Ref.TH.3118-CERN

COSMIC STRINGS AND DOMAINS IN UNIFIED THEORIES

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A B S T R A C T

Unified theories predict that the Universe passes through a sequence of phase transitions as it cools after the big bang. Topological structures (domain walls, strings and monopoles) may be created in these transitions. The production of domains of different zero temperature vacua (separated by massive walls) excludes some popular scenarios for the breaking of $SU(5)$ and $SO(10)$.

Massive strings appear in other schemes. They may be open strings or closed loops. We exhibit an $SO(10)$ example with string formation. $SO(8)$ supergravity may deliver an effective $SU(5) \times SU(2)$ unified theory below the Planck mass scale. This can also lead to string production. We present the rather restrictive conditions under which string producing theories are harmless to conventional cosmology. Even then, strings do not seem to provide the initial density perturbations that evolved into galaxies.

Ref.TH.3118-CERN

7 July 1981

1. - INTRODUCTION

Unified theories ^{1),2)} of the fundamental interactions permit us to study the thermal and other properties of the early Universe at temperatures of order of the unification mass. This is particularly important as, after the big bang, the cooling Universe is expected to pass through a series of transformations of phase. These phase changes occur as the high degree of symmetry at temperatures close to the Planck mass $G^{-\frac{1}{2}} = 1.22 \times 10^{19}$ GeV is reduced in stages down to the present zero temperature symmetry, $SU(3) \times U(1)$. The precise nature of the phase transformation depends on the surviving symmetry at a given temperature. It also depends on the Higgs potential which breaks this symmetry on further cooling through a critical temperature. Depending on the form and parameters of the Higgs potential, a given phase transformation will be of first or second order.

In general, a system undergoing a phase transformation develops topological structures or textures ³⁾. These may be points, lines or surfaces where the higher temperature symmetric phase is preserved - e.g., by a topological conservation law. If no conservation law exists, the structure will dissipate. However, it may be long-lived and therefore produce observable effects.

We thus expect monopoles ⁴⁾, strings ⁵⁾ and domain walls ⁶⁾. Phenomena of this sort are familiar as boojums ⁷⁾ in He^3A (a monopole and its attached flux string which have migrated to a container wall), vortex strings in type II superconductors and the well-known domains in ferromagnets.

Such structures can appear in grand unified theories. They may be produced in the phase transitions we have remarked on. The simplest grand unified theory ²⁾ - $SU(5)$ - leads to topologically stable magnetic monopoles ⁸⁾. This occurs independently of the exact form of the Higgs potential which breaks the symmetry. The only requirement is that the unbroken subgroup at zero temperature be $SU(3) \times U(1)$. Deciding whether strings or domains are formed requires a more careful examination of the symmetry breaking pattern.

The possible cosmological importance of these structures is clear from simple arguments. If the vacuum expectation value of the Higgs field is ϕ_0 , then the monopole mass M is of order $g^{-1}|\phi_0|$. The string mass per unit length σ is of order $|\phi_0|^2$ and the domain wall mass per unit area μ is of order $g|\phi_0|^3$. (g is the gauge coupling at the mass scale $g|\phi_0|$.) The expectation value will be of order 10^{12} to 10^{15} GeV, depending on the theory and the symmetry breaking. If we take $\phi_0 \sim 10^{15}$ GeV then the monopole mass is $\sim 10^{16} \approx 10^{-8}$ gram. Also,

$\sigma \gtrsim 10^{20}$ gm/cm and $\mu \gtrsim 10^{48}$ gm/cm². The radius (or thickness) of all these structures is $\lesssim 10^{-29}$ cm. Monopoles will be cosmologically unimportant if there are less than one per 10^{16} baryons or $\lesssim 10^{63}$ monopoles in the observed Universe. However, even one string passing through our horizon (or Hubble sphere) will have $\gtrsim 10^{-6}$ of its total mass. A domain wall of the dimensions of our horizon would have a mass at least 50 orders of magnitude larger than the observed mass of the Universe. Our very existence is enough to exclude the presence of such a wall.

In Section 2 we consider the first step in the symmetry breaking of SU(5), using the Coleman-Weinberg⁹⁾ potential or any other Higgs potential with a discrete symmetry. We also discuss the breaking of SO(10)¹⁰⁾ down to SU(3)×SU(2)×U(1) with a minimal Higgs system. In both cases we show that the vacuum manifold consists of two connected components. Thus, a domain structure emerges as the Universe cools below some critical temperature. We expect that domain walls are formed with non-zero density in the big bang. These symmetry breaking patterns are therefore most probably excluded. We do, however, remark on a simple and viable symmetry breaking scheme^{11),12)} for SU(5). It is amusing that these difficulties with the popular symmetry breaking mechanisms have not been noticed until now^{*)}.

