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INTERNAL ELECTRON-POSITRON PAIR PRODUCTION FROM ELECTRIC
MONOPOLE TRANSITIONS⁺

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Abstract:

Nuclear EO-transitions above the pair production threshold of $\omega=2m_e c^2 \sim 1\text{MeV}$ are considered. The ratio η of the electron-positron pair creation probability to the K-shell conversion probability is calculated. For ${}_{92}\text{U}$ and $\omega=1450\text{ keV}$ a typical value of $\eta=2 \cdot 10^{-3}$ is obtained. Good agreement with experimental data is found. Our numerical results for finite size nuclei are compared with analytical expressions utilizing relativistic electron wavefunctions for point nuclei. The energy distributions of emitted positrons are presented for various nuclear charge values and transition energies. The possibility of monoenergetic positron emission is discussed. Of particular interest is the relation of the theoretical results to the recently observed structure in the positron spectra, which have been measured in high energy collisions of very heavy ions.

1. Introduction

Nuclear EO-transitions are characterized by the absence of single photon emissions, because a photon must carry at least one unit of angular momentum. Therefore EO-transitions predominantly occur by bound state electron conversion or by internal electron-positron pair creation. The latter process requires nuclear transition energies ω larger than twice the electron rest mass m . The basic processes under investigation are depicted schematically in fig. 1. The nucleus which undergoes the EO-transition is labeled by its initial and final state angular momenta $J_i, J_f = J_i$ and eigen-energies $E_i, E_f = E_i - \omega$. Process a) describes the electron-positron pair creation. An electron of the negative energy continuum ($\epsilon = E < -m$) with Dirac quantum number κ is lifted to the positive energy continuum. The final state energy obviously amounts to $E' = \epsilon + \omega$ whereas the angular momentum quantum number remains unchanged. Since neither the initial electron state energy nor the final state energy is fixed, one expects an energy distribution for the emitted positron. Part b) indicates the conversion of a K-shell electron ($n=1, l=0, j=\frac{1}{2}, \kappa=-1$) with energy eigenvalue $E_{1s_{1/2}}$. Process c) symbolizes monoenergetic positron production. Here a negative energy electron is excited to a bound state, e.g. to the $1s_{1/2}$ -state. This represents a rather rare process since it requires a vacant bound state with a large overlap with the nuclear interior. In the conversion processes b) and c) bound states with definite energies are involved. Energy conservation then simply causes monoenergetic lepton emission for a fixed

nuclear transition energy ω .

The calculation of the nuclear EO-transition probability implies the precise knowledge of the initial and final state nuclear wavefunction, which clearly is model dependent. Therefore it seems to be more advantageous to perform model independent evaluations by considering the ratio of two different conversion probabilities. In this ratio the nuclear transition matrix element drops out, thereby avoiding difficulties with the specific nuclear structure. Our theoretical investigations were basically motivated by the recently observed peak structure ^{1,2)} in the energy distribution of emitted positrons which have been produced in collisions of very heavy ions like ^{92}U - ^{92}U , ^{92}U - ^{96}Cm and ^{92}U - ^{90}Th . These systems are overcritical and in principle may lead to spontaneous positron production. Two different explanations for this peculiar structure were proposed: a) It is assumed that a small fraction of the projectile and target nuclei come into nuclear contact. Both nuclei stick together for a certain time of about $T \sim 4 \cdot 10^{-20}$ s, which is long enough to allow for a large yield of spontaneous positron emission. The kinetic energy distribution of the created spontaneous positrons is localized around 300 - 400 keV. Its contribution must be coherently superimposed on a generally dominant dynamical portion of produced positrons which are formed by the time varying Coulomb field during the collision. The calculated total positron spectrum finally exhibits a similar structure as observed experimentally. For a detailed discussion of the underlying ideas and the results we refer to the work of Reinhardt et al.³⁾. b) Intrinsic nuclear excitations may take place either by Coulomb excitation or by transfer reactions. The nuclear transition can be accompanied by

internal electron-positron pair creation^{4,5,6)}. Contrary to EO-processes nuclear transitions of multipolarity E1 or E2 should also be observable in the emitted photon spectra provided that proper Doppler shift corrections are performed as in ref.⁷⁾.

