Large muon electric dipole moment from flavor?

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We study the prospects and opportunities of a large muon electric dipole moment (EDM) of the order \(10^{-24} - 10^{-22}\) ecm. We investigate how natural such a value is within the general minimal supersymmetric extension of the standard model with \(CP\) violation from lepton flavor violation in view of the experimental constraints. In models with hybrid gauge-gravity-mediated supersymmetry breaking, a large muon EDM is indicative for the structure of flavor breaking at the Planck scale, and points towards a high messenger scale.

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### I. INTRODUCTION

\(CP\) violating phenomena receive strong and continuous interest because they provide a gateway to physics beyond the standard model (SM). While \(CP\) violation is observed in the quark sector and is currently believed to be predominantly stemming from the Kobayashi-Maskawa mechanism, further searches are being pursued to test this (SM) picture where \(CP\) and flavor violation are intimately linked. No breakdown of \(CP\) symmetry has been seen so far among leptons, however, there has been great progress made in neutrino masses and mixing.

We consider here lepton electric dipole moments (EDMs) \(\mathcal{L}_{\text{EDM}} = d_l (-i/2) \bar{\psi}_l d_{\mu l} \gamma_5 F_{\mu \nu} \psi_l\) [1], specifically for muons, as probes of \(CP\) and lepton flavor violation. Lepton EDMs in the SM appear first at four loop order, and are tiny, e.g., \(d_{\text{SM}} \lesssim 10^{-38}\) ecm [2]. The current experimental limits are [3,4]

\[
\begin{align*}
    d_e &= (6.9 \pm 7.4) \times 10^{-28} \text{ ecm}, \\
    d_\mu &= (-0.1 \pm 0.9) \times 10^{-19} \text{ ecm}. 
\end{align*}
\]

While the bound on the electron EDM severely constrains flavor-blind \(CP\) phases, lepton flavor violating couplings are subject to the constraints from the branching ratios of rare lepton decays. At 90% C.L. [3,5]

\[
\begin{align*}
    B(\mu \to e\gamma) &< 1.2 \times 10^{-11}, \\
    B(\tau \to \mu \gamma) &< 4.4 \times 10^{-8}, \\
    B(\tau \to e \gamma) &< 3.3 \times 10^{-8}. 
\end{align*}
\]

This situation can change significantly due to dedicated experimental initiatives in addition to the upcoming direct searches up to multi-TeV energies at the LHC: The MEG collaboration expects a reach in the \(\mu \to e\gamma\) branching ratio of order \(10^{-13}\) in the next few years [6]; The bounds on the radiative tau decays can be improved at a possible future super flavor factory up to \(2 \times 10^{-9}\) [7] (with 75 ab\(^{-1}\)); there is a recent proposal to improve the current limit on \(d_\mu\) by 3 orders of magnitude, even as low as \(5 \times 10^{-25}\) ecm [8].

We ask here whether \(d_\mu\) can be large, i.e., not many orders of magnitude below its current bound and if \(d_\mu\) is large, what are the requirements and implications for flavor physics?

We address these questions within the minimal supersymmetric standard model (MSSM). New sources of lepton flavor violation arise from supersymmetry (susy) breaking contributions causing intergenerational slepton mixing, for earlier works, see, e.g., [9–11], and [12] for flavor nonuniversal but diagonal effects. With current data, and depending on the mass scale of the susy spectrum, for sizable muon EDMs some fine-tuning is involved. We measure its amount within the general MSSM considering scenarios where \(CP\) violation is genuinely linked to lepton flavor violation only. In this framework, there is no model-independent connection between hadronic and leptonic \(CP\) violation, and we do not impose the \(CP\) constraints from the hadron sector. Specifically, nuclear effects in the extraction of the electron EDM from the thallium EDM, see, e.g., [1,13], and the two-loop effects from [13] are not included here. The direct link to weak \(CP\) violation highlights the importance of the muon EDM with respect to the nucleon ones.

Models with gauge mediation being the dominant source of susy breaking but with an additional contribution from Planck-scale gravity have recently been studied for their nonminimal flavor properties [14,15], but also the vacuum structure [16]. We work out the conditions for a large muon...
EDM for such a realistic hybrid model and argue that an observation in the range of the anticipated reach could be explained with very specific solutions to the flavor problem only.

The plan of the paper is as follows: In Sec. II we introduce the basic MSSM parameters relevant for the evaluation of the leptonic EDMs and rare decays, and discuss the relation between these observables. A detailed numerical study in the general MSSM is presented in Sec. III, where we also access the fine-tuning required for a large muon EDM. Models based on hybrid gauge-gravity mediation are investigated in Sec. IV. More conventions and formulas are given in the Appendix.

II. THE MUON EDM WITH FLAVOR

In case of flavor-blind CP violation only the muon EDM is constrained by the tight limit on the electron one, Eq. (1) to be below, at 90% C.L.,
\[ d_\mu \sim m_\mu / m_e d_e < 3.9 \times 10^{-25} \text{ ecm}, \]  
(3)
where \( m_l \) denotes the lepton masses. We are thus led to consider CP violation in flavor violation to obtain larger values of \( d_\mu \). For the purpose of this work, we set all CP phases not related to flavor to zero. To ease the notation we use \( d_i \) interchangeably for both the EDM and its magnitude throughout this work.

In Sec. II A we introduce the susy slepton flavor sector and define the requisite mass insertion parameters. Constraints from rare decays of leptons [Eq. (2)] put constraints on the amount of flavor violation. In Sec. II B we discuss the interplay between the muon EDM and the rare decays. In Sec. II C we investigate the Higgsino contributions.

A. Slepton flavor

Genuine susy flavor violation enters through the structure of the soft terms in generation space. This concerns the mass-squared matrix of the charged sleptons, which is given by
\[ M^2 \tilde{l} = \begin{pmatrix} M^2_{\tilde{l} L} & M^2_{\tilde{l} R} \\ M^2_{\tilde{l} R} & M^2_{\tilde{l} R} \end{pmatrix}, \]  
(4)
and which connects left-chiral \( \tilde{l}_L \) and right-chiral sleptons \( \tilde{l}_R \) as \( \tilde{l}_N M^2_{\tilde{N}M} \tilde{l}_M \), with the chiral projectors \( N, M = L, R \). The \( 3 \times 3 \) submatrices read as
\[ M^2_{\tilde{l} L, i,j} = M^2_{\tilde{l} L, i,j} + (F, D \text{ terms}), \]
\[ M^2_{\tilde{l} R, i,j} = v_1 A_{E,i,j} - \mu v_2 Y_{e,i,j}, \]
\[ M^2_{\tilde{l} R, i,j} = M^2_{\tilde{l} L, i,j} + (F, D \text{ terms}). \]
(5)
Here, \( i, j = 1, 2, 3 \) denote generational indices and \( Y_e \) is the Yukawa matrix of the charged leptons. The Higgs vacuum expectation values \( v_{1,2} \) obey \( v = \sqrt{v_1^2 + v_2^2} \sim 174 \text{ GeV} \) and \( v_2 / v_1 = \tan \beta \). The \( \mu \) parameter denotes the Higgs mass term from the MSSM superpotential.