After this misfortune, we go on in Section 3 to consider a non-minimal SO(10) breaking pattern. It leads to monopoles and strings but no domains. It turns out not to be possible to break a simple or semisimple unified group in such a way as to produce only closed loops of strings. (We implicitly include infinite endless strings when referring to closed loops.) The string arises when a U(1) symmetry is broken. To get the U(1) one must first break the simple or semisimple unified group. This, however, produces monopoles with a U(1) charge. If the U(1) is then broken at a lower temperature, the U(1) monopole flux is squeezed into a tube running to an antimonopole. (Of course, closed loops of string may also be formed at this stage.) It is important to keep in mind that these monopoles are not the stable electromagnetic ones of SU(5). Section 3 also contains an esoteric but possibly important case. The superunification of all interactions through a conjectured hidden SU(8) symmetry¹³⁾ of SO(8) supergravity might lead^{14),15)} to an effective SU(5)×SU(2) symmetry at or somewhat below the Planck mass scale. Breaking this symmetry down to SU(3)×SU(2)×U(1) via SU(5)×U(1) will lead to the formation of open and closed strings.

*) Rumor has it that this has been noted independently for SU(5) by A. Guth and S.Y. Pi.

This scheme need not produce any domain structure. The presence of the original $SU(2)$ factor in $SU(5) \times SU(2)$ may have other interesting consequences.

We next investigate the cosmology of strings in unified theories. It can happen that such string producing theories conflict with conventional cosmology. The relic gravitational radiation from the strings can dominate the expansion during nucleosynthesis. (It is even possible that such theories do not lead to any picture of the Universe like the real one.) We find a scenario mild enough to avoid such problems. But then it leads to no interesting cosmological effects at all. Even granting that this scenario is realistic and that string producing theories are allowed by cosmology, we do not see how they can lead to galaxy formation. Zeldovich¹⁶⁾ and Vilenkin¹⁷⁾ have advocated strings as a source of cosmic density perturbations which can evolve into galaxies. However, they make specific assumptions about the strings which do not agree with what we find. The presence of monopoles leads to a cut-off in the length of the strings. Then the strings live too short a time to be of any relevance to galaxy formation.

2. - COSMOLOGICAL DOMAINS OF $SU(5)$ AND $SO(10)$

In this section, we take up the usual choices of symmetry breaking for $SU(5)$ and $SO(10)$ and consider the vacuum state. We then go on to the cosmology of these models.

$SU(5)$

Consider the symmetry breaking via a single real 24 Higgs field ϕ . This is the smallest representation which breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. In general, the renormalizable and $SU(5)$ invariant potential can include terms $(\text{tr } \phi^2)^2$, $\text{tr } \phi^4$, $\text{tr } \phi^3$, and $\text{tr } \phi^2$. An elegant choice would be to exclude all terms which involve dimensional couplings^{9),18)}. Then the only dimensional parameter is the vacuum expectation value of ϕ . Alternatively, we can just impose on the potential $V(\phi)$ the discrete symmetry $\phi \rightarrow -\phi$. Then

$$V(\phi) = \frac{a}{4} (\text{tr } \phi^2)^2 + \frac{b}{2} \text{tr } (\phi^4) - \frac{\mu^2}{2} \text{tr } (\phi^2) + \text{radiative corrections} \quad (1)$$

Requiring no dimensional couplings would mean $\mu^2 = 0$.

The potential (1) leads to two vacuum states with the same energy density and vacuum expectation values

$$\phi_0 = \pm |\phi_0| = \pm |y| \text{diag} \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right) \quad (2)$$

where $|y| \sim 10^{15}$ GeV. Regions with $\phi_0 = +|\phi_0|$ will be separated from $\phi_0 = -|\phi_0|$ regions by a wall of thickness $\sim 10^{-29}$ cm, where the Higgs field passes through zero. The two vacua in (2) belong to two different connected components of the vacuum manifold. This is because no SU(5) transformation can go from one to the other. The expression $\text{tr } \phi^3$ is SU(5) invariant but not invariant under $\phi \rightarrow -\phi$. This is thus not an SU(5) transformation.

The degeneracy of the two vacua (2) would be removed by a $\text{tr } \phi^3$ term in $V(\phi)$. If the dimensional coefficient of this term is of order $g^2 |\phi_0|$ then there is in fact only one vacuum state.

SO(10)

The conventional minimal breaking of this theory is

$$SO(10) \xrightarrow[M_S]{16} SU(5) \xrightarrow[M_X]{45} SU(3) \times SU(2) \times U(1) \quad (3)$$

where M_S and M_X are the vector boson masses generated at each stage. The spinor 16 of SO(10) is denoted χ and the antisymmetric tensor 45 is Φ_{ij} . Provided $M_S \geq 10^{17}$ GeV, the left-handed neutrino masses are compatible with the cosmological bound¹⁹⁾. M_X is of order 10^{15} GeV and the model reproduces the usual SU(5) results for $\sin^2 \theta_w$ and the proton lifetime.