In this paper we will prove the following assertion: If the observed structure in the positron distribution originates from EO-pair production one should definitely observe also a peak in the high energy part ($E_e > 1\text{MeV}$) of the δ -electron distribution originating from K-shell conversion. EO-conversion probabilities related to e^+ , e^- - pair creation and K-shell ionization have been calculated earlier by Thomas⁸⁾, Church and Weneser⁹⁻¹⁰⁾, Reiner¹¹⁾, Achleser and Berestezki¹²⁾ and by Lombard^{13,14)}. As approximation these authors mainly utilized electron wavefunctions that are solutions of the Dirac equation for point like nuclei. We shall compare our results with these calculations. The monoenergetic positron creation (process c) of fig. 1) has been first proposed by Sliv¹⁵⁾, see also ref.¹²⁾. The experimental detection is reported in refs.¹⁶⁻¹⁹⁾. Our paper is organized as follows: After the introduction we briefly describe the theoretical formalism, clearly emphasizing some convenient approximations. We shall show, that EO-pair conversion probabilities are simply determined by electron density distributions at the nuclear origin. Employing relativistic wavefunctions for point like nuclei, we derive an analytical expression for the differential conversion ratio with respect to the kinetic positron energy.

In section 4 we present the numerical results and draw our conclusions. Systematic investigations of the positron energy distribution are performed as function of both, the nuclear charge Z and transition energy

ω . The computed results are then compared with the available experimental data. We extended our calculations also to the region of superheavy nuclei with $Z > 107$. The EO-conversion probabilities per unit time increase by 4 orders of magnitude when the nuclear charge is increased from $Z=92$ to $Z=169$.

2. Theoretical description of EO-conversion

It is our intention in this section to derive a handy formula for the ratio η of the two conversion probabilities P_{e^+}, e^- for electron-positron pair creation and P_{e^-} for the ionization of bound state electrons

$$\eta = \frac{P_{e^+e^-}}{P_{e^-}} \quad (2.1)$$

The differential ratio with respect to the positron energy E obviously is

$$\frac{d\eta}{dE} = \frac{\frac{dP_{e^+e^-}}{dE}}{P_{e^-}} \quad (2.2)$$

and consequently ($\hbar = m = c = 1$)

$$\eta = \int_1^{\omega-1} \frac{d\eta}{dE} dE \quad (2.3)$$

The nuclear EO-transition is characterized by the transition energy ω , parity change $\pi_n = +1$ and angular momenta $J_i = J_f = J$ for the initial and final state. For the special case $J_i = J_f = 0$, one-photon transitions as well as EL- or ML conversion processes with multipolarities $L \geq 1$ are strictly forbidden. Therefore, we expect as dominant transition modes the e^+e^- -pair conversion and the bound state electron conversion.

It follows simply

$$P_{e^+e^-} = \int_1^{\omega-1} \frac{dP_{e^+e^-}}{dE} dE \quad (2.4)$$

Due to energy conservation the electron energy E' is given by

$$E' = \omega - E. \quad (2.5)$$

The variables belonging to the final state electron are always indicated by a prime. According to refs. 4,10,12,14 we can write

$$\frac{dP_{e'e}}{dE} = \frac{2\pi}{2j+1} \sum_{M_i, M_f=-j}^j \sum_{k, \mu} \sum_{k', \mu'} |U_{if}|^2, \quad (2.6)$$

$$P_e = \frac{2\pi}{2j+1} \sum_{M_i, M_f=-j}^j \sum_{k, \mu} \sum_{k', \mu'} |U_{if}|^2. \quad (2.7)$$

Here the index m runs over all occupied bound states with energy E_m , Dirac quantum number κ_m and z -projection of the total angular momentum μ_m . The energy of the corresponding continuum state electron then is

$$E' = \omega + E_m. \quad (2.8)$$

The second order transition matrix U_{if} in general reads ¹²⁾

$$U_{if} = -i \int_0^\infty d\tau \int_0^\infty d\tau' e^{i\omega(\vec{r}_n - \vec{r}_e)} \frac{e^{i\omega(\vec{r}_n - \vec{r}_e)}}{|\vec{r}_n - \vec{r}_e|} \quad (2.9)$$

where $j_n(\vec{r}_n)$ and $j_e(\vec{r}_e)$ are the transition currents of the nucleus and the electron, respectively. For EO-transitions ($L=0$) eq. (2.9) may be transformed into ¹²⁾

$$U_{if}^{L=0} = -i \int_0^\infty d\tau \int_0^\infty d\tau' e^{i\omega(\vec{r}_n - \vec{r}_e)} \left(\frac{1}{r_n} - \frac{1}{r_e} \right). \quad (2.10)$$