The sneutrino masses cause intergenerational mixing as well. In the presence of left-handed neutrinos only, the mass-squared matrix is written as
\[ M^2_{\tilde{\nu}, i,j} = M^2_{\tilde{\nu}, L} + (F, D \text{ terms}). \]  
(6)
In \( M^2_{\tilde{\nu}_{R, L}, i,j} \) and \( M^2_{\tilde{\nu}} \) we did not spell out explicitly the flavor diagonal \( F \) and \( D \) terms whose effect on the diagonal entries is suppressed as \( m^2_{\tilde{\nu}} / M_{\tilde{\nu}, L} \) and \( v^2 / M_{\tilde{\nu}, L} \), respectively. In the full numerical analysis these terms are included.

To make contact with the low energy phenomenology, we need to evaluate the soft terms at the weak scale, \( m_Z \). Further, we go over from the flavor eigenstates to the basis where the sleptons are mass eigenstates and the neutralino interactions are diagonal in generation space. We denote the respective unitary matrix of slepton-type \( A = L_L, \nu_L, \nu_R, E \) by \( V_A \), and the corresponding slepton soft terms by a tilde, \( M^2_{\tilde{l} L,R} = V^*_A M^2_{\tilde{l} L,R} V^*_A \) and \( M^2_{\tilde{\nu}} = V^*_A M^2_{\tilde{\nu}} V^*_A \). From the latter the mass insertions can be read off as
\[ \delta_{ij} = (M^2_{\tilde{l} L})_{ij}/M^2_{\tilde{l} A}, \]
\[ \delta_{ij} + \mu = (M^2_{\tilde{\nu}})_{ij}/M^2_{\tilde{\nu} A} \]  
(7)
where we introduced an average slepton mass \( M_{\tilde{l} A} \). The \( \delta \) parameters [Eq. (7)] induce flavor changing neutral currents (FCNCs) and, if complex, CP violation.

B. \( d_\mu \) versus \( B(\tau \rightarrow \mu \gamma) \)

To understand the interplay between a large muon EDM but small enough lepton flavor violating branching ratios, we employ the following approximations: We restrict ourselves to stau-smuon flavor mixing only, and neglect all lepton masses except for the one of the tau; we set to zero the flavor violating chirality-flipping couplings which are contained in the \( A_E \) terms of Eq. (5). Furthermore—note that this only matters for the rare decays—we assume that the leading contributions are due to photino exchange. We use the formulas from Ref. [17], which are obtained in the mass insertion approximation, i.e., for perturbative deltas sufficiently smaller than 1.

In this approximation, the leading contribution to \( d_\mu \) can be obtained at one-loop from
\[ d_\mu / e = \frac{\alpha}{2\pi} \frac{M_\mu}{M^2_{\tilde{l} A}} G_1(y) \text{Im}(\delta_{22}), \]  
(8)
where \( y = M^2_{\tilde{l} A} / M^2_{\tilde{l} \tilde{l}}, \) and \( M_\mu \) denotes the relevant gaugino mass, that is, here the photino. The loop function \( G_1(y) \) obeys \( G_1(1) = 1/12 \), drops rapidly with increasing \( y \), and is given in the Appendix.

To estimate the EDM from flavor we approximate \( \delta_{22} \) by its effective value \( \delta_{22} \sim \delta_{23} \delta_{23} \delta_{23} \), see also Fig. 1, where the bino contribution is shown. Then, for \( y = 1 \),
expected to illustrate the qualitative features only. We used
in these figures obtained from extrapolations beyond the

tributing to the muon EDM. A cross denotes a mass insertion.

FIG. 1. Flavor and chirality flow of the leading diagram con-

tributing to the muon EDM. A cross denotes a mass insertion.

via a geometric vector

The photon is attached wherever possible.

\[
\frac{d_\mu}{e} \sim 1 \times 10^{-20} \text{ cm} \left( \frac{200 \text{ GeV}}{M_A} \right) \text{Im} (\delta_{23}^{L_E} \delta_{33}^{L_E} \delta_{23}^{\nu \bar{\nu}}). \tag{9}
\]

We allow for maximal CP phases of \( \delta_{23}^{L_E} \) and \( \delta_{32}^{L_E} \). The factor \( \delta_{33}^{L_E} \), see Eq. (7), parametrizes the left-right (LR) mixing

down to the
de the staus, and is taken to be real here. In our analysis

dependence on \( A_{E,33} \), \( \mu \), \( \tan \beta \), and \( M_A \).

The \( \tau \to \mu \gamma \) branching ratio can we written using the

same approximations as

\[
\mathcal{B}(\tau \to \mu \gamma) = \kappa (|\delta_{23}^{L_E}|^2 + |\delta_{23}^{\nu \bar{\nu}}|^2), \tag{10}
\]

with

\[
\kappa = \left( 1 + \frac{M_1 G_1(y)}{m_\tau} \frac{\delta_{33}^{L_E}}{G_3(y)} \left( \frac{\alpha^3}{G_F^2 M_A^3 G_3(y)} \right)^2 \right) \mathcal{B}(\tau \to \mu \nu \bar{\nu}). \tag{11}
\]

We approximated \( \delta_{23}^{L_E} \) and \( \delta_{32}^{L_E} \) by their effective values

\( \delta_{23}^{L_E} \delta_{33}^{L_E} \) and \( \delta_{32}^{L_E} \delta_{33}^{L_E} \), respectively. The loop function \( G_3(y) \) satisfies \( G_3(1) = 1/40 \) and is given in the Appendix.

The maximal value for the muon EDM allowed by the
upper limit on the branching ratio \( \mathcal{B}(\tau \to \mu \gamma)_{\text{max}} \) is then determined by \( 9 \)

\[
\text{max} |\delta_{23}^{L_E} \delta_{33}^{L_E}| = \mathcal{B}(\tau \to \mu \gamma)_{\text{max}} / (2\kappa). \tag{12}
\]

The resulting reach is shown in Figs. 2 and 3. The curves in these figures obtained from extrapolations beyond the
validity of the mass insertion approximation (thin lines) are
expected to illustrate the qualitative features only. We used
\( \mathcal{B}(\tau \to \mu \nu \bar{\nu}) = 17.36\% \) \[3\].