The Higgs potential for χ need not concern us for the moment. The potential for Φ alone contains no $\text{tr } \Phi^3$ term due to the antisymmetry of Φ . The potential exhibits an accidental $\Phi \rightarrow -\Phi$ discrete symmetry. We have

$$V_2(\Phi) = V_2(-\Phi) = \frac{a}{16} (\text{tr } \Phi^2)^2 + \frac{b}{4} \text{tr } \Phi^4 + \frac{\mu^2}{4} \text{tr } \Phi^2 \quad (4)$$

There are again two vacua degenerate in energy with expectation values $\Phi_0 = \pm |\Phi_0|$. They cannot be connected by an SO(10) transformation because the SO(10) invariant expression

$$\epsilon_{i_1 i_2 \dots i_{10}} \Phi_{i_1 i_2} \Phi_{i_3 i_4} \dots \Phi_{i_9 i_{10}} \quad (5)$$

changes sign under $\Phi \rightarrow -\Phi$, which is therefore not an $SO(10)$ transformation. [Note, however, that $\Phi \rightarrow -\Phi$ is an $O(10)$ transformation.] The two vacua belong to the two different connected components of the vacuum manifold. In the case where this discrete symmetry is imposed on the full Higgs potential $V(\Phi, \chi)$, the symmetry breaking leads to a similar vacuum domain structure as did our $SU(5)$ example.

It might appear that the inclusion in the Higgs potential of a term

$$-\beta \sum_{i,j=1}^{10} \chi^\dagger \frac{\sigma^{ij}}{2} \chi \bar{\Phi}_{ij} \quad (6)$$

will destroy the $\Phi \rightarrow -\Phi$ symmetry and eliminate the domain structure of the vacuum. [In the above, $\sigma^{ij} = (\Gamma^i, \Gamma^j) / 2i$ where the Γ^i are generalized by Dirac matrices.] However, this does not lead to a physically interesting symmetry breaking. Including (6) in the Higgs potential leads to the following unpleasant result : either $SU(5)$ only breaks down to $SU(4) \times U(1)$ or $M_\chi \sim M_S$, giving unacceptably large neutrino masses. We can accept neither case.

Consider the most general $SO(10)$ invariant potential $V(\Phi, \chi) = V_1(\chi) + V_2(\Phi) + V_3(\Phi, \chi)$. $V_1(\chi)$ and $V_3(\chi, \Phi)$ have the form ²⁰⁾

$$\begin{aligned} V_1(\chi) &= -\mu_\chi^2 \chi^\dagger \chi + \frac{\lambda}{2} (\chi^\dagger \chi)^2 + \frac{\kappa}{2} \sum_{i=1}^{10} (\chi^\dagger \Gamma^i \chi)^2 \\ V_3(\chi, \Phi) &= -\frac{\alpha}{4} (\chi^\dagger \chi) \text{tr} \bar{\Phi}^2 + \gamma \sum \chi^\dagger \frac{\sigma^{ij}}{2} \frac{\sigma^{kl}}{2} \chi \bar{\Phi}_{ij} \bar{\Phi}_{kl} \\ &\quad - \beta \sum \chi^\dagger \frac{\sigma^{ij}}{2} \chi \bar{\Phi}_{ij} \end{aligned} \quad (7)$$

We expect the quartic couplings to be $\sim g^2$, $\beta \sim g M_S$ and $\mu_\Phi, \mu_\chi \sim M_S$. We can perform an $SO(10)$ transformation so as to write Φ in the form

$$\bar{\Phi} = \left(\frac{s}{5} + \phi \right) \otimes i\tau^2 \quad (8)$$

where s is an $SU(5)$ singlet, ϕ is a diagonalized $SU(5)$ 24 plet and τ^2 is the usual Pauli matrix. At a critical temperature $T_{c1} \sim M_S/g$, the field χ starts developing an expectation value $\chi_0 \sim M_S/g$ and $SO(10)$ breaks down to $SU(5)$. In addition to χ , s will in general also develop a non-zero vacuum expectation value. In fact, the s dependent terms of V are

$$V_{eff}(s) = \left(\frac{a}{100} + \frac{b}{250} \right) s^4 + \left[-\frac{\mu_\Phi^2}{10} + \left(\delta + \frac{\alpha}{10} \right) \chi_o^2 \right] s^2 - \beta \chi_o^2 s \quad (9)$$

and only for $\beta = 0$ can we have a minimum at $s = s_o = 0$. Otherwise, the minimum of (9) is at $s_o \sim M_S/g$. ($\beta = 0$ can only be assured by imposing the $\Phi \rightarrow -\Phi$ symmetry which we wished to avoid.)

The effective potential for the subsequent $SU(5)$ breaking at scale M_X is now

$$V_{eff}(\phi) = \frac{a}{4} (\text{tr } \phi^2)^2 + \frac{b}{2} (\text{tr } \phi^4) + \frac{c}{3} \text{tr } \phi^3 - \frac{\mu_\phi^2}{2} \text{tr } \phi^2$$

$$\mu_\phi^2 = \mu_\Phi^2 - \alpha \chi_o^2 - a s_o^2/5 - 6 b s_o^2/25 \quad (10)$$

where $c = 6 s_o b/5 \sim g M_S$.