The angular integrations yield the Kronecker symbols $\delta_{M_i M_f} \delta_{\kappa \kappa'} \delta_{\mu \mu'}$. Furthermore the matrix elements do not depend on $\mu = \mu'$ and $M_i = M_f$. Therefore one obtains

$$\frac{dP_{e'e}}{dE} = 2\pi \sum_{\kappa} (2j+1) |U_{if}^{L=0}|^2 \quad (\kappa' = \kappa) \quad (2.11)$$

$$\{\mu' = \mu, M_i = M_f \text{ arbitrarily}\}$$

and

$$P_e = 2\pi \sum_m (2j_m+1) |U_{if}|^2. \quad (2.12)$$

L' denotes the sum over all occupied states (E_m, j_m) excluding the summation over magnetic substates, which already has been performed. Hence the summation extends only over all different radial wave-functions. We take over the nomenclature of ref. 20). Because of

$$\int d\Omega \chi_{\kappa}^{\mu'} \chi_{\kappa}^{\mu} = \delta_{\kappa \kappa'} \delta_{\mu \mu'} \quad (2.13)$$

where the χ_{κ}^{μ} are the usual Dirac spinors ²⁰⁾, the angular integration yields

$$\begin{aligned} \int d\Omega g_e(\vec{r}) &= \int d\Omega \{ g' g \chi_{\kappa}^{\mu'} \chi_{\kappa}^{\mu} + g' g \chi_{-\kappa}^{\mu'} \chi_{-\kappa}^{\mu} \} \\ &= (g' g + g' g) \delta_{\kappa \kappa'} \delta_{\mu \mu'}. \end{aligned} \quad (2.14)$$

The radial wavefunctions f, g belong to the negative energy $\epsilon = -E$ in the case of pair creation or to the bound state electron. f', g' are the radial wavefunctions for the final continuum state with positive energy E' . The radial wavefunctions are chosen to be real. The asymptotic behaviour ($r \rightarrow 0$) of the radial wavefunctions for a potential belonging to an extended nucleus is²⁰⁾

$$\begin{aligned} g &\sim r^{|k|} & f &\sim r^{|k|-1} & \text{for } k > 0 \\ & & & & (2.15) \end{aligned}$$

and

$$\begin{aligned} g &\sim r^{|k|-1} & f &\sim r^{|k|} & \text{for } k < 0. \\ & & & & (2.16) \end{aligned}$$

An extended nucleus precisely means a homogeneously charged sphere with radius R . The numerical values are computed by $R = 1.2 A^{1/3}$ fm. For the transition density therefore holds

$$ff' + gg' = C r^{2j-1} \left(1 + \sum_{k=1}^{\infty} C_k r^{2k} \right), \quad (2.17)$$

with the normalization factor

$$C = \lim_{r \rightarrow 0} \frac{ff' + gg'}{r^{2j-1}} = \begin{cases} \lim_{r \rightarrow 0} \frac{ff'}{r^{2j-1}} & \text{for } k > 0 \\ \lim_{r \rightarrow 0} \frac{gg'}{r^{2j-1}} & \text{for } k < 0 \end{cases} \quad (2.18)$$

There occur two radial integrals ($k=k', \mu=\mu'$)

$$\frac{d\rho_{\epsilon\epsilon'}}{dE} = \frac{\pi d^2}{9} |M|^2 \sum_{k=\pm 1} |\langle e^+ e^- \rangle|^2, \quad (2.24)$$

$$\begin{aligned} \int_0^{r_n} d\tau_e g_e(\tilde{r}_e) / r_n &= \frac{1}{r_n} \int_0^{r_n} (ff' + gg') r^2 d\tau \\ &= \frac{C}{r_n} \int_0^{r_n} d\tau r^{2j+1} \left(1 + \sum_{k=1}^{\infty} C_k r^{2k} \right) = \frac{C}{2j+2} r_n^{2j+1} + O(r_n^{2j+3}), \end{aligned} \quad (2.19)$$

$$\begin{aligned} \int_0^{r_n} d\tau_e g_e(\tilde{r}_e) / r_e &= \int_0^{r_n} (ff' + gg') r d\tau \\ &= C \int_0^{r_n} d\tau r^{2j} \left(1 + \sum_{k=1}^{\infty} C_k r^{2k} \right) = \frac{C}{2j+1} r_n^{2j+1} + O(r_n^{2j+3}). \end{aligned} \quad (2.20)$$