We learn the following:

(i) Values of \( d_\mu \) as large as \( O(10^{-22}) \) ecm are consistent

with current FCNC constraints. For this to happen it

requires \( O(1) \) intergenerational mixing. (The mass

insertion approximation breaks down).

(ii) The maximal allowed value of the muon EDM given

by Eq. (12) grows with increasing \( M_A \) and \( y \).

(iii) The muon EDM vanishes for vanishing stau

LR mixing \( \delta_{33}^{L_E} \) in our approximation. Large

LR mixing, however, suppresses \( d_\mu \) because of the

enhancement of the coefficient \( \kappa \). For our

parameters we find \( \delta_{33}^{L_E} \sim 10^{-23} \) to give

the largest EDM, depending on the slepton mass.

(iv) The sign of the stau chirality mixing matters:

Negative values of \( \delta_{23}^{L_E} \) allow for larger EDMs by

suppressing \( \mathcal{B}(\tau \to \mu \gamma) \), see Eq. (11), at the price

of increased tuning. This is illustrated in Fig. 3.

(v) Within the mass insertion approximation, \( |\delta| \leq 1 \),

values of \( d_\mu \) up to \( O(10^{-23}) \) ecm are possible if

the sleptons have masses below a few hundred GeV,

\( \delta_{33}^{L_E} \sim 10^{-23} \) and \( y \leq O(1) \).

(vi) The maximal value for \( d_\mu \) is proportional to the

upper limit on \( \mathcal{B}(\tau \to \mu \gamma) \). The anticipated future

bound on \( \mathcal{B}(\tau \to \mu \gamma) \) from super flavor factories of

\( 2 \times 10^{-9} \) \[7\] has a significant impact on the maxi-

dmal value of \( d_\mu \).
C. Including Higgsinos

Since we neglect the mass of the muon, the Higgsinos (and winos) do not contribute to $d_\mu$ to the order we are working, and Fig. 1 represents the leading contribution to the EDM. However, Higgsino contributions have been found to be of importance in the calculation of $B(\tau \to \mu \gamma)$ [9]. More specifically, for a light $\mu$ term $\mu \sim M_1, M_2$ the lightest neutralino has a substantial Higgsino fraction and the photino-only approximation in the rare $\tau$ decays [Eq. (10)] receives large corrections. Consequently, the bounds on flavor violation change in the presence of the Higgsinos, which then affects the maximal value of $d_\mu$ from flavor.

The Higgsino contribution to the $\tau \to \mu \gamma$ amplitude involves directly the $\tau$ Yukawa coupling $y_\tau \propto \tan \beta$. Hence, keeping $\delta^L_{33}$ fixed while enhancing $\tan \beta$, the Higgsino loops get scaled up with respect to the bino ones and the maximal value of the muon EDM drops with respect to the analysis of the previous section. The same holds if the $A$ term dominates the stau LR mixing, $\delta^L_{11} \sim \nu_{A E,33}/\tan \beta$. If the $\mu$ term is large and dominates, $\delta^L_{33} \sim -\mu m_\tau \tan \beta$, then all bino and Higgsino contributions grow with $\tan \beta$. Since the EDM depends only linearly on $\delta^L_{33}$, whereas $B(\tau \to \mu \gamma)$ does quadratically, the maximal value of the EDM drops with increasing $\tan \beta$ also in this case. Therefore, large values of $d_\mu$ favor a low $\tan \beta$.

In Fig. 4 the maximal muon EDM as a function of the common slepton (snuon) mass is presented for the same values of $\delta^L_{33}$ as in Fig. 2, however with the Higgsino and wino contributions in $B(\tau \to \mu \gamma)$ included. We use $\tan \beta = 3$, $\mu = 2$ TeV, $M_1 = M_\tilde{A}$, and $M_2 = (g_\tau^2/g_\mu^2)M_1$. As expected, a reasonably high value of $\mu$ suppresses the contribution from the Higgsino loops, and, in general, the increase in $B(\tau \to \mu \gamma)$ is small. Therefore, the photino analysis of Sec. II B gives a good approximation of the maximum value of the muon EDM. However, for low values of $\delta^L_{33}$ the leading bino graphs in the $\tau \to \mu \gamma$ amplitude, which are proportional to $\delta^L_{33}$, are suppressed, and external chirality flip graphs $\propto m_\tau$, which are usually subleading, become more important at the low slepton mass range. Therefore, a cancellation against the Higgsino graphs occurs, which can be seen in Fig. 4 as a spike in the allowed maximal EDM curves. For larger values of $\delta^L_{33}$, the cancellation does not happen, since the external flip contribution cannot compete with the leading contributions.

In Fig. 5 we show the maximal muon EDM for a Higgsino mass parameter $\mu$ which equals the slepton mass $M_\tilde{A}$. Consequently, the smaller $\mu$ term enhances the Higgsino mediated loop contributions in $B(\tau \to \mu \gamma)$ and the maximal EDM is not as large as in Figs. 2 or 4. With increasing slepton mass also $\mu$ increases, and the characteristics of Figs. 4 and 5 become alike.

Here, we assumed that $\delta^L_{33}$ is equal to $\delta^L_{13}$. Typically the sneutrino loop gives the largest single contribution to the branching ratio, unless the Higgsinos are decoupled, in which case the bino loops dominate.

To summarize, the inclusion of Higgsino contributions modifies the correlation between the maximal value of the muon EDM from flavor and $B(\tau \to \mu \gamma)$ given in the previous section for low values of the $\mu$ term or when cancellations occur. Values of $d_\mu$ at order $10^{-22}$ ecm are possible with current $B(\tau \to \mu \gamma)$ constraints for order one flavor mixings. Values of order $10^{-23}$ ecm can be reached within the mass insertion approximation, $|\delta| \lesssim 1$.

![FIG. 4 (color online). The maximal value of the muon EDM from flavor as a function of the average slepton mass based on [9] for $\tan \beta = 3$, $M_1 = M_\tilde{A}$, and $\mu = 2$ TeV. The three solid curves correspond to the current upper limit on $B(\tau \to \mu \gamma)$ [Eq. (2)] and are obtained for $\delta_{33}^L = 10^{-2}$, $10^{-3}$, and $10^{-4}$ (from top to bottom at $M_\tilde{A} = 1000$ GeV). The dashed line refers to the hypothetical limit $B(\tau \to \mu \gamma) < 2 \times 10^{-9}$ for $\delta_{33}^L = 10^{-3}$. The thick curves have mass insertions $|\delta| < 1$.](https://example.com/fig4)

![FIG. 5 (color online). The same as in Fig. 4 but with $\mu = M_\tilde{A}$, hence lighter and more important Higgsinos.](https://example.com/fig5)
III. NUMERICAL ANALYSIS IN THE GENERAL MSSM

We study the muon EDM in the general MSSM neglecting CP phases unrelated to lepton flavor mixing. Formulas used are given in the Appendix.