This is not an acceptable symmetry breaking potential because of the magnitude of c . The vacuum expectation value of ϕ is of the form $\phi_o = y \text{diag}(1,1,1,-\frac{3}{2},-\frac{3}{2})$ where

$$y = \frac{c}{2b} (15\eta+7)^{-1} \left(1 + [1-8\xi(15\eta+7)]^{1/2} \right)$$

$$\xi = -\mu_\phi^2 b/c^2, \quad \eta = a/b > -7/15 \quad (11)$$

In order to obtain $\phi_o \sim M_X/g \ll \chi_o$ with $M_\phi^2 \sim M_S^2$ (i.e., without tuning the parameters in μ_ϕ^2) we require $|\xi|$ to be of order of unity and therefore η of order of 10^3-10^4 from (11). This drastically violates the condition that $SU(5)$ breaks down to $SU(3) \times SU(2) \times U(1)$ as the Universe cools. This requirement can be written as $\xi < -(10\eta+7)/8$ ¹²⁾. [The actual symmetry breaking is to $SU(4) \times U(1)$.] This unacceptable result of $c \sim g M_S$ cannot be easily avoided, because even tuning parameters to get $\mu_\phi^2 \ll M_S^2$ forces us to $|\xi| \ll 1$ which is compatible with $SU(5)$ breaking to $SU(3) \times SU(2) \times U(1)$ only in the small range $-7/15 \ll \eta \lesssim -3/10$ ^{11),12)}. In this range, however, we get $\phi_o \sim M_S/g$ [see Eq. (11)], which is also unacceptable.

We conclude that the symmetry breaking pattern (3) requires the discrete symmetry $V(\Phi, \chi) = V(-\Phi, \chi)$ and therefore leads to a vacuum domain structure.

Cosmology

For two vacua separated by a wall in the $x y$ plane, the time independent solution of the equations of motion is [see Eq. (11)] $\phi = y(z) \text{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$,

$$y(z) = |\phi_0| \tanh(z\mu_\phi) \quad (12)$$

The mass density per unit area is $\mu = g|\phi_0|^3$ on dropping constants of order unity and on assuming that a and b are of order g^2 .

The energy momentum tensor for an array of slowly moving walls will, to good accuracy, take the form ⁶⁾

$$T_{\mu\nu} = S_\mu \text{diag}(1, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}) \quad (13)$$

where S is the wall area per unit volume. (Off diagonal elements of $T_{\mu\nu}$ will be of order v/c , where v is the wall velocity perpendicular to its surface.)

At the phase transition temperature T_c , the original symmetric vacuum will be replaced by a domain structure as the Universe falls into different $(\pm|\phi_0|)$ vacua in different places. We denote the characteristic size of the domains at the phase transition temperature to be ℓ_c . Up to the time when the domain walls dominate the expansion (call it $t = t_0$) we can assume that the surrounding gas is relativistic and in equilibrium. Then

$$\rho \propto R^{-4}, \quad R \propto t^{1/2} \quad (t < t_0) \quad (14a)$$

where R is the cosmic scale factor. The radius of the particle horizon or Hubble sphere is $2t$. After t_0 the domain walls dominate. They stretch conformally and

$$\rho = T_{00} \propto R^{-1}, \quad R \propto t^2 \quad (t > t_0) \quad (14b)$$

The horizon radius grows as

$$R(t) \int_0^t \frac{dt'}{R(t')} \sim 3t^2/t_0 \quad (15)$$

After t_0 no new domain walls enter the horizon, since the scale factor and the horizon radius keep pace with one another. The expansion is permanently dominated by the domain walls.

We estimate t_0 as follows. Assume several walls in the horizon. A domain wall of dimension ℓ has mass $\sim \ell^2$. The gas in it has mass $\sim \ell^3 T^4$. The walls then dominate the expansion for temperature or time

$$T \lesssim (\rho / T_c \ell_c)^{1/3}$$

$$t \gtrsim t_0 \sim t_c (\ell_c T_c)^{2/3} \quad (16)$$

where t_c is the time $T = T_c(t_c)$ and we used $\ell \propto R$. If the distance between walls at $T = T_c$ is large, t_0 may be much larger than t_c . However, we do not expect t_0/t_c to be more than a few orders of magnitude.

This argument assumed walls in causal contact within the horizon. If the domain size is very large and the walls come into causal contact very late, then it is only after a few walls enter the horizon that the local expansion is affected by the walls.

Once the walls are within one another's horizon, part of the structure can undergo gravitational collapse. However, even this cannot lead to less than one wall per horizon volume if the original domains were smaller than the horizon ³⁾.

What is the characteristic size ℓ_c of the domains at the phase transition temperature? One can argue that at T_c different regions not in causal contact should fall at random into one of the two vacuum states. Then $\ell_c \lesssim 2t_c$. (At $T_c \sim 10^{14}$ GeV we have $2t_c \sim 10^{-24}$ cm and a cosmic scale factor $R \sim 10^2$ cm with $R_{\text{now}} = H_0^{-1} = 10^{28}$ cm.) It appears not at all easy to escape this bound on ℓ_c , because it is hard to imagine that the sign of ϕ_0 is correlated over distances bigger than the horizon. The walls between different vacua are then forced on us. This situation is qualitatively different from that involving monopole or string formation, where it is probably easier to speculate about mechanisms to suppress their production.

Our past light cone contains no wall. We remarked in the Introduction that such a wall would be too massive to have escaped notice. The absence of a domain wall in our horizon means $\ell_c/2t_c \gg 10^{26}$ at $T = T_c$. For the reason we

just mentioned, it seems quite improbable to us that the domain sizes could be so large. This means that theories which produce domain structure conflict with standard cosmology and must most probably be excluded.

