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THE SCATTERING OF POSITIVE KAONS BY NUCLEI†

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Abstract

We examine the sensitivity of low energy K^+ -nucleus scattering to both the poorly known K^+ -nucleon phase shifts, and to the neutron distribution. By using the technique of isotope comparison which has worked well for low energy pions we find that there seem to be "windows" where one could already learn something quantitative about the neutron distributions in 4_0 , ${}^8_0\text{Ca}$ for example.

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I. Introduction

Strong hopes have been expressed recently that K^+ -nucleus scattering will be a sensitive tool for studying neutron density distributions in nuclei (1). For a discussion of the alternative probes available for neutron distributions, and the theoretical significance we refer to the review at the recent Berkeley conference (2). Prerequisites to the success of K^+ scattering in this role are that: a) the K^+ -nucleon (K^+ -N) scattering amplitudes are known precisely; b) the K^+ -nucleus multiple scattering theory is sufficiently unambiguous to provide a reliable microscopic optical potential; c) K^+ -nucleus scattering shows some sensitivity to the neutron distribution. The optimism thus far is based on the success on point (b), in comparison with other projectiles. In fact, only very low energy pions have a mean free path comparable to that of the K^+ . The latter has a mean free path longer than a typical nuclear radius, and thus single scattering dominates. This was recognised and studied as long ago as 1957 (3). However, there is even now considerable uncertainty on point (a), and therefore also on point (c), because of our lack of knowledge of the isospin zero K^+ -N phase shifts.

In the present work we present some case studies of K^+ -nucleus elastic scattering in the "low" energy region. We place particular emphasis on its sensitivity to neutron densities in order to provide some guidance for future experimental work in this area. For these calculations we use both the standard K-N phase shifts of Martin (4), and the more recent analysis of Watts *et al.* (5). [Watts *et al.* (5) compare their own phases with the latest analysis of the BGRT group(6).] The difference between the older BGRT phases (7) and those of Martin (4)

has already been demonstrated in ref. (1).

In order to make our case studies as useful as possible we have made calculations for three isotopic pairs $^{12,13}\text{C}$, $^{16,18}\text{O}$ and $^{40,48}\text{Ca}$, which have been extensively studied using other probes (2,8). We also make extensive use of the technique of looking only at isotopic differences (rather than absolute sizes), because it has proven so successful in reducing ambiguities (2) in the analysis both of low energy π^- scattering (9), and of high energy proton scattering (2). The calculations have been performed at K^+ laboratory kinetic energies of 50, 100 and 200 MeV.

II. Construction of the K^+ -nucleus optical potential

In many respects the construction of the K^+ -nucleus optical potential in terms of the basic K^+N interaction is even simpler than the pionic case (10,11,12). For example, there is nothing analogous to the true absorption contribution for pions. The p-wave interaction is much less important for low energy kaons than for pions. This is extremely important, as we shall see, in reducing the importance of non-linearities in the optical potential, such as the Lorentz-Lorenz effect (10). In addition it reduces the importance of Fermi-averaging in the energy region considered here. The only modification of the KN amplitudes in the nuclear medium which we include, is the Pauli blocking in the absorptive part of the potential.

The potential then has the following terms (10,13,14):

$$[1] \quad 2\bar{w} U_{\text{opt}} = 4\pi \left[-b(r) - \frac{P_1-1}{2} \nabla^2 c(r) + \vec{\nabla} L(r) c(r) \vec{\nabla} \right] + 2\bar{w} V_C(r);$$

where

$$[2] \quad b(r) = P_1 (\bar{b}_{0\rho}(r) - \kappa \bar{b}_1 \delta\rho(r));$$

$$[3] \quad c(r) = \frac{1}{P_1} (\bar{c}_{0\rho}(r) - \kappa \bar{c}_1 \delta\rho(r));$$

$$[4] \quad L(r) = \left[1 + \frac{4\pi\lambda}{3} \left(\frac{A-1}{A} \right) c(r) \right]^{-1};$$

with

$$[5] \quad \delta\rho(r) = \rho_n(r) - \rho_p(r);$$

$$[6] \quad P_1 = \frac{1 + \varepsilon}{1 + \varepsilon/A}.$$

The electric potential is given by:

$$[7] \quad V_C = \kappa e^2 \int \frac{\rho_C(r')}{|r-r'|} d^3r';$$

where $\kappa = \pm 1$ gives the kaon charge, and the charge distribution $\rho_C(r)$ is normalized such that $\int \rho_C(r) d^3r = Z$. The kinematical factor P_1 contains $\varepsilon = w/M$, with w being the initial relativistic kaon energy, and M the average nucleon mass in the nucleus (931.5 MeV).

