DEEP INELASTIC SUM RULES AT FIXED HADRON ENERGY

C. Lopez

Universidad Autonoma de Madrid

and

F.J. Yndurain ^{*)}

CERN - Geneva

A B S T R A C T

Experimentally, it is much easier to measure structure functions at fixed hadron energy E_h , rather than at fixed Q^2 . We prove the sum rules

$$\begin{aligned} \int_0^1 dx F_3 |_{\text{fixed } E_h} &= 3 \left\{ 1 - (1+h_{NS}) \frac{\alpha_s(s)}{\pi} \right\}, \\ \int_0^1 dx F_2 |_{\text{fixed } E_h} &= \frac{3n_f}{16+3n_f} \left\{ 1 - \left(\frac{171+44n_f}{9(67-12n_f)} + h_s \right) \right. \\ &\quad \times \frac{\alpha_s(s)}{\pi} + K \left[\frac{\alpha_s(s)}{\pi} \right]^{0.75} \left. \right\}, \\ h_{NS} &= \frac{4}{3} \left(\frac{\pi^2}{6} - \frac{5}{8} \right), \\ h_s &= \frac{4}{16+3n_f} \left(2\pi^2 + \frac{\pi^2}{6} n_f - \frac{65}{12} - \frac{5}{72} n_f \right). \end{aligned}$$

Here $s = 2m_p E_h$ and $F_{2,3}$ are the standard structure functions for scattering of neutrinos on an isoscalar target. K is an unknown constant, and the corrections to the sum rules are $O(\alpha_s^2)$, $O(\alpha_s^{1.75})$, respectively.

^{*)} Permanent address : Departamento de Fisica Teorica, C-XI, Universidad Autonoma de Madrid, Canto Blanco, Madrid-34, Spain.

$$I_{NS}(s) = \int_0^1 dx \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn x^{-n} \left(\log \frac{xs}{\Lambda^2} \right)^{d_{NS}(n)} \{1 + C^{(1)}(n) \alpha_s(s)\} A_n,$$

and we have substituted $Q^2 \rightarrow s$ in the $C^{(1)}\alpha_s(Q^2)$ term as the error thereby committed is $O(1/\log^2 s/\Lambda^2)$. Expanding in powers of $1/\log^2/\Lambda^2$ and keeping only zero and first-order terms,

$$I_{NS}(s) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn \left(\log \frac{s}{\Lambda^2} \right)^{d_{NS}(n)} \times \int_0^1 dx x^{-n} \left\{ 1 + C^{(1)}(n) \alpha_s(s) + d_{NS}(n) \frac{\log x}{\log s/\Lambda^2} \right\} A_n.$$

Note that the expansion is absolutely convergent inside the dx integral ; no problem attaches to exchange of orders of integration or summation. The integral over dx is trivial, as is the dn for the $C^{(1)}\alpha_s$ term. We get

$$I_{NS}(s) = A_1 \{1 + C^{(1)}(1) \alpha_s(s)\} - \frac{1}{\log s/\Lambda^2} \cdot \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn \frac{d_{NS}(n) A_n}{(1-n)^2}. \quad (4)$$

We have used the fact that $d_{NS}^{(1)}$ vanishes due to its being the anomalous dimension of a conserved operator. A_1 and $A_1 C^{(1)}\alpha_s$ give the leading term ³⁾ and correction ⁴⁾ to the standard (Q^2 fixed) Gross-Llewellyn Smith sum rule. The last term in (4) is the correction due to the fact that we are integrating at fixed E_h rather than at fixed Q^2 . For a general moment this correction would not be calculable as it would involve $d_{NS}(n) dA_n/dn$; but in our case, because $d_{NS}^{(1)} = 0$, this unknown term drops. We can write, with d'_{NS} given in Eq. (2b),

$$d_{NS}(n) \underset{n \rightarrow 1}{=} (1-n) d'_{NS},$$

hence perform the integral in (4) :

$$I_{NS}(s) = A_1 \left\{ 1 + C^{(1)}(1) \alpha_s(s) - \frac{d'_{NS}}{\log s/\Lambda^2} \right\}.$$

This yields Eq. (1) after substitution of the known values of $C^{(1)}(1)$, d'_{NS} .

