Ref.TH.3071-CERN

QCD CORRECTIONS TO P WAVE QUARKONIUM DECAYS

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A B S T R A C T

We give a detailed account of the calculation of the one loop QCD corrections to the annihilation widths into hadrons and into two photons of P wave quarkonium states of $J^{PC} = 0^{++}, 2^{++}$.

*) Supported in part by the Swiss National Science Foundation

1. INTRODUCTION

In an attempt to provide for a rather complete set of QCD predictions which might be compared to a corresponding set of experiments on quarkonium properties¹⁾, we give here a detailed account of the calculation of the strong QCD corrections to P wave quarkonium decays. The final results of such calculations have already been presented in Ref. 2). They have to be considered as an extension of the analogous calculation³⁾ for the S wave pseudoscalar state. In fact, the underlying physical frame is the same: at least up to first order corrections in the strong coupling constant α_s , the ratio of quarkonium decay rates into definite channels is computable as the ratio of the annihilation cross-sections into the same channels of free heavy (constituent) quarks in the static limit. In such a way, unknown parameters related to the bound state properties cancel out, allowing for possibly testable predictions, whose significance increases with the mass of the constituent quark.

From a general point of view, the calculation for the P wave annihilation differs from that for the S wave³⁾ in that the P wave amplitudes are proportional to the first derivative of the wave function in the relative co-ordinate R, since the wave function itself vanishes at $R = 0$. This in turn requires the expansion of the annihilation kernel up to the first power of the relative momentum. The expansion is the source of infra-red divergences, appearing in the final results as a logarithmic dependence on the binding energy (or on a confinement radius). These divergences are of the same nature as those already found⁴⁾ in the lowest order 1^{++} annihilation. At the same time, the expansion introduces considerable technical complications which are not present in the S wave problem. The reader interested in a detailed description of the way they are dealt with is referred to Ref. 5).

Here we are mainly concerned with the presentation and the discussion of the complete results, relative to the states $J^{PC} = 0^{++}, 2^{++}$. As a by-product and a cross check of the calculations described here, the 0^{-+} and the 1^{++} results of Refs 3) and 6) have been reproduced.

In Section 2 we present the separate results for the different perturbative "channels": three gluons, one gluon plus a light quark-antiquark pair and the virtual corrections to two gluons. For the latter case, to provide information for possible related future work, we show the results for the individual diagrams.

In Section 3 the over-all unrenormalized analytic results are given. We also briefly comment on the way they can be converted into direct physically sensible predictions.

2. UNRENORMALIZED CONTRIBUTION

The lowest order rates $\Gamma^{(0)}(J^{PC} \rightarrow \text{hadrons})$ for the $0^{++}, 2^{++}$ states, relative to the two photons annihilations, are

$$\Gamma^{(0)}(J^{PC} \rightarrow \text{hadrons}) = \Gamma^{(0)}(J^{PC} \rightarrow 2g) = \frac{2}{9Q^4} \left(\frac{\alpha_s}{\alpha}\right)^2 \Gamma^{(0)}(J^{PC} \rightarrow 2\gamma) \quad (1)$$

where Q is the charge of the constituent quark in units of the electron charge.

As is well known, the relative ratios⁷⁾

$$\frac{\Gamma^{(0)}(2^{++} \rightarrow \text{hadrons})}{\Gamma^{(0)}(0^{++} \rightarrow \text{hadrons})} = \frac{\Gamma^{(0)}(2^{++} \rightarrow 2\gamma)}{\Gamma^{(0)}(0^{++} \rightarrow 2\gamma)} = \frac{15}{4} \quad (2)$$

agree with the unfortunately rather scanty experimental information.

The diagrams contributing in next-to-leading order are the same as for the 0^{++} annihilation. They are given in Fig. 1.

The over-all three gluon cut contribution gives

$$\Gamma^{(1)}(0^{++} \rightarrow 3g) = \Gamma^{(0)}(0^{++} \rightarrow 2g) C_A \frac{\alpha_s}{\pi} \left[L + \frac{521}{162} - \frac{7}{48} \pi^2 \right] \quad (3)$$

$$\Gamma^{(1)}(2^{++} \rightarrow 3g) = \Gamma^{(0)}(2^{++} \rightarrow 2g) C_A \frac{\alpha_s}{\pi} \left[L + \frac{1741}{216} - \frac{883}{1152} \pi^2 \right]$$

where $C_A = 3$ and

$$L = 2 \lg^2 \frac{\lambda}{2m} + \frac{11}{3} \lg \frac{\lambda}{2m} + \sqrt{3} \frac{\pi}{6}, \quad (4)$$

which contains all the infra-red singular dependence on the regularizing gluon mass λ . m is the constituent quark mass.