3. - COSMIC STRINGS OF SO(10) AND OF SU(8) SUPERGRAVITY

We now go on to scenarios for symmetry breaking which lead to cosmic string formation. Our first example is a variant of SO(10) breaking to SU(5) in stages. The second example is more adventurous. We consider a conjectured theory based on a local SU(8) gauge invariance found in SO(8) supergravity. This theory at least offers some ideas on how the usual gauge interactions could be unified with gravity at the Planck scale. The gauge group below the Planck scale could be SU(5)×SU(2). We discuss this theory and how it might lead to strings. Finally, we consider the cosmology of the strings, and the credibility of theories which produce them.

SO(10)

We can illustrate here in a specific case why the breaking of a simple or semi-simple group cannot lead to the formation of only loops of string. (By loop we also mean to include the case of a string of infinite extent without ends.) Consider the chain to SU(5)

$$SO(10) \xrightarrow[M_1]{210} SU(5) \times U(1) \xrightarrow[M_2]{126} SU(5) \quad (17)$$

where we choose to break SO(10) down to SU(5)⊗U(1) with an antisymmetric 210 dimensional Higgs Φ_{ijkl} . (Had we chosen the 45 dimensional representation, we would have got domain structure again. It would tend to disappear at the next stage of symmetry breaking, but we prefer to avoid this complication.)

Monopoles will be produced at the first stage of symmetry breaking, because an U(1) factor appears. The monopoles have U(1) charge as well as SU(5) charge. The SU(5) quantum numbers of these monopoles depend only on the way SU(5) and U(1) are "patched" together in SO(10). [More precisely, certain elements of SU(5)⊗U(1) will have to be identified with the identity in Spin(10), the covering group of SO(10). This also explains the product notation ⊗; SU(5)⊗U(1) is not a simple Cartesian product.]

The following pedestrian procedure will work in general and has the advantage of simplicity. It gives the minimal U(1) magnetic charge and the SU(5) magnetic flux of the stable monopole.

To find how $SU(5) \otimes U(1)$ is imbedded in $Spin(10)$, consider the decomposition of the fundamental spinorial 16 with respect to $SU(5)$, $16 = 10 + \bar{5} + 1$. The $U(1)$ hypercharges of the $SU(5)$ multiplets are +1, -3 and +5, respectively. A $U(1)$ rotation $\exp(iQ\theta)$ transforms them as follows

$$\psi_{10} \rightarrow e^{i\theta} \psi_{10}, \quad \psi_{\bar{5}} \rightarrow e^{-3i\theta} \psi_{\bar{5}}, \quad \psi_1 \rightarrow e^{5i\theta} \psi_1, \quad (18)$$

Clearly, for $\theta = 2\pi/5$ or a multiple thereof, the singlet is unchanged. Take $\theta = 2\pi/5$. Then we can see that a pure $SU(5)$ transformation which is the identity times $\exp(4\pi i/5)$ can exactly compensate the remaining $U(1)$ transformations on the 10 and $\bar{5}$. This pair of $[SU(5), U(1)]$ transformations $[\exp(4\pi i/5), \exp(2\pi i/5)]$ must be identified with the identity in $spin(10)$, as it leaves the fundamental 16 representation (and hence all others) invariant. The monopoles appear with charges corresponding to the patching together of $SU(5)$ and $U(1)$ in $spin(10)$. The monopole with minimal $U(1)$ charge (say, $m = 1$) then has two units of $SU(5)$ flux. By iterating the $U(1)$ transformation we can find all pairs of $SU(5)$ and $U(1)$ transformations which are to be identified with the identity in $spin(10)$. The $m = 1$ monopole turns out to be the one with minimal charge and mass.

These monopoles will be produced during the phase transformation at temperature $T_{C1} \sim M_1/g$ [Eq. (17)]. They are zeros of the 210 dimensional Higgs field, and have mass $m \sim g^{-2} M_1$. (M_1 and M_2 are vector boson masses.) At $T_{C2} \sim M_2/g$ the $U(1)$ symmetry is broken. The $U(1)$ magnetic flux is confined to strings of radius $\sim M_2^{-1}$ for temperatures $T < T_{C2}$. The centre of a string is a line where the 126 dimensional Higgs field vanishes. The flux strings usually run from a monopole to an antimonopole. [It is, however, possible to form closed $U(1)$ flux strings at $T \sim T_{C2}$ from pure 126 Higgs field fluctuations.] Note that the monopoles or antimonopoles at the ends of flux strings still have an $SU(5)$ flux which is not confined to the string. It extends to infinity.

$SU(8) \supset SU(5) \times SU(2)$

Our next example is totally unlike the preceding ones. The $SU(8)$ here is imagined to be a dynamically realized local gauge symmetry derived from $SO(8)$ supergravity. This theory unifies all interactions including gravity at the Planck mass scale.

It was realized by Cremmer and Julia¹³⁾ that $SO(8)$ supergravity contains a local $SU(8)$ symmetry associated with composite fields. They conjectured that a corresponding $SU(8)$ gauge theory could be dynamically realized. A concrete example has been proposed by Ellis et al.¹⁴⁾ It has anomalies and is therefore not renormalizable (this need not be worrissome at the Planck mass). They suggest that this theory should break down to an effective grand unified theory of the non-gravitational interactions which is renormalizable and anomaly free. This effective grand unified theory would be relevant below the Planck mass scale. It can be argued that the only satisfactory grand unified theory of this sort is $SU(5)$ with three generations of fermions. We take a slightly different point of view here. We ask : what is the maximal subgroup of $SU(8)$ which is anomaly free and contains $SU(5)$ with three families ?

