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SLOW COLLECTIVE NUCLEAR MOTION

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MEMORY EFFECTS IN THE ENERGY DISSIPATION FOR SLOW COLLECTIVE NUCLEAR MOTION

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Abstract

The energy dissipation in slow collective nuclear motion is considered as a combined effect of the diabatic production of particle-hole excitations and the subsequent equilibration by two-body collisions. Memory effects due to the long mean free path of the nucleons are treated analytically for an interacting Fermi gas within moving walls leading to a friction kernel (frequency dependent friction coefficient) in the classical equation of collective motion.

Deeply inelastic heavy-ion collisions have initiated remarkable activities on the theory of dissipative nuclear phenomena. Although considerable progress has been achieved in the last few years, we are still far from a complete physical understanding of these processes. Current transport theories make use of an assumed separation of the slow collective variables from the fast equilibrating intrinsic degrees of freedom [1]. These approaches are confronted by one-body theories [2], like the self-consistent time-dependent Hartree-Fock (TDHF) theory [3]. Whereas in the transport theories the residual interaction is the major agent for the assumed fast equilibration within the intrinsic degrees of freedom, residual interactions are completely ignored in the one-body theories.

The *mean free path* of a nucleon with energy ϵ close to the Fermi energy ϵ_F and for small nuclear temperatures $T \ll \epsilon_F$ is approximately given by [4]

$$\lambda = \alpha \frac{1 + \exp(-|\epsilon - \epsilon_F|/T)}{(\epsilon - \epsilon_F)^2 + \pi^2 T^2} \quad (1)$$

The factor α is $\approx 540 \text{ fmMeV}^2$ when estimated from the Fermi gas model [5] and $\approx 850 \text{ fmMeV}^2$ when estimated from the imaginary part of the optical potential (cf. Fig. 23 of [6]). With an average value $\alpha \approx 700 \text{ fmMeV}^2$ we expect the mean free path λ to vary from large values at the initial stage of a heavy-ion collision to $\approx 15 \text{ fm}$ at the completely damped stage ($T \approx 2.5 \text{ MeV}$). Therefore we expect TDHF to be realistic at the initial stage only, because the mean free path is not large enough to justify TDHF at the later stages. First steps to include two-body collisions in TDHF have been undertaken [7]. However, because of the numerical difficulties, these attempts are still rudimental. On the other hand, the current transport theories are restricted to the com-

pletely damped stage of heavy-ion collisions. This becomes apparent from the local equilibration times which follow from the mean free paths (1). Assuming three subsequent collisions to be sufficient for establishing equilibrium for fixed values of the collective variables ("local" equilibrium), we have

$$\tau_{loc} \approx 3\lambda/v_F \quad (2)$$

for the *local equilibration time* (v_F = Fermi velocity). For typical temperatures around 2.5 MeV in the completely damped stage we find $\tau_{loc} \approx 0.5 \cdot 10^{-21}$ s. In the completely damped stage of the process the collective velocities might be small enough, such that the system is always close to local (i.e. for fixed values of the collective variables) equilibrium and hence, that current transport theories are applicable. However, at the initial stage of heavy-ion collisions and for fission at small and intermediate energies the local equilibration time is considerably larger (of the order of several 10^{-21} s). Therefore we expect large memory effects in the approach phase of heavy-ion collisions (duration about $0.3 \cdot 10^{-21}$ s) but also for the low energy fission process.

This letter is intended to draw attention to and stimulate discussion on the memory effects due to the long mean free path of nucleons. In order to study this new feature in the dissipation mechanism for collective nuclear motion, an interacting Fermi gas model is introduced which analytically describes the main mechanism of energy dissipation as the result of diabatic excitations of particles and holes and subsequent equilibration by two-body collisions. The treatment presents a generalization of the classical one-body dissipation mechanism [8] and clarifies the role of giant vibrations [9] in the dissipation mechanism. The analytic study may also guide future developments of more realistic formulations of transport theories which bridge the gap between TDHF and current transport theories.

We consider *slow collective motions* such that the Fermi velocity v_F is large compared to the collective velocity v_{coll} . In this limit the individual nucleons can adjust to the collective motion in a time-reversible manner. A particular problem is encountered by the quasi-crossing of two adiabatic single-particle levels as functions of the collective variable (Landau-Zener crossing [10], cf. Fig. 1). Due to some (usually small) parts of the single-particle Hamiltonian, a crossing of adiabatic levels is forbidden. The neglect of this coupling leads to the definition of corresponding *diabatic* levels which cross each other. If the collective velocity is not too small, a nucleon occupying the lower level before the crossing of an unoccupied level will stay in the diabatic level and finds itself in the upper level after the crossing. Thus, this diabaticity leads to the excitation of a particle-hole state. The condition of more than 90% *diabaticity* in a Landau-Zener crossing leads to the condition $v_{coll} \gtrsim 25|V|^2_R/(Mc_F)$ where R and V denote the radius of the nucleus and the interaction matrix element between the diabatic levels. With a typical value of 0.5 MeV for V we obtain $v_{coll} \approx v_F/40$ for the minimum collective velocity. Combining this with $v_{coll} \ll v_F$ we find

$$\frac{1}{40} \text{ MeV} \lesssim E_{coll}/A \ll 40 \text{ MeV} \quad (3)$$

for the range of collective kinetic energy per nucleon where the diabatic approximation is applicable. These estimates are well supported by numerical studies on fission [11] and by TDHF calculations [12] on heavy-ion collisions. In general, the definition of diabatic single-particle states appropriate for the solution of a time-dependent problem is not straight forward. A simple example is given below for the single-particle motion in a box with moving walls.

