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INFORMATION ON GLUON DISTRIBUTIONS
FROM NEUTRINO DEEP INELASTIC SCATTERING

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A B S T R A C T

Recent data on nucleon structure functions from neutrino induced deep inelastic scattering are used to compute moments. A QCD analysis of the non-singlet moments, including $O(\alpha_s)$ corrections, indicates $\Lambda_{\overline{MS}} \approx 0.4 \rightarrow 0.5$ GeV. A small higher twist contribution cannot be excluded but such a term would probably increase, rather than decrease, Λ . A novel procedure of analyzing the singlet moments demonstrates consistency of theory with data. Moments of a gluon distribution are then extracted which show a clear Q^2 evolution over the range $5 \rightarrow 90$ GeV². Comparisons with gluon distributions from ψ , T production (via the fusion model) are made.

1. INTRODUCTION

There is now a new generation of experiments on deep inelastic scattering^{*)}, which cover a wide range in Q^2 and x , in which the structure functions of the nucleon are being measured with great precision. The recent results of the CDHS collaboration²⁾ offer the possibility of examining whether both non-singlet and singlet quantities show the type of scaling violations expected from QCD. By analyzing the moments of xF_3 we can learn (a) whether there is evidence for the $O(\alpha_s)$ corrections to the leading formulae of conventional (i.e, twist-two) QCD, (b) whether higher twist terms can explain away much of the Q^2 behaviour and (c) estimate the size of the scale parameter Λ , referred to some renormalization scheme.

From the moments of F_2 we can hope to extract information on the gluon density in the nucleon. A previous attempt at such an exercise^{3),4)} for the combined electron data from SLAC and muon data from Fermilab revealed that the observed singlet moments had a Q^2 behaviour which actually was inconsistent with asymptotic freedom and only very limited information on the gluon density could be obtained.

In this paper we examine the singlet moments using a novel procedure which demonstrates in a graphic way the consistency of the data with theory. Furthermore we determine the moments of the gluon distribution from $Q^2 = 5 \text{ GeV}^2$ to 90 GeV^2 albeit with significant uncertainty - mainly due to the well-known sensitivity to the choice of Λ . This choice is decided by the results of our analysis of the non-singlet moments but still the final gluon density cannot be determined with great precision at a given Q^2 . However, the variation of the distribution over the whole Q^2 range is observed and in line with our expectations. If we parametrize $xG(x, Q^2)$ as $\sim (1-x)^{N_G}$ we see N_G varying from around 5 at $Q^2 = 5 \text{ GeV}^2$ to around 11 at $Q^2 = 90 \text{ GeV}^2$, although such a simple parametrization is an approximation only in a limited region of x .

A knowledge of the gluon distribution is essential to applications of perturbative QCD to processes such as hadroproduction of dileptons or direct photons at large p_T as well as charmed particle production in photo- (or hadro-) production. In the last example data on ψ and T production have been described in terms of the photon-gluon (or gluon-gluon) fusion model and used to extract gluon distributions corresponding to $Q^2 = m_\psi^2$ and m_T^2 . Bearing in mind the ambiguities involved in defining a gluon density function, we compare the results from deep inelastic scattering with those from the fusion model.

*) For a survey of these experiments, see the review talk of Sciulli¹⁾.

2. NON-SINGLET ANALYSIS

Our procedure for computing the Nachtmann moments is the same as that described in Ref. 4). Because the errors on xF_3 are relatively large we prefer to use the data on F_2 for $x > 0.4$. Furthermore the low Q^2 region is supplemented by data from SLAC⁵⁾ on $g_2(F_2^{\text{ED}})$ for which $x \geq 0.45$. Fermi motion effects are included, using the tables of Bodek and Ritchie⁶⁾. To calculate the non-singlet moments, the data points on F_2 are weighted by the ratio of the Nachtmann factors w_2/w_3 appropriate to F_2 and xF_3 . To compute the corrections at the ends of the integrals we extrapolate to $x = 0$ and $x = x_{\text{TH}}$ (the inelastic threshold) assuming a form $xF_3 \sim \sqrt{x} (x_{\text{TH}} - x)^3$. These corrections are then given an arbitrary error of 10% and if the correction amounts to more than 25% of the total moment it is rejected by the analysis. Finally the elastic contribution is added.

