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HIGHER ORDER CORRECTIONS IN QCD

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A B S T R A C T

Some processes for which higher order corrections in the strong coupling constant have been calculated are reviewed.

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INTRODUCTION

Eight years ago it was discovered ^{1, 2} that the non-Abelian theory of quarks and gluons now called QCD has the unique property among renormalizable theories ³ of being asymptotically free. The coupling constant decreases as a logarithm of the typical scale size of the interaction. As a consequence of this discovery we now believe that we can describe many hard hadronic processes as a perturbation series in α_s . Our confidence in the perturbative treatment is based to a large extent on its successes in QED, but in fact perturbation theory in QCD emphasizes different features of the perturbative method.

Foremost among these new features is the crucial rôle played by the higher order terms in the perturbation series. As always the calculation of higher order terms gives us control on the precision of the perturbative estimate, but in addition in the case of a theory with a running coupling constant it gives meaning to the lower order corrections. The behaviour of the coupling constant is controlled by the beta function which appears in the renormalization group equation. Defining $\kappa = g^2/4\pi^2 = \alpha_s/\pi$ we have that,

$$\frac{d\kappa}{dt} = \beta(\kappa) = -(\beta_0\kappa^2 + \beta_1\kappa^3 + \beta_2\kappa^4). \quad (1)$$

where ^{1,2,4,5}

$$\begin{aligned} 4\beta_0 &= (11 - \frac{2}{3}n_f), & 16\beta_1 &= (102 - \frac{38}{3}n_f), \\ 64\beta_2 &= (\frac{2857}{2} - \frac{5033}{18}n_f + \frac{325}{54}n_f^2) \end{aligned}$$

For the particular case of four flavours we find that,

$$\frac{d\kappa}{dt} = -\beta_0\kappa^2(1 + c\kappa + d\kappa^2) \quad (2)$$

with $\beta_0 = 2.083$, $c = 1.540$, $d = 3.047$.

SCHEMES AND PRESCRIPTIONS

In general the values of the coefficients in any perturbative expansion such as Eq. (2) depend on the particular renormalization prescription used to remove the ultra-violet divergences. If we consider

for example, the three gluon coupling, the bare and renormalized couplings are related by

$$g_R = Z_1^{-1} Z_3^{\frac{3}{2}} g_B \quad (3)$$

Differing definitions of the renormalization constants Z , chosen subjects to the constraints of the Ward Identities, are

$$Z'_1 = Z_1(1 + \sigma_1 \kappa + \dots), \quad Z'_3 = Z_3(1 + \sigma_3 \kappa + \dots). \quad (4)$$

These expressions lead to coupling constants which are related by a perturbation expansion

$$\kappa' = \kappa(1 + (3\sigma_3 - 2\sigma_1) \kappa + \dots) \quad (5)$$

In fact the first two terms in Eq. (1) denoted by β_0, β_1 are independent of such changes of scheme, whereas the third, (β_2) will in general depend on the renormalization prescription. As a consequence the significance of β_2 alone (calculated in MS scheme, Feynman gauge) is limited. Its implementation will require the calculation of a complete physical quantity through to this order. It will therefore be ignored in the following discussion.

In the literature physical quantities have been calculated in several renormalization schemes which are detailed in Table 1.

Table 1 Subtraction schemes

Subtraction Scheme	Definition	Advantages	Qualifications
MS ⁶	Subtraction of poles $\frac{1}{\epsilon}; \quad \epsilon = \frac{4-n}{2}$	Simple gauge invariant	Unphysical definition requires exact specification of extension to n dimensions Spinor Trace $\text{Tr } 1 = 4$ $\int d\Omega = (4\pi)^{1-\epsilon}$ $\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$
$\overline{\text{MS}}$ ⁷	Subtraction of $\frac{1}{\epsilon}$ $\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \ln 4\pi - \gamma_E$		
MOM ^{8, 9}	Determination of (e.g.) three gluon vertex in Landau gauge at symmetric spacelike point.	Allows physical intuition of correct scale ?	Vertex dependent Gauge dependent

For the particular case of four flavours we find

$$\kappa_{\overline{\text{MS}}} = \kappa_{\text{MOM}} (1 - 7.29\kappa + \dots) \quad (6)$$

$$\kappa_{\overline{\text{MS}}} = \kappa_{\text{MOM}} (1 - 3.22\kappa + \dots) \quad (7)$$

where the momentum scheme (MOM) refers to the renormalization of the three gluon vertex in the Landau gauge at the symmetric spacelike point ⁸.