We have chosen to use the notation which is standard in the field of pion-nucleus interactions—e.g. ref. (14). Notice especially that the coefficient b_1 of ref. (1) corresponds to our c_0 , and that our b_1 and c_1 are the isovector s- and p-wave scattering length, and volume, which have not been used in many kaon-nucleus studies. These isovector terms are of course essential for the study of neutron distributions. Equation [1] also contains the relativistic reduced energy $\bar{w}$ of the kaon-nucleus system. The Ericson-Ericson factor (10), $L(r)$, which weakens the p-wave interaction to an extent determined by the parameter λ (or ξ) (13), arises from the inclusion of short range correlations between nucleons. This term inhibits second order scattering (15) completely for $\lambda = 1$, a value which will be adopted for our calculation for K^+ —apart from one comparative calculation in order to show its influence.

The above specified optical potential, inclusive of the electric part, is then introduced in the semi-relativistic wave-equation:

$$[8] \quad (\nabla^2 + k^2 - 2\bar{w} U_{opt})\phi(r) = 0.$$

The linear behaviour in the potential is chosen because of the cleaner connection to multiple scattering as far as the strong interaction part is concerned, and with respect to the electric potential, because we match to Coulomb-wavefunctions outside the nucleus. The momentum k of the kaon is to be taken in the kaon-nucleus c.m. system.

The isoscalar and isovector s- and p-wave parameters b_0 , b_1 , c_0 , and c_1 are deduced from the K^+N partial wave amplitudes according to the parametrization of Martin (4). Since the notation changes so widely in the literature, we first define these parameters with respect to the amplitudes of Martin:

$$[9] \quad f = f(w, \theta) = \frac{1}{q} \sum_{\ell=0}^{\infty} \left[(\ell+1) F_{\ell+}(w) + \ell F_{\ell-}(w) \right] P_{\ell}(\cos \theta),$$

where w and q are the relativistic c.m. energy and initial c.m. momentum with respect to the *nucleon*. Introducing now the isospin index one writes:

$$f(K^+p \rightarrow K^+p) \equiv f_p = f^1,$$

$$f(K^+n \rightarrow K^+n) \equiv f_n = \frac{1}{2}(f^1 + f^0),$$

$$f(K^+n \rightarrow K^0p) \equiv f_x = \frac{1}{2}(f^1 - f^0) = f_p - f_n,$$

$$[10] \quad f_p + f_n \equiv f_d.$$

In the single scattering approximation in an uncorrelated (Z+N) system (with identical point neutron and proton densities) the total amplitude is proportional to:

$$[11] \quad Zf_p + Nf_n = (Z+N) \frac{f_p + f_n}{2} - (N-Z) \frac{f_p - f_n}{2},$$

and we see that our isoscalar and isovector parameters correspond (apart from a separate momentum behaviour) to half of the deuteron and charge exchange amplitudes. In more detail the potential parameters in eq.

[1] are defined by: (the superscript on F denotes isospin, the subscript the orbital and total angular momentum)

$$b_0 = \frac{1}{q} \frac{1}{2} \left[F_0^1 + \frac{1}{2}(F_0^1 + F_0^0) \right] = \frac{1}{q} \frac{1}{4} \left[F_0^0 + 3F_0^1 \right],$$

$$b_1 = \frac{1}{q} \frac{1}{2} \left[F_0^1 - \frac{1}{2}(F_0^1 + F_0^0) \right] = \frac{1}{q} \frac{1}{4} \left[F_0^1 - F_0^0 \right],$$

$$c_0 = \frac{1}{q} \frac{1}{4} \left[6F_1^1 + 3F_1^1 + 2F_1^0 + F_1^0 \right],$$

$$[12] \quad c_1 = \frac{1}{q} \frac{1}{4} \left[2F_1^1 + F_1^1 - 2F_1^0 - F_1^0 \right].$$

These "bare" parameters, b_0 , b_1 , c_0 and c_1 , being scattering lengths (respectively volumes) at zero energy, are generally not constant at momenta different from zero, and the additional momentum dependence is not transferred into the gradient structure of the potential but rather introduced in a simple, functional way.