First we fit the xF_3 moments for $N = 2 \rightarrow 6$ at 13 values of Q^2 (only 9 values for $N = 2$; 12 for $N = 3$) using only the twist-two contribution of QCD:

$$M_N^{\text{NS}}(Q^2) = A_N \left(\frac{\bar{g}^2}{16\pi^2} \right)^{x_N/2\beta_0} \left[1 + \frac{C_N}{\beta_0 \ln(Q^2/\Lambda^2)} \right] \quad (1)$$

with

$$C_N = B_{3,\text{NS}}^N + \frac{\gamma_1^N}{2\beta_0} - \frac{\beta_1 \gamma_2^N}{2\beta_0^2} \quad (2)$$

with $\bar{g}^2$ related to Λ by

$$\frac{16\pi^2}{\beta_0 \bar{g}^2} - \frac{\beta_1}{\beta_0^2} \ln \left[\frac{16\pi^2}{\beta_0 \bar{g}^2} + \frac{\beta_1}{\beta_0^2} \right] = \ln(Q^2/\Lambda^2) \quad (3)$$

$B_{3,\text{NS}}^N$, the $O(\bar{g}^2)$ term in the expansion of the coefficient function relevant to xF_3 , is connected to the corresponding $B_{2,\text{NS}}^N$ for F_2 by

$$B_{3,\text{NS}}^N = B_{2,\text{NS}}^N - \frac{4}{3} \cdot \frac{(4N+2)}{N(N+1)} \quad (4)$$

We choose to work in the so-called $\overline{\text{MS}}$ scheme⁷⁾ in which the term $(\ln 4\pi - \gamma_E)$ is dropped in the expression for $B_{2,NS}^N$. In this scheme (also for the MOM scheme⁸⁾) the $O(\bar{g}^2)$ terms are small; the approximate connection between the scale parameters being

$$\Lambda_{\text{MOM}} \approx 2 \Lambda_{\overline{\text{MS}}} \approx 6 \Lambda_{\text{MS}} \quad (5)$$

The results for $\Lambda_{\overline{\text{MS}}}$ obtained by fitting each moment in turn are shown in Fig. 1. There is no evidence for any significant variation with N and we would conclude that $\Lambda_{\overline{\text{MS}}} = 0.46 \pm (0.04) \text{ GeV}$. The value of $\Lambda_{\overline{\text{MS}}}$ for $N = 2$ is determined more precisely from the singlet moment - in this case the $N = 2$ moment gives $0.50 \pm 0.11 \text{ GeV}$. A more graphic way of displaying the effect of the $O(\bar{g}^2)$ corrections is to use the Λ_N scheme⁷⁾. In this scheme, the C_N term in Eq. (2) is absorbed into a new scale parameter which varies with N . Choosing

$$\Lambda_N = \Lambda \exp [C_N / \bar{g}_N^2] \quad (6)$$

gives an expression for the non-singlet moment which assumes a form similar to the leading order expression:

$$M_N^{NS}(Q^2) = A_N \left(\frac{\bar{g}_N^2}{16\pi^2} \right)^{N/2} \quad (7)$$

where $\bar{g}_N$ is related to Λ_N by an expression exactly similar to Eq. (3). The effect of the higher order corrections is then reflected in the N dependence of Λ_N . Figure 1 also shows the resulting values of Λ_N compared with Eq. (6). The remarkable agreement between the points and the curve demonstrates the evidence for the presence of $O(\bar{g}^2)$ corrections in the data.

Next we investigate the effect of possible higher twist contributions. Such terms could, in principle, be sufficiently important to account for a significant amount of the observed scaling violations in which case Λ could be considerably smaller than our first estimate. Since higher twist terms are most strongly felt at low Q^2 , we repeat our analysis with only the twist-two term [Eq. 1)] but varying the range of Q^2 to see if the resulting estimate of $\Lambda_{\overline{\text{MS}}}$ is affected. Choosing $Q_{\text{min}}^2 = 10 \text{ GeV}^2$ instead of 5 GeV^2 , we find

$\Lambda_{\overline{\text{MS}}} = 0.52 \pm 0.07 \text{ GeV}$, - a value which is entirely consistent with the value when $Q_{\text{min}}^2 = 5 \text{ GeV}^2$. There is no indication for $\Lambda_{\overline{\text{MS}}}$ to fall as Q_{min}^2 increases, even for the $N = 5, 6$ moments; hence no suggestion of a significant higher twist term.