The choice of renormalization scheme does not fix the one parameter ambiguity implicit in Eq. (1), since we have still not selected the scale at which we determine the strong interaction coupling constant. Consider the case of the e^+e^- total cross-section which has been calculated through to two loops.

$$R = N_c \sum_f e_f^2 [1 + S(Q)] \quad (8)$$

$$S(Q) = \kappa(Q) [1 + A(Q) \kappa(Q)] \quad (9)$$

In the various schemes defined above the coefficient A^* has the values ¹⁰

$$A_{\overline{\text{MS}}}(Q) = 5.60 \quad A_{\overline{\text{MS}}}(Q) = 1.52 \quad A_{\text{MOM}}(Q) = -1.69 \quad (10)$$

Should the running coupling constant in Eq. (9) be evaluated at Q or some multiple of Q given by (xQ) ? If we knew the whole perturbation series such haggling would be irrelevant; because perturbation series are normally only available truncated to the first two terms such rescalings may significantly alter the estimate for the size of the function S as well as the apparent rate of convergence of the perturbation series. At a rescaled value of Q

$$S(xQ) = \kappa(xQ) [1 + A(xQ) \kappa(xQ) + \dots] \quad (11)$$

where

$$A(xQ) = A(Q) + 2\beta_0 \ln x \quad (12)$$

A strong dependence of the size of the perturbatively evaluated quantity on the value chosen for x is first and foremost an indication that the series is unlikely to be trustworthy. It has been suggested ¹¹ that an optimum choice for x may be obtained by requiring the truncated series to be invariant under small changes of x . To apply this condition it is convenient to define $\tau = \beta_0 \ln(x^2 Q^2 / Q_0^2)$ where Q_0^2 is some reference momentum at which we assume the value of α_s is known. We impose the condition that,

$$\left. \frac{\partial S(\tau)}{\partial \tau} \right|_{\tau = \tau^*} = 0 \quad (13)$$

¹¹) The notation here may be somewhat confusing; A is a number whose value depends on the choice of scale Q in the running coupling constant.

where τ^* corresponds to the "optimum" x^* . Applying Eq. (13) in Eq. (9) we obtain

$$A^* = - \frac{c}{2(1+c\kappa^*)} \quad S^* = \kappa^* \frac{1+c\kappa^*/2}{(1+c\kappa^*)} \quad (14)$$

where κ^* is the solution of the equation

$$\int_{\kappa(Q_0)}^{\kappa^*} \frac{d\kappa'}{\kappa'^2(1+c\kappa')} + \frac{c}{2(1+c\kappa^*)} = \beta_0 \ln \frac{Q^2}{Q_0^2} - A(Q^2) \quad (15)$$

which in the limit $c\kappa < 1$ gives

$$x^* = \exp - \left[\frac{(A(Q)+c/2)}{2\beta_0} \right] \quad (16)$$

There are corresponding formula for perturbative series which begin at order κ^n for $n \neq 1$. Equation (16) makes it clear that the numerical difference between the scale for which $A = 0$ and the "optimum" scale is small.

The only other piece of information which we need to make concrete predictions is the actual magnitude of the coupling constant. This is normally specified through a parameter Λ defined as an approximate solution of

$$\beta_0 \ln \frac{Q^2}{\Lambda^2} = \int^{\kappa(Q)} \frac{d\kappa'}{\kappa'^2(1+c\kappa')} \quad (17)$$

where traditionally the constant of integration is specified such that

$$\kappa(Q) \sim \frac{1}{\beta_0 \ln Q^2/\Lambda^2} - \frac{c \ln(\ln Q^2/\Lambda^2)}{\beta_0^2 \ln^2 Q^2/\Lambda^2} + O\left(\frac{1}{\ln^3 Q^2/\Lambda^2}\right) \quad (18)$$

This procedure has the disadvantage that a further expansion in inverse powers of $\ln(Q^2/\Lambda^2)$ is being performed. At moderate values of Q^2 the numerical difference retaining one or two inverse powers of $\ln(Q^2/\Lambda^2)$ is substantial. We are therefore introducing another expansion parameter into the theory which expressed in terms of the coupling constant has large corrections.