The left-handed (helicity $-\frac{1}{2}$) fermions transform as a $216 + \overline{504}$ of $SU(8)$. Their decomposition into $SU(5) \times SU(3) \times U(1)$ shows that neither the $SU(3)$ nor the $U(1)$ can be made anomaly free. The maximal anomaly free subgroup is in fact¹⁵⁾ :

$$SU(5) \times SU(2) \subset SU(8) \quad (19)$$

with the fermion content

$$\begin{aligned} (45 + \overline{45}) + 4(24) + 9(10 + \overline{10}) \\ + 3(5 + \overline{5}) + 9(1) + 3(\overline{5} + 10) \end{aligned} \quad (20)$$

Various scenarios for $SU(5) \times SU(2)$ breaking can be imagined. One nice possibility is

$$SU(5) \times SU(2) \xrightarrow{M_1} SU(5) \times U'(1) \xrightarrow{M_2} SU(3) \times SU(2) \times U(1) \quad (21)$$

At temperature $T_{c1} \sim M_1/g$ the first stage of breaking occurs and leads to 't Hooft-Polyakov monopoles. They carry $U(1)$ magnetic flux. Both the $SU(5)$ and the $U(1)$ are broken at $T_{c2} \sim M_2/g$ by a complex 24 plet which carries a non-zero $U(1)$ charge. In this phase transition, magnetic flux strings are produced. There will be open strings connecting monopoles to antimonopoles and there may be also closed loops of string.

Remarkably, in this case we can escape the domain problem of $SU(5)$ with a discrete $\phi \rightarrow -\phi$ symmetry for the 24 plet. This symmetry is now a part of the continuous $U(1)$ symmetry of the complex 24 plet.

In this scheme, the left-handed neutrinos are light enough ($\lesssim 10^{-3}$ eV) to be cosmologically unimportant.

The extra $SU(2)$ factor in (19) is notable because it relates different families of fermions, something which a direct breakdown of $SU(8)$ to $SU(5)$ cannot do. Although we used (21) as an example of a string-producing theory, it may well have other interesting implications.

Cosmology

Before discussing the detailed cosmology of strings, we consider their possible disintegration by monopole-antimonopole formation in the string field, and also various modes of string energy loss.

Long strings are not stable. They can break by monopole-antimonopole pair creation in their magnetic field. We estimate the rate of $M\bar{M}$ tunnellings to be ²¹⁾

$$dN/dt \approx \sigma \exp(-2\pi M^2/\sigma) \quad (22)$$

per unit length and time. This can be ignored if it is small enough. It will be entirely negligible if there is less than one tunnelling in a string of the length of the present horizon in the age of the Universe $\sim H_0^{-1} \approx 3 \times 10^{17}$ sec. Thus $\exp(\pi M^2/\sigma) \gtrsim H_0^{-1} \sqrt{\sigma}$ or $M/\sqrt{\sigma} = M_1/M_2 \gtrsim 10$. Both our sequential symmetry breaking scenarios can satisfy this condition. We neglect instability against $M\bar{M}$ tunnelling.

After the cosmic strings are formed, open segments or closed loops entirely within one horizon (and in causal contact) will start to oscillate. They lose energy, either by friction with the surrounding medium or by emission of gravitational radiation.

For strings much longer than the horizon, irregularities larger in scale than the horizon will be stretched conformally by the Hubble expansion ¹⁷⁾. Waves on such strings with wavelengths smaller than the horizon will be redshifted ¹⁷⁾, as well as losing energy by viscosity or radiation. These strings will then tend to straighten out on horizon scales.

Viscous loss has been considered by Kibble ³⁾ and Everett ²²⁾ and gravitational radiation by Vilenkin ^{17),23)}. Viscous loss can be shown ³⁾ to be unimportant for times $t > 10^{-34}$ sec. To estimate gravitational radiation loss we applied Einstein's quadrupole loss formula to the small amplitude vibrations of a classical (non-relativistic) string. Setting the tension equal to the mass per unit length should then give a satisfactory order of magnitude estimate of the energy loss for a grand unification string. We find that waves of scale λ are damped out after a number of oscillations

$$N_G \sim (G\sigma)^{-1} \sim 10^6 \quad (23)$$

where N_G is independent of λ . Clearly, closed loops or open segments of string survive for a long time τ after their entire length appears within the horizon, $\tau \sim (G\sigma)^{-1} \ell$ where ℓ is the size of the loop or the open string.

Strings will disturb the evolution of the Universe if their total length within the horizon exceeds $\sim (G\sigma)^{-1}$ (the ratio of the mass of a single string through the horizon to the mass in it is $\sim G\sigma$). In particular, such a large density of string during nucleosynthesis (at $t \sim 10^1$ - 10^2 sec) is unacceptable. Even if the string density is zero during nucleosynthesis, the density of gravitational radiation then must not exceed a few times the photon energy density. This can happen in principle if the strings ever dominate over radiation and then deposit their entire energy in the form of gravitational radiation. The density of gravitational radiation would then always exceed the radiation density due to all other particle species. This would, of course, destroy conventional nucleosynthesis.

