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AND CRITICAL DIMENSIONS FOR STRING MODELS.

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QUANTIZATION OF TWO-DIMENSIONAL
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FOR STRING MODELS

by

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Abstract

We consider the quantization of $O(2)$ two-dimensional supergravity corresponding to the model of string with the charge and spin. The effective action is obtained and it is shown that anomalies are absent for the space-time dimension $2n = 4$.

Recently Polyakov [1] proposed the method of calculating the path integrals for bosonic (fermionic) models of the string based on the quantization of the two-dimensional gravity (supergravity), interacting with n multiplets of scalar (scalar and spinor) fields (for the corresponding lagrangians see [2,3]). In both cases the effective action can be explicitly found by "integrating the anomalies" due to the conformal (and super-conformal) invariance of the classical action. The theory is free from the anomalies for the well-known "critical dimensions" $n = 26$ ($n = 10$), see e.g. [4]. In this work we consider the last realistic supersymmetric model of the string - the string with a charge and spin [5] with the lagrangian presented in [6]. We also discuss in passing a number of principal as well as technical points connected with the first two models of the string.

The lagrangian of the $O(2)$ two-dimensional supergravity (one "graviton" e_μ^a , two Majorana or one Dirac "gravitino" χ_μ , one vector field A_μ) interacting with the n complex $(0, \frac{1}{2})$ matter multiplets (φ^i, ψ^i, F^i) ($i = 1, \dots, n$, φ is the complex scalar, ψ is the Dirac spinor and F is the complex scalar auxiliary field) is given by [6]

$$\mathcal{L} = \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi^* + \frac{i}{2} \bar{\psi} \gamma^\mu \mathcal{D}_\mu \psi + A_\mu \bar{\psi} \gamma^\mu \psi \right. \\ \left. + (\partial_\mu \varphi^* + \bar{\psi} \chi_\mu) \bar{\chi}_\mu \gamma^\mu \gamma^\rho \psi + (\partial_\mu \varphi + \bar{\chi}_\mu \psi) \bar{\psi} \gamma^\mu \gamma^\rho \chi_\mu \right] \sqrt{g} \\ - \frac{1}{2} F F^* \sqrt{g},$$

(we use the notation of [6]: $\gamma_a \gamma_b = \gamma_{ab} + \gamma_5 \epsilon_{ab}$,

$\gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3$, $\epsilon^{01} = 1$, $\gamma_{ab} = \{+1, -1\}$). The action

is invariant under the following transformations

$$\delta e_\mu^a = \xi^\lambda \partial_\lambda e_\mu^a + e_\mu^a \partial_\lambda \xi^\lambda + \ell \epsilon_\mu^a e_\mu^b + 2i(\bar{\chi}_\mu \gamma^a \alpha - \bar{\alpha} \gamma^a \chi_\mu) + \Omega e_\mu^a, \quad (2)$$

$$\delta \chi_\mu = \xi^\lambda \partial_\lambda \chi_\mu + \chi_\mu \partial_\lambda \xi^\lambda - \frac{1}{2} \ell \gamma_5 \chi_\mu + \bar{D}_\mu \alpha + \frac{1}{2} \Omega \chi_\mu + i(E - \gamma_5 \theta) \chi_\mu + \delta_\mu \Lambda, \quad (3)$$

$$\delta A_\mu = \xi^\lambda \partial_\lambda A_\mu + A_\mu \partial_\lambda \xi^\lambda + E^{\lambda\rho} (\gamma_\lambda \gamma_\rho \bar{D}_\mu \chi + h.c.) + \partial_\mu E + E_{\mu\rho} \partial^\rho \theta + (\bar{\chi}_\mu \gamma^\lambda \delta_\lambda \Lambda + h.c.), \quad (4)$$

$$\delta \psi = \xi^\lambda \partial_\lambda \psi - 2\bar{\alpha} \psi, \quad \delta \psi = \xi^\lambda \partial_\lambda \psi - \frac{1}{2} \ell \gamma_5 \psi + i(\hat{\partial} \psi + 2\delta^\mu \bar{\chi}_\mu \psi) \alpha - \frac{1}{2} \Omega \psi + i(E + \gamma_5 \theta) \psi + F \alpha. \quad (5)$$

