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DAMPING OF NUCLEAR COLLECTIVE MOTION IN THE
EXTENDED MEAN FIELD THEORY

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ABSTRACT

The dissipation mechanism in slow nuclear collective motion is studied in the frame of the extended mean-field theory. The collective motion is explicitly treated by employing a travelling single-particle representation in the semi-classical approximation. The rate of change of the collective kinetic energy is determined by (i) One-body dissipation which reflects uncorrelated particle-hole excitations as a result of the collisions of particles with the mean field, (ii) Two-body dissipation which consists of simultaneous 2 particle- 2 hole excitations via direct coupling of the residual two-body interactions, and (iii) Potential dissipation due to the redistribution of the single-particle energies as a result of the random two-body collisions. In contrast to the first two processes the potential dissipation exhibits memory effects due to the large values of the local equilibration times.

1. Introduction

Recently there are attempts to extend the mean-field theory by including two-body collisions caused by the residual interactions¹⁻⁵. The action of two-body collisions randomizes the single-particle momentum distribution and consequently drives the system towards the thermal equilibrium^{6,7}. An additional energy dissipation is brought in by two-body collisions on top of the one-body dissipation contained in standard time-dependent Hartree-Fock theory. However within the fully microscopic description of the extended mean-field theory it is difficult to understand the dissipation and fluctuation mechanism of the collective motion. In order to achieve a better understanding of the dissipation process, it seems that inspite of loosing the detailed microscopic description, the collective motion should be explicitly treated in the frame of the extended mean-field theory. In this case it appears that, dissipative forces are acting on the collective motion in addition to the force resulting from the mean field. From the structure of the dissipative forces, the different mechanisms contributing to the dissipation then become transparent. The appearance of dissipative forces is a general property of statistical theories⁸ and it is connected with fluctuating forces acting on the part of the system which is explicitly considered. In the extended mean-field theory the dissipative forces acting on the collective motion are connected with the mean-field fluctuations which are imposed by the fluctuating component of the one-body density matrix as a result of the random behaviour of the residual interactions.

We identify the collective kinetic energy of the system by employing the travelling single-particle representation in the semi-classical approximation and calculate the rate of change of the collective kinetic energy using the extended mean-field equations and taking into account both the thermalization and the mean-field fluctuations. As a result of this calculations it comes out that the damping of the collective energy is simultaneously governed by three different mechanisms; (i) the dissipation of the dynamical potential energy by redistributions of the single particle energies via two-body collisions^{9,10}, (ii) the one-body dissipation due to the random collisions of the particles with the mean field¹¹, and (iii) the two-body dissipation due to the direct coupling of the single particle motion via the two-body residual interactions.

In section 2, a reminder of the extended mean-field equations is given. The rate of change of the collective kinetic energy is evaluated and the relations with the existing theories are discussed in section 3. The results are summarized in section 4.

2. Extended Mean-Field Equations

In a recent work¹⁾ we have studied the extension of the mean-field theory by including the residual interactions. Here we give a short summary and remind some of the basic equations. For details see ref. 1). In the extended mean-field theory the one-particle density matrix $\rho^{(1)}(t) \equiv \rho(t)$ is determined from

$$i\frac{\partial}{\partial t} \rho(t) = [\mathcal{H}(t), \rho(t)] - i K(\rho(t)) \quad (2.1)$$

where $\mathcal{H}(t)$ denotes the mean-field Hamiltonian and is defined as

$$\mathcal{H}(t) = \frac{1}{2m} p^2 + \text{Tr } v \rho(t) \equiv \frac{1}{2m} p^2 + v(t) \quad (2.2)$$

via the two-body interactions v . The second term on the r.h.s. of (2.1) is usually referred to as the collision term which, in general, is non-local in time and a complicated functional of $\rho(t)$. In the limit of long mean free path (weak-coupling) the collision term can be simplified to a large extent by neglecting the memory effects. Within the Markov approximation $K(\rho(t))$ is given by (see eq. (3.6) in ref. 1),

$$K(\rho(t)) = \int_0^\infty dt \text{Tr} [V(t), [V_I(t, \tau), \rho_0^{(4)}(t)]] \quad (2.3)$$

where $\rho_0^{(4)}(t)$ is the anti-symmetrized product of four one-particle density matrices,

$$\rho_0^{(4)}(t) = \frac{1}{4!} \sum_{k=1}^4 \rho(\vec{r}_k, \vec{r}_k; t) \quad (2.4)$$