A better procedure¹² would be to choose the integration constant in Eq. (17) such that the lower and higher orders are matched at $Q^2 = Q_0^2$

$$\kappa(Q) \sim \frac{1}{\beta_0 \ln Q^2/\Lambda^2} - \frac{c(\ln \ln Q^2/\Lambda^2 - \ln \ln Q_0^2/\Lambda^2)}{\beta_0^2 \ln^2 Q^2/\Lambda^2} \quad (19)$$

where Q_0^2 is some four momentum squared in the physically measured range.

This is equivalent to forgetting about Λ altogether and specifying the value of κ at a given Q_0^2 . Experimentalists naturally measure the quantity κ at some Q_0^2 ; expression of their results in terms of Λ leads to an unappealing magnification of their error. The value of κ at any Q^2 is given by

$$\beta_0 \ln \frac{Q^2}{Q_0^2} = \int_{\kappa(Q_0^2)}^{\kappa(Q^2)} \frac{d\kappa'}{\kappa'^2(1+c\kappa')} \quad (20)$$

No further expansion need be made so the ambiguities and pitfalls associated with Λ are avoided. I therefore recommend that Λ should be expunged from the literature. For the numerical estimates in this paper I choose,

$$\kappa_{\overline{MS}}(Q_0^2) = 0.0551 \quad \alpha_s^{\overline{MS}}(Q_0^2) = 0.173 \quad Q_0^2 = 10 \text{ GeV}^2 \quad (21)$$

For the unconvinced this corresponds to $\Lambda_{\overline{MS}} = 100 \text{ MeV}$ in Eq. (18). From the initial condition Eq. (21) I have plotted the behaviour of κ as a function of Q in Fig. 1.

Fig. 1 - $\kappa = \alpha_s/\pi$ and α_s plotted as a function of Q starting from initial condition Eq. (21). The dotted line is the prediction of Eq. (18) and is indicative of the influence of higher order terms.

The predictions for the e^+e^- total cross-section using this value of the coupling constant are shown in Table 2 at $Q^2 = 900$. The perturbative corrections are small at this energy and give a correction of about

4%; the quoted ¹³ experimental uncertainties in the normalization of the e^+e^- cross-section are of the order of 10%. The prediction of perturbative QCD has a much smaller uncertainty. The value of the coupling constant in the momentum scheme is obtained by imposing Eq. (7) at $Q_0^2 = 10 \text{ GeV}^2$.

Table 2 Predictions for correction S to parton model result in e^+e^- annihilation at $Q^2 = 900 \text{ GeV}^2$. Quantities S and A are given in Eq. (9).

κ	A	S
$\kappa_{\overline{\text{MS}}} = 0.036$	1.53	0.038
$\kappa_{\text{MOM}} = 0.039$	-1.694	0.037
$\kappa^* = 0.039$ $x_{\overline{\text{MS}}}^* = 0.58$	-0.726	0.038

DECAYS OF $Q\bar{Q}$ BOUND STATES

Another extremely interesting set of processes for which radiative corrections have been calculated are the decays of bound states composed of heavy quarks. If the mass of the heavy quark is much larger than the characteristic size of the state,

$$m_Q \gg \frac{1}{r_c} \sim \Lambda \quad (22)$$

the annihilation into gluons occurs at a scale m_Q and the binding effects factor. The decays of heavy $Q\bar{Q}$ states with $J^{PC} = 0^{-+}$ such as η_c, η_b have been calculated. The radiative corrections to the two photon width of the η_c are,

$$\Gamma^{(1)}(Q\bar{Q} \rightarrow 2\gamma) = \Gamma^{(0)}(Q\bar{Q} \rightarrow 2\gamma) \left[1 + c_F \kappa \left(\frac{\pi^2}{4v} - 5 + \frac{\pi^2}{4} \right) \right] \quad (23)$$