As a further probe of higher twist effects we allow an explicit higher twist term of arbitrary strength in addition to the twist-two term. We write

$$M_N^{NS}(Q^2) = A_N \left(\frac{\bar{q}^2}{16\pi^2} \right)^{\gamma_N^{NS}/2\beta_c} \left[1 + \frac{C_N}{\beta_c \ln(Q^2/\Lambda^2)} \right] \left[1 + \frac{\mu^2 N}{(1+Q^2/m^2)} \right] \quad (8)$$

since it has already been demonstrated that such a parametrization of the higher twist will, on its own, give a very good description of the low Q^2 nucleon structure function⁹⁾. We take $m \sim 1 \text{ GeV}$ and allow μ^2 to vary (not necessarily positive) together with $\Lambda_{\overline{\text{MS}}}$. There is no significance attached to the factor N multiplying μ^2 in Eq. (8) since we analyze each moment in turn. Such a factor yields values of $\Lambda_{\overline{\text{MS}}}$, as μ^2 varies, which are practically independent of N . The quality of the resulting fits is shown in Fig. 2. For each moment, the minimum χ^2 occurs for values of μ^2 less than zero, for which $\Lambda_{\overline{\text{MS}}} > 0.5 \text{ GeV}$. We conclude that there is no suggestion in the data of a significant higher twist contribution which would, in turn, lead to a decrease in the estimate of Λ . If higher twists were important they would be most strongly felt at larger N and it is for $N = 5, 6$ that the χ^2 variation most significantly discourages a positive contribution. What evidence there is tends to suggest a higher twist term with a negative coefficient - this is interesting in the light of recent suggestion by Luttrell et al.¹⁰⁾ who calculate the coefficient functions of four-quark operators and conclude that such terms are likely to be negative.

3. SINGLET ANALYSIS

Provided that the singlet structure function F_2^{VN} has a Q^2 behaviour consistent with the theory, we can exploit the mixing between the quark singlet and gluon operators to obtain information on the gluon distribution in the nucleon.

We write the Q^2 evolution of the singlet moment as

$$M_N^S(Q^2) = A_N(Q^2, Q_0^2; \Lambda) M_N^S(Q_0^2) + B_N(Q^2, Q_0^2; \Lambda) G_N(Q_0^2) \quad (9)$$

where the coefficient A_N has the form^{11)*}

$$\begin{aligned} A_N(Q^2, Q_0^2; \Lambda) &= \left[A_{11}^N + A_{12}^N \bar{g}^2(Q^2) + A_{13}^N \bar{g}^2(Q_0^2) \right] \exp\left(\frac{-\gamma_+^N}{2\beta_0} \bar{s}\right) \\ &+ \left[A_{21}^N + A_{22}^N \bar{g}^2(Q^2) + A_{23}^N \bar{g}^2(Q_0^2) \right] \exp\left(\frac{-\gamma_-^N}{2\beta_0} \bar{s}\right) \end{aligned} \quad (10)$$

with an analogous expression for B_N and where $\bar{s} = \ln[(\ln Q^2/\Lambda^2)/(\ln Q_0^2/\Lambda^2)]$. Because we have included $O(\bar{g}^2)$ corrections there is ambiguity in the definition of the gluon distribution^{13),14)}. This means that there is some freedom in the choice of the coefficients $A_{13}^N, A_{23}^N, B_{13}^N, B_{23}^N$ corresponding to different definitions of the gluon distribution. The choice we make corresponds to the definition of $xG(x, Q^2)$ adopted by Altarelli et al.¹⁴⁾.

We can use Eq. (9) to examine whether the experimental singlet moments are indeed consistent with QCD. Let us label the values of Q^2 where we have computed $M_N^S(Q^2)$ as Q_i^2 ($i = 1, 13$ in our case) and then invert Eq. (9) to

$$G_N(Q_j^2) = \left[M_N^S(Q_i^2) - A_N(Q_i^2, Q_j^2; \Lambda) M_N^S(Q_j^2) \right] / B_N(Q_i^2, Q_j^2; \Lambda) \quad (11)$$

Since the left-hand side of Eq. (11) is independent of Q_i^2 ($\neq Q_j^2$) as we vary i , then we just fix a value of j and plot the right-hand side of Eq. (11), which is directly obtained from the experimental values of the singlet moments, and inspect whether the data are constant with Q_i^2 . If so, then the value of the constant will be simply the value of the gluon moment at $Q^2 = Q_j^2$. In this way we can hope to extract gluon moments over the whole Q^2 range.