In (2.3) trace is taken over all the coordinates except one so that $K(\rho(t))$ is a one-body operator. The residual interactions are the traceless part of the two-body interaction v and denoted by $V(t) = v - v(t)$ and $V_I(t, \tau)$ is the residual interactions in the interaction representation,

$$V_I(t, \tau) = g(\tau) V(t) g^\dagger(\tau) \quad (2.5)$$

with the modified mean-field propagator $g(\tau)$ which is given by (ref. 1) eq. (3.7))

$$g(\tau) = \exp \{ -i\mathcal{H}(t) \tau - i\Gamma(t) \} \quad (2.6)$$

Here the first factor is the mean-field propagator and the second factor is the modification arising from the residual interaction which determines the decay of the propagator. In relation with the Markov approximation it should be pointed out that the collision term (2.3) essentially involves two different characteristic times. These are the duration time $\tau_\Delta \sim \hbar/\Delta$ of a two-body collision and the decay time $\tau_\lambda \sim \hbar/\Gamma$ of the propagator which corresponds to the mean free path of the nucleons. In low-energy heavy-ion collisions τ_λ is larger than the nucleon transit time R/v_F (R = nuclear dimension, v_F = Fermi velocity) and it varies from several $\times 10^{-21}$ sec. to a few $\times 10^{-22}$ sec. as temperature increases from $T = 0$ to $T = 2-3$ MeV¹⁰⁻¹²). The duration time of a two-body collision is determined by the energy width Δ which describes the strength distributions of the matrix elements of the residual interactions as a function of the off-shell energy. The on-shell matrix elements $V_{\alpha\beta, \gamma\delta}$ with $\omega = \epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta = 0$ have the largest strength and the strength of the matrix elements decreases as $|\omega|$ increases. From a numerical calculation of the matrix elements performed by employing the harmonic oscillator wave functions¹³⁾ we estimate that for a heavy nucleus $\Delta \hbar \sim 7$ MeV (ω_0 is the oscillator frequency), which gives $\tau_\lambda \sim 10^{-22}$ sec. and indicates that the weak-coupling limit ($\tau_\Delta \ll \tau_\lambda$) is a good approximation in low-energy nuclear reactions. In the weak-coupling limit τ_Δ is the smallest characteristic time in the collision term and introduces an implicit cut-off to the τ -integration in (2.3). In this case memory effects can be ignored (Markov approximation) by neglecting the variations in $\mathcal{H}(t)$, $\Gamma(t)$ and $\rho(t)$ during the time interval of the order $\tau \sim \tau_\Delta$, and consequently, the more complicated expression for the collision term takes the form of (3.2).

The solution of (2.1) for the density matrix is usually considered in the time-dependent Hartree-Fock representation¹⁻³⁾ which however is not very well suited for a separation of the collective and the intrinsic energies

of the system. Instead we introduce the travelling representation which approximately provides an identification of the collective kinetic energy in the semi-classical limit. The travelling single-particle representation is defined by the eigenstates of the self-consistent cranking Hamiltonian $\tilde{\mathcal{H}}(t) = \mathcal{H}(t) - i\frac{\partial}{\partial t}$,

$$(\mathcal{H}(t) - i\frac{\partial}{\partial t})|\tilde{\psi}_\alpha(t)\rangle = \tilde{\epsilon}_\alpha(t)|\tilde{\psi}_\alpha(t)\rangle. \quad (2.7)$$

In the semi-classical approximation the time dependence of the mean-field is determined by a set of collective parameters $q(t)$ (here we consider one such parameter) which may be specified by the expectation values of some deformation operators $\hat{q}$, $q(t) = \langle \hat{q} \rangle$. Consequently the travelling wave functions and the energies depend on time through $q(t)$ and all higher order derivatives, $\dot{q}(t)$, $\ddot{q}(t)$, ... In the limit of slow collective motion (by neglecting $\ddot{q}(t)$ and the higher order derivatives) equation (2.7) may be replaced by

$$(\mathcal{H}(q) - i\dot{q}\frac{\partial}{\partial q})|\tilde{\psi}_\alpha(q, \dot{q})\rangle = \tilde{\epsilon}_\alpha(q, \dot{q})|\tilde{\psi}_\alpha(q, \dot{q})\rangle. \quad (2.8)$$