On the basis of the diabatic single-particle motion the *mechanism of energy dissipation* is two-fold (cf. Fig. 1):

- (1) Due to the direct coupling of the collective motion to the diabatic shift of single-particle levels, the energy is transformed in a *time-reversible* fashion to the single-particle motion. In this way collective kinetic energy is temporarily stored as a conservative potential which results from the diabatic excitation of particle-hole states.

(ii) Due to two-body collisions which lead to single-particle levels with random slopes, the diabatic potential is destroyed. This *time-irreversible* process thus leads finally to the dissipation of the collective energy.

In the following this process is treated analytically for an *interacting nuclear Fermi gas* within moving walls. In this model the diabatic level crossings are built in by high symmetries. We consider a box located around the origin with the surfaces defined by $\bar{x}(t) = \frac{1}{2} \bar{y}(t) = \frac{1}{2} a[1+\gamma(t)]^{-1/2} = -\bar{x}(t) = -\bar{y}(t)$ and $\bar{z}(t) = \frac{1}{2} a[1+\gamma(t)] = -\bar{z}(t)$ filled with a Fermi gas of A nucleons. The collective variable $\gamma(t)$ defines a collective motion of the system *without* compression. Using periodic boundary conditions (thus neglecting surface effects) we obtain the moving diabatic single-particle wave functions

$$\psi_j(\vec{r}, t) = a^{-3/2} \exp \{ i [k_j^+(t) \cdot \vec{r} - \hbar^{-1} \int_0^t \epsilon_j(t') dt' - \hbar^{-1} M \phi(\vec{r}, t)] \} \quad (4)$$

which satisfy the Schrödinger equation except for terms proportional to $\dot{\gamma}^2$ and $\dot{\gamma}$. The quantities $\epsilon_j(\gamma)$, M and ϕ denote the energy $\hbar^2 k_j^2(\gamma)/(2M)$ of the state j, the nucleon mass and the velocity potential

$$\phi(\vec{r}, t) = -\frac{1}{2} \dot{\gamma} r^2 P_2(\cos \theta) \quad (5)$$

of the incompressible irrotational flow. The motion of the walls determine also the diabatic change of the momenta

$$\vec{k}_j(\gamma) = \{k_x^{(j)}(1+\gamma)^{1/2}, k_y^{(j)}(1+\gamma)^{1/2}, k_z^{(j)}(1+\gamma)^{-1}\} \quad (6)$$

which leads to a deformation of the Fermi distribution (Fermi sphere).

The *adiabatic deformation* of the Fermi sphere gives rise to a restoring force

$$F = -\frac{3}{2\gamma} \sum_j \epsilon_j(\gamma) n_j = \frac{1}{2} \frac{\hbar^2 Q}{M} \quad (7)$$

where n_j denotes the occupation probability of the moving diabatic state ψ_j and

$$Q(t) = \sum_j \{ 3 [k_z^{(j)}(t)]^2 - k_j^2(t) \} n_j \quad (8)$$

is the quadrupole moment of the momentum distribution. Thus the *classical equation of motion* for the collective variable $\gamma(t)$ reads

$$m \ddot{\gamma}(t) = \frac{1}{2} \hbar^2 Q(t)/M \quad (9)$$

where the collective mass $m = \frac{1}{8} a^2 A M$ is determined from the moving wave functions (4).

Up to here we have considered the diabatic approximation where the occupation probabilities n_j do not change in time. Such changes can be induced by *two-body collisions* between the nucleons. We approximate the collision integral by a relaxation ansatz

$$\frac{\partial}{\partial t} n_j(t) = -\frac{1}{\tau_{loc}} \{ n_j(t) - \bar{n}_j(\gamma, \mu, T) \} \quad (10)$$

where $\bar{n}_j$ denotes the equilibrated occupation probability for fixed values of the collective variable γ . The chemical potential $\mu(t)$ and the temperature $T(t)$ are determined by the conservation of the total number of particles and of the total excitation energy. The local equilibration time τ_{loc} introduced in (2) is taken as the measure for the time necessary to reach this (local) equilibrium. Evaluating the time derivative of (8) with (6) and (10) we obtain

$$2Q(t)/\partial t + 2 \langle k^2 \rangle_0 \dot{\gamma} = -Q(t)/\tau_{loc} \quad (11)$$

with $\langle k^2 \rangle_0 = (3/5) A k_F^2$ and k_F the Fermi momentum (equal for protons and neutrons). In deriving (11) the momenta $\vec{k}_j$ have been replaced by a continuous variable and the limits $T \ll \epsilon_F$ (Fermi energy) and $Q \ll k_F^2$ have been taken.