Here ξ , ℓ , α , Ω , E , θ , Λ are parameters of the coordinate, Lorentz, supersymmetry, Weyl, phase, chiral and "accidental" (or superconformal) transformations; $E^{\mu\nu} = \frac{\epsilon^{\mu\nu}}{\sqrt{g}}$, $\bar{D}_\mu = D_\mu - i A_\mu$, $D_\mu = \partial_\mu + \frac{1}{2} \omega_\mu \gamma_5$, and ω_μ contains the torsion $S_\mu = i \bar{\chi}_\mu \gamma^\rho \chi_\rho + h.c.$. These symmetries allow one to choose the following classical gauge

$$e_\mu^a = \delta_\mu^a, \quad \chi_\mu = 0, \quad A_\mu = 0 \quad (6)$$

(4 parameters ξ , ℓ , Ω fix e_μ^a , 2 Dirac spinors α and Λ fix χ_μ , while in two dimensions E and θ fix A_μ 1/).

1/ let us note that θ -invariance of (1) was not mentioned in [6] and so the statement in [6] about the possibility to choose (6) without the use of the γ_5 -invariance is incorrect (observe also that due to the identity $\epsilon_{ab} \gamma_a = \epsilon_{ab} \gamma_b$, A_μ is simultaneously the gauge field for E - and θ -groups).

In this gauge we recover the linearized form for the equations of motion and constraints of the charged spinor string [5] (see also [7]).

Quantization of the theory is based on the averaging with the following measure (we use the euclidean formulation)

$$Z = \int d\epsilon d\chi dA dF \Phi[\epsilon, \chi, A] X[\epsilon, \chi, A] e^{-W_m} \quad (7)$$

$$e^{-W_m[\epsilon, \chi, A, F]} = \int d\psi d\psi^\dagger e^{-\int \psi d\psi^\dagger}$$

Here Φ stands for the quantum gauge conditions and X

is the corresponding gauge determinant. The theory is sensible only after the choice of a regularization (i.e. those of the symmetries in (2)-(5) which are conserved at the quantum level). We shall assume that the regularization preserves the coordinate (and Lorentz) symmetry, supersymmetry and phase invariance but that the conformal, superconformal and chiral symmetries may be absent in the regularized version of (7) (and thus one may confront the corresponding anomalies). As a such kind of a regularization one can use the ordinary dimensional regularization or the regularization by dimensional reduction (see [8,9] and references therein).

It is possible to use different regularizations in the discussion of the Ω , V and θ anomalies (e.g. the proper time regularization for the Ω and θ ones). In the case the following superinvariant Pauli-Villars regularization is useful: $\chi = \frac{1}{2} F(\epsilon) F(\epsilon) e^{\frac{1}{2} V + \sum c^k M^k V^2}$ (continued on the next page)

As a result, we take the following quantum gauges (fixing (ξ, ℓ) , α and E -symmetries)

$$\begin{aligned} \Phi &= \delta(\bar{e}_\mu^a) \delta(\xi_\mu) \delta(A_\mu'') \\ \bar{e}_\mu^a &= e_\mu^a - \frac{1}{2} \delta_\mu^a (e_\lambda^b \delta_b^\lambda), \quad \xi_\mu = \delta^\lambda \delta_\mu^\lambda \chi_\lambda, \\ A_\mu'' &= \nabla_\mu \nabla^{-2} \nabla_\lambda A^\lambda, \end{aligned} \quad (8)$$

i.e.

$$\begin{aligned} e_\mu^a &= \phi \delta_\mu^a \equiv e^\sigma \delta_\mu^a, \quad \chi_\mu = \frac{1}{2} \delta_\mu^\lambda \chi_\lambda, \quad A_\mu = A_\mu^\perp = \\ &= \frac{1}{2} E_{\mu\nu} \partial^\nu \rho, \end{aligned} \quad (9)$$

$$Z = \int d\sigma d\lambda d\rho e^{-\bar{W}}, \quad \bar{W}[\sigma, \lambda, \rho] = W_m - \log X. \quad (10)$$

It is important to stress that the regularized values of X and W_m in (7) are no longer Ω , Λ and θ -invariants, i.e. they anomalously depend on σ , λ and ρ .^{3/}

(here V and E are the superfields and $\mathcal{L}'$ coincides with (1) when $M_k = 0$, $A_\mu = 0$ and $h \rightarrow 2h$, see [10]).

^{3/} the auxiliary fields are absent in (2)-(4) and therefore in X and can be simply integrated away (here is no analog of the "quartic ghost interaction" present in the case of the four dimensional supergravity).

