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**A METHOD FOR COIL DESIGN OF SUPERCONDUCTING QUADRUPOLES
BASED ON SECTOR COILS AND FOURIER SERIES**

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Abstract

In this paper we present a novel method to optimize quadrupole coil cross-sections based on the $\cos^2 \theta$ -type. We first approximate real coil blocks made of cables with concentric sectors powered with a constant current density to make use of analytical expressions of the magnetic field based on the Fourier series development. This allows a rapid evaluation of the field harmonics which permits an almost exhaustive scan on sectors position and dimensions for coil layouts made of up to five sectors. The coil layouts having a larger short-sample gradient can be further explored, approximating them with blocks of Rutherford cables. This second step requires a second fine tuning of the cross-section around the previously found optimum. The method has been used to design a proposed coil cross-section for the new inner triplet quadrupole for the LHC.

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A Method for Coil Design of Superconducting Quadrupoles Based on Sector Coils and Fourier Series

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Index Terms— Magnetic field computation, superconducting quadrupoles, coil optimization.

I. INTRODUCTION

THE first step in the design of the coil transverse cross-section of a superconducting magnet consists in finding, for a given aperture, cables width, and number of layers, the coil blocks arrangements generating a field of high homogeneity. They are solutions of an inverse problem and cannot be derived explicitly. Due to the complexity of the coil, i.e. coil blocks made of cables and cables made of strands, it is usually not possible, for a given number of blocks, to perform a scan through all possible coil block configurations. Coil optimization methods usually rely on optimization algorithms [1] which gives a collection of local solutions on which a further analysis of decisional parameters such as the short-sample gradient and current allows choosing the coil layout which best fits specifications.

Here we propose a staged approach to the electromagnetic design of $\cos 2\theta$ -type coil cross-sections. We first approximate real coil layouts made of cables with pure sector blocks (sector delimited by radial straight lines), having uniform current density (Fig. 1) so as to use explicit equations of the magnetic field derived from Fourier series.

Fig. 1. Quadrupole coil cross-section made of pure sector blocks (left), and made of Rutherford cable (right).

This simplified representation of the coil allows a very fast computation of the field harmonics (of the order of 30 μ s), and therefore permits a fine scan on a very large number of possible coil layouts. For the coil layouts satisfying the field quality requirements, the analytical knowledge of the magnetic field [2] makes possible a fast computation of relevant parameters such as the short sample current and gradient. This allows to easily spot the cross-section which best suits the specifications. Once the coil cross-section is chosen one can build the “real” coil by approximating sectors with blocks of Rutherford cables. A further optimization on the position of the blocks by means of magnet design software [3] allows reducing the unavoidable field harmonic distortion due to the discrete cable size.

This method can be used as a complementary tool at the early stage of a quadrupole coil design. For a reasonable computational time of at max one day, it can substantially increase the chance to spot the global optimum for coils made up to five blocks. This method has been used for a proposed design for the coil cross-section of the new inner triplet quadrupole of the LHC [4].

II. COIL LAYOUT BASED ON SECTOR COILS

A. The Sector Coil Model: Analytical Computation

A superconducting quadrupole coil made of P sectors can be considered as the sum of P sub-quadrupoles with coils made of one sector (see Fig. 2).

Fig. 2. Decomposition of a quadrupole with coils made of 3 sectors (left) into sub-quadrupoles with coils made of one sector.

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Due to the linearity of the Maxwell equations, the magnetic field produced in the coil and in the air is obtained by summing up the contribution of each sub-quadrupole. The x - and y -components of magnetic field at the position (x, y) within the coil aperture can be expressed as

$$B_y + iB_x = \sum_{n=2,6,10,\dots} \left(\sum_{p=1}^P B_{n,p} \right) \left(\frac{x + iy}{R_{ref}} \right)^{n-1} \quad (1)$$

where $B_{n,p}$ [T] is the non-normalized field harmonic of order n allowed by the quadrupolar symmetry, given at the reference radius R_{ref} , and provided by the sub-quadrupole p . Please note that we do not use normalized multipoles because we would lose the linearity sketched in Fig. 2. For $n=2$, the coefficient $B_{n,p}$ is given by

$$B_{2,p} = -R_{ref} \mu_0 j_p \frac{2}{\pi} \ln \left(\frac{R_{2,p}}{R_{1,p}} \right) [\sin(2\theta_{2,p}) - \sin(2\theta_{1,p})] \quad (2)$$

and for $n > 2$ it reads

$$B_{n,p} = -R_{ref}^{n-1} \frac{\mu_0 j_p}{n(n-2)\pi} \left(\frac{1}{R_{2,p}^{n-2}} - \frac{1}{R_{1,p}^{n-2}} \right) \times [\sin(n\theta_{2,p}) - \sin(n\theta_{1,p})] \quad (3)$$

where $R_{1,p}$ and $R_{2,p}$ are the inner and outer radius of the coil of the sub-quadrupole p delimited by the angles $\theta_{1,p}$ and $\theta_{2,p}$ and powered with a uniform current density j_p (see Fig. 3).