A. Input and constraints on susy parameters

We generate sets of data points by randomly sampling susy parameters at the weak scale. We consider two scenarios, a “light” and a “heavy” one with their respective parameter ranges defined in Table I.

As can be seen from Table I, we use different ranges for the flavor diagonal and off-diagonal mass parameters. The former contain the gaugino masses $M_1$ and $M_2$, the $\mu$ term, its susy breaking companion, $B_\mu$, and the diagonal entries in the slepton soft terms $\sqrt{M_{L,ii}}$, $\sqrt{M_{E,ii}}$, $A_{E,ii}$. For simplicity we restrict our analysis to degrade first and second generation masses $M_{L,ij} = M_{L,ij,22}$. Diagonal $A$ terms are assumed to follow the corresponding lepton Yukawa couplings $A_{E,ii} = A_0 y_i$. Furthermore, we include slepton flavor mixing between the second and third generation only. The respective off-diagonal parameters are therefore $A_{E,23}, A_{E,32}$, $\sqrt{M_{L,23}}$, and $\sqrt{M_{E,23}}$. We allow for an order of magnitude suppression of the off-diagonal soft terms with respect to the diagonal entries in the light scenario, whereas in the heavy scenario we allow the off-diagonal terms to be of the same order of magnitude as the diagonal ones. In both scenarios $\tan \beta$ is sampled over the range 2–50.

For each generated point we check the scalar potential against unboundedness from below, and the existence of charge or color breaking minima [18]. These bounds constrain the off-diagonal soft contributions of the $A$ terms. The soft masses of the Higgs fields are solved for with the vacuum condition $\partial V/\partial \phi = 0$. The resulting spectrum at each generated point is checked against experimental constraints from direct searches [3]. To generate a muon EDM we introduce random CP phases in all flavor off-diagonal soft terms between 0 and $2\pi$. Since we assume here the $\mu - e$ and $\tau - e$ slepton mixing terms and all flavor-blind $CP$ violation to be zero, the experimental constraint from the electron EDM is automatically fulfilled. Contributions to $\mu \rightarrow e \gamma$ decays from sneutrino loops are also suppressed and neglected.

For each sample point, we calculate the contribution to the muon anomalous magnetic moment, $\Delta a_\mu$, the muon electric dipole moment $d_\mu$, and the branching ratio for the decay $\tau \rightarrow \mu \gamma$. $\Delta a_\mu$ is constrained to be less than $10^{-9}$ corresponding to roughly two sigma of the experimental and theoretical uncertainty [19]. The bound on $\Delta a_\mu$ is always fulfilled in the heavy scenario.

B. Random walk analysis

The interplay between the muon EDM and the $\tau \rightarrow \mu \gamma$ branching ratio for our sampled points can be seen in Fig. 6. For both the light and the heavy spectrum the value of the EDM for points satisfying $B(\tau \rightarrow \mu \gamma) < 10^{-8}$ is typically less than $10^{-24}$ ecm (squares and circles). This value is, however, not a hard upper bound on the muon EDM, because it stems from the stochastic nature of our parameter selection. Indeed, and as we will see, one can find parameters such that the constraint on the branching ratio is fulfilled and $d_\mu$ is large. The question then is how fine-tuned such points are.

To obtain points with $d_\mu > (10^{-22} - 10^{-24})$ ecm obeying the $\tau \rightarrow \mu \gamma$ constraint we use a random walk process. Starting from a point $p$ in parameter space, we randomly vary the parameters by a small amount, $p \rightarrow p' = p + \delta p$, and check for all the constraints as well as for a larger value of the muon EDM. If $p'$ passes the test, the procedure is repeated with $p'$ as the new starting point. If we start out with a point $p$ that has $B(\tau \rightarrow \mu \gamma) > 10^{-8}$, we additionally require a decrease in the branching ratio before accepting $p'$. The requirements on the branching ratio and the muon EDM thus drive a point into the desired direction. Unless we exceed a predetermined number of failed attempts, this procedure eventually yields a point with $B(\tau \rightarrow \mu \gamma) < 10^{-8}$ and a large value of $d_\mu$ as requested, after which the random walk is terminated.

In Fig. 6 we show the result of this random walk in the heavy and the light scenario. We see a tendency of

TABLE 1. The sampling ranges for the flavor diagonal and off-diagonal mass parameters in the light and the heavy scenario.

<table>
<thead>
<tr>
<th></th>
<th>Flavor diagonal</th>
<th>Flavor off-diagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0–1 TeV</td>
<td>0–100 GeV</td>
</tr>
<tr>
<td>Heavy</td>
<td>3–5 TeV</td>
<td>0–3 TeV</td>
</tr>
</tbody>
</table>

FIG. 6 (color online). The distribution of the muon EDM $d_\mu$ vs the branching ratio $B(\tau \rightarrow \mu \gamma)$. Shown is the initial distribution in the light scenario (circles) and the heavy scenario (squares), as well as the distribution after the random walk process in the light (triangles up) and the heavy scenario (triangles down).
points that start out with very low branching ratios to migrate to larger ones. This is expected, as the branching ratio has similar contributions as the EDM, which is being forced to increasing values. It should be noted that for the fixed number of iterations, we find that in the heavy scenario only about 2% of the original points achieved \( d_\mu = 10^{-22} \) ecm, whereas in the light scenario this fraction is 35%.

For each scenario we consider in the following two data sets, an inclusive one containing all the points that went through the random walk routine and an exclusive one containing only points that actually pass the EDM constraint \( d_\mu \geq 10^{-22} \) ecm for the light scenario and \( d_\mu \geq 0.5 \times 10^{-22} \) ecm for the heavy scenario.

The random walk process affects the distribution of the parameters. For example, in the light scenario there is a transition from a near flat distribution of \( \tan \beta \) to one which is peaked at small values, see Fig. 7. The latter behavior is in agreement with the findings of Sec. II C. In the next section we use this selection effect to construct a measure of fine-tuning.

C. Measures of fine-tuning

In order to better understand the fine-tuning needed for achieving an experimentally interesting \( d_\mu \) while respecting the experimental \( B(\tau \rightarrow \mu \gamma) \) constraint, we first note that the subamplitudes in \( a_{22}^{L/R} \), see Eq. (A9), corresponding to neutralino and chargino 1-loop Feynman diagrams, need to be small or cancel each other to great accuracy. We find that, however, there is usually only one subamplitude which contributes a dominant part of the branching ratio. When we increase the EDM via our random walk routine this disparity is lessened, but all individual subamplitudes remain small enough for there to be no need for large cancellations.