The simplest way to establish the general form of $\overline{W}$ is to use the residual supersymmetry present in the theory under the conditions (9). One has from (2)-(4) for the global ($\alpha = \text{const}$) linear in the fields part of this algebra

$$\delta \phi = i(\bar{\lambda} \alpha - \alpha \lambda), \quad \delta \rho = -(\bar{\lambda} \gamma_5 \alpha + \alpha \gamma_5 \lambda), \quad (11)$$

$$\delta \lambda = (\hat{\partial} \phi - i \hat{\partial} \rho \gamma_5) \alpha + \mathcal{F} \alpha, \quad \delta \mathcal{F} = i \alpha \hat{\partial} \lambda$$

(we formally introduced the complex auxiliary field $\mathcal{F}$ to extend the symmetry off the mass shell). For the corresponding invariant we have

$$\overline{W} = \gamma \int d^2 x \left[\frac{1}{2} (\partial_a \phi)^2 - \frac{1}{2} (\partial_a \rho)^2 + \frac{i}{2} \bar{\lambda} \overleftrightarrow{\partial} \lambda - \frac{1}{2} \mathcal{F} \mathcal{F}^* \right], \quad (12)$$

($\gamma = \text{const}$ and we assume that in (10) $\mathcal{F} = 0$). The first and the third terms in (12) are the same as in the case of the neutral fermionic string [1] (excluding the strange disagreement in the coefficient before the spinor term: $\frac{1}{2}$ in [1] instead of 1 in our notations (9); note that here

$\lambda = \frac{\lambda_1 + i \lambda_2}{\sqrt{2}}$, $\lambda_{1,2}$ are the Majorana spinors), while the second ($\sim (A_\alpha^\perp)^2$) has the same origin as in the Schwinger model [11]. It is important to stress the opposite signs of the conformal (ϕ) and chiral (ρ) anomalous terms in (12) (i.e. the presence of a ghost in the theory for every $\gamma \neq 0$) which are dictated by (11) (and will be confirmed below by the explicit calculation). Let us also point out that (contrary to the neutral string case) the residual algebra (11) differs from the global super-algebra for $\hbar = 1$

(see [7]) localization of which gives the whole theory (1)-(5).

To calculate $\mathcal{V}$ as the coefficient before the first term in (12) one can "forget" about the " Λ " and " θ "-anomalies and take the gauge

$$e_\mu^a = \delta_\mu^a, \quad A_\mu = 0, \quad \chi_\mu = 0, \quad (13)$$

in which (cf. [1])

$$X = \frac{[\det \Delta_{1/2}]^{1/2}}{[\det \tilde{\Delta}_{1/2}]^2} \det \Delta_0, \quad e^{-W_m} = \left[\frac{\det \Delta_{1/2}}{\det \Delta_0} \right]^n, \quad (14)$$

$$\Delta_{1/2}{}^\mu{}_\nu = -\delta_\nu^\mu \left(\frac{R}{2} \right), \quad \Delta_0 = -\nabla_\mu \nabla^\mu, \quad (15)$$

$$\Delta_{1/2} = -\frac{1}{\sqrt{3}} \partial_\mu (\sqrt{3} \delta^\mu{}_\nu \partial^\nu) = -\tilde{\Delta}_\mu \partial^\mu + \frac{R}{4}, \quad (16)$$

$$\tilde{\Delta}_{1/2} = -\frac{1}{\sqrt{3}} \partial_\mu (\sqrt{3} \tilde{\delta}^\mu{}_\nu \partial^\nu) = -\tilde{\Delta}_\mu \partial^\mu - \frac{R}{4}, \quad (17)$$

where $\tilde{\delta}_\mu^\nu = \delta_\mu^\nu$, $\tilde{\delta}_\mu^\nu = \delta_\mu^\nu - \frac{1}{3} \delta_\mu^\nu$, $\partial_\mu = D_\mu (S=0)$, R is the curvature scalar and $\Delta_{1/2}$ and $\tilde{\Delta}_{1/2}$ are defined on the Majorana spinors (e.g. $[\det \Delta_{1/2}]^{1/2}$ corresponds to the one fermion degree of freedom). In the derivation of (14) the following simple observation is useful: if the transformed gauge condition is $C^\alpha(\epsilon) = C_A^\alpha \epsilon^A$ (where the indices α and A have the same range of variation but different tensor "substructure")

one can multiply two "unities" (following Faddeev-Popov and t'Hooft)

$$X \int d\epsilon \delta(C^\alpha(\epsilon) - \eta^\alpha) = 1, \quad \int d\eta e^{-\frac{1}{2} \eta^\alpha \eta_\alpha} = 1 \quad (18)$$

and obtain

$$X^{-1} = \int d\epsilon e^{-\frac{1}{2} C_\alpha(\epsilon) C^\alpha(\epsilon)} = [\det C^T C]^{s/2} \quad (19)$$

where $C^T C = (C_\alpha^T C_\beta)$ and $s = \pm 1$ if E^A commute or anti-commute. For example, using (18), (19) we get $\int d\epsilon d\theta \delta(A_\mu - \partial_\mu \epsilon - \frac{1}{2} E_\mu, \partial^\nu \theta) = [\det \Delta_0]^{-1}$. Note also that in the gauge (13) X factorizes into the separate (ξ, ℓ) , α and (ϵ, θ) contributions.

