boson fields and define new S boson states such that the interactions between these redefined fields and the fermion mass eigenstates are exactly the same as the interactions between the original S boson states and fermion gauge eigenstates. Thus the CP conserving property of the S boson interactions is preserved even after fermion mixing, provided the diagrams contain only fermions of the same charge. If the right hand fermion mixing matrix is the product of a diagonal phase matrix times the left hand mixing matrix, processes forbidden before fermion mixing remain forbidden after fermion mixing. However, processes allowed before fermion mixing, like the one in fig. 2, now pick up complex factors. Thus fig. 2 contributes to the neutron dipole moment in our model unlike in ref. [3]. The present model has the required magnitude of CP violation in K^0 - $\bar{K}^0$ for $m_t = 20$ GeV, $g_S = g_W$ and $M_S = 3000 M_W$. The neutron dipole moment is then of order 10^{-26} e-cm [3] and is below present limits.

For cases (i) and (ii) all diagonal flavour-violating processes are suppressed. The only difference between the cases is in the Higgs bosons which give masses to the charged leptons. In case (i) these bosons are the same ones which give masses to the charge $-1/3$ quarks. This means that the mixing angles in the lepton sector are the same as

the angles in the d-quark sector. The equality of mixing angles suppresses the process $K^+ \rightarrow \pi^+ \bar{\mu} e$ by a factor $\Delta M_S/M_S$ where ΔM_S is the S boson mass degeneracy breaking. The reason is the same as for the suppression of diagonal processes, namely, the rotation taking the fermion gauge eigenstates to the mass eigenstates can be cancelled by a rotation of the S boson fields if the latter are degenerate [1]. For the same reason the μe current coupling to the $\bar{d}d$ current is also suppressed. Thus, μe conversion takes place only due to the coupling to the $\bar{u}u$ current. The dependence of the μe conversion rate on atomic number Z is then uniquely fixed [8], but there is no other phenomenological consequence. The discrete symmetry D_1 gives one mass relation: $\frac{m_{\tau} m_\mu}{m_t m_e} = \frac{m_b m_s}{m_b m_d}$. The derivation of the mass relation requires cancelling two large quantities to get a small one. Hence the uncertainties introduced by renormalisation of the coupling constants, higher-order contributions to the mass matrix, etc. as well as the uncertainties in the quark mass definitions due to strong interaction renormalisation, due to contributions from chiral symmetry-breaking, etc. [9] make a test of this mass relation difficult.

For case (ii) the charged leptons get their masses from the Higgs which give masses to the up-type quarks. Hence the mixing angles of the two sectors are equal. The process $K^+ \rightarrow \pi^+ \bar{\mu} e$ is now suppressed only by mixing angles. Table 2 presents the upper limits predicted for flavour-violating processes. As in ref. [1], M_S was fixed by μe conversion to be heavier than $3000 M_W$ (this is an order of magnitude estimate), and $\Delta M_S/M_S$, the S boson mass degeneracy breaking, was fixed by the K_L - K_S mass difference to be less than 0.004. One does not get any

mass relations in case (ii) because of the extra parameter introduced by the phase of the VEV of w_b . One could have let w_a develop a complex VEV rather than w_b . In that case the model has similar consequences to the four quark model of refs. [3] and [1]. CP violation would be due to the S bosons alone and is suppressed by $\Delta M_S/M_S$. For $M_S = 3000 M_W$ and $\Delta M_S/M_S = 10^{-5}$ it is consistent with present phenomenology. Unlike the four quark version of ref. [3] the contribution of fig. 2 to the neutron dipole moment is not suppressed. The neutron dipole moment is of order $10^{-27} e\text{-cm}$.

We now come to cases (iii)-(vi). One of the main differences between these and the previous cases is in the CP-violation phenomenology. In cases (iii)-(vi) the Higgs multiplet ξ_b is assumed to get complex VEVs. The mass matrix of the u-type quarks is no longer diagonalizable by a unitary transformation (except for special choices of the VEVs). The biunitary transformation which diagonalizes the mass matrix is generally complex, and the weak interactions can pick up a complex CP-violating phase in the KM matrix. Hence the phenomenology of CP violation is essentially the same as in the standard model with a complex phase in the KM matrix, except that now there are other sources of CP violation also. For the $K^0-\bar{K}^0$ process the only contribution comes from the KM matrix. For the neutron dipole moment there can be additional contributions from the S bosons and from Higgs particles. CP violation in the $D^0-\bar{D}^0$ process could also get contributions from these sources.

Since the mass matrix of the u-type quarks is now complex and not necessarily diagonalized by a unitary transformation, the tracelessness condition does not imply that $m_t = m_c + m_u$. Hence one can dispense with

the w_b Higgs field, and this is what is done in case (iii). The consequences of case (iii) for flavour violation are presented in table 2.

In this case we get the mass relations $\frac{m_e}{m_u} = \frac{m_\mu}{m_c} = \frac{m_\tau}{m_t}$ and the mixing angles in the charged lepton and charged 2/3 quark sectors are equal.

The mixing angles would show up in flavour-violating processes. As discussed by Georgi and Jarlskog [9], the quark masses and their momentum dependences are very poorly understood for the light quarks. The best we can do with the mass relation is to calculate the mass of the t quark using the second of the two equations, and hope that there will not be large renormalisation corrections and other corrections, specially for the mass of the charmed quark. A naive calculation gives $m_t = 26 \text{ GeV}$. The same mass relation as the above was postulated by Glashow [10], but in the grand unification regime. One could choose not to throw away the Higgs field w_b . None of the consequences would change except that the mass relations and mixing angle relations would no longer be valid.