Similarly, Eq. (A7) suggests that the imaginary part of \( a_{22}^{R/L} \) needs to be as large as possible for a large EDM.
complex phases of the off-diagonal soft mass terms. Let 
\[ 2\phi_{X_{33}} = \arg(M^2_{X_{23}}) \] for \( X = L, E \). While we find for each 
\( \phi_{L_{23}} \) and \( \phi_{E_{33}} \) separately the ratio \( r \approx 1 \), their difference 
\( \Delta \phi = \phi_{L_{23}} - \phi_{E_{33}} \) has \( r_{\Delta \phi} \approx 0.47 \) with sharp peaks around \( \pi/4 \) and \( 3\pi/4 \) (heavy scenario), as can be seen in 
Fig. 8. This correlation can be understood from Eq. (9), 
with \( \text{Im} \left( \delta^L_{23} \delta^E_{33} \delta^{E}_{23} \right) \) maximized.

We investigate the phase differences of \( M^2_{X_{23}}, X = L, E \) and \( A_{E,23}, A_{E,32} \), and assign the product of the individual 
volume changes \( r_{\Delta \phi} \), as

\[ d_{\phi} = \prod_i r_{\Delta \phi_i}. \] (13)

Furthermore, we consider various ratios of mass parameters. We examine the following ratios containing 
off-diagonal entries:

\[ \frac{M^2_{L_{23}}}{M^2_{L,33}}, \quad \frac{M^2_{E_{23}}}{M^2_{E,33}}, \quad \frac{|A_{E,23}|}{A_0}, \quad \frac{|A_{E,32}|}{A_0}. \] (14)

The product of their individual volume fractions is denoted 
by \( d_{vd} \). We look at some general ratios of parameters in the 
slepton sector and the mass parameters \( M_1, M_2, \mu \):

\[ \frac{\sqrt{M^2_{L,22} - M^2_{L,33}}}{\sqrt{M^2_{E,22} - M^2_{E,33}}}, \quad \frac{\sqrt{M^2_{L,22} + \sqrt{M^2_{L,33}}}}{\sqrt{M^2_{E,22} + \sqrt{M^2_{E,33}}}}, \]
\[ \frac{\sqrt{M^2_{L,33}M^2_{E,33}}}{A_0}, \quad \frac{\sqrt{M^2_{L,33}M^2_{E,33}}}{M^2_{L}}, \quad \frac{\sqrt{M^2_{L,33}M^2_{E,33}}}{M^2_{E}}. \] \] (15)

The product of their individual volume fractions is denoted 
by \( d_v \). We then define a total change of volume of parameter space, \( d_v \), as

\[ d_v = d_{\phi} \cdot d_{vd} \cdot d_G \cdot r_{\tan \beta}. \] (16)

![FIG. 8 (color online). The distribution of \( \phi_{L_{23}} - \phi_{E_{33}} \) before (solid line) and after (dashed line) the random walk process in the heavy scenario (exclusive data set).](image)

One would expect that the volume changes \( d_v \) depend on 
the value of the fraction of points, \( fr \). In Fig. 9 we show 
this dependence for \( d_v \). As can be seen, \( d_v \) is very stable over a 
large range of \( fr \). The unstable behavior for values of \( fr \approx 0.8 \) is due to the way statistical outliers affect our analysis. 
A more sophisticated approach to measuring the width of 
parameter distributions by fitting, e.g., Gaussian curves 
could improve the stability. We also verify that the values 
of the \( r_{\chi} \) (and consequently \( d_v \)) remain stable under variations of spurious quantities such as changes in data binning.

**D. Discussion**

The various volume fractions for the two scenarios and 
data sets are shown in Table II for \( fr = 0.8 \). Comparing the 
different entries, we get an idea about fine-tuning caused 
by the random walk process.

Notably, the effect of the \( CP \) phases becoming correlated 
as shown in Fig. 8) emerges in the exclusive heavy 
scenario only. Also we see that a tuning of \( \tan \beta \) occurs in 
the light scenario only (see Fig. 7).

The ratios of the off-diagonal to diagonal mass elements 
which we have collected into \( d_{vd} \) have values close to unity 
in the inclusive set but spread out, by up to an order of magnitude 
in the case of heavy scenario, when we look at

![FIG. 9 (color online). The parameter space volume fraction \( d_v \) [Eq. (16)] as a function of \( fr \) in the light inclusive (solid line), 
light exclusive (dotted line), heavy inclusive (dashed line), and heavy exclusive (dash-dotted line) scenario.](image)

| TABLE II. The volume fractions of parameter space after the random walk for \( fr = 0.8 \) for the light and the heavy scenario. Smaller numbers correspond to larger tuning. |
|---|---|---|---|---|
| Inclusive | Light | Heavy | Exclusive | Light | Heavy |
| \( r_{\tan \beta} \) | 0.25 | 0.96 | 0.16 | 0.95 |
| \( d_{vd} \) | 1.3 | 0.88 | 2.9 | 10 |
| \( d_{\phi} \) | 0.98 | 0.96 | 0.98 | 0.49 |
| \( d_G \) | 1.8 | 69 | 0.39 | 33 |
| \( d_v \) | 2.2 | 59 | 1.1 | 170 |
the exclusive set. This means that the more fine-tuned the points get, the wider the range of acceptable mass ratios is. These ratios enter the loop contributions of the branching ratio and EDM and their increase in size, which leads to the spreading out of the distribution, is in line with the observations from the beginning of Sec. III C.

Finally, we see that for the set of flavor diagonal mass ratios in $d_{G}$, there is a fine-tuning effect in the light scenario that emerges for the exclusive set, whereas in the heavy scenario there is instead again a spreading out.

It is apparent that the ranges for the mass ratios are more constrained in the light scenario. In the heavy scenario, the individual loop contributions (which partly depend on these mass ratios) start out much smaller than in the light scenario and thus there is more “room” to spread out. The phases $\phi_{L_{3}}$ and $\phi_{E_{3}}$ becoming correlated is also a sign of insufficient size of the loop contributions.

It needs to be emphasized that the change in parameter space volume is a relative measure of the amount of fine-tuning and useful for comparisons between different data sets. It is not an absolute measure of the ease of fine-tuning. Even comparing different rows in Table II is not trivial, e.g., fine-tuning the phases can have a greater or lesser effect on the EDM than fine-tuning the mass ratios.

We find that both heavy and light scenarios appear to be not particularly tuned to give a large value of the muon EDM while respecting existing constraints. While most of the volume fractions $d_{X}$ in the light scenario are smaller than in the heavy one, from looking at Fig. 6 and noting that a large percentage (~98%) of points in the heavy scenario cannot be tuned to high enough values of $d_{\mu}$, we conclude that the heavy scenario is quantitatively harder to fine-tune.