Fig. 3. Quadrupole with coils made of one pure sector.

The normalized multipole of order n produced in the aperture of a quadrupole composed of P sub-quadrupoles is

$$b_n = 10^4 \frac{\sum_{p=1}^P B_{n,p}}{\sum_{p=1}^P B_{2,p}} \quad (4)$$

The field homogeneity is usually considered as acceptable when b_6 , b_{10} and b_{14} are all within one unit, higher orders being naturally small with respect to one unit.

B. Method to explore the possible coil layouts

When designing a quadrupole coil, the aperture radius, cables width and number of layers are usually fixed parameters. Therefore, to find out all possible coil layouts

built with a given number of sectors and providing a field quality satisfying specifications, one has to perform a scan through all the possible combinations of sector angular positions and thickness. This can be achieved by implementing (4) into a numerical code and following the four-step, time-optimized approach outlined below:

- Fix $2P-1$ angles and vary with a preliminary chosen angular step (see Section I (C)) the remaining angle and keep only the coil layout which generates the smallest quadratic sum of b_6 and b_{10} (it exists only one angle which can possibly cancel both b_6 and b_{10}).
- Store the coil layout if it generates b_6 and b_{10} within one unit. Discard it otherwise.
- Repeat the same analysis for all possible $2P-1$ angle combinations.
- Among the retained solutions, keep only those generating b_{14} within one unit.

The mid-plane angles are fixed to zero, but an open mid-plane can be also analyzed. Anyway, these mid-plane angles are usually fixed a priori and therefore the space of the phase angles is not $2P$ but rather $2P-q$, q being the number of layers.

Practically, superconducting quadrupole coils for accelerator are often made of one or more layers, each one made of one or two blocks, as it is the case for instance for the LHC MQXA [5] and for the Tevatron MQ [6]. In the following we will only consider the four coil cross-sections shown in Fig. 4 made from two to five blocks. They provide from three to seven angles to optimize.

Fig. 4. Coil layouts considered.

C. Choice of the angular step

To generate a large gradient, we propose to set the minimal angular dimension of the sectors in contact with the mid-plane to 15° . In that case, the maximal variation of the multipole $\Delta b_n^{(k)}$ due to an angular variation $\Delta\theta$ in the layer k of a quadrupole coil of inner radius R_i and made of q -layers of width w_L and given at $R_{ref} = 2/3 R_i$ is

$$\Delta b_n^{(k)} \approx \Delta\theta \frac{2}{(n-2)} \left(\frac{2}{3} \right)^{n-2} \left[\ln \left(1 + q \frac{w_L}{R_i} \right) \sin \left(\frac{\pi}{6} \right) \right]^{-1} \times \left[\left(1 + k \frac{w_L}{R_i} \right)^{2-n} - \left(1 + (k-1) \frac{w_L}{R_i} \right)^{2-n} \right] \quad (5)$$

From (5) and for a given angular step one observes that:

- The largest multipole variation is on b_6 ($|\Delta b_6^{(k)}| > |\Delta b_{10}^{(k)}| > |\Delta b_{14}^{(k)}|$).
- The variation $|\Delta b_6^{(k)}|$ increases when increasing the ratio width of the coil/magnet aperture ($q w_L / R_i$).
- In a multilayer coil, $|\Delta b_6^{(k)}|$ is larger when it is generated by an angular variation in the inner layers than in the outer ones.

Therefore, the angular step corresponding to a given maximal variation of b_6 (i) has to be smaller when decreasing the coil aspect ratio, and (ii) has to be larger in the outer layers of a multilayer coil than in the inner ones. If the angular step is too thick, one wastes a lot of computer time to find very similar solutions. On the other hand, if the step is too coarse, one risks to miss some interesting solutions. The two cases are shown in Fig. 5. An analysis performed on a two and a three layer coil showed that a step size corresponding to a maximal b_6 variation of about 15 units is a good choice for the scan. The step size satisfying this criterion is plotted in Fig. 6 and 7 for a one, two and three layers coil.

Fig. 5. Angles in a one layer coil made of two blocks giving b_6 , b_{10} and b_{14} within one unit. The angular step considered is of 0.15° (left) and of 0.05° (right). The angle numbering corresponds to the first coil layout from the left in Fig. 4

Fig. 6. Angular step to use in the innermost layer of a one, two and three layer coil.

Fig. 7. Angular step to use in the 2nd and 3rd layer of a two and three-layer.

D. Computational time

Estimations of the time needed to evaluate one coil cross-section with Matlab® R2009a running on a PC with two 2.8 GHz Pentium 4 processors are given in Table 1 for the two, three, four, and five-block coils of Fig. 4. It is of the order of tens of microseconds, to be compared to 0.1-1 s that can be

reached in usual codes that calculate a coil lay-out made of blocks of Rutherford cables. This means that we can scan at least a 3000 times larger set of solutions.