Let us first establish the infinite part of the effective action $\bar{W}_\infty$ corresponding to (14). One has for the elliptic 2-nd order differential operator on a vector bundle over M^2

$$W = \frac{1}{2} \log \det \mu^{-2} \Delta = -\frac{1}{2} \int_{\mathbb{R}^2} \frac{ds}{s} \text{tr} e^{-s\Delta}, \quad (20)$$

$$W_\infty = -\frac{1}{2} (A_0 L^2 + 2A_1 L + A_2 \log \frac{L^2}{\mu^2}), \quad L \rightarrow \infty$$

where $A_p = \int_M b_p \sqrt{g} d^2x + \int_{\partial M} c_p \sqrt{\delta} dx$ are the corresponding Seeley coefficients. In the case of $\Delta = -\mathcal{D}^2 + X$ (see e.g. [12])

$$\bar{b}_0 = \text{tr} 1, \quad \bar{b}_2 = \text{tr} \left(1 \cdot \frac{R}{6} - X \right), \quad (21)$$

$$\bar{c}_0 = 0, \quad \bar{c}_1 = \frac{\sqrt{\pi}}{2} \text{tr} 1, \quad \bar{c}_2 = \frac{1}{6} 2K \text{tr} 1 \quad (22)$$

($\bar{A}_p = 4\pi A_p$) corresponds to the Dirichlet and Neumann boundary conditions, γ and K are the metric and the trace of the second fundamental form of the boundary; let us remind also that the Euler number of M^2 is $\chi = \frac{1}{4\pi} \left(\int_M R \cdot \sqrt{g} d^2x + \int_{\partial M} 2K \sqrt{\delta} dx \right)$. As a consequence,

$$W_\infty = \int_M d^2x \sqrt{g} (\Lambda_\infty + \alpha_\infty R) + \int_{\partial M} (p_\infty + 2K q_\infty) \sqrt{\sigma} dx, \quad (23)$$

$$\Lambda_\infty = -\frac{N}{8\pi} L^2, \quad p_\infty = \pm \frac{N}{8\sqrt{\pi}} L, \quad N = \text{tr } 1, \quad (24)$$

$$q_\infty = -\frac{N}{48\pi} \log \frac{L^2}{\mu^2}, \quad \alpha_\infty = -\frac{N}{16\pi} \log \frac{L^2}{\mu^2}.$$

Using (23) and (24) it is easy to obtain that in $\bar{W}_\infty$
 $N=h-2, \hat{N}=h-8$ for the bosonic string, $N=0, \hat{N}=\frac{3}{2}(h-2)$
for the fermionic string and $N=0, \hat{N}=3h$ for the charged
fermionic string. The absence of the quadratic divergences
in the latter two cases is the consequence of the zero total
number of degrees of freedom in the supersymmetric theory.
Therefore, there is no necessity for the introduction of a
bare "quantum" cosmological term in (7) though one needs
the bare Euler number term (let us point out that the L^2 -
divergences are absent also in the case of the bosonic string
under the correct choice of the measure in the path integral
over metrics, see [13]). We conclude that in the bosonic
(fermionic) case $\bar{W}$ is the action of the free scalar
field ϕ (scalar ϕ and spinor λ fields) and not of the
Liouville model (its supersymmetric extension) as it was pro-
posed in [1]. Thus the interaction terms ($\sim e^\phi$) are absent
in (12).

The finite part $\bar{W}_f$ can be obtained by integrating $\frac{\delta W}{\delta \phi}$
for all operators in (14) (cf. [1]). Suppose the validity
of the following representation

$$\text{tr} (s_\Delta e^{-s\Delta}) = \text{tr} (a s \hat{s} \Delta e^{-s\Delta} + \hat{a} s \hat{s} \hat{\Delta} e^{-s\hat{\Delta}}), \quad (25)$$

where a and $\hat{a}$ are numbers and $\hat{\Delta}$ is some auxiliary operator.