The quark anti-quark gluon cuts give (Fig. 1j)

$$\Gamma^{(1)}(0^{++} \rightarrow g \bar{q}_i q_i) = \Gamma^{(0)}(0^{++} \rightarrow 2g) \frac{\alpha_s}{\pi} \left[-6 \lg \frac{m_i}{2m} - \frac{16}{27} + \frac{4}{27} \lg \frac{4m^2}{|4m^2 - M^2|} \right] \quad (5)$$

$$\Gamma^{(1)}(2^{++} \rightarrow g \bar{q}_i q_i) = \Gamma^{(0)}(2^{++} \rightarrow 2g) \frac{\alpha_s}{\pi} \left[-\frac{2}{3} \lg \frac{m_i}{2m} - \frac{11}{18} + \frac{5}{9} \lg \frac{4m^2}{|4m^2 - M^2|} \right]$$

where $m_i \ll m$ is the mass of the quark q_i , and M the mass of the decaying bound state.

The singularity in the "binding energy" shows up in (5). Numerically one may take

$$\lg \frac{4m^2}{|4m^2 - M^2|} \simeq \lg m R_c$$

where $R_c \simeq (400 \text{ MeV})^{-1}$ is a typical confinement radius.

The two-gluon cuts for the individual diagrams of Fig. 1 are given in Tables 1 and 2. The contribution from the diagram (Fig. 1a), with the exchange of a gluon between the incoming quark legs, contains the the expected universal singularity in the relative velocity $\sim \pi^2/v^3$, to be reabsorbed in the wave function normalization and in any case disappearing in the appropriate ratio of widths. Since renormalization has not yet been performed, the terms in

$$D \equiv \frac{1}{n-4} - \frac{1}{2} \lg(4\pi) + \frac{\gamma}{2} + \lg \frac{m}{\mu} - 1, \quad \gamma = 0,577 \quad (6)$$

contain the ultra-violet singularity $\sim (4-n)^{-1}$ regularized by going to n dimensions. Tables 1 and 2 also give the leading strong corrections to $\Gamma(J^{PC} \rightarrow 2\gamma)$ which are read by taking $C_A = 0$ in the QED-like diagrams of Fig. 1a,b,c,d. This exhausts the list of the partial contributions.

3. RESULTS

The over-all sum of the partial unrenormalized contributions is

$$\frac{\Gamma(0^{++} \rightarrow \text{hadrons})}{\Gamma(0^{++} \rightarrow 2\gamma)} = \frac{\Gamma^{(0)}(0^{++} \rightarrow \text{hadrons})}{\Gamma^{(0)}(0^{++} \rightarrow 2\gamma)} \left[1 + \frac{\alpha_s}{\pi} \left(-2 \left(\frac{11}{6} C_A - \frac{1}{3} \beta \right) D \right) \right] \quad (7)$$

$$+ C_A \left(\frac{157}{81} - \frac{11}{3} \lg 2 - \frac{\pi^2}{144} \right) + (f-1) \left(\frac{2}{27} + \frac{2}{3} \lg 2 + \frac{4}{27} \lg m R_c \right) + \frac{2}{3} \Big]$$

$$\frac{\Gamma(2^{++} \rightarrow \text{hadrons})}{\Gamma(2^{++} \rightarrow 2\gamma)} = \frac{\Gamma^{(0)}(2^{++} \rightarrow \text{hadrons})}{\Gamma^{(0)}(2^{++} \rightarrow 2\gamma)} \left[1 + \frac{\alpha_s}{\pi} \left(-2 \left(\frac{11}{6} C_A - \frac{1}{3} f \right) D \right. \right. \quad (7)$$

$$\left. \left. + C_A \left(\frac{1447}{216} - 2 \lg 2 - \frac{337}{384} \pi^2 \right) + (f-1) \left(\frac{1}{18} + \frac{2}{3} \lg 2 + \frac{5}{9} \lg m R_c \right) + \frac{2}{3} \right] .$$

f stands for the total number of quarks including the bound quark and μ in D , [Eq. (6)], is the mass scale introduced when going to n dimensions through the standard replacement of the bare coupling constant $g_0 \rightarrow g_0 \mu^{(4-n)/2}$. At this point we leave the reader free to choose his preferred way of converting the above results into a renormalized form. The straightforward way is to introduce the renormalized coupling α_s^R through

$$\alpha_s = \alpha_s^R \left[1 + \frac{\alpha_s^R}{\pi} \left(\left(\frac{11}{6} C_A - \frac{1}{3} f \right) D + k \right) \right] , \quad (8)$$

with an appropriately chosen constant k . Different values of k correspond to the various popular renormalization schemes. We prefer here to get rid of the divergences by considering those ratios which are D independent, because their expansion in α_s starts from α_s to the zeroth power. This was done in Ref. 2), to which the reader is referred to for the numerical results for charmonium and bottonium.