How long are the strings formed in the phase transition? We assume that there is less than one monopole per horizon volume t_c^3 , and that the strings are approximately straight on horizon scales even at $T = T_{c2}$. Their direction will be random on larger scales. The strings therefore execute a random walk with step length of order the horizon length t_c . We imagine this walk to occur on a cubic lattice with lattice spacing t_c . Some vertices of this lattice are occupied by monopoles and others by antimonopoles. (We also suppose that the average monopole charge in some sufficiently large volume is zero.) A typical string starts from a monopole and ends on an antimonopole.

To get a feeling for the behaviour of these random walks, we simplify the problem as follows: we assume that different walks do not influence one another. In particular, we ignore the constraint that only one walk can end on a given antimonopole. (This should not be a serious limitation if the monopole density is low.)

Consider a walk which starts from a given monopole. We ask for the probability that this walk survives k steps without hitting and stopping at an anti-monopole. It is $(1-p)^k$, where $p = t_c^3/\ell_M^3$, the density of vertices occupied by antimonopoles (ℓ_M is the average distance between monopoles). For small p the probability to survive k steps is thus

$$\begin{aligned} P_k &= \exp [-k |\ln(1-p)|] \\ &\approx \exp (-kp) \end{aligned} \quad (24)$$

The mean number of steps to an antimonopole is thus $\langle k \rangle \sim 1/p$. It is thus clear that the walks collectively traverse essentially all links of the lattice. In other words, there is of the order of one string segment per horizon. The mean end-to-end distance of a string is

$$\langle r \rangle \approx \langle k \rangle^{1/2} t_c \approx \ell_M (\ell_M/t_c)^{1/2} \quad (25)$$

Long strings are exponentially suppressed. For the same reason, even closed loops cannot be formed with sizes much exceeding $\langle r \rangle$. (The above problem has been solved by Montroll ²⁴⁾ in a different context. Our results agree qualitatively with his.)

We consider two possible scenarios for the subsequent evolution of the system of strings. In the first, we ignore the interaction of the strings with one another. Specifically, two strings pass through one another without effect. The second scenario assumes the opposite. The strings interact strongly in such a way that there tends to be only one string per horizon.

The first scenario leads to difficulties. This can be seen as follows. Since strings tend to straighten out on horizon scales, the string trajectory will be a random walk at all times, but with step length equal to the horizon size t . (We assume that this walk occurs on a cubic lattice of spacing t .) The end points of a given string are always attached to the same monopole-antimonopole pair. In the radiation dominated era, the end-to-end distance of any string grows like

$$r(t) = r(t_c) (t/t_c)^{1/2} \quad (26)$$

until the ends come within the horizon. This corresponds to $r(t)/t = r(t_c)/(t t_c)^{1/2}$ steps. The string then consists of $k(t)$ steps,

$$k(t) = k(t_c) (t_c/t) \quad (27)$$

The total length of the string $\ell(t) = k(t)t$ remains constant. Since its ends follow the cosmological expansion, the string strenghtens out.

Let t_H be the time when the string of initial mean end-to-end separation $\langle r \rangle$ comes entirely within the horizon. From Eq. (26),

$$t_H = \langle r \rangle^2 / t_c \quad (28)$$

For $t > t_H$, such strings cease following the cosmological expansion. They lose energy by emitting gravitational radiation. They disappear at a time

$$t_d \sim (G\sigma)^{-1} t_H \quad (29)$$

Now we estimate the energy density of the collection of strings for times $t_c \lesssim t \lesssim t_d$. Call this density $\rho_s(t)$. We note that the total length of string in a comoving volume is time-independent. Thus (in the radiation dominated era)

$$\begin{aligned} \rho_s(t) &= \rho_s(t_c) (t_c/t)^{3/2} \\ &= \sigma / (t_c t^3)^{1/2} \end{aligned} \quad (30)$$

Requiring that the strings never dominate, the expansion of the Universe means that they must die out by a time $(G\sigma)^{-2} t_c$. This leads to a constraint on the initial monopole density,

$$\ell_M/t_c \lesssim (G\sigma)^{-1/3} \sim 10^2 \quad (31)$$

and the strings die away before $t \sim 10^{-23}$ sec. They produce no interesting cosmic density perturbations.

The bound (31) does not appear a priori implausible. It becomes so when we consider the cosmological bound on the density of stable SU(5) monopoles²⁵⁾. This bound can be written as $\ell_M/t_c \gtrsim 10^5 - 10^6$. It does not appear credible that one mechanism for monopole production can give a high density of pre-SU(5) monopoles and simultaneously an acceptably low density of SU(5) monopoles.

Violation of the bound (31) means that the strings dominate over the radiation density for times larger than $(G\sigma)^{-2} t_c$. However, these strings deposit their entire energy density in the form of gravitational radiation by time t_d [Eq. (29)]. Subsequently, the gravitational radiation density always exceeds the

radiation density of all other species. (The two follow the same $\rho \propto R^{-4}$ law.) This is unacceptable, as we have already indicated.