where v is the relative velocity of the quark-antiquark. On taking ratios of different rates the divergence drops out

$$\frac{\Gamma(Q\bar{Q} \rightarrow 2G)}{\Gamma(Q\bar{Q} \rightarrow 2\gamma)} = \frac{2\pi^2}{9e_q^2\alpha^2} S(2m_Q) ; \quad (24)$$

$$S(2m_Q) = \kappa^2(2m_Q) [1 + A(2m_Q)\kappa]$$

where

$$A_{\overline{MS}}(2m_Q) = \left(\frac{61}{2} - \frac{2}{3}\ln 2 - \frac{13\pi^2}{8}\right) = 14.00$$

In Table 3 we illustrate the predictions for the η_b resulting from this formula. The results vary significantly ($\sim 20\%$) depending on the scheme and scale for the interaction which is chosen. It has been suggested in the literature ⁹ that the modest nature of the correction in the momentum subtraction scheme at scale m_Q provides a simple physical picture of the decays of states containing heavy quarks. Is the smallness of the correction at scale m_Q fortuitous?

Table 3 Results from Eq. (24) for $2m_Q = 9.4$ GeV

$\kappa_{\overline{MS}}(2m_Q) = 0.043$	$A_{\overline{MS}}(2m_Q) = 14.0$	$S_{\overline{MS}}(2m_Q) = 0.0030$
$\kappa_{\overline{MS}}(m_Q) = 0.050$	$A_{\overline{MS}}(m_Q) = 8.22$	$S_{\overline{MS}}(m_Q) = 0.0035$
$\kappa_{\text{MOM}}(2m_Q) = 0.049$	$A_{\text{MOM}}(2m_Q) = 7.56$	$S_{\text{MOM}}(2m_Q) = 0.0033$
$\kappa_{\text{MOM}}(m_Q) = 0.058$	$A_{\text{MOM}}(m_Q) = 1.78$	$S_{\text{MOM}}(m_Q) = 0.0038$

We can test this conjecture by looking at the ratios of rates which are free from ambiguities in the choice of scale. Following Ref. 14 we define

$$B(J^{PC}) = \frac{\Gamma(J^{PC} \rightarrow \gamma\gamma)}{\Gamma(J^{PC} \rightarrow \text{hadrons} (2G))} \quad (25)$$

The results for these ratios are

$$\frac{B(0^{--})}{B(0^{++})} = (1 + 0.9\kappa) \quad (26)$$

$$\frac{B(2^{++})}{B(0^{++})} = (1 + 6.5\kappa) \quad (27)$$

$$\Gamma(0^{++} \rightarrow \gamma\gamma) / (2^{++} \rightarrow \gamma\gamma) = \frac{15}{4}(1 + 5.5\kappa) \quad (28)$$

$$\Gamma(0^{++} \rightarrow \text{hadrons}) / (2^{++} \rightarrow \text{hadrons}) = \frac{15}{4}(1 + 12\kappa) \quad (29)$$

The above results are given for the case of bound states containing charmed quarks. The corresponding results for states containing heavier quarks can be obtained by reference to the original paper. The fact that the corrections continue in some cases to be large is

in contradiction with the philosophy that the correct choice of scale in the momentum subtraction scheme is a panacea for all convergence ills. The matrix elements clearly contain more information than just the number of gluons to which the state decays.

The definitive result ¹⁶ for the radiative corrections to the experimentally most important decay $J/\psi, T \rightarrow 3G$ will also help us to understand whether the smallness of the correction to η_c at scale m_Q is fortuitous.

THREE JET STRUCTURE IN e^+e^- ANNIHILATION

The calculation of the higher order corrections to the three jet structure in e^+e^- annihilation is a matter of some experimental importance. Values for α_s derived using the lowest order formula ¹⁷ [i.e., $O(\alpha_s)$] are subject to systematic errors of the order of 10% ¹⁸. The extraction of the value of α_s from the raw data requires a model for the fragmentation of quarks and gluons into hadrons (normally the Feynman-Field model ¹⁹) but the value of α_s obtained is independent of the values of the parameters taken in this particular model. One should however worry whether this is true in all models and also whether the separation which has been made between the calculation of the cross-section for the production of on-shell quarks and gluons and their subsequent fragmentation into hadrons is really a meaningful one; such a procedure runs the risk of double counting.

