EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/PS/AA/81-23

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B. Autin, A. Bellanger, R. Billinge, R. Brown, C.D. Johnson,
E. Jones, R. Sherwood, H.H. Umstätter

Paper presented at the 7th International Conference
on Magnet Technology
March 30-April 3, 1981
Karlsruhe, Federal Republic of Germany

Geneva - April 1981

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Abstract - The Antiproton Accumulator is a very large aperture storage ring in which antiprotons are collected over a wide momentum range, stochastically cooled and stacked. The optical properties must be precisely realised and require a variety of high order field components. The very wide aperture and a lack of available space, preclude the use of separate sextupoles and octupoles to provide these corrections. One of the sextupole components was incorporated into the pole profile of the wide elliptic quadrupoles the left-right symmetry of which was thus broken. The other non linear components were produced by variation of the effective length of the dipoles and quadrupoles and achieved in practice with detachable end shims.

INTRODUCTION

The Antiproton Accumulator (AA) is the heart of the antiproton physics programme under current development at CERN. It is designed to collect $\sim 6 \times 10^{11}$ antiprotons per day. The antiprotons are produced in a target by an intense 26 GeV/c proton beam and injected into the accumulator at 3.5 GeV/c (near the production peak) where they are stacked and their density is increased using a special technique called "stochastic cooling" [1].

The shape of the ring is shown in Fig. 1. A characteristic of this machine is the strong modulation of the horizontal envelope of the beam which varies from 6 cm in certain regions to 68 cm in others. For obvious economical and technical reasons, the horizontal aperture of the magnet conforms the beam shape and the lattice is composed of a regular arrangement of dipoles and quadrupoles which vary in width. Moreover, the magnet system must be as compact as possible to have the greatest number of free long straight sections. Therefore the bending and focussing functions are separated and the multipole fields are integrated in the main magnets instead of being produced by dedicated lumped lenses such as sextupoles or octupoles as it is the usual rule. The desired multipole component can be obtained by including a special term in the equation of the pole profile as in the AA combined quadrupole-sextupole magnet or by varying the effective magnetic length of the magnet. In the latter case, a convex or a concave end face gives mainly a sextupole-like component in a dipole and an octupole-like component in a quadrupole; the sense of curvature is determined by the focussing sign: a convex face is defocussing horizontally and focussing vertically, a concave face has the converse effect. The distribution of the multipole fields respects the symmetry of the machine.

THE QUADRUPOLE-SEXTUPOLE MAGNET

Theory

In a two-dimension geometry, the magnetic field theory can be treated in the framework of the analytic functions of a complex variable

Manuscript received

The authors are with the European Organization for Nuclear Research (CERN), 1211 Geneva 23, Switzerland

Fig. 1 - Antiproton Accumulator Layout

$$z = x + iy \quad (1)$$

where x and y are the horizontal and vertical coordinates respectively. The complex potential $w(z)$ is

$$w(z) = u(x,y) + iv(x,y) \quad (2)$$

with u and v satisfying Cauchy's conditions:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (3)$$

The complex field B is the derivative of $w(z)$ with respect to z :

$$B(z) = \frac{dw}{dz} = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \quad (4)$$

Its real and imaginary parts are the vertical and horizontal components respectively:

$$B(z) = B_y + iB_x \quad (5)$$

With these definitions, $u(x,y)$ is interpreted as the axial component of the vector potential and $v(x,y)$ as the scalar potential. A particular equipotential line is the pole profile where v is given by the Ampère-turns, assuming a nul reluctance of the iron:

$$v = \text{Im} \int_{z=0}^{z_{\text{pole}}} B(z) dz = \mu_0 nI \text{ (Ampère's law)} \quad (6)$$

The complex potential is a power expansion of z

$$w(z) = \sum_{n=1}^N a_n z^n \quad (7)$$

where $a_1, a_2, a_3, \dots$ represent the dipole, quadrupole, sextupole, ... coefficients respectively.

The equation of the pole profile can be obtained by solving the complex equation

$$a_1 z + a_2 z^2 + \dots + a_n z^n = u + i(\mu_0 n I) \quad (8)$$

where u is a parameter. Using a root finder routine [2], a point $(x(u), y(u))$ of the pole is determined and the complete pole profile is obtained by varying u within a known range of values.