**IV. A FLAVORED EDM IN HYBRID MODELS**

After analyzing the muon EDM within the general MSSM, we now consider the situation in an explicit model with a hybrid mechanism of susy breaking, gauge-gravity mediation. These models have recently received attention because they provide insights into the nature of flavor breaking due to the appearance of flavor structures beyond the Yukawa couplings [14,15,21,22]. Viable gauge-gravity models have the dominant source of suzy breaking from the Yukawa couplings [14,15,21,22], generating also the Yukawa matrices.

At the scale of gauge mediation, $m_{M}$, we parametrize the soft terms of the sleptons as [14]

$$\begin{align*}
M_{i}^{2}(m_{M}) &= \tilde{m}_{L_{i}}^{2}(1 + x_{L}X_{L}), \\
M_{E}^{2}(m_{M}) &= \tilde{m}_{E}^{2}(1 + x_{E}X_{R}).
\end{align*}$$

(17)

The new flavor structure from gravity with respect to gauge mediation is encoded in the Hermitian matrices $X_{L,R}$, following from the charge assignments of the FN symmetry. The factors $x_{L}$ and $x_{E}$ implicitly defined by Eq. (17) quantify the relative size of the gravity versus the gauge contribution to the $SU(2)$ singlet and doublet soft masses, respectively. In this way, the $x_{i}$ are measures of the separation between the messenger and the Planck scale, $m_{Pl}$. In minimal gauge mediation with $N_{M}$ messengers (see, e.g., [24]),

$$x_{i} \sim \left( \frac{m_{M}}{m_{Pl}} \right)^{2} \frac{4\pi}{\alpha_{i}(m_{M})} x_{S}, \quad c_{1} = \frac{5}{6}, \quad c_{2} = \frac{2}{3}, \quad (18)$$

where we assumed that the $F$ terms from $m_{Pl}$ couple also to gauge mediation. The ratio $x_{1}/x_{2}$ as in Eq. (18) is of order one, and increases for lower values of the messenger scale, see Fig. 10.

In Sec. IVA we give formulas for the flavor violating low energy parameters in gauge-gravity models. A flavor model with explicit FN charges is studied in Sec. IV B. We investigate the impact of trilinear $A$ terms in Sec. IV C.

**A. Hybrid gauge gravity**

Starting from the soft slepton masses at the scale $m_{M}$ of the form given in Eq. (17) and solving the MSSM renormalization group (RG) equations [25], the soft slepton masses at the weak scale can be written in the following approximate form:

$$\begin{align*}
M_{i}^{2}(m_{Z}) &\sim \tilde{m}_{L_{i}}^{2}(r_{2} + c_{L}Y_{e}Y^{\dagger}_{e} + x_{2}X_{L}), \\
M_{E}^{2}(m_{Z}) &\sim \tilde{m}_{E}^{2}(r_{1} + c_{E}Y^{\dagger}_{e}Y_{e} + x_{1}X_{R}).
\end{align*}$$

(19)

In the spirit of the FN flavor models, Eq. (19) and the following estimations are only accurate up to numbers of order one, which we indicate by “~.” The running of the gravity-induced terms is such an order one effect. The coefficients $c_{L}, c_{E}$ are negative, obey $|c_{L}| < |c_{E}|$, and are of the order $-1/(16\pi^{2})\ln(m_{M}/m_{Z})$, which are at most of order one for $m_{M}$ near the GUT scale.
The $r_i$ coefficients include the leading RG correction to the flavor diagonal elements. Neglecting contributions from the subleading gauge couplings, one obtains an analytical expression for $r_i$ at one-loop order (see, e.g., [24]):

$$r_i = 1 + \frac{1}{c_i} \left( \int_{\ln(m_{\nu})}^{\ln(m_{\mu})} dt \frac{\Delta^2(t)}{\alpha^2(t)} \right) \frac{M^2_i(m_M)}{\bar{m}^2_i(m_M)}.$$  \hspace{1cm} (20)

Here, $M_i$ denotes the respective gaugino mass and $\bar{m}^2_i$ equals $\bar{m}^2_2$ for $i = 2$ and $\bar{m}^2_E$ for $i = 1$. In messenger models of gauge mediation, the ratio $M^2_i/\bar{m}^2_i$ is determined by a simple formula at the scale of mediation including only the leading gauge contributions:

$$\frac{M^2_i(m_M)}{\bar{m}^2_i(m_M)} = c_i N_i.$$  \hspace{1cm} (21)

The RG parameters $r_i$ from Eqs. (20) and (21) obey $r_3 > r_1 > 1$ and are of order one for not too many messengers. Since $r_3 > 1$, the RG effect strengthens the flavor-blind entries, hence, reduces the flavor violation. The properties of the $r_i$ can be different in general gauge mediation [26] which does not exclude negative soft sfermion masses squared at the mediation scale.

In the basis where the leptons are mass eigenstates and the neutralino interactions are diagonal in generation space, we obtain

$$M^2_{L_{\ell}}(m_Z) \sim m^2_{\nu}(r_2 1 + c_L D^2 E + x_2 V_{L_{\ell}} X_{L_{\ell}} V_{L_{\ell}}^\dagger),$$

$$M^2_{r_{\nu}}(m_Z) \sim m^2_{\nu}(r_2 1 + c_L U^2 D E + x_2 V_{L_{\nu}} X_{L_{\nu}} V_{L_{\nu}}^\dagger),$$

$$M^2_{E_{\nu}}(m_Z) \sim m^2_{\nu}(r_1 1 + c_{E} D^2 E + x_1 V_{E_{\nu}} X_{E_{\nu}} V_{E_{\nu}}^\dagger),$$

where $U = V_{L_{\ell}} V_{L_{\ell}}^\dagger$ denotes the Maki-Nakagawa-Sakata matrix and $D_{e} = \text{diag}(y_{e}, y_{\mu}, y_{\tau}) = V_{L_{\ell}} Y_{e} V_{L_{\ell}}^\dagger$. Note that the parametrization [Eq. (22)] holds beyond minimal gauge mediation with perturbative messengers.