Case (iv) merely states that the Higgs field w_b has not been thrown away. The mass and mixing angle relations are lost because there are now two Higgs fields ξ_b and w_b , and the ratio of the Yukawa couplings to these Higgs multiplets can be different for the charged leptons and the u-type quarks. Note that the addition of a multiple of the unit matrix to a non-hermitian mass matrix can significantly change mixing angles and masses.

In cases (iii) and (iv) charged leptons got their masses from ξ_b (and also w_b). One could arrange it such that they get their masses from ξ_a and w_a , and this is what is done in case (v). The only difference from case (i) is in the CP violation, resulting from different assumptions regarding the VEVs of the Higgs. The flavour-violation phenomenology and

mass relations of case (v) are the same as for case (i). In case (vi) an extra Higgs multiplet ξ_c with properties similar to ξ_a is introduced. Since the Yukawa coupling of ξ_c introduces an extra parameter, the mass relations and mixing angle relations of case (i) are lost. The absence of mixing angle relations between the lepton and d-quark sectors removes the suppression of $K^+ \rightarrow \pi^+ \bar{\nu}_e$ and $(\bar{e}\mu)(\bar{d}d)$ processes that occurs in case (v). The lepton number phenomenology is exactly the same as in ref. [1]. Note that for cases (v) and (vi) the presence or absence of w_b makes no difference to the phenomenology.

The models considered above do not by any means exhaust the possibilities of extending the mechanism of ref. [1]. For example, if the left-handed fermions are put into $SU(2)_H$ singlets, then one can relax the condition that mixing angles in the right and left sectors be equal. A model in which the right-handed particles are in an $SU(2)_H$ doublet and a singlet (and the left-handed fermions are in singlets), and in which a discrete symmetry permits the coupling of each left-handed fermion to only one right-handed multiplet, has the following features: (1) There is one massless fermion in lowest order. This will pick up a mass in higher order due to flavour-violating Higgs particles. (2) In lowest order one can get a non-trivial but real KM matrix. (3) Diagonal flavour-violating interactions due to the S bosons will be naturally suppressed. It is not obvious to what extent the latter two properties would be preserved in higher order. The higher-order terms cannot be too small because they have to give mass to the massless fermion. Theories in which the masses of the lightest fermions are calculable higher-order effects have intrigued many physicists [11]. If we introduce a fourth generation of fermions and consider two right-handed $SU(2)_H$ doublets (say, $\{e_R, \tau_R\}$).

$\{\mu_R, \nu_{\mu}\}$, M a new heavy lepton), then in lowest order there will be two massless particles which pick up mass in higher order. In such a model the masses of the muon and electron may be calculated in terms of the other lepton masses and Higgs sector parameters. These are interesting possibilities but will not be followed up here.

Conclusions

We have constructed six quark horizontal gauge models with natural suppression of diagonal flavour-changing processes. We have studied the phenomenology of CP violation, flavour violation and fermion mass and mixing angle relations in these models.

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Table 1
Transformation properties of the Higgs particles.

Particle	needed for case:	Quantum numbers			D_1 parity
		$SU(2)_H$ isospin	$SU(2)_W$ isospin	$U(1)$ hypercharge	
ω_a	i-vi	0	1/2	1	even
ξ_a	i-vi	2	1/2	1	even
ξ_b	i-vi	2	1/2	1	odd
χ	i-vi	1/2	0	0	even
ω_b	i, ii, iv	0	1/2	1	odd
ξ_c	vi	2	1/2	1	even

Figure captions

Fig. 1. Contribution to CP violation in $K^0-\bar{K}^0$ transition.

Fig. 2. Possible contributions to neutron dipole moment.

Fig. 1

Fig. 2

Process	Cases (I), (II), (V), (VI)	order of suppression magnitude for Rthe	order of suppression magnitude for Rthe	order of suppression magnitude for Rthe	Experimental upper limit
$\Gamma(u\gamma)$	$\frac{\Gamma(u\gamma)}{\Delta M_S}$	2×10^{-16}	mixing angles	10^{-11}	2×10^{-10}
$\frac{\Gamma(u\gamma)}{\Delta M_S}$	$\frac{\Gamma(u\gamma)}{\Delta M_S}$	3×10^{-16}	mixing angles	2×10^{-11}	2×10^{-9}
$\frac{\Gamma(u\gamma)}{\Delta M_S}$	$\frac{\Gamma(u\gamma)}{\Delta M_S}$	determines	mixing angles	determines	7×10^{-11}
$\frac{\Gamma(u\gamma)}{\Delta M_S}$	$\frac{\Gamma(u\gamma)}{\Delta M_S}$	mixing angles	mixing angles	M_S	(suphur)
$\frac{\Gamma(u\gamma)}{\Delta M_S}$	$\frac{\Gamma(u\gamma)}{\Delta M_S}$	none	none	4×10^{-15}	2×10^{-9}
$\frac{\Gamma(u\gamma)}{\Delta M_S}$	$\frac{\Gamma(u\gamma)}{\Delta M_S}$	none	none	6×10^{-13}	5×10^{-9}
$\frac{\Gamma(u\gamma)}{\Delta M_S}$	$\frac{\Gamma(u\gamma)}{\Delta M_S}$	10^{-17}	mixing angles	6×10^{-13}	7×10^{-9}

Lepton number violation in the model. M_S is fixed at $3000 M_W$ and $\Delta M_S/M_S$, the S boson degeneracy breaking, is fixed at 0.004 .

Table 2