The radiative corrections to the three jet structure have been calculated in terms of two variables. The first of these comes from the consideration of the 3×3 infra-red finite tensor,

$$\theta^{ij} = \sum_a \frac{P_a^i P_a^j}{|\vec{P}_a|} / \sum_a |\vec{P}_a| \quad (30)$$

where P_a^i is the centre-of-mass three momentum of the a^{th} particle and the sum on a runs over all final state particles. From the eigenvalues of this infra-red finite tensor we can define the variable ²⁰,

$$C = 3(\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1) \quad (31)$$

For a two-jet event $C = 0$; for a planar event $0 \leq C \leq 3/4$. General multijet events span the range from zero to one. In terms of the variable C it is found that the corrections to the multijet fraction are large ($Q^2=4E^2$)

$$\frac{1}{\sigma_0} \int_{\frac{1}{2}}^1 dC \frac{d\sigma}{dC} = 1.4 \kappa(Q^2) [1 + A(Q^2) \kappa(Q^2)] \quad (32)$$

where $A^{\overline{MS}}(Q^2) = 18$, $A^{\text{MOM}}(Q^2) = 15$. By direct examination of the formula for the jet cross-section we find terms which suggest that a more appropriate scale in the three-jet region is $(S_{13} S_{23}/Q^2)$ where S_{13} (S_{23}) is the invariant mass squared of the gluon and the quark (antiquark). Choosing a reduced scale we find

$$A^{\overline{MS}}\left(\frac{Q^2}{9}\right) = 13.4, \quad A^{MOM}\left(\frac{Q^2}{9}\right) = 10.4.$$

The second variable is the thrust²¹ defined as

$$t = 2 \max_n \sum_i^{\text{hemisphere}} n \cdot p_i / \sum_i |p_i| \quad (33)$$

The thrust distribution has been calculated by several authors. Similar results have been found by Kunszt²², who used the analytic results of Ref. 20), and by Vermaseren, Gaemers and Oldham²³ who regularize the infra-red and mass singularities by giving the gluons a mass. Also in the case of the thrust distribution the corrections are found to be large. For example integrating over a region which excludes two jets yields²⁴

$$\frac{1}{\sigma} \int_{0.5}^{0.85} dt \frac{d\sigma}{dt} = \rho \kappa(Q) (1 + A\kappa), \quad A^{\overline{MS}}(Q) = 16.6 \pm 0.4 \quad (34)$$

Although the correction is large the distribution of events after exclusion of four jet events is very similar to the distribution which would be predicted by a lowest order calculation indicating that the large terms have the same kinematic structure as the lowest order. This is illustrated in Fig. 2 where $O(\alpha_s)$ and $O(\alpha_s^2)$ contribution to the thrust are plotted separately.

Fig. 2 - Contributions of both orders to thrust distributions from Ref. 23.

$$O(\alpha_s) : (1/\sigma_0 \kappa)(d\sigma/dt) ; O(\alpha_s^2) : (1/\sigma_0 \kappa^2)(d\sigma/dt)$$

A dissenting view of the corrections to the thrust distribution is given in Ref. 25 where it is found that the correction is small and negative in the region of thrust giving the dominant contribution to three jet events. This problem clearly requires further study to pin down the source of the large corrections and to attempt to resum them ²⁶ so that we can find a reliable estimate of the multijet fraction in e^+e^- annihilation.