The mass insertions of the charged leptons are then given as $(i \neq j)$

$$\delta^{L_{\ell}}_{ij} \sim \frac{x_2}{r_2} (V_{L_{\ell}} X_{L_{\ell}} V_{L_{\ell}}^\dagger)_{ij}, \quad \delta^{E}_{ij} \sim \frac{x_1}{r_1} (V_{E_{\nu}} X_{E_{\nu}} V_{E_{\nu}}^\dagger)_{ij}.$$  \hspace{1cm} (23)

Here, we neglected contributions in the denominators from the tau Yukawa of the order $\gamma^2 \sim 10^{-4} \tan^2 \beta$ with respect to the $r_i$ for mixings involving the third generation. The sneutrino mass insertions, which matter for the rare decays only to the order we are working, $m_{\mu} = 0$, are given as $(i \neq j)$

$$\delta^{\nu}_{ij} \sim \frac{1}{r_2} (c_L \gamma^2_{L_{\ell}} U_{3i}^* U_{3j} + x_2 (V_{r_{\nu}} X_{r_{\nu}} V_{r_{\nu}}^\dagger))_{ij}.$$  \hspace{1cm} (24)

receiving two independent competing contributions. Note that the second term in Eq. (24) equals $U_{3i}^* U_{3j}$. Since its sign/phase is not fixed by the FN symmetry, there can be cancellations in $\delta^{\nu}_{ij}$.
or flavor violation beyond the Yukawa matrices, and do not introduce CP violation at the weak scale, however, such terms do not introduce CP violation beyond the Yukawa matrices, and do not matter for flavor phenomenology.

We obtain in the flavor model [Eq. (25)] for the maximal value of the muon EDM

\[ \frac{d_\mu}{e} \lesssim 5 \times 10^{-24} \text{ cm}, \]  

in agreement with existing data, specifically the FCNC constraints [Eq. (2)]. The other FN models of Ref. [14] have smaller values of \( \delta_{32} \), hence yield smaller values of \( d_\mu \), either due to a smaller 2–3 mixing, or by a stronger constraint on \( x_i/r_i \) effective for less suppressed 1–2 mixing.

We conclude that an observation of the muon EDM at the level of order 10\(^{-23}\)–10\(^{-24}\) cm would imply the following in context of gauge-gravity models with a FN symmetry:

(i) The flavor structure needs to be very specific, i.e. not every FN symmetry correctly reproducing the SM lepton masses and mixings works.

(ii) CP is broken together with flavor, and the CP phases are unsuppressed.

(iii) The SUSY spectrum is not too heavy, of the order of a few 100 GeV, and \( y \lesssim 1 \).

(iv) The messenger scale needs to be very high, not far from \( m_\text{M} \sim \alpha/(4\pi)m_\text{Pl} \).

Note that analogous studies in the quark sector have found that FCNC data constrain the messenger scale in typical FN models to be about 3 orders of magnitude below \( m_\text{Pl} \) [15].

**C. Including \( A \) terms**

The gauge mediated contribution to the trilinear \( A \) terms is of higher order and usually set to zero \( A_E(m_M) = 0 \). This has also been employed implicitly in the previous section. While the MSSM RG equations do induce finite \( A \) terms at the weak scale, however, such terms do not introduce CP or flavor violation beyond the Yukawa matrices, and do not matter for flavor phenomenology.

However, in our hybrid setup, there can be a finite contribution from gravity to the \( A \) terms [22]:

\[ A_E(m_M) \sim \sqrt{m^2 x Y_e}, \]  

where we used that the gravity-induced \( A \) terms receive the same parametric suppressions from the flavor symmetry breaking as the corresponding Yukawa matrices. Here, \( m^2 \), \( x \) are of the order of \( m_\text{P}^2 \) and \( m_\text{P}^2 \), \( x_1 \), and \( x_2 \), respectively.

We then evolve according to the MSSM RG equations. The result can be recast in the following approximate form:

\[ A_E(m_Z) \sim Y_e(a_E + \sqrt{m^2 x} + b_E Y_e^2 Y_e). \]  

The dominant contribution to \( a_E \) and \( b_E \) stems from the gaugino masses, which drives them to magnitudes of electroweak size for large enough \( m_\text{M} \). Note that \( a_E < 0 \) and \( b_E > 0 \). Unless \( \tan \beta \) is very large, the double Yukawa suppression makes the \( b_E \) term negligible. As indicated in Eq. (31), the gravity contribution evolves with a coefficient of order one.

In the basis with lepton mass eigenstates and diagonal neutralino couplings, we obtain

\[ A_E = V_{L,E} A_E V_{L,E}^\dagger \sim D_x(a_E + \sqrt{m^2 x} + b_E Y_e^2 Y_e ). \]  

Under the RG evolution the \( A \) terms mix into the soft masses squared and induce also corrections to the soft masses in Eq. (22). Because of the Yukawa texture, this results effectively into a correction of the order \( x \) of the \( c_{L,E} \) coefficients, and will be absorbed therein for the purpose of this work.

The strongest bound on the hybrid model including \( A \) terms [Eq. (30)] is from the electron EDM, stemming from \( \delta_{11L,E} \) and its imaginary part. An approximate expression for \( d_e \) reads as [see, Eq. (8) for the corresponding formula for \( d_\mu \)]

\[ \frac{d_e}{e} = \frac{\alpha}{2\pi} \frac{M_i}{M_\text{A}} G_1(y) \text{Im}(\delta_{11L,E}), \]  

\[ \lesssim 2.6 \times 10^{-26} \text{ cm} \left( \frac{200 \text{ GeV}}{M_\text{A}} \right)^2 \sqrt{\frac{x}{r}} , \]  

where we assumed \( y = 1 \) in the second line. Here, CP violation is induced by the gravity-mediated \( A \) terms, Eq. (30), which are fixed by the FN symmetry up to order one, in general complex numbers, only. One obtains from Eqs. (1) and (33)

\[ x/r \lesssim 5 \times 10^{-3}. \]  

This bound implies a separation between the Planck scale and the messenger scale of about up to 4 orders of magnitude, see Fig. 12. Information on the hadronic sector, that is, the neutron EDM in a similar setup with Yukawa-like \( A \) terms and a viable quark FN symmetry gives somewhat milder constraints [22].
FIG. 12 (color online). The ratio \( x/r \) in minimal gauge mediation [Eq. (18)] as a function of the messenger scale. The curves correspond to \( N_M = 1, 3, \) and 7 (from top to bottom). The horizontal line denotes the constraint from the electron EDM, Eq. (34).

The bound Eq. (34) excludes a muon EDM from flavor since the mass insertions are subsequently suppressed as

\[
\delta^{k}_{23} g^{2}_{523} \sim \lambda^k (x/r)^2 \leq \lambda^k 10^{-5} \leq 10^{-5},
\]

where \( k \) denotes the non-negative integer power from the FN charges of a given model.