Figure 2 illustrates the equation of a combined quadrupole-sextupole profile

$$a_2 z^2 + a_3 z^3 = u + i(\mu_0 n I) \quad (9)$$

for which a closed form of the solution exists. The vertical symmetry plane of the pure quadrupole is replaced by a hyperbola. Left- and right-hand profiles are images of each other in a hyperbolic mirror through a conformal mapping.

Fig. 2 - Combined quadrupole-sextupole profile

Design

The wide quadrupole-sextupole magnet of the anti-proton accumulator embodies many of the peculiarities which can be contemplated in accelerator magnet technology. As its length (54 cm) is smaller than its aperture (~ 1 m), the contribution of the stray fields to the integrated gradient is evaluated by using three-dimension field computations [3]. On the other hand, the beam is much wider than high inside the magnet ($68 \times 10 \text{ cm}^2$), it is thus economical to have an aperture with elliptic symmetry and, with respect to a circular symmetry, the elliptic configuration improves the distribution of the flux in the pole and alleviates iron saturation problems. A real scale prototype containing the quadrupole components only was built. After magnetic measurements [4], the high order coefficients in the pole profile equation were re-adjusted to get a better gradient uniformity and a sextupole term such that

$$\frac{\int G(x) ds}{\int G(0) ds} = 0.14 x$$

was added. As a correction in the pole profile does not extend to the whole field region, the coefficient a_3 in (8) was equated to 0.14 multiplied by the effective length to iron length ratio (.75/.54). The asymmetry due to this term affects the pole profile only; the rest of the magnet has a quadrupole structure: the die set used for lamination blanking was the same

for right- and left-hand laminations with the exception of the pole tip segment [5] and the type of coils which equipped the prototype was used for the series.

Measurement

Figure 3 shows the form of the gradient variation across the horizontal aperture of the bare magnet before shimming. This is to be compared to the designed shape (pure quadrupole + sextupole) shown dotted. Using the method of least squares the best fit to the measured curve in Fig. 3 of the power series (Eq. 7) containing the first ten coefficients, over the range $\pm .4$ m, was computed. This fitted curve followed the measured curve to within a gradient error of ± 0.001 . Decomposed into its odd and even parts it is shown by the faint lines in Fig. 3. The presence of an important decapole term together with the even octupole and duodecapole terms had to be taken into account during subsequent shimming operations.

Fig. 3 - Gradient error curve of the quadrupole-sextupole magnet

SEXTUPOLE COMPONENT AT THE END OF A DIPOLE

System of Coordinates

The optics of bending magnets including sextupole effects due to second degree curves for the magnet end faces are well known [6]. The magnet face, in the hard edge model, is taken as the magnet length measured from the centre of the magnet as shown in Fig. 4. If the end face is curved with the form

$$\eta = a \xi^2 \quad (10)$$

then the incremental effective length as felt by the beam is given as a function of x by

$$y = x \tan \frac{\phi}{2} + \frac{ax^2}{\cos \frac{3\phi}{2}} + 2 \frac{\tan \frac{\phi}{2}}{\cos \frac{\phi}{2}} a^2 x^3 + \dots \quad (11)$$

For practical reasons, measuring coil translation was along the ξ direction. Apart from an insignificant asymmetry due to the coil being slanted along the beam direction, only the sextupole component of a correctly shimmed magnet is observed.

Some bending magnets are sufficiently close to nearly inclined quadrupole magnets (Fig. 4) that their influence had to be measured. Gradient-pair coils were used when measurements were made with both types of magnets in place to allow a better subtraction of the

quadrupole gradient. For required values of "a" (Eq. 11) calculated coil responses were plotted and compared with measured results as shimming progressed.

Fig. 4 - Magnet coordinate systems

Natural End Effect and Correction

The dipole magnets for the AA ring are of two types [5]: the BST or short magnets and the BLG or long magnets.

Magnets delivered from the manufacturer had their pole ends recessed as shown in Fig. 5. It was planned

Fig. 5 - Perspective view of a recessed pole end

to fill these spaces with thick parallel shims, thin sheet shims and an outer selection of prisms to approximate any required curve. The thick shims and prisms were of Armco steel and the sheet shims were cut out of 1.5 mm thick magnet laminations.