We expand the one-particle density matrix in terms of the complete set of the travelling states, $|\tilde{\alpha}(t)\rangle \equiv |\tilde{\psi}_\alpha(t)\rangle$,

$$\rho(t) = \sum_{\alpha\beta} \tilde{e}^{-i\tilde{\epsilon}_\alpha(t)} |\tilde{\alpha}(t)\rangle \langle \tilde{\beta}(t)| \tilde{e}^{i\tilde{\epsilon}_\beta(t)} \quad (2.9)$$

with $\tilde{e}^{-i\tilde{\epsilon}_\alpha(t)} = \int dt' \tilde{e}^{-i\tilde{\epsilon}_\alpha(t')}$. Theoretical approaches up to now have neglected the off-diagonal elements $\rho_{\alpha\beta}(t)$ ($\alpha \neq \beta$) and derived a master equation for the diagonal elements $\rho_{\alpha\alpha}(t) \equiv \rho_\alpha(t)$ which correspond to the occupation probabilities²⁻⁵. The master equation reads (see eq. (3.10) in ref.¹),

$$\frac{\partial}{\partial t} \rho_\alpha(t) = -2\Gamma_\alpha(t) \rho_\alpha(t) + \sum_{\alpha\beta, \gamma\delta} W_{\alpha\beta, \gamma\delta}(t) \rho_\gamma(t) \rho_\delta(t) \bar{\rho}_\alpha(t) \bar{\rho}_\beta(t) \quad (2.10)$$

where $\bar{\rho}_\alpha(t) = 1 - \rho_\alpha(t)$, $W_{\alpha\beta, \gamma\delta}(t)$ is the transition probabilities given in terms of the residual interactions and

$$\Gamma_\alpha(t) = \frac{1}{2} \sum_{\alpha\beta, \gamma\delta} W_{\alpha\beta, \gamma\delta}(t) \rho_\beta(t) \bar{\rho}_\gamma(t) \bar{\rho}_\delta(t) \quad (2.11)$$

is the decay width of the particle state $|\tilde{\alpha}(t)\rangle$ into all possible 2 particle-1 hole configurations.

The expectation value of a one-body operator $\hat{\mathcal{O}}$ can then approximately be evaluated as

$$\langle \hat{\mathcal{O}} \rangle = \text{Tr } \hat{\mathcal{O}} \rho(t) \approx \frac{1}{N} \langle \tilde{\alpha}(t) | \hat{\mathcal{O}} | \tilde{\alpha}(t) \rangle_{\rho_{\alpha\alpha}(t)} \quad (2.12)$$

which reflects the neglect of the off-diagonal elements $\rho_{\alpha\beta}(t) = (\delta\rho(t))_{\alpha\beta}$ and consequently the neglect of the mean-field fluctuations. In this contribution we want to stress that the approximation (2.12) should be refined in order to include the effects of the mean-field fluctuations connected with the off-diagonal elements of the density matrix. The off-diagonal part $\delta\rho(t)$ of the density matrix, which may be considered a fluctuating quantity with zero mean, introduces a fluctuating component to the mean field, $\delta v(t) = \text{Tr } v \delta\rho(t)$, according to (2.2). In this case the single-particle wave functions determined by (2.7) have a fluctuating component $|\tilde{\delta\alpha}(t)\rangle \propto \delta v(t)$ which is proportional to $\delta v(t)$ in the lowest order. The quadratic contributions in $\delta\rho(t)$ to the expectation value (2.12) do not vanish and may in fact grow in time. For large mean-field fluctuations (or off-diagonal elements) (2.12) then will no longer be a good approximation to the expectation values.

One way to take account of the off-diagonal elements is to calculate the expectation values in the spirit of the time-dependent RPA¹⁴ or to calculate the density matrix in the path integral approach¹⁵. This corresponds to the inclusion of quantal fluctuations of the mean field via virtual phonon excitations. In contrast to these approaches we are here mostly interested in the statistical fluctuations of the mean field. Although it is difficult to calculate the off-diagonal elements $\rho_{\alpha\beta}(t)$ explicitly, they in principle are contained in the equation of motion (2.1). Instead of calculating the expectation values directly, we prefer to calculate the rate of the expectation values $\frac{d}{dt} \langle \hat{\mathcal{O}} \rangle$ using the extended mean-field equation (2.1). In this way the effects of the thermalization and the mean-field fluctuations on the expectation values are implicitly included.