An example is shown in Fig. 3, where we have chosen $Q_j^2 = 5.5 \text{ GeV}^2$ and $\Lambda_{\overline{\text{MS}}} = 0.46 \text{ GeV}$ and computed the right-hand side of Eq. (11) for $Q_i^2 = 7 \rightarrow 90 \text{ GeV}^2$ and $N = 3 \rightarrow 5$. For Q_i^2 close to Q_j^2 the errors are expected to be large and so it is essential to have as wide a range as possible in Q^2 . The plots of Fig. 3 show consistency with a constant value, as do the corresponding plots at higher values of Q_j^2 . Extracting the fitted values of the constants for each N and Q_j^2 gives the resulting estimates for the gluon moments $G_N(Q_j^2)$ for the

^{*)} Recently, Furmanski and Petronzio¹²⁾ have claimed discrepancies with the results of Ref. 11). Numerically we find these lead to almost negligible ($< 1\%$) changes for the coefficients A_N, B_N .

particular choice of $\Lambda_{\overline{MS}}$ (in this case $\Lambda_{\overline{MS}} = 0.46$ GeV). The results are shown in Fig. 4 for $N = 2, 3, 4$. Although the errors on the extracted gluon moments are not small, it is possible to see a dependence on Q^2 . In order to compare whether the observed evolution in Q^2 is consistent with the choice of $\Lambda_{\overline{MS}} = 0.46$ GeV, we also plot in Fig. 4 the expected dependence of $G_N(Q^2)$ with this choice of Λ . To normalize the gluon moments at one value of Q^2 , we assume that at $Q_0^2 = 5.5$ GeV², we can write

$$x G(x, Q_0^2) = A_G (1-x)^{N_G} \quad (12)$$

where the gluon distribution $G(x, Q)$ is given by

$$G_N(Q^2) = \int_0^1 dx w_2^N(x, Q^2) x G(x, Q^2) \quad (13)$$

and $w_2^N(x, Q^2)$ is the standard Nachtmann weighting factor. The curves in Fig. 4 correspond to the choice $N_G = 5.5$ at $Q^2 = 5.5$ GeV² with $\Lambda_{\overline{MS}} = 0.46$ GeV and indicate that the extracted values of the gluon moments show a consistent evolution with Q^2 .

Choosing a simple parametrization such as Eq. (12) for the gluon distribution is permissible at one value of Q^2 , at most. The evolution up in Q^2 is expected to produce a sharp "spike" at very low x , for instance. However, to get a feel for the steepening of $xG(x, Q^2)$ as Q^2 increases, we assume a parametrization $\sim(1-x)^{N_G}$ and use the $N = 2, 3$ moments to extract the exponent. Clearly, the value of this exponent is relevant only in a very limited x region - around the mean $\langle x \rangle$ for the distribution, which turns out to be $0.08 \leq x \leq 0.15$. The result is shown in Fig. 5. Thus we see a "shrinking" of the gluon density from $\sim(1-x)^{5 \rightarrow 6}$ at $Q^2 = 5$ GeV² to $\sim(1-x)^{9 \rightarrow 10}$ at $Q^2 = 50$ GeV². This is just another way of saying that the mean x of $xG(x, Q^2)$ moves from $\langle x \rangle \sim 0.13$ at $Q^2 = 5$ to $\langle x \rangle \sim 0.09$ at $Q^2 = 50$. However, there is quite considerable uncertainty in the value of N_G at a given Q^2 from this analysis. This is true even for a specific value of Λ and the errors on the moments $G_N(Q^2)$, or on N_G , are further increased by the uncertainty in the value of Λ . Although our non-singlet analysis indicated a value $\Lambda_{\overline{MS}} = 0.46$ GeV, any value between 0.43 and 0.51 cannot be ruled out. We have re-computed the gluon moments using values of $\Lambda_{\overline{MS}}$ in this range and find the resulting estimate for $G_{N=3}(Q^2)$ can vary by as much as 30% from the values shown in Fig. 4.