Appropriate averaging procedures have eventually to be applied, and this is one reason why we have "effective" parameters $\bar{b}_0$, $\bar{b}_1$, $\bar{c}_0$ and $\bar{c}_1$ in the meson-nucleus potential. In order to be able to decide about the importance of these averaging procedures, we display the real and imaginary parts of b_0 and the real parts of b_1 and c_0 (in units of pion Compton wavelength) versus the c.m. momentum q , from 90 to 720 MeV/c, in Figs. 1 to 4. The curves extending over the whole range represent the phase-shift analysis of Martin (4), and the lines starting at about 430 MeV/c correspond to the recent phase-shift analysis of Watts *et al.* (5).

We think that for our c.m. momenta (≤ 440 MeV/c) we need not Fermi-average the amplitudes. However, Pauli-blocking will be taken into account for the imaginary parts of the parameters by applying the Goldberger-Clementel-Villii suppression factor, developed for pions by Landau and McMillan (16). Since the momentum of a 50 MeV incident K^+ is already above the typical Fermi momentum of a finite nucleus ($k_{lab} = 228$ MeV $\geq k_F$), this reduces to a simple multiplicative factor for the imaginary part of the potential—namely $\{1 - 0.4\} (k_F/k)^2$. We have further simplified the procedure by choosing $k_F = 200$ MeV/c for all three isotopic pairs considered.

III. Results

A naive inspection of the relative size of the isovector to isoscalar pieces of the K^+N interaction tells us that the sensitivity to isotopic differences should be lower for kaons than for pions. [For pions at 50 MeV, $\bar{b}_1/\bar{b}_0 \sim 2.5$ and $\bar{c}_1/\bar{c}_0 \sim 1$ —the latter was exploited particularly in ref. (9).] However, the complications of true absorption cloud the situation a little for pions. Whereas, the angular distribution of low energy pion scattering shows strong interference effects even for light elements, for kaons we find essentially smoothly decreasing cross-sections. To illustrate this, we compare in Fig. 5, the elastic angular distributions for 50 MeV (T_{lab}) pions and kaons on ^{16}O .

The absence of diffractive behaviour for kaons seems to make the study more difficult, since it is experimentally more difficult to measure absolute values of cross-sections. If, however, ratios of differential elastic cross-sections are measured—as was done for pions

by the TRIUMF group (9)—we may expect to extract quite sensitive information from kaon-nucleus scattering. In the following we discuss our calculations on the basis of such ratios between isotopes.

Figure 6 shows the ratio of the elastic differential cross-sections between ^{12}C and ^{13}C . The solid line represents "standard" parameters—i.e. the potential strength parameters $\bar{b}_0, \bar{b}_1, \bar{c}_0$ and $\bar{c}_1$ are calculated from Martin's amplitudes (4) according to the procedure specified above (eqs. [9]–[12]). The dashed line has the same strength parameters, the only difference being the density distribution of the neutrons which has an rms radius different by 2% from that in the ^{12}C . We see that the ratio varies by only 10% and that the present level of accuracy is a fraction of this ratio, actually a fifth of the variation itself, probably too small to be experimentally significant at the present time. We recall that at lowest-energy pion scattering (30 MeV π^-) the variation of the ratio for the carbon isotopes was roughly 70% (9). However, the situation for K^+ improves rapidly with increasing energy.

We now turn to the oxygen isotopes $^{16}O, ^{18}O$, and show first, in Fig. 7, the angular distribution of the elastic scattering for ^{16}O at 200 MeV lab energy. The solid line is again with the standard parameters and root mean square radii of $(r_p)_{16} = (r_n)_{16} = (r_p)_{18} = 2.60$ fm and $(r_n)_{18} = 2.81$ fm for the modified Gaussian shape. The dashed line is for 200 MeV incident K^+ once more but with Watts parameters (5). We see at 200 MeV, which is in a region of overlap of the analyses of Martin and Watts, that there is a high uncertainty coming into play because of the still very different phase-shift analysis of KN data. Looking then at the ratios $\sigma_{el}(^{16}O)/\sigma_{el}(^{18}O)$, we find in Fig. 8 that

at 200 MeV the curves are sufficiently different between both phase-shift analyses so that a kaon nuclear cross-section ratio measurement should even help to discriminate between different sets of KN amplitudes—albeit in a very gross way. This is clearly not a case to study nuclear structure yet, since changes from nuclear structure differences are generally smaller. We show in the dashed line the ratio using the standard set of Martin, but with a changed neutron distribution for 180 —i.e. a decrease of the radius by 5% to $(r_n)_{18} = 2.67$ fm.