3. Damping of Collective Energy

A differential equation for the rate of change of the expectation value of a one-body operator $\mathcal{O}(t)$, which might explicitly depend on time, can be derived by taking time derivative on both sides of (2.12)

$$\frac{d}{dt} \langle \mathcal{O}(t) \rangle = \text{Tr } \mathcal{O}(t) \frac{\partial}{\partial t} \rho(t) + \text{Tr } \frac{\partial \mathcal{O}(t)}{\partial t} \rho(t) \quad (3.1)$$

Using (2.1) and the cyclic invariance of trace, (3.1) can be written as

$$\begin{aligned} \frac{d}{dt} \langle \mathcal{O}(t) \rangle &= \frac{\partial \langle \mathcal{O}(t) \rangle}{\partial t} + \langle -i[\mathcal{O}, H] \rangle_0 \\ &\quad - \int_0^\infty d\tau \langle [[\mathcal{O}, V(t)], g(\tau) V(t) g^+(\tau)] \rangle_0 \end{aligned} \quad (3.2)$$

with H is the total Hamiltonian. Here we introduce the averages $\langle A \rangle_0$ defined by

$$\langle A \rangle_0 \equiv \text{Tr } A \rho_0(k)(t) \quad (3.3)$$

where k is the rank of the operator A and $\rho_0(k)(t)$ is the uncorrelated k -particle density matrix defined similar to (2.4). We want to apply equation (3.2) in order to calculate the rate of change of the collective kinetic energy. For this aim we have to define the operator corresponding to the collective kinetic energy of the system. In the semi-classical approximation, the operator corresponding to the collective kinetic energy may be identified with the help of the travelling representation (2.8) in the following way. We consider the quasi-static single-particle wave functions defined by

$$\mathcal{A}(t) |\psi_\alpha(t)\rangle = \varepsilon_\alpha(t) |\psi_\alpha(t)\rangle \quad (3.4)$$

and introduce the operator $\hat{T}$ of infinitesimal displacement of the quasi-static states $|\alpha(t)\rangle \equiv |\psi_\alpha(t)\rangle$ as

$$\hat{T} |\alpha(t)\rangle = + i \frac{\partial}{\partial t} |\alpha(t)\rangle \quad (3.5)$$

In the semi-classical approximation the time dependence of the mean-field is determined by a set of parameters $q(t)$ (see eq. (2.8)) and consequently the quasi-static wave functions and energies depends parametrically on time via $q(t)$. In this case the infinitesimal displacement operator becomes as $\hat{T} = i\dot{q}(t) \frac{\partial}{\partial q}$. The travelling wave functions and the energies may be approximately given in terms of the quasi-static representation^{10,16,17},

$$|\alpha(t)\rangle = e^{ip\dot{q}} |\alpha(q)\rangle \quad (3.6)$$

$$\varepsilon_\alpha(t) = \varepsilon_\alpha(q) - \frac{p^2}{2} \langle \alpha(q) | \hat{M}^{-1} | \alpha(q) \rangle \quad (3.7)$$

with

$$\hat{M}^{-1} = [-\ddot{q}, [\ddot{q}, H]] \quad (3.8)$$

Here the parameter $p(t)$ is identified as the conjugate momentum and it is related to $q(t)$ via the condition

$$\dot{q}(t) = \sum_\alpha \langle \alpha(t) | [-i\dot{q}, H] | \alpha(t) \rangle \rho_\alpha(t) \quad (3.9)$$

which can be evaluated using (3.7) and considering the expansion

$$e^{-ip\dot{q}[-i\dot{q}, H]} e^{ip\dot{q}} = [-i\dot{q}, H] + p \hat{M}^{-1} + \dots \quad (3.10)$$

in powers of p . The expectation value of the first term vanishes due to time-reversal symmetry and the higher order terms do not contribute. Hence (3.10) becomes

$$\dot{q}(t) = p(t) \hat{M}^{-1}(q) \quad (3.11)$$

with the mass parameter

$$\hat{M}^{-1}(q) = \sum_\alpha \langle \alpha(q) | \hat{M}^{-1} | \alpha(q) \rangle \rho_\alpha(q) \quad (3.12)$$

Employing the approximate expression (3.6) for the travelling wave-functions and assuming that $[\ddot{q}, i\dot{q}] = 1$, $\beta = i \frac{\partial}{\partial \dot{q}}$, it can be seen that the expectation value $\text{Tr}(t)$ of $\hat{T}$ over the travelling states

$$\text{Tr}(t) = \sum_\alpha \langle \alpha(t) | \hat{T} | \alpha(t) \rangle \rho_\alpha(t) \quad (3.13)$$