PROCESSES INVOLVING PARTON DISTRIBUTION AND DECAY FUNCTIONS

Predictions have also been worked out for a wide variety of processes involving distribution and decay functions. In the context of the parton model the cross-section for a process involving one external incoming parton is

$$d\sigma(p) = \sum_j \int d\sigma_j(yp) f_j^0(y) dy \quad (35)$$

where $d\sigma_j$ is the cross-section for the j^{th} parton and f_j is a bare parton distribution function. As a consequence of the factorization theorem ²⁷ the parton cross-section may be written as

$$d\sigma_j(p) = \sum_k \int_0^1 dz d\tilde{\sigma}_k(zp, M^2) \Gamma_{kj}(z, M^2, \epsilon) \quad (36)$$

where $d\tilde{\sigma}_k(zp, M^2)$ is a finite rescaled parton cross-section. If the factorization scale M is chosen equal to the ultra-violet renormalization scale the moments of $d\tilde{\sigma}$ satisfy a simple renormalization group equation.

As before the absorption of infinities into renormalized parameters (in this case IR renormalized distribution functions) have ambiguities associated with the constant piece which one chooses to lump in with the infinities. A partial solution to this problem is to define quark densities beyond the leading order in terms of deep inelastic scattering ²⁸. (This is in some sense equivalent to a choice of prescription in the case of UV divergences; it makes no statement about the correct choice of scale.)

For deep inelastic scattering we find (using dimensional regularization to control the mass singularity)

$$\begin{aligned} \frac{F_2(x, Q^2)}{x} = & \int_0^1 dy \int_0^1 dz \delta(zy-x) \left[\delta(z-1) - \kappa \frac{C_F}{2} P_{qq}(z) \right. \\ & \left. \left(\frac{1}{\epsilon} + \ln 4\pi - \gamma_E - \ln \frac{M^2}{\mu^2} \right) \right. \\ & + \kappa \frac{C_F}{2} (f_{q,2}'(z) + P_{qq}(z) \ln \frac{Q^2}{M^2}) q_0(y) \\ & \left. + (\text{terms involving incoming gluons}) \right] \end{aligned} \quad (37)$$

The parton densities defined in terms of F_2 are thus

$$q(x, Q^2) = \frac{F_2(x, Q^2)}{x} \quad (38)$$

which corresponds to the choice,

$$\Gamma_{qq}(z, \epsilon, Q^2) = \left[\delta(z-1) - \frac{\kappa}{2} C_{F^{P_{qq}}}(z) \left(\frac{1}{\epsilon} + \ln 4\pi - \gamma_E - \ln \frac{Q^2}{\mu^2} \right) + f_{q,2}(z) \right] \quad (39)$$

Using this choice the valence value inside a particular hadron is preserved

$$\int_0^1 dx [q^h(x, t) - \bar{q}^h(x, t)] = V^h \quad (40)$$

DRELL-YAN CROSS-SECTION

Using this prescription for the quark distribution functions the Drell-Yan cross-section becomes ($z = Q^2/x_1 x_2 S$)

$$\begin{aligned} \frac{d\sigma^{DY}}{dQ^2} = & \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \left[q(x_1, Q^2) \bar{q}(x_2, Q^2) + (1 \leftrightarrow 2) \right] \\ & \left[\delta(z-1) + \kappa(Q^2) [f_{q,DY}(z) - 2f_{q,2}(z)] \right] \\ & + [(q(x_1, Q^2) + \bar{q}(x_1, Q^2)) G(x_2, Q^2) + (1 \leftrightarrow 2)] \\ & \kappa(Q^2) (f_{G,DY}(z) - f_{G,2}(z)) \end{aligned} \quad (41)$$

The differences of finite pieces in Eq. (41) are independent of the method of regularization and are given by ^{28, 29}

$$\begin{aligned} (f_{q,DY} - 2f_{q,2}(z)) = & \frac{2}{3} \left[\frac{3}{(1-z)_+} - 6 - 4z + 2(1+z) \left(\frac{\ln(1-z)}{1-z} \right) + \right. \\ & \left. + (1 + \frac{4}{3}\pi^2) \delta(z-1) \right] \end{aligned} \quad (42)$$

$$\begin{aligned} (f_{G,DY} - f_{G,2}(z)) = & \frac{1}{4} \left[(z^2 + (1-z)^2) \ln(1-z) + \frac{9}{2}z^2 - \right. \\ & \left. - 5z + \frac{3}{2} \right] \end{aligned} \quad (43)$$