Figure 9 represents a test of the sensitivity to the Ericson-Ericson factor $L(r)$, by comparing a standard calculation ($\lambda=1$) for $^{160}/^{180}$ at 100 MeV (solid line), with a nearly standard calculation but for $\lambda=0$ (dashed line). The difference is remarkably small, confirming both the relative absence of double scattering corrections, and the small sensitivity to p-wave interaction. Comparison is also given to a calculation where the neutron rms radius in 180 has been changed by 5% (curve marked "change").

Finally we display the equivalent situation to Fig. 8 for the calcium isotopes $^{40}, ^{48}\text{Ca}$ in Fig. 10. We find it interesting that the situation has changed there, and that the ratio is more sensitive to nuclear structure than it is to the difference in the present phase-shift analyses. There is a 5% change in the rms radius of the neutron distribution of ^{48}Ca between the solid and the dashed line. Thus, there seem to be "windows", where K^+ -nucleus scattering can discriminate between either different sets of KN amplitudes (in a gross way), or between different nuclear matter distributions, without the one being too much dependent on the other. A more systematic study of those

windows is indicated for experimental groups planning experiments in the K^+ -nucleus field. Of course, in testing K^+ -N amplitudes there is no substitute for much better two-body experiments—which may have to await a kaon factory.

IV. Conclusion

In conclusion, we may state that in comparison with, for example pion-nucleus scattering in the (3,3) resonance region, low energy K^+ -nucleus scattering (although experimentally difficult to measure and therefore less precise) will be easier to analyse theoretically. The assumed absence of double scattering corrections is even reinforced by a relative insensitivity to the Lorentz-Lorenz factor $L(r)$. The angular distribution of the elastic cross-section, although rather uninformative at very low energies, contains quite sensitive information at slightly higher momenta, say around 500 MeV/c. These energies are still small enough to avoid minor controversies which might come into the discussion because of Fermi-averaging of amplitudes, or the importance of inelastic channels. At these energies there seem to exist "windows" where experimental data could already yield information about matter densities—with little sensitivity to the poorly known K-N phase-shifts.

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Figure Captions

- Fig. 1. The real part of the isoscalar s-wave scattering parameter, $\text{Re}(b_0)$ (in units of pion Compton wavelength m_π^{-1}) as a function of KN c.m. momentum. The long line corresponds to the analysis of Martin (4), and the short line to that of Watts *et al.* (5).
- Fig. 2. Same description as in Fig. 1, but for the imaginary part of the isoscalar s-wave scattering parameter, $\text{Im}(b_0)$.
- Fig. 3. Same description as in Fig. 1, but for the real part of the isovector s-wave scattering parameter, $\text{Re}(b_1)$.
- Fig. 4. Same description as in Fig. 1, but for the real part of the isoscalar p-wave scattering parameter, $\text{Re}(c_0)$. (Unit here is m_π^{-3} .)
- Fig. 5. Differential elastic cross section for K^+ (solid line) and π^+ (dashed line) scattering on ^{16}O at $E_{\text{lab}} = 50$ MeV. Parameters are as described in the text for K^+ , and from ref. (14) for π^+ .
- Fig. 6. Ratio of differential elastic cross-sections of carbon 12 to carbon 13 for K^+ at 50 MeV. The solid line represents the standard set of parameters and densities [refs. (4) and (9)], the dashed curve has a 2% change in the neutron density radius of ^{12}C .
- Fig. 7. Differential elastic cross-section for K^+ scattering on ^{16}O at $E_{\text{lab}} = 200$ MeV. The solid line shows the result of the standard set of parameters, and the dashed line has its parameter basis from Watts *et al.* (5).

Fig. 8. Ratios analogous to those of Fig. 6, but for ^{16}O to 180 and 200 MeV lab energy of the K^+ . The solid line marked "Martin" represents the standard set of parameters; the solid line marked "Watts" shows the same ratio but for parameters from Watts (5) instead of Martin. The dashed line is for the standard parameter set of Martin, but has a decrease in the neutron density radius of 5% for 180 .

Fig. 9. The ratios of the differential elastic scattering of K^+ on ^{16}O to 180 at 100 MeV. The solid curve is for the standard set (including $\lambda=1$), the dashed line for standard set but $\lambda=0$, and the long dash line marked "change" has a 5% smaller rms neutron radius for 180 .

Fig. 10. Ratios analogous to those of Fig. 8, but for ^{40}Ca to ^{48}Ca . The dashed line marked "change" has a decrease in the neutron rms radius of 5% for ^{48}Ca .

Fig. 1

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Fig. 3

Fig. 4

Fig. 5

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Fig. 7

Fig. 8

Fig. 9

Fig. 10