So far, we assumed the \( CP \) phase \( \phi \) of \( A_{E,11} \) to be fully unsuppressed. The bound Eq. (34) weakens by a factor \( 1/\sin^2 \phi \) if \( \phi \) is not maximal. However, for \( k = 2 \) as in the model from the previous section, even with a phase suppression as low as \( \sin \phi \) at the percent level, the prediction for \( \delta_{\mu} \) is as in the linear mass scaling case, Eq. (3).

V. SUMMARY

Observing a large muon EDM \( \delta_{\mu} \approx O(10^{-24}) \) ecm requires \( CP \) violation from flavor violation and favors a light mass spectrum. The heavier the spectrum and the larger the Higgsino admixture, the larger is the requisite tuning to avoid the most relevant flavor constraint, \( B(\tau \rightarrow \mu \gamma) \).

In the context of hybrid gauge-gravity models a measurement of such large values of the muon EDM would point towards a specific Planck-scale flavor structure and a very large messenger scale not far from \( \sim \alpha/(4\pi) m_{\gamma} \). In the presence of Yukawa-textured \( A \) terms at the level of Eq. (30) with \( CP \) phases, however, the muon EDM is limited by the electron EDM data to follow the linear mass scaling constraint Eq. (3), which is below the current experimental sensitivity.

We conclude that the muon EDM is informative on various aspects of the underlying fundamental physics.

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APPENDIX: THE RELEVANT MSSM PARAMETERS

The notation follows closely Refs. [10,27].

1. Mixing matrices

The slepton mass-squared matrices [Eqs. (4) and (6)] are diagonalized by the matrices \( R_{ij/E}^{\overline{M}/E} \):

\[
R_{ij/E}^{\overline{M}/E} (R_{ij/E}^{\overline{M}/E})^{-1} = \text{diag}(M_{\chi_i}^2).
\]

The neutralino and chargino mass matrices are as usual

\[
M_{\chi_0} = \\
\begin{pmatrix}
M_1 & 0 & -\frac{g' v_1}{\sqrt{2}} & \frac{g' v_2}{\sqrt{2}} \\
0 & M_2 & \frac{g v_1}{\sqrt{2}} & -\frac{g v_2}{\sqrt{2}} \\
-\frac{g' v_1}{\sqrt{2}} & \frac{g v_1}{\sqrt{2}} & 0 & -\mu \\
\frac{g' v_2}{\sqrt{2}} & -\frac{g v_2}{\sqrt{2}} & \mu & 0
\end{pmatrix},
\]

where \( M_1 \) and \( M_2 \) are the \( U(1) \) and \( SU(2) \) gaugino soft masses, respectively, and \( g' = e/\cos \theta_W = \sqrt{3}/5g_1 \), \( g = e/\sin \theta_W = g_2 \). The neutralino and chargino mass matrices are diagonalized by matrices \( N \) and \( U \), \( V \) as follows:

\[
U^* M_{\chi^+} V^{-1} = \text{diag}(M_{\chi^+}), \quad N^* M_{\chi^0} N^{-1} = \text{diag}(M_{\chi^0}).
\]

2. Leptonic observables

The relevant Lagrangian involving sleptons, neutralinos, and charginos is as follows:

\[
\mathcal{L} = \bar{l}_i (n_{ijL}^L + n_{ijR}^R R_{ij}^R) a_{ij}^L \tilde{\nu}_k + \bar{l}_i (c_{ijL}^L + c_{ijR}^R R_{ij}^R) a_{ij}^R \tilde{\nu}_k + H.c.,
\]

where

\[
n_{ijL}^R = -g V_{i1j} R_{ki}^{\overline{M}}, \quad c_{ijL}^R = y_i U_{j2}^R R_{ki}^{\overline{M}},
\]

\[
n_{ijR}^L = -g' \sqrt{2} N_{ijL} R_{ki}^{\overline{M}}, \quad y_i N_{ijL}^{R} R_{ki}^{\overline{M}},
\]

\[
n_{ijR}^R = (g' N_{ijL} + g N_{ijL}) R_{ki}^{\overline{M}}/\sqrt{2} - y_i N_{ijL} R_{ki}^{\overline{M}}.
\]

The susy contributions to the radiative lepton decay width, the lepton EDM, and the magnetic moment can be written at one loop as [10]

\[
\Gamma(l_j \to l_i \gamma) = \frac{\alpha m_{l_j}}{16} (|a_{ij}^L|^2 + |a_{ij}^R|^2),
\]

\[
d_i = \frac{e}{4m_{l_i}} \text{Im}(-a_{ij}^L + a_{ij}^R).
\]
\[ \Delta a_i = \frac{1}{2} \text{Re}(a_{ij}^L + a_{ij}^R), \]  
(A8)

where terms of order \( O(m_i/m_j) \) have been dropped. The coefficients \( a_{ij}^{L,R} \) are given as

\[
a_{ij}^L = \frac{1}{16\pi^2} \sum_{k=1}^4 \sum_{r=1}^4 \left[ \left( n_{ikr}^L n_{jkr}^L m_{k}^Z \right) \chi_k^0 - n_{ikr}^R n_{jkr}^R m_{k}^Z \right]
\]

\[ \times F_1 \left( \frac{m_{k}^Z}{m_{k}^Z} \right) + n_{ikr}^R n_{jkr}^R m_{k}^Z \chi_k^0 F_3 \left( \frac{m_{k}^Z}{m_{k}^Z} \right) \]

\[ + \frac{1}{16\pi^2} \sum_{k=1}^4 \sum_{r=1}^4 \left[ \left( c_{ikr}^L c_{jkr}^L m_{k}^Z \right) \chi_k^0 - c_{ikr}^R c_{jkr}^R m_{k}^Z \right]
\]

\[ \times F_2 \left( \frac{m_{k}^Z}{m_{k}^Z} \right) + c_{ikr}^R c_{jkr}^R m_{k}^Z \chi_k^0 F_4 \left( \frac{m_{k}^Z}{m_{k}^Z} \right) \right],
\]

\[ a_{ij}^R = a_{ij}^L (L \leftrightarrow R), \]  
(A9)

where

\[ F_1(x) = \frac{-2 + 3x - 6x^2 + x^3 + 6x \log x}{6(1 - x)^4}, \]  
(A10)

\[ F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x}{6(1 - x)^4}, \]  
(A11)

\[ F_3(x) = -\frac{1 - x^2 + 2x \log x}{(1 - x)^3}, \]  
(A12)

\[ F_4(x) = \frac{1 - 4x + 3x^2 - 2x^2 \log x}{(1 - x)^3}. \]  
(A13)

The functions \( G_{1,3} \) relevant for the calculation in the mass insertion approximation are obtained from Eqs. (A10) and (A12) as \( G_1(x) = (x^2/2) \cdot \partial F_1 / \partial x \) and \( G_1(x) = (x^2/2) \cdot \partial F_3 / \partial x \).