Then according to (20) ^{4/}

$$\delta W = \frac{1}{2} \int d^2x \delta \epsilon(x) [a b_2(x) + \hat{a} \hat{b}_2(x)] \sqrt{g}. \quad (26)$$

As a result, ^{5/}

$$W_f = - \frac{\alpha}{12\pi} \int d^2x \left[\frac{1}{2} (\partial_a \epsilon)^2 \right] + \text{const}, \quad (27)$$

where

$$\alpha = -\frac{1}{2} (a\beta + \hat{a}\hat{\beta}), \quad \bar{b}_2 = \beta \cdot \frac{R}{6}, \quad \hat{\bar{b}}_2 = \hat{\beta} \cdot \frac{R}{6}. \quad (28)$$

Eq.(25) takes place if $\Delta = A \cdot B$ and $\delta \Delta = a' \delta \epsilon \Delta + a'' \Delta \delta \epsilon + \hat{a} A \cdot \delta \epsilon \cdot B$, where $a' + a'' = a$ and A and B are first order differential operators (then $\hat{\Delta} = BA = A^{-1} \Delta A$). This is valid for all operators

^{4/} for simplicity we shall omit the boundary terms in $\overline{W}_f$ (their treatment depends on the boundary conditions in the theory); for the results in the case of Δ_0 see [14].

^{5/} if $e_\mu^a = \phi \delta_\mu^a$ we have: $R = -\phi^{-2} \square \log \phi^2$,
 $\partial_\mu = \partial_\mu + \frac{1}{2} \delta_\mu^\nu \epsilon_{\nu\alpha} \partial^\alpha \phi$, $\Delta_0 = -\phi^{-2} \square$,
 $\Delta_{1/2} = -\phi^{-3/2} \partial_a \phi^{-1} \partial_a \phi^{1/2}$, $\tilde{\Delta}_{1/2} = -\phi^{-5/2} \partial_a \phi \partial_a \phi^{-1/2}$,
 $\Delta_{1ab} = -\phi^{-4} \partial^c \phi^2 (\delta_{ab} \partial_c + \delta_{bc} \partial_a - \delta_{ac} \partial_b)$.

in (14), i.e. they all have the calculable "through the anomaly" effective action (this is the consequence of the conformal invariance of X and W_m in (7) before the regularization). Using (21) and the fifth footnote we get

$$\begin{aligned}\Delta_0: a &= -2, \hat{a} = 0, \beta = 1, \alpha_0 = 1, \\ \Delta_{1/2}: a &= -1, \hat{a} = -1, \beta = -\frac{1}{2}, \hat{\beta} = -\frac{1}{2}, \alpha_{1/2} = -\frac{1}{2}, \\ \tilde{\Delta}_{1/2}: a &= -3, \hat{a} = +1, \beta = \frac{5}{2}, \hat{\beta} = -\frac{7}{2}, \tilde{\alpha}_{1/2} = \frac{11}{2}, \\ \Delta_{1\text{aff}}: a &= -4, \hat{a} = +2, \beta = 8, \hat{\beta} = -10, \alpha_1 = 26.\end{aligned}\quad (29)$$

Finally, the coefficient before the first term in the effective action $\overline{W}_f$ (12) is $\gamma = -\frac{d}{12\pi}$ where for the three string models

$$\begin{aligned}d_{\text{bose}} &= h d_0 - d_1 = h - 26, \\ d_{\text{fermi}} &= h d_0 - h d_{1/2} - d_1 + 2 \tilde{\alpha}_{1/2} = \frac{3}{2}(h - 10), \\ d_{\text{charged}}^{\text{fermi}} &= 2h d_0 - 2h d_{1/2} - d_1 + 4 \tilde{\alpha}_{1/2} - 2 d_0 = 3(h - 2).\end{aligned}\quad (30)$$

The results for the first two models coincide with those of [1] (here one must put $\lambda = 0, \rho = 0$ or $\rho = 0$ in (12)) excluding the fact that we do not see the arguments for the introduction of the Liouville interaction terms in (12). In the case of the charged spinor string we thus obtain the correspondence with the well-known result for the critical dimension $h = 2$.

Let us remark that if there exists the supersymmetric $SU(2)$ generalization of (1) (see discussion in [7]), the obvious modification of (30) gives $\gamma \sim h + g, g > 0$

in agreement with the known fact of the presence of ghosts in this theory for all dimensions n .