gives twice the collective kinetic energy¹⁷⁾

$$T(t) = p^2 M^{-1}(q) \quad (3.14)$$

which indicates that the operator of the collective kinetic energy may be identified with $\tilde{T}$ and the rate of change of the collective kinetic energy may be calculated using (3.2) with $\theta = \tilde{T}$. It should be stressed that the mean-field propagator $g(\tau)$ in the collision term of (3.2) is not diagonal in the travelling representation (3.6) due to the fact that the travelling states are not eigenstates of $\mathcal{H}(t)$. In order to take the off-diagonal contributions into account we expand the collision term in powers of the off-diagonal part of the mean-field Hamiltonian, $\mathcal{H}_{off}^{(0)}(t) = \sum_{\alpha \neq \beta} |\alpha(t)\rangle \mathcal{H}_{\alpha\beta}^{(0)}(t) \langle \beta(t)|$, using

$$g(\tau) V(t) g^\dagger(\tau) = g_d(\tau) \{V(t) - i\tau[\tilde{T}, V(t)] + \dots\} g_d^\dagger(\tau) \quad (3.15)$$

Here $\mathcal{H}_{off}^{(0)}$ is replaced by $\tilde{T}$ since according to (2.8) their off-diagonal elements are equivalent and $g_d(\tau)$ is the propagator corresponding to the diagonal part of the Hamiltonian, $\mathcal{H}_d^{(0)}(t) = \sum_{\alpha} |\alpha(t)\rangle \mathcal{H}_{\alpha\alpha}(t) \langle \alpha(t)|$ and defined via (2.6) with $\mathcal{H}(t)$ replaced by $\mathcal{H}_d^{(0)}(t)$. Then the equation for $T(t) = \langle \tilde{T} \rangle$ up to the second order in $\tilde{T}$ (or collective velocity $\dot{q}$) becomes

$$\begin{aligned} \frac{d}{dt} T(t) = & \langle \frac{\partial \tilde{T}}{\partial t} \rangle_0 + \langle -i[\tilde{T}, H] \rangle_0 \\ & - \int_0^\infty d\tau \langle [[\tilde{T}, V(t)], g_d(\tau) V(t) g_d^\dagger(\tau)] \rangle_0 \\ & + \int_0^\infty d\tau i\tau \langle [[\tilde{T}, V(t)], g_d(\tau) [\tilde{T}, V(t)] g_d^\dagger(\tau)] \rangle_0 \end{aligned} \quad (3.16)$$

The terms on the r.h.s. can be explicitly evaluated in the travelling representation (3.6) by noting that $\tilde{T} = \dot{q}\tilde{p}$ is acting on the wave functions only and by observing¹⁷⁾

$$\begin{aligned} \langle \tilde{p}_n(t) | [\tilde{p}, H] | \tilde{p}_n(t) \rangle &= -i \frac{\partial}{\partial q} \langle \tilde{p}_n(q) | e^{-i p q} H e^{i p q} | \tilde{p}_n(q) \rangle \\ &= -i \frac{\partial}{\partial q} \langle \tilde{p}_n(q) | \{H + \frac{p^2}{2} \tilde{M}^{-1}\} | \tilde{p}_n(q) \rangle \end{aligned} \quad (3.17)$$

and

$$\begin{aligned} \langle \tilde{p}_n(t) | [\tilde{p}, V(t)] | \tilde{p}_m(t) \rangle &= -i \frac{\partial}{\partial q} \langle \tilde{p}_n(q) | V(t) | \tilde{p}_m(q) \rangle \\ &+ i \langle \tilde{p}_n(q) | \frac{\partial V}{\partial q} | \tilde{p}_m(q) \rangle \end{aligned} \quad (3.18)$$

where $|\tilde{p}_n(q)\rangle$ and $|\tilde{p}_m(q)\rangle$ are Slater determinants of the quasi-static single-particle states $|\alpha(q)\rangle$. The second line of (3.17) follows the expansion in powers of p , the higher order terms do not contribute, the term linear in p vanishes due to the time-reversal symmetry and only the mass contribution remains. It is important to note that although $V(t) = v \cdot v(t)$ is a traceless two-body operator, i.e. $\text{Tr } V(t) \rho(t) = 0$, this is not true for $[\tilde{p}, V(t)]$, and as can be seen from the second term on the r.h.s. of (3.18) it has a non-vanishing one-body part. Collecting the terms together (3.16) becomes

$$\frac{d}{dt} [p^2/2M(q)] = -\frac{\partial}{\partial q} [U(q, t) + \Delta U(q, t)] - [\dot{q}_1(t) + \dot{q}_2(t)] \quad (3.19)$$

where in the first term $U(q, t)$ is the potential energy (intrinsic energy)