For the singlet, the path is similar. The complications due to the matrix character of the anomalous dimension $\tilde{D}(n)$, are solved by diagonalizing it. The analogue of the Gross-Llewellyn Smith sum rule is the relation, to leading ⁵⁾ and next-to-leading ²⁾ order,

$$\int_0^1 dx F_2(x, Q^2) = \frac{3n_f}{16+3n_f} \left\{ 1 - \frac{4}{9} \cdot \frac{171+44n_f}{67-12n_f} \cdot \frac{1}{\beta_0 \log Q^2/\Lambda^2} + K \left[\frac{\alpha_s(Q^2)}{\pi} \right]^{-d^{(2)}} \right\}. \quad (5)$$

The term in K appears because only one combination of the two that are involved in the singlet equations (viz., the sum of the gluon and quark singlet operators) is conserved.

When modifying (5) to an integral at fixed E_h we get corrections involving $\int dn d_+(n)/(2-n)^2$, where $d_+(n)$ is the eigenvalue of $\tilde{D}(n)$ that vanishes at $n = 2$. Therefore we write

$$d_+(n) \underset{n \rightarrow 2}{=} (2-n) d'_+,$$

and finally obtain Eq. (2).

An interesting remark is that, because $d_{NS}(n)$ only vanishes at $N = 1$, and $\det \tilde{D}(n)$ is zero only for $N = 2$, Eqs. (1), (2) are the only ones where one can replace integrals at fixed Q^2 by integrals at fixed s with corrections which are known to second order.

We conclude this note with a comparison of the sum rules with data ⁶⁾. Although Eqs. (1)-(3) agree qualitatively with experiment, the data are not yet good enough to provide stringent tests. However, an increase of a factor ~ 3 in the experimental accuracy (as we understand the CDHS collaboration is about to produce) should yield reasonably accurate quantitative results. This is particularly true for the non-singlet xF_3 . While qualitative agreement is good, the present experimental precision is not sufficient to say that the QCD corrections $O(\alpha_s)$ in Eqs. (1a) are quantitatively necessary.

In the Figure we show the experimental data. The necessity for the $K \alpha_s^{-d_-}$ term is clearly seen : when it is included we find excellent agreement with $K = 2.8 \pm 0.2$ letting Λ vary in the range 0.2 to 0.4 GeV. A non-trivial consistency check for this value of K will be possible when the μFe data

become available. The sum rule (3), which is independent of K , may also be tested, although the errors in the published CDHS data ⁶⁾ makes this test not very meaningful. Using experimental data we find the left-hand side of Eq. (3) of the order of -0.025 ; the experimental errors do not allow us to extract the $\log Q^2$ dependence. The theoretical prediction [the right-hand side of Eq. (3)] gives values varying between -0.10 and -0.07 , for $\Lambda \sim 0.2$ to 0.4 GeV. We conclude that the sum rule is qualitatively satisfied and, probably, quantitative tests may be possible when the new improved data become available.

ACKNOWLEDGEMENTS

We are grateful to H. Wahl for interesting discussions, which in particular triggered our interest in this problem.

REFERENCES

- 1) A. Gonzalez-Arroyo, C. Lopez and F.J. Yndurain - Nuclear Phys. B159 (1979) 512 ; B174 (1980) 474 ;
R.M. Barnett, D. Schlatter and L. Trentadue - Phys.Rev.Letters 46 (1981) 1659 and references therein.
- 2) C. Lopez and F.J. Yndurain - Nuclear Phys. B183 (1981) 157.
- 3) D. Gross and C.H. Llewellyn Smith - Nuclear Phys. B14 (1969) 337.
- 4) W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta - Phys.Rev. D18 (1978) 3998.
- 5) C.H. Llewellyn Smith - Physics Reports C3 (1972) 261 ;
D. Gross and F. Wilczek - Phys.Rev. D9 (1974) 980.
- 6) J.G. De Groot et al. - Z.Phys. C1 (1979) 143.

FIGURE CAPTION

Comparison of the sum rule for the singlet. Data from Ref. 6).
Broken line : without the $K \alpha_s^{-d}$ term. Continuous line : with
this term included.

FIG. 1