Roughly speaking, this increases the error on N_G just one unit at any Q^2 in the range considered. This only serves to demonstrate that analysis of deep inelastic structure functions is not the most precise method for determining the gluon distribution.

4. COMPARISON WITH PHENOMENOLOGICAL GLUON DENSITIES

Recently, data on elastic ψ photoproduction has been described in terms of the photon-gluon fusion mechanism^{*)}. In this model the ν dependence of $\sigma(\psi)$ directly determines the x dependence of $xG(x, Q^2 = m_\psi^2)$ and we may hope to compare such a gluon density with our determination. Strictly speaking this is only allowed provided that precisely the same definition of the gluon density is used in each case since, as we have already remarked, inclusion of $O(\bar{g}^2)$ terms introduces an ambiguity into the definition, and as yet the higher order terms have not been included into the fusion model. But if the changes to $xG(x, Q^2)$ introduced by including the $O(\bar{g}^2)$ terms in Eq. (9) are numerically small then we may hope that so are the differences due to alternative definitions of $xG(x, Q^2)$. In our analysis we can examine the effects of switching off the $O(\bar{g}^2)$ terms and typically one finds the value of $G_N(Q^2)$ falls by around 10% and insofar that this can be regarded as a small effect, then a comparison between the two determinations is not meaningless. The shapes of the gluon distributions obtained from the fusion model for $\gamma^*N \rightarrow \psi N$ and $NN \rightarrow TN$ (the normalization is undetermined) are shown in Fig. 6. Both are consistent with a simple power law $\sim (1-x)^{N_G}$ over the range shown, with $N_G \approx 5, 6$ for ψ, T production respectively.

From our previous discussion, a value of $N_G \approx 5$ at $Q^2 \approx 10 \text{ GeV}^2$ seems quite reasonable but a value of $N_G \approx 6$ at $Q^2 \approx 100 \text{ GeV}^2$ would appear, at first sight, to be ruled out (see Fig. 5). However as pointed out earlier, such values are meaningful only for $0.07 \lesssim x \lesssim 0.15$ whereas the data for $Q^2 \approx 100$ have $x > 0.15$. To make a more realistic comparison we do the following exercise:

- (i) Choose a gluon distribution which $\sim (1-x)^5$ at $Q^2 = 10 \text{ GeV}^2$, i.e., consistent with the ψ data. Such a distribution follows from the singlet data for $\Lambda_{\overline{MS}} \approx 0.51 \text{ GeV}$, a value which is not excluded by our non-singlet analysis;
- (ii) Take a value $\Lambda_{L.O.} = 0.5 \text{ GeV}$ and evolve the gluon distribution, using the Altarelli-Parisi equations, up to $Q^2 = 100 \text{ GeV}^2$. The result is shown in Fig. 6.

^{*)} For a survey of the phenomenology of new particle production, see the review by Phillips¹⁵⁾.

We see that the distribution behaves like $(1-x)^{N_G}$ only in limited regions of x . For $x \approx 0.07$, the slope corresponds to $N_G \approx 10$ while for $x > 0.15$, the slope gives $N_G \approx 6.4$ and is quite consistent with the data on T production. Thus while the gluon distribution shows a clear sharpening as Q^2 increases, this takes place predominantly at small x values (< 0.1) leaving only a small increase in slope for $x > 0.1$ which is consistent with the "observed" change seen in the ψ , T data. The $N = 2, 3, 4$ moments of the distributions shown in Fig. 6 correspond very closely to the values which result from plotting Fig. 4 but with $\Lambda_{\overline{MS}} = 0.51$ GeV.

5. DISCUSSION AND CONCLUSIONS

The structure functions F_2^{VN} and xF_3^{VN} measured with such precision over a wide range of Q^2 by the CDHS collaboration²⁾ allow a detailed comparison with the predictions of perturbative QCD. Whether the analysis is performed in terms of moments or the structure functions themselves is a matter of taste. We prefer to work with moments because it is then possible to examine in a graphic way (a) whether there is evidence for higher order corrections and (b) whether the singlet data are in fact consistent with asymptotic freedom.