The contribution of the $q\bar{q}$ annihilation term, Eq. (42) is large and positive. At present values of Q^2 and τ the correction is approximately independent of τ for reasonable choice of parton distribution function for the proton and the pion.

$$K = \frac{\text{Drell-Yan section through to } O(\alpha_s)}{\text{Naive parton model cross-section}} \sim 1.8 - 2.0 \quad (44)$$

The correction to the rapidity differential cross-section is of the same order of magnitude ^{28, 30}. Note that in this order in perturbation theory one is not sensitive to the running of the coupling constant (renormalization prescription) and the scale of the running coupling constant in Eq. (41) has been inserted by hand.

The conjectured form ³¹ of the resummed Drell-Yan cross-section is mainly based on simple arguments coming from QED. The quark form factor is believed to have the asymptotic form

$$F(q^2) \sim \exp \left[-\frac{2}{3} \int^{Q^2} dq_T^2 \kappa(q_T^2) \frac{\ln(q_T^2)}{q_T^2} \right] \quad (45)$$

which leads to a form for the squares of the form factor in timelike and spacelike regions

$$\lim_{Q^2 \rightarrow \infty} \left| \frac{F(Q)}{F(-Q)} \right|^2 = \exp \left[\frac{2}{3} \kappa(Q^2) \pi^2 \right] \quad (46)$$

In similar vein the log n pieces constant inherent in Eq. (42) can be summed so that for the moments of the factor K we have

$$K^{(n)} = \left| \frac{F(Q^2)}{F(-Q^2)} \right|^2 \left[\frac{\exp 8/3 \kappa(Q^2) \ln^2 n}{\exp 4/3 \kappa(Q^2) \ln^2 n} \right] \quad (47)$$

Equation (45) and hence Eq. (46) are presumably correct. However, in Eq. (42) there is no clear-cut distinction between the terms which have been summed and the terms which have been neglected, since only some of the π^2 terms in Eq. (42) have been included. For this reason it is hard to envisage a consistent approximation scheme which justifies Eq. (47).

HIGH p_T HADRON-HADRON SCATTERING

The cross-section for one particle inclusive hadron-hadron scattering at high p_T is given by

$$\frac{E_3 d^3 \sigma}{d^3 p_3} = \sum_{i,j,k} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 \frac{dz}{z} D_k(z, M^2) \frac{p_3^0 d^3 \sigma^{ijk}}{d^3 p_3} \bigg|_{\substack{p_1 = x_1 P_1 \\ p_2 = x_2 P_2 \\ p_3 = P_3/z}} f_i(x_1, M^2) f_j(x_2, M^2) \quad (48)$$

where the factorized parton cross-section is calculable as a perturbation series in $\alpha_s(M^2)$. The $O(\alpha_s^2)$ contributions to all the contributing subprocesses corresponding to various values of i, j and k are well known ³². The corrections to these basic subprocesses have been calculated ³³ only for the case of quark-quark scattering whereas experimentally the processes involving initial gluons are also important. All the contributing processes should be calculated. However, for the case of incoming gluons we have no clear-cut method of defining the gluon distribution beyond the leading order. This ambiguity is present even in the calculation which has already been performed with incoming quarks since the radiation of gluons from these incoming lines renormalizes the gluon distribution.

Since the cross-section begins in order $\alpha_s^2(M^2)$ and involves three parton distribution/decay functions the size of the calculated corrections is very sensitive to the choice which is made for M^2 . In Fig. 3 we have plotted the ratio R of the cross-section through to $O(\alpha_s^3)$ to the $O(\alpha_s^2)$ result. Distribution and decay functions have been defined from other physical processes where possible. The two curves correspond to the choices $M^2 = s$ and $M^2 = p_T^2 = ut/s$ and the dotted curve illustrates another choice for the gluon distribution function.

Fig. 3 - The ratio R giving the corrections to one hadron inclusive scattering from Ref. 33.