Table 1

Partial contributions to $\Gamma^{(1)}(0^{++} \rightarrow 2g)$ from individual diagrams of Fig. 1.

Diagram ^{a)}	Contribution to imaginary part ^{b)}
a	$C_F \left[-\frac{\pi^2}{v} - \frac{4}{9} + 2\lg \frac{\lambda}{2} + \frac{32}{9} \lg 2 + \frac{\pi^2}{12} \right]$
b	$C_F \left[D - \frac{4}{9} + \frac{19}{4} \lg 2 \right]$
c	$C_F \left[D - 1 - 2\lg \frac{\lambda}{2} - 2\lg 2 \right]$
d	$(C_F - \frac{1}{2}C_A) \left[2D - \frac{4}{9} - \frac{28}{9} \lg 2 + \frac{\pi^2}{6} \right]$
e	$C_A \left[-3D + \frac{7}{9} - \frac{20}{9} \lg \frac{\lambda}{2} - \frac{10}{3} \lg 2 - \frac{\pi^2}{27} - \frac{5}{18} \sqrt{3} \pi - \frac{2}{3} \lg^2 \frac{\lambda}{2} \right]$
f	$C_A \left[\frac{1}{9} - \frac{2}{9} \lg \frac{\lambda}{2} - \frac{10}{3} \lg 2 - \frac{5}{54} \pi^2 + \frac{1}{18} \sqrt{3} \pi + \frac{4}{3} \lg^2 \frac{\lambda}{2} \right]$
g	$C_A \left[\frac{7}{27} + \frac{4}{9} \lg \frac{\lambda}{2} + \frac{14}{27} \lg 2 + \frac{19}{54} \pi^2 - \frac{\sqrt{3}}{9} \pi - \frac{8}{3} \lg^2 \frac{\lambda}{2} \right]$
h	$C_A \left[-\frac{19}{27} + \frac{70}{27} \lg 2 \right]$
i+i'+i''	$C_A \left[-\frac{5}{3} D - \frac{35}{18} - \frac{5}{3} \lg \frac{\lambda}{2} - \frac{5}{3} \lg 2 + \frac{1}{6} \sqrt{3} \pi \right]$

a) The proper multiplicity factor is included

b) Normalized to $\Gamma^{(0)}(0^{++} \rightarrow 2g) \frac{\alpha_s}{\pi}$

Table 2

Partial contributions to $\Gamma^{(1)}(2^{++} \rightarrow 2g)$ from individual diagrams of Fig. 1.

Diagram ^{a)}	Contribution to imaginary part ^{b)}
a	$C_F \left[-\frac{\pi^2}{v} - \frac{5}{3} + 2\lg \frac{\lambda}{2} + \frac{7}{3}\lg 2 + \frac{\pi^2}{8} \right]$
b	$C_F \left[D + \frac{5}{6} + \frac{17}{6}\lg 2 \right]$
c	$C_F \left[D - 1 - 2\lg \frac{\lambda}{2} - 2\lg 2 \right]$
d	$(C_F - \frac{1}{2}C_A) \left[-2D - \frac{13}{6} - \frac{19}{2}\lg 2 - \frac{\pi^2}{8} \right]$
e	$C_A \left[-3D - \frac{13}{12} - \frac{4}{3}\lg \frac{\lambda}{2} - 3\lg 2 - \frac{\pi^2}{48} - \frac{3}{8}\sqrt{3}\pi - \frac{3}{8}\lg^2 \frac{\lambda}{2} \right]$
f	$C_A \left[-\frac{1}{12} + \frac{2}{3}\lg \frac{\lambda}{2} - \frac{11}{4}\lg 2 - \frac{17}{36}\pi^2 - \frac{1}{24}\sqrt{3}\pi + \frac{13}{8}\lg^2 \frac{\lambda}{2} \right]$
g	$C_A \left[\frac{11}{9} - \frac{4}{3}\lg \frac{\lambda}{2} + \frac{89}{18}\lg 2 + \frac{23}{72}\pi^2 + \frac{\sqrt{3}}{12}\pi - \frac{13}{4}\lg^2 \frac{\lambda}{2} \right]$
h	$C_A \left[-\frac{5}{9} - \frac{10}{9}\lg 2 \right]$
i+i'+i''	$C_A \left[-\frac{5}{3}D - \frac{35}{18} - \frac{5}{3}\lg \frac{\lambda}{2} - \frac{5}{3}\lg 2 + \frac{1}{6}\sqrt{3}\pi \right]$

a) The proper multiplicity factor is included

b) Normalized to $\Gamma^{(0)}(2^{++} \rightarrow 2g) \frac{\alpha_s}{\pi}$

REFERENCES

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FIGURE CAPTION

Fig. 1 : Representative diagrams whose imaginary parts contribute to the annihilation cross-section. Trivial permutations of the intermediate gluons (photons) are not explicitly indicated.