$$U(q, t) = \sum_{\alpha} \langle \alpha | -\nabla^2/2m | \alpha \rangle \rho_{\alpha}(t) + \frac{1}{2} \sum_{\alpha\beta} V_{\alpha\beta}(q) \rho_{\alpha}(t) \rho_{\beta}(t) \quad (3.20)$$

or

$$U(q, t) = \sum_{\alpha} \epsilon_{\alpha}(q) \rho_{\alpha}(t) - \frac{1}{2} \sum_{\alpha\beta} V_{\alpha\beta}(q) \rho_{\alpha}(t) \rho_{\beta}(t)$$

and $\Delta U(q, t)$ is a correction to the potential energy due to the virtual transitions imposed by the residual interactions

$$\Delta U(q, t) = \frac{1}{4} \sum_{\alpha\beta, \gamma\delta} V_{\alpha\beta, \gamma\delta}(q) \frac{\sum_{\alpha\beta, \gamma\delta} \frac{\omega_{\alpha\beta, \gamma\delta}}{\omega_{\alpha\beta}^2 + \Gamma^2} \rho_{\alpha}(t) \rho_{\beta}(t) \bar{\rho}_{\gamma}(t) \bar{\rho}_{\delta}(t)}{\omega_{\alpha\beta, \gamma\delta}^2 + \Gamma^2} \quad (3.21)$$

with $\bar{\rho}_{\gamma}(t) = 1 - \rho_{\gamma}(t)$, arising from the third term on the r.h.s. of (3.16). The quantities $\dot{q}_1(t)$ and $\dot{q}_2(t)$ are given by

$$\dot{q}_1(t) = \dot{q} \sum_{\alpha} \langle \alpha | \frac{\partial}{\partial q} v(q) | \beta \rangle^2 \frac{\sum_{\alpha\beta} \frac{\omega_{\alpha\beta}}{\omega_{\alpha\beta}^2 + \Gamma^2} \rho_{\alpha}(t) \rho_{\beta}(t)}{\sum_{\alpha\beta} \frac{\omega_{\alpha\beta}}{\omega_{\alpha\beta}^2 + \Gamma^2}} \quad (3.22)$$

and

$$\dot{q}_2(t) = \frac{\dot{q}^2}{4} \sum_{\alpha\beta, \gamma\delta} V_{\alpha\beta, \gamma\delta}(q) \frac{\sum_{\alpha\beta, \gamma\delta} \frac{\omega_{\alpha\beta, \gamma\delta}^2}{\omega_{\alpha\beta}^2 + \Gamma^2} \rho_{\alpha}(t) \rho_{\beta}(t) \bar{\rho}_{\gamma}(t) \bar{\rho}_{\delta}(t)}{\sum_{\alpha\beta, \gamma\delta} \frac{\omega_{\alpha\beta, \gamma\delta}^2}{\omega_{\alpha\beta}^2 + \Gamma^2}} \quad (3.23)$$

with $\Gamma_{\alpha\beta}(t) = \Gamma_{\alpha}(t) + \Gamma_{\beta}(t)$, $\omega_{\alpha\beta}(t) = \mathcal{E}_{\alpha}(t) - \mathcal{E}_{\beta}(t)$,

$$\Gamma_{\alpha\beta\gamma\delta}(t) = \Gamma_{\alpha}(t) + \Gamma_{\beta}(t) + \Gamma_{\gamma}(t) + \Gamma_{\delta}(t) \text{ and } \omega_{\alpha\beta\gamma\delta}(t) = \mathcal{E}_{\alpha}(t) + \mathcal{E}_{\beta}(t) - \mathcal{E}_{\gamma}(t) - \mathcal{E}_{\delta}(t)$$

where $\Gamma_{\alpha}(t)$ and $\mathcal{E}_{\alpha}(t) = \epsilon_{\alpha}(q) + \frac{1}{2} p_{\alpha}^2 |\tilde{M}^{-1}|_{\alpha}$ are the width and the

energies of the single particle states. According to (3.19) the rate of

change of the collective kinetic energy is determined by two different

contributions. The first term describes the reversible changes in the

potential energy ($U + \Delta U$), which is reversible in a sense that if the occu-

pation probabilities are held constant, the energy flows back into the

kinetic energy. On the other hand the second term ($\dot{Q}_1 + \dot{Q}_2$) determines

the irreversible changes of the collective kinetic energy as a result

of the coupling of the particle motion with the fluctuating mean-field.

The damping mechanism due to the random collisions of the particles with

the mean-field, which results from the uncorrelated particle-hole excitations

around the Fermi surface, is reflected by the quantity $\dot{Q}_1(t)$ in (3.19).