Table I Computational time to evaluate b_6 and b_{10}

	2 blocks	3 blocks	4 blocks	5 blocks
Nb of angles	3	4	6	7
Time (μs)	~25	~30	~40	~45

The number N_c of coil layouts to evaluate in a two-block coil as the one shown in Fig. 4 is of

$$N_c = \frac{n_s^3}{6} + \frac{n_s^2}{2} + \frac{n_s}{3}, \quad (6)$$

where n_s is the number of angular subdivision in the range one has to perform the analysis. In Fig. 8 is plotted the computational time needed to perform a complete scan on all possible cross-sections obtained with angular steps chosen according to what is discussed. Curves have been obtained by means of Table I and of (6) and considering angles varying within $[15^\circ-41^\circ]$.

Fig. 8. Computational time needed to perform an angular scan in coil layouts made of 2, 3, 4 and 5 blocks (Fig. 4) versus the coil aspect ratio.

For a reasonable computational time of at maximum of one day, one can perform a scan on up to a five-block coil and for aspect ratios larger than 0.6. The corresponding number of analyzed cross-section is 2.3 billion.

III. COIL LAYOUT WITH RUTHERFORD CABLES

We now dispose of a list of several tens of set of angles corresponding to coil-layouts which provide a good field quality *i.e.* b_6 , b_{10} and b_{14} within 1 unit. In order to determine which coil cross-section fits the best with the requirements (usually high gradient and low current), one can calculate the short-sample gradient and current by means of analytical equations of the magnetic field truncated at a sufficiently high order ($n > 100$). One can find these equations for ironless or yoked quadrupoles elsewhere [2].

To pass from an analytical solution which consists into a set of angles to a coil made of Rutherford cables, we first compute for each coil block the number of turns of cable which best fits with the sector coil angles. Then the

unavoidable field harmonic distortion due to the discrete cable size and its slight variation from a purely radial sector block are compensated by tuning the block positions and allowing small block tilts. The optimization is done through a sensitivity matrix built around the first guess and takes into account the saturation of the iron yoke. This is achieved by means of numerical code [3].

IV. APPLICATION TO THE ELECTROMAGNETIC DESIGN OF THE NEW INNER TRIPLET QUADRUPOLE FOR THE LHC

The coil for the new inner triplet quadrupole for the LHC had been specified to have an inner radius of 60 mm and to be made of four blocks based on the 15 mm wide Nb-Ti spare cables of the LHC main dipole [7] [8] [9]. The search of cross-sections of good field quality has been done following the approach described in Section II. The scan performed with Matlab® has been accomplished within 5 hours with angular step in the inner and outer layer of 0.3° and 0.9° (according to Fig. 6 and 7 and for a coil width to aperture radius ratio of 0.5). Among the 440 million of analyzed coil layouts, one got about 15000 solutions providing b_6 , b_{10} and b_{14} within one unit. To further restrict the space of solutions, we put more stringent constraints on b_{10} and b_{14} ($|b_6| < 1$, $|b_{10}| < 0.2$ and $|b_{14}| < 0.05$ units), reducing the number of coil cross-sections to 200 only.

Fig. 10. Short-sample gradient versus short sample current.

Fig. 11. Short-sample gradient versus the cross-sectional area of one coil.

For these cross-sections, the short-sample gradient and current and the amount of cables were calculated using analytical equations [2]. In Fig. 10 are plotted the short-sample gradient versus the short-sample current for each one of the 200 coil layouts. In Fig. 11 is plotted the short-sample gradient versus the amount of superconductor. One clearly sees the Pareto-optimal solutions lying on the two dotted lines. We

want to achieve the largest short-sample gradient while minimizing the amount of superconductor: the fill marker corresponds to the chosen cross-section. The calculated coil cross-section and the real coil after fine tuning with Roxie are shown in Fig. 12. Note that the inner and outer layers of the coil are not made from the same cables (current grading method). Another cable arrangement consisting in using the inner layer cable to wound the upper block of the outer layer has also been studied but did not exhibit higher gradient solutions [4].

Fig. 12. Chosen coil cross-section made of pure sector block (left) and cross-section made of real cables and fitting the best pure sector block angles.

In the framework of the same project, an independent work aiming at optimizing the same magnet and carried out using genetic algorithms obtained as a best option a 6-blocks solution with a 1.1 % larger short-sample gradient. This confirms that, due to the large number of analyzed coil layouts, our method provides a global optimum.

V. CONCLUSION

We proposed as a first step in the design of a superconducting quadrupole coil to consider coil models made of concentric sectors: this allows using analytical expressions of the magnetic field and a very fast computation of the field harmonic with respect to conventional magnet design software ($\sim 10^4$ times faster). The improved speed permits to perform a much larger scan over the sectors angular position and dimensions (up to few billion configurations) and thus to increase the chance to spot the coil layout which best fits the specifications. The sector coil solution is then approximated with a coil made of blocks of Rutherford cables. The method was applied to a proposed design of the coil cross-section of the new inner triplet quadrupole for the LHC.

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