With (9) and (11) we have derived a *closed set of two coupled equations* for the collective variable $\gamma(t)$ and the quadrupole moment $Q(t)$ of the momentum distribution. The solution of these coupled equations is now discussed for various cases.

(i) We consider the stationary limit of the motion *driven with constant collective velocity* $\dot{\gamma}$. From (11) the stationary value $Q_{st} = -2 \langle k^2 \rangle_0 \tau_{loc} \dot{\gamma}$

of the quadrupole moment is obtained. Since the driving force has to neutralize the force $\frac{1}{2} \pi^2 Q_{3t}/M$ on the r.h.s. of (9), we find for the rate of energy dissipation

$$\dot{E}_{\text{diss}} = \dot{\gamma} F_{\text{driving}} = \frac{6}{5} A \epsilon_F \tau_{10c} \dot{\gamma}^2. \quad (12)$$

Since the *friction coefficient*

$$\xi = \frac{6}{5} A \epsilon_F \tau_{10c} \quad (13)$$

is proportional to the local equilibration time, i.e. proportional to the mean free path (1), its temperature dependence becomes proportional to T^{-2} for n_j close to the equilibrium distribution. The rate (12) of energy dissipation can be compared with the wall formula of one-body dissipation[8]. Application of [8] to our model yields $\xi_W = (9/8) A \epsilon_F (a/v_F)$ with the Fermi velocity v_F . This agrees with (13) for $\tau_{10c} = (15/16) a/v_F \approx a/v_F$, i.e. for the local equilibration time given by the time of flight across the box. Indeed, it is assumed in [8] that a particle which has changed momentum by a collision with the moving wall, equilibrates its momentum with the other particles in the subsequent collision with another wall. Therefore, in contrast to our result (13) the wall friction coefficient ξ_W is independent of the temperature. It should also be noted that the expression (13) gives considerably larger friction since $\tau_{10c} \gg a/v_F$ (cf. discussion below (2)).

(ii) Solving (11) formally for $Q(t)$ and inserting this result in (9) we find the equation

$$\ddot{\gamma}(t) + \int_0^t dt' \xi(t-t') \dot{\gamma}(t') = \frac{1}{2} \frac{\pi^2 Q(0)}{Mm} \exp \left[-\frac{t}{\tau_{10c}} \right] \quad (14)$$

for the collective variable $\gamma(t)$ alone. The friction coefficient of current friction models is replaced here by the *friction kernel*

$$\xi(t-t') = \omega_0^2 \exp \left[-\frac{t-t'}{\tau_{10c}} \right] \quad (15)$$

with $\omega_0^2 = (6/5)(A/m)\epsilon_F$. The non-locality of the friction kernel is determined

by the local equilibration time. The corresponding frequency dependent friction coefficient which is defined by the Fourier transform $\tilde{\xi}(v)$ of $\xi(t-t')$ exhibits a pole at $v = -i/\tau_{10c}$.

(iii) For $\tau_{10c} \gg t$ and $Q(0) = 0$, eq. (14) describes a harmonic vibration with energy

$$\hbar \omega_0 = 66 A^{-1/3} \text{ MeV} \quad (16)$$

which is readily identified with the *isoscalar giant quadrupole vibration* [13] (experimental energy $63 A^{-1/3} \text{ MeV}$). The direct coupling of the collective motion to the diabatic single-particle motion results in pure giant vibrations. The use of the giant vibrations of the colliding nuclei by Broglia et al. [9] as agents for the dissipation of energy in heavy-ion collisions can be regarded as an example of the dissipation mechanism described here. However, the general concept goes beyond [9] because it is not restricted to small amplitude oscillations around the equilibrium shapes of the colliding nuclei.

(iv) A crude approximation to the friction kernel in heavy-ion collisions has been put forward by assuming that the approach of the nuclei is largely diabatic. This fast approach gives rise to a *diabatic potential* which then decays in time [14]. Since this diabatic potential arises from the diabatic single-particle motion and not from nuclear compression it is conceptually different from other sudden potentials [15].

In conclusion we can summarize that the combined effect of diabatic excitation of particle-hole states and subsequent equilibration by two-body collisions can be regarded to be the major mechanism of energy dissipation in the wide range (3) of collective kinetic energies. An important feature of this dissipation mechanism is the large memory arising in the friction kernel. Thus, for example, the dissipation of relative kinetic energy in heavy-ion collisions is expected to be strongly influenced by memory effects.

Such memory effects are not included in current transport theories [1] where the coherent (diabatic) single-particle motion is not considered explicitly.

Numerical studies are necessary, however, in order to see how these memory effects show up in observable quantities.

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Fig. 1. Illustration of the dissipation mechanism as diabatic excitation of particle-hole states and subsequent equilibration by two-body collisions.