Despite the fact that R is the ratio of two rapidly varying functions of x_T it is seen to be remarkably constant so that R can be approximately parametrized as,

$$R \sim 1 + A(p_T^2)\kappa$$

where $A^{\overline{MS}}(p_T^2) = 25$, $A^{MOM}(p_T^2) = 19$. To illustrate the extreme sensitivity to the choice of M^2 , we note that for $M^2 = p_T^2/4$ the corrections can be reduced to less than 20% throughout most of the range in x_T . It would be interesting to know whether all parton subprocesses are small at this scale. To summarize the calculation of the quark-quark scattering subprocesses indicates that the $O(\alpha_s^3)$ predicts a much bigger cross-section than the $O(\alpha_s^2)$ calculation and there are serious doubts about the convergence of the perturbation series.

DOUBLE MOMENT RATIOS IN NEUTRINO SCATTERING

In the parton model the cross-section for one hadron inclusive neutrino hadron scattering

$$W(q) + H(P) \rightarrow H'(P') + x \quad (49)$$

may be written as

$$\frac{dW^h}{dz_h} = \int_{x_h}^1 \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} D\left(\frac{z_h}{z}\right) \frac{dW(x, z, Q^2)}{dz} F\left(\frac{x_h}{x}\right) \quad (50)$$

where

$$x_h = \frac{Q^2}{2P \cdot q}, \quad z_h = \frac{P \cdot P'}{P \cdot q} \quad (51)$$

To lowest order in α_s the parton cross-section has a simple product form

$$\frac{dW}{dz} = A \delta(1-x) \delta(1-z) + O(\alpha_s) \quad (52)$$

so that the double moments of the hadronic cross-section

$$\Sigma^{nm} = \int_0^1 dx_h x_h^{n-1} \int_0^1 dz_h z_h^{m-1} \frac{dW}{dz_h} \quad (53)$$

have a simple factorized form

$$\Sigma^{nm} = A D^{(m)} F^{(n)} + O(\alpha_s) \quad (54)$$

The difference of the ratio R from 1

$$R^{nm\ell\kappa} = \frac{\Sigma^{nm} \Sigma^{\ell\kappa}}{\Sigma^{n\kappa} \Sigma^{\ell m}} \quad (55)$$

measures the deviation from this simple factorized form. Furthermore, all dependence on parton distribution and fragmentation functions and all mass singularities cancel in this ratio. The perturbation series for this quantity is therefore similar in character to the perturbation series for the e^+e^- annihilation cross-section.

Higher order corrections³⁴ have been calculated to the perturbative cross-section where the W boson indices are saturated with the tensor $-g^{\mu\nu}$. In lowest non-vanishing order the parton cross-section is given by³⁵

$$\frac{dW}{dz} = \kappa \frac{C_F}{2} \left[\frac{1 + (1-x-z)^2}{(1-x)(1-z)} \right] \quad (56)$$

Radiative corrections to the ratio R have been calculated numerically and denoting the expansion coefficients by A and B we find that

$$R^{nm\ell\kappa} = 1 + \kappa(Q) A^{nm\ell\kappa} + \kappa^2 B^{nm\ell\kappa} \quad (57)$$

where for the lower values of n, m, ℓ, κ which are accessible experimentally the ratio B/A is roughly constant and given by

$$\left. \frac{B}{A} \right|_{\overline{MS}} \sim 10 \quad \left. \frac{B}{A} \right|_{MOM} \sim 7 \quad (58)$$

which lead to sizeable corrections at presently accessible energies.

CONCLUSIONS

As mentioned in the introduction the calculation of higher order corrections is important to control the precision of perturbative estimates. The basic conclusion of this review must be that in many cases our estimates are rather imprecise. This is in part the result of the calculations themselves but also in part due to the ambiguities inherent in an asymptotically free perturbation series. This is particularly true for processes which begin in higher order in the running coupling constant.

The pattern which emerges is that the classic processes, such as the e^+e^- total cross-section and the moments of the deep inelastic scattering, appear to be theoretically more reliable than the processes which have been calculated more recently. This is of little importance to experimentalists who prefer for example the Drell-Yan process where a large (but theoretically unsound) correction provides a significant difference between the parton model and QCD. It thus appears more fruitful to concentrate theoretical effort on making the predictions for processes having large corrections reliable, because the ambiguities of the theory make small corrections so elusive.

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