The $\dot{Q}_1(t)$ dissipation is equivalent to the rate of energy dissipation

formula of the one-body dissipation theory^{11,18,19} and it may be attributed

to the shape fluctuations of the mean-field. The quantity $\dot{Q}_2(t)$ in (3.19)

describes the dissipation as a result of the simultaneous 2 particle-2 hole

excitations via direct coupling of the residual interactions and it is

referred to as the two-body dissipation. In contrast to $\dot{Q}_1(t)$ which is a

surface effect, $\dot{Q}_2(t)$ describes a volume dissipation and it may be con-

sidered as if arising from the local fluctuations of the mean-field. The

dissipation mechanism contained in (3.19) is further complicated by

the fact that the force $F(q,t) = -\frac{\partial}{\partial q} U(q,t)$ evolves in a time irreversible

manner as a result of the time dependence of the occupation probabilities

(cf. master eq. (2.10)). The irreversible behaviour of $F(q,t)$ means that

the energy stored in the dynamical potential can be dissipated into the

intrinsic degrees of freedom by redistributions of the occupation probabili-

ties^{9,10}. This dissipation mechanism can be understood better by explicitly

considering the coupling between the rate equation (3.19) and the master

equation (2.10). Using the master equation, an equation for the force,

which may be approximated by $F(q,t) = -\sum \frac{\partial}{\partial q} \epsilon_{\alpha}(q) \rho_{\alpha}(t)$, can be derived.

Denoting the average value of $\frac{\partial^2}{\partial q^2} \epsilon_{\alpha}(q)$ by $\epsilon''(q)$ we have

$$\frac{d}{dt} F(q,t) + \dot{q} \epsilon''(q) = -2 \Gamma(t) [F(q,t) - F_{\text{eq}}(q,t)] \quad (3.24)$$

where the r.h.s. is obtained from the master equation replacing $\Gamma_{\alpha}(t)$ by an average decay width $\Gamma(t)$ and approximating the gain term by its

equilibrium value. This approximation is equivalent to a relaxation ansatz

with the local equilibration time $\tau_{\text{loc}}(t) = 2/\Gamma(t)$ ^{6,10}. Assuming

$F_{\text{eq}}(q,t)$ is a smooth-function of time, the solution of (3.24) can be

given as

$$F(t) = - \int_0^t dt' \chi(t,t') \dot{q}(t') e''(t') + F_{\text{eq}}(t) [1 - \chi(t,0)] + F(0) \chi(t,0) \quad (3.25)$$

with

$$\chi(t,t') \approx \exp \{ -(t-t')/\tau_{\text{loc}}(t) \} \quad (3.26)$$

The local equilibration times in low-energy dissipative heavy-ion collisions are estimated using (2.11) within a Fermi gas model in ref.¹⁰. According

to these estimates $\tau_{\text{loc}}(t)$ is very large during the initial phase of the

collision, $\tau_{\text{loc}}(t) \sim \text{several } 10^{-21} \text{ sec.}$, and it becomes considerable smaller

during the strongly damped phase, $\tau_{\text{loc}}(t) \sim 5 \times 10^{-22} \text{ sec.}$ Therefore during

the initial phase, the occupation probabilities remain unchanged for a

long time specified by $t \ll \tau_{\text{loc}}(t)$ and consequently (3.25) acts as a

conservative force $F(q) = -\sum \frac{\partial}{\partial q} \epsilon_{\alpha}(q) \rho_{\alpha}(0)$ with no explicit time dependence.

The energy dissipation during this time interval is mainly determined by

the uncorrelated particle-hole excitations and the correlated 2 particle-

2 hole excitations as described by $\dot{Q}_1(t)$ and $\dot{Q}_2(t)$ respectively, see (3.22)

and (3.23). For later times $F(q,t)$ contributes to the damping via dissipation

of the dynamical potential energy which is stored during the initial phase.

The potential dissipation exhibits memory effects due to the large value

of the local equilibration times. This gives a non-local friction force,

the first term on the r.h.s. of (3.25), as pointed out in a recent work

of Nörenberg¹⁰, in contrast the local friction forces contained in $\dot{Q}_1 + \dot{Q}_2$.

Various dissipation mechanisms described by (3.19) are illustrated in

Fig. 1.

4. Conclusions

In the extended mean-field theory the inclusion of the residual interactions modifies the motion of the particles in two ways.