Our first scenario therefore appears quite implausible.

The second scenario assumes strong string-string interactions. (Leading for example, to a frequent exchange of ends when two strings cross.) We therefore assume that each expansion time t leads to a rearrangement of strings so that we have a new set of random walks. These walks occur on a lattice of spacing t and with a density of vertices occupied by monopoles (or antimonopoles) $p(t) = p(t_c) (t_c/t)^{\frac{3}{2}}$. The conditions on the walks are the same as they were for the original set of walks at $t = t_c$. In particular, there is of order one segment of string per horizon for times which are not too large. The new end-to-end distance of a typical string is

$$\langle r(t) \rangle \sim \langle r(t_c) \rangle (t/t_c)^{1/4} \quad (32)$$

- quite unlike the previous case, Eq. (26). (The mean total length of string in a comoving volume decreases monotonically with time.)

The above analysis can only hold up to the time when entire strings appear within the horizon. This occurs at a time $t_H' = \langle r(t_c) \rangle^{\frac{4}{3}} t_c^{-\frac{1}{3}}$. We then have one monopole-antimonopole pair (plus string) inside the horizon. After this time, more and more such pairs appear within the horizon. This goes on until the time $(G\sigma)^{-1} t_H'$, when they disappear. Between t_H' and $(G\sigma)^{-1} t_H'$, the string density behaves as $\rho_s(t) \sim \sigma / (t_H' t^3)^{\frac{1}{2}}$.

At the time when the strings disappear, the ratio of their relic gravitational radiation density to the total density is of order

$$\frac{\rho_{\text{grav}}}{\rho_{\text{tot}}} \sim (G\sigma)^{1/2} \sim 10^{-3} \quad (33)$$

This is acceptably small.

This scenario thus avoids any conflict with the standard cosmology. We now see, for both scenarios, that the presence of monopoles and antimonopoles prevents the appearance of arbitrarily long strings. As a result, the lifetime of the strings is limited. To make a galaxy, strings have to persist at least until

$t \sim 10^8$ sec, when a galactic mass in baryons enters the horizon¹⁷⁾. This would require a very low monopole density, or $\ell_M/t_C \sim 10^{22}$. This would be quite unnatural and we reject it. We conclude that strings do not lead to galaxy formation. This result is, of course, more general than our specific scenarios are.

The above analysis rests on an assumption which we now wish to question. It seems reasonable to assume, as we have done, that the total monopole charge in a sufficiently large volume is zero. However, the production of monopoles and antimonopoles in the phase transition is not necessarily correlated. (They are even produced with spacelike separations of $\ell_M > t_C$.) As a consequence, the total magnetic charge in a given volume need not be zero.

Suppose we return to our lattice example. We collect lattice blocks which contain a large number of monopoles and antimonopoles. There will be random walks which pair most monopoles to antimonopoles provided the block size much exceeds $\langle r \rangle$. However, a slight excess of monopoles due to fluctuations will mean that some walks must be longer than the block size. Some will end in a nearby block where a fluctuation of opposite sign in the total monopole charge has ensured appropriate stopping places for the walks. If we collect together many such small blocks (whose size, as mentioned, must still much exceed $\langle r \rangle$), then this ensemble can still exhibit a total monopole charge due to a fluctuation. Walks will proceed out of it to nearby blocks of the same size whose fluctuations are of similar magnitude. This process can in principle continue as long as there is some dimensional scale on which fluctuations in the total monopole charge can exist. We end up with some strings which are arbitrarily long, or which at least have lengths up to this dimensional scale. If there is no scale to the charge fluctuations, some walks extend to infinity.

If - purely for illustration - we take a $\sqrt{N}$ law for the charge fluctuation in a given volume (where N is the number of monopoles in it), then we expect of order $\sqrt{N}$ strings to exit this volume, independent of its size and depending only on N . But this is clearly a disaster. The present volume of the Universe would have had so many monopoles and antimonopoles in it that $\sqrt{N} \gg 1/G\sigma$, destroying the conventional cosmology again. Even much smaller fluctuations would produce the same trouble.

Thus our apparently harmless second scenario, which appears to reconcile strings with conventional cosmology has, in fact, a worm in it. If the assumption of no fluctuations in the total monopole charge in large volumes is rejected, then we think that symmetry breaking with string production is excluded independent of any detailed scenario as to how the strings interact.

4. - CONCLUSION

Straightforward cosmological arguments allow us to rule out some symmetry breaking mechanisms for grand unified theories. These are the ones which lead to a domain structure of the vacuum.

Strings can also appear in some grand unified theories. However, such theories necessarily also involve monopoles. They prevent the presence of arbitrary long strings, provided the net monopole charge of large volumes is required to be zero. Doing this, the density perturbations caused by the strings do not persist long. They cannot be responsible for galaxy formation. If large scale fluctuations in the total monopole charge are allowed, then a large fraction of all strings could become arbitrarily long. Their density would be so great as to lead to a cosmology totally unlike the observed one. This would mean that theories with strings are excluded.

ACKNOWLEDGEMENTS

We thank D.B. Abraham, J.P. Derendinger, J. Ellis and C. Savoy for discussions.

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