Our analysis of the non-singlet moments clearly shows the behaviour expected from the $O(\bar{g}^2)$ corrections and furthermore we can confidently estimate $\Lambda_{\overline{MS}}$ to be around $0.4 \rightarrow 0.5$ GeV. There seems to be little indication that possible higher twist effects might reduce the size of Λ .

Turning to the singlet moments, graphs such as Fig. 3 show that the singlet data are consistent with the predictions of asymptotic freedom which is somewhat reassuring following the failure of the singlet moments of the electron-muon data below $Q^2 = 20$ to display such consistency^{3),4)}. However, a precise determination of the gluon moments is not possible but a significantly varying gluon density is clearly observed as Q^2 goes from 5 to 90 GeV².

There has been some discussion about the "hardness" of the gluon distribution at low Q^2 . Recently Gluck et al.¹⁷⁾ argued that including heavy quark thresholds in the analysis of electron + muon data could affect the extracted gluon distribution. They arrived at a gluon density, determined to a very high accuracy, which was considerably harder than the conventional counting rule distribution at $Q^2 = 4$ GeV², corresponding to a value for $G_{N=3}(Q^2)$ of 0.104 compared with 0.07 from a $(1-x)^5$ distribution. Such accuracy in the determination of the gluon distribution from deep inelastic data is impossible in our view. The errors shown in Fig. 4, together with the uncertainty in $\Lambda_{\overline{MS}}$ suggest that at $Q_0^2 = 4$ GeV², $G_{N=3}(Q^2)$ could be anywhere from 0.06 to 0.10.

While we do not claim to have much precision in our determination of the gluon density at a particular value of Q^2 , we do see an unmistakable sharpening of the distribution as Q^2 increases. This behaviour is expected to be most dramatic for small x . A comparison with the gluon distributions extracted from elastic ψ and T production using the fusion model are quite consistent with the apparent small Q^2 dependence since the latter data cover only the larger x region.

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FIGURE CAPTIONS

- Fig. 1 : Values of the scale parameter $\Lambda_{\overline{MS}}$ extracted from the moments of xF_3 and also Λ_N as a function of N . The curve shows the prediction for Λ_N from Eq. (6); normalized arbitrarily. The $N = 2$ moment is better determined from the singlet moments:
 $\Lambda_{\overline{MS}} = 0.50 \pm 0.11$.
- Fig. 2 : Variation of χ^2 and $\Lambda_{\overline{MS}}$ with the strength of the higher twist contribution. These are based on fits to the non-singlet moments according to Eq. (8).
- Fig. 3 : Plots of the right-hand side of Eq. (11) at $Q_j^2 = 5.5 \text{ GeV}^2$ for $N = 3 \rightarrow 5$. The horizontal lines show the fitted constant value corresponding to the best estimate of $G_N(Q_0^2 = 5.5)$. A value of $\Lambda_{\overline{MS}} = 0.46$ was used.
- Fig. 4 : The estimates of the Nachtmann gluon moments $G_N(Q^2)$ for $N = 3, 4$ as determined from Eq. (11), using $\Lambda_{\overline{MS}} = 0.46 \text{ GeV}$ and for $N = 2$ directly from $1 - M_{N=2}^S(Q^2)$. The curves show the Q^2 dependence expected with such a value of Λ assuming $xG(x, Q_0^2) \sim (1-x)^{5.5}$ at $Q^2 = 5.5 \text{ GeV}^2$.
- Fig. 5 : Assuming $xG(x, Q^2) \sim (1-x)^{N_G}$ at each Q^2 , the value of N_G versus Q^2 extracted from the $N = 2, 3$ moments of Fig. 4. The curve corresponds to values extracted from the curves of Fig. 4.
- Fig. 6 : Gluon densities extracted from fusion model¹⁶⁾ analyses of " $\gamma^*N \rightarrow \psi N$ and $NN \rightarrow Tx$ ". The curves correspond to choosing a $(1-x)^5$ distribution at $Q_0^2 = 10 \text{ GeV}^2$ and evolving up to $Q^2 = 100$ using the Altarelli-Parisi equations with $\Lambda_{L.O.} = 0.5 \text{ GeV}$. Note that the x axis is logarithmic in $(1-x)$.

FIG. 1

FIG. 2

FIG. 3

FIG.6