(i) The action of two-body collisions randomizes the single particle momentum distributions and hence drives the system towards the thermal equilibrium. (ii) The residual interactions introduce a fluctuating component to the mean field via the off-diagonal elements of the density matrix. In the present work, in order to bring out these effects, the collective motion is treated explicitly by employing the travelling single-particle representation which allows an identification of the collective variables in the semi-classical approximation. The rate of change of the collective energy is calculated using the extended mean-field equations. For the rate of energy dissipation a simple expression is obtained (see eq. (3.19)),

$$\frac{d}{dt} [P^2/2M(q)] = -q \frac{\partial}{\partial q} U(q,t) - \dot{Q}_1(t) - \dot{Q}_2(t) \quad (4.1)$$

in which ΔU is neglected and $U, \dot{Q}_1, \dot{Q}_2$ are given by (3.20), (3.22), (3.23) respectively. The quantity $\dot{Q}_1(t)$ is the rate of energy dissipation due to the particle-hole excitations around the Fermi surface and it is essentially equivalent to the rate calculated in the one-body dissipation theory^{18,19} (Fig. 1). On the other hand $\dot{Q}_2(t)$ describes the dissipation as a result of the direct coupling of the single-particle states via the residual interactions (2-particle-2-hole excitations) and it is identified as the two-body dissipation. The two-body dissipation is expected to be small in the low-energy heavy-ion collisions and may be neglected. The first term on the r.h.s. of (4.1) determines the rate of change of the potential energy $U(q,t)$ which also contributes to the dissipation due to the fact that at every action of random two-body collisions some part of the correlated intrinsic energy is converted into the uncorrelated energy (heat). The potential dissipation has memory effects as a result of the large value of the local equilibration times and consequently it appears as a non-local friction force¹⁰, see eq. (3.25). The numerical calculations of the one-body and the two-body friction coefficients based on (3.22) and (3.23) are presently in progress.

The remaining problem to be studied is connected with the fluctuations of the collective variables. There is some evidence that the random two-body collisions are not responsible for the large dispersions in the dissipative heavy-ion collisions. The present work however indicates that the mean-field fluctuations should be included in order to achieve a better understanding of the fluctuations of the collective variables. This point will be studied in a forthcoming paper.

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References

1. S. Ayik, Z. Physik A 298 (1980) 83
2. C.Y. Wong and H.F. Tang, Phys. Rev. Lett. 40 (1978) 1070
3. H. Orland and R. Schaeffer, Z. Physik A 290 (1979) 191
4. G. Mantzouranis and H.C. Pauli, Z. Physik A 281 (1977) 165
5. P. Grangé, H.A. Weidenmüller and G. Wolschin, preprint MPI - H - 1981 - V8
6. H.S. Köhler, Nucl. Phys. A 343 (1980) 315
7. P. Grangé, J. Richert, G. Wolschin and H.A. Weidenmüller, Nucl. Phys. A 356 (1981) 260
8. H.A. Weidenmüller, in: Progress in Particle and Nuclear Physics, vol. 3, ed. D.H. Wilkinson (Pergamon Press, Oxford, 1980)
9. W. Nörenberg and C. Riedel, Z. Physik A 290 (1979) 385
10. W. Nörenberg, Proceedings of International Workshop IX on Gross properties of Nuclei and Nuclear Excitations, Hirschegg, Austria (1981) and Phys. Lett. in press
11. S.E. Koonin, Proceedings of the International school of Physics "Enrico Fermi", Varenna, Italy (1979)
12. P. Morel and P. Nozières, Phys. Rev. 126 (1962) 1909
13. G. Mantzouranis and H.C. Pauli, Phys. Rev. C 20 (1980) 1550
14. P.G. Reinhard, K. Goeke, and K.Y. Cusson, Z. Physik A 295 (1980) 45
15. H. Reinhardt, Nucl. Phys. A 346 (1980) 1
16. H.C. Pauli and L. Wilets, Z. Physik A 277 (1976) 83
17. F. Villars, Nucl. Phys. A 285 (1977) 269
18. S.E. Koonin, R.L. Hatch and J. Randrup, Nucl. Phys. A 283 (1977) 87
19. H. Hofmann and P.J. Siemens, Nucl. Phys. A 257 (1976) 165 and Nucl. Phys. A 275 (1977) 464

Figure 1. (a) Uncorrelated particle-hole excitations (one-body dissipation due to collisions of particles with the mean field).
 (b) Correlated 2 particle - 2 hole excitations (two-body dissipation) via the residual interactions.
 (c) Potential dissipation due to redistributions of the intrinsic energies via two-body collisions.