Figures 6 and 7 show the effective length of a magnet at different values of x. The marked asymmetry of this curve arises because the measuring coils, slanted in the beam direction, sample the flux in the fringe field at different values of x and includes contributions from x even in excess of the pole iron width. The required curve is obtained by selecting and attaching prisms as needed. An example of a final shim shape for a BLG magnet is shown in Fig. 8.

The shimming procedure was carried out to a precision determined by the addition of a single 1.5 mm under a prism. It was considered that a piece of iron as small as this should not significantly alter the optical performance of these 3 and 5 m long magnets.

Fig. 6 - Radial variation of the effective lengths in an H-type magnet

Fig. 7 - Radial variation of the effective length of a window frame magnet

Fig. 8 - Typical shape of dipole end shims

MULTIPOLE SYNTHESIS AND QUADRUPOLE END SHIMMING

To correct a machine up to the sextupole term is not sufficient. Already at the design stage octupole corrections were made and higher order multipole corrections were foreseen. The solution which has been adopted consists of tailoring the end faces of the quadrupoles with washer packs distributed on the end faces along the pole edges; in this way, the field defined by the pole shape does not change but the magnetic length is varied according to a specific law. Using this method, we save extra magnets, power supplies and software control.

Field Multipole Synthesis

Narrow quadrupoles - Quadrupole shim adjustments were made by adding or subtracting soft iron in the form of 1.5 mm thick washers disposed along the end faces of the magnets near to the pole tips (see Fig. 9). The perturbations to measured integrated gradients produced by individual packs of adjacent washers were measured on prototype magnet of the QN type. The

gradient error data was analysed in terms of allowed quadrupole harmonics and used to solve the matrix equation:

$$[C_{ij}] \times [R_j] = [A_i] \quad (12)$$

where R_j = the number of washer packs in position j required to produce a gradient error distribution described by allowed multipole coefficients A_i and C_{ij} is the i th coefficient in the allowed fit to the perturbation produced by the j th washer pack.

This gave the number of washer packs and hence the total number of washers needed to synthesize a desired gradient error distribution which for the narrow quadrupoles had the form

$$\frac{\int G(x) ds}{\int G_0 ds} = 1 + A_1 x^2 \quad (13)$$

with all coefficients after A_1 set to zero.

The success of this method depended on selecting washer packs giving gradient perturbations that could be accurately represented by a few low-order natural multipole components.

Fig. 9 - Washer packs in a narrow quadrupole

F-Quadrupoles - The multipole shimming of the narrow F-quadrupoles was treated in the same way as for the D-quadrupoles. However, although similar techniques were developed for the asymmetric wide quadrupole, it became necessary to define the gradient distribution of these quads in terms of three different expansions of the type (13) for different regions of the quadrupole aperture. This is illustrated in Fig. 10. Such gradient curves were produced with nothing more elegant than trial and error based on the known contributions from previously measured washer packs. In the end this method proved to be remarkably fast, a desired gradient curve could be achieved after two to four iterations.

Accuracy of the Correction and Actual Integrated Gradient Distributions

The reproducibility of measured gradient error curves was ± 2 parts in 10^3 and the possibilities for adjustment allowed by the washer packs permitted

Fig. 10 - Example of a gradient error curve in a wide quadrupole

measured gradients to be fitted to desired gradients to better than 1 part in 10^3 . In practice the sensitivity of the AA optics to changes in quadrupole gradient was such that a fit to ± 1 part in 10^3 was considered satisfactory. As fine tuning of the AA continues these fits will be improved as required.

Conclusion

All the multipole fields of the Antiproton Accumulator are produced by an alteration of the pole of the dipoles and quadrupoles. This approach which could be called the "iron doctrine" imposed severe constraints on the computation, the fabrication and the operation of the magnet system but it had the advantage of stimulating the research of non-classical solutions which turned out to be quite satisfactory. When the machine came into operation, the beam proved to be the most sensitive probe of the magnetic fields. The behaviour of the orbit and betatron tunes substantiated the validity of the sextupole components and all higher multipole corrections could be made in situ.

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