It is interesting to note that there is a more simple way of calculating $\mathcal{V}$ in (12): omitting the conformal and superconformal anomalies (i.e. taking the gauge $e_\mu^a = \delta_\mu^a, \chi = 0, A_\mu = A_\mu^\perp$) we obtain for the A_μ -dependent part of $\bar{W}$ (here all operators are defined on the Dirac spinors)

$$\exp(-\bar{W}) = \frac{[\det \hat{\mathcal{D}}(A)]^n}{\det \tilde{\Delta}_{\frac{1}{2}}(A)}, \quad (31)$$

$$\hat{\mathcal{D}}(A) = \partial_\mu - i A_\mu, \quad \tilde{\Delta}_{\frac{1}{2}}(A) = -\partial_\mu^+ \tilde{\gamma}^\mu \tilde{\gamma}^\nu \partial_\nu.$$

Using the well-known formula of the Schwinger model [11]

$$\log \det \hat{\mathcal{D}}(A) = - \int d^2x \left[\frac{1}{2\pi} (A_\alpha^\perp)^2 \right], \quad (32)$$

we get $\bar{W}_2 = - \frac{(n-2)}{4\pi} \int d^2x [-2 (A_\alpha^\perp)^2] + \dots$, in the agreement with (12) and (30). Let us also mention that the independent calculation (i.e. do not appealing to the residual symmetry (11)) of the coefficient before the $\bar{\lambda} \hat{\mathcal{D}} \lambda$ term in (12) can be carried out in the gauge $e_\mu^a = \delta_\mu^a, A_\mu = 0, \chi = 0$ using the regularization of the second footnote.

Let us now remark on the interpretation of the string model (1). The presence of the $2n$ real scalar fields indicates that (1) describes the string in the $d=2n$ -dimensional space-time (scalar fields define the map $M^2 \hookrightarrow \mathbb{R}^d$) while for the bosonic and fermionic strings $d=n$. Hence for the string in the four dimensional space-time (corresponding to $n=2$) we obtain the absence of the anomalies in the theory ($\mathcal{V}=0$ in (12)). This is a strong argument in

favour of the claim that charged spinor string with $h = 2$ is the most realistic string model. (It is interesting to note that in the last variant of the fermi string model describing QCD at $N=\infty$ [15] there is just two Dirac spinors). The fact of the absence of the anomalies at $h = 2$ shows that in this case the classical gauge (6) is valid also on the quantum level, i.e. all amplitudes obtained with help of (7) coincide with the ordinary dual amplitudes.

In conclusion we want to point out that $O(1)$ [2,3] and $O(2)$ [6] models do not exhaust all local superinvariant 2-dimensional theories. Really, one can construct the variant of a 2-dimensional supergravity using the dimensional reduction of the ordinary 4-dimensional supergravity to two dimensions (just like it was done in [16] for the reduction from 11 to 4). The bosonic part of the lagrangian has the form

$$\mathcal{L} = -R_{\mu\nu} g^{\mu\nu} \sqrt{g} \sqrt{\varphi} + \frac{1}{4} \sqrt{\varphi} \varphi_{mn} F_{\mu\nu}^m F_{\rho\sigma}^n g^{\mu\rho} g^{\nu\sigma} \sqrt{g} + \\ + \frac{1}{4} \varphi^{mn} \varphi^{\rho\sigma} \sqrt{\varphi} g^{\mu\nu} \sqrt{g} (\partial_\mu \varphi_{np} \partial_\nu \varphi_{\rho\sigma} - \partial_\mu \varphi_{mn} \partial_\nu \varphi_{\rho\sigma}), \quad (33)$$

where $\left[\begin{array}{c|c} g_{\mu\nu} + \varphi_{mn} A_\mu^m A_\nu^n & \varphi_{mn} A_\nu^m \\ \hline \varphi_{mn} A_\mu^m & \varphi_{mn} \end{array} \right]$ is the 4-metric, $\varphi = \det \varphi_{mn}$,

A_μ^m are two abelian vector fields with $F_{\mu\nu}^m$ as the curls and φ_{mn} is the set of the three scalar fields. A non-trivial feature of the reduction to two dimensions is the impossibility to get rid of the φ in the first term in (33) ("graviton" here is not trivial as compared to the case of the ordinary 2-dimensional gravity). The theory

contains also two gravitini and four Majorana spinors (again reminding the $h=2$ model (1)). However, the non-polynomial nature of (33), the presence of the A -kinetic term and the lack of the conformal invariance (the effective action here will not be simply "calculable") apparently precludes the connection of this theory with a string model.

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