In summary it results

$$\int_0^{r_n} d\tau_e g_e(\tilde{r}_e) \left(\frac{1}{r_n} - \frac{1}{r_e} \right) = \frac{-C}{(2j+1)(2j+2)} r_n^{2j+1} + O(r_n^{2j+3}). \quad (2.21)$$

Now we employ an important approximation for the nuclear matrix elements, namely

$$M = \int_0^{\infty} d\tau_n g_n(\tilde{r}_n) r_n^2 \neq 0 \quad (2.22)$$

whereas all other nuclear integrals are assumed to vanish

$$M_1^{(k)} = \int_0^{\infty} d\tau_n g_n(\tilde{r}_n) r_n^k = 0 \quad (2.23)$$

with $k \geq 4$.

As a consequence only electrons states with $j = \frac{1}{2}$ ($k = \pm 1$) have to be considered in the matrix elements. In addition only the first term in the series expansion of $ff' + gg'$ contributes. This leads to

$$P_e = \frac{\pi d^4}{9} |M|^2 \sum_w |C_e|^2 \quad (2.25)$$

and for the ratio

$$\frac{d\eta}{dE} = \frac{\sum_{k \neq \pm 1} |C_{e',e}|^2}{\sum_w |C_e|^2} \quad (2.26)$$

Thus the ratio η of pair conversion to bound state electron conversion is completely determined by the density of the electron wavefunctions at the nuclear origin. Provided the wavefunctions for the potential of an extended nucleus are known the numerical values of C_{e^+} , C_{e^-} and C_e can be directly obtained from the definitions (2.18). We computed the wavefunctions for bound and continuum states as described extensively in ref. 21).

3. Analytical expressions for electric monopole conversion probabilities

In this section we shall derive analytical expressions for the differential pair creation probability $dp_{e^+e^-}/dE$, the K-shell conversion probability P_{e^-} as well as for the ratio $d\eta/dE$. Consequently, we assume that the only occupied state with $|K|=1$ is the $1s_{1/2}$ -state.

For this purpose we utilize electron wavefunctions that are solutions of the Dirac equation with a point-nucleus potential. In the case of extended nuclei we considered the following densities

$$C = \lim_{r \rightarrow 0} (f f' + g g') \quad (3.1)$$

$$= \begin{cases} \lim_{r \rightarrow 0} f f' & \text{for } \kappa > 0 \\ \lim_{r \rightarrow 0} g g' & \text{for } \kappa < 0 \end{cases}$$

for electron states with angular momenta $j = \frac{1}{2}$. Far in the nuclear interior ff' or gg' remain almost constant as function of r . In analogy we introduce for the wavefunctions according to a point nucleus potential

$$S(r) = \begin{cases} f(r) f'(r) & \text{for } \kappa > 0 \\ g(r) g'(r) & \text{for } \kappa < 0 \end{cases} \quad (3.2)$$

Due to the singular behaviour of f and g at the origin ($r=0$) we now take the numerical values of the radial wavefunctions at the nuclear surface ($r=R$). For $r \rightarrow 0$ the asymptotic behaviour of the point nucleus wavefunctions is determined by²⁰⁾

$$\lim_{r \rightarrow 0} f, g \sim r^{s-1} \quad (3.3)$$

$$\text{with } s = (\kappa^2 - (Z\alpha)^2)^{1/2}.$$

In consequence it follows for the density

$$S(R) = D R^{2s-2} \quad (3.4)$$

with the normalization factor

$$D = \lim_{r \rightarrow 0} S(r) / r^{2s-2}. \quad (3.5)$$

Hence the conversion probability in the point-nucleus approximation can be written as

$$\frac{dP_{e^+e^-}}{dE} = \frac{\pi \alpha^2}{9} |M|^2 R^{4s-4} \sum_{\kappa=2,1} |D_{e^+e^-}|^2, \quad (3.6)$$

$$P_{e^-} = \frac{\pi \alpha^2}{9} |M|^2 R^{4s-4} |D_{e^-}|^2 \quad (3.7)$$

and

$$\frac{d\eta}{dE} = \frac{\sum_{\kappa=2,1} |D_{e^+e^-}|^2}{|D_{e^-}|^2} \quad (3.8)$$

where D_{e^-} corresponds to the K-shell conversion.

As next step the quantities D_{e^+} , e^- and D_{e^-} must be evaluated. The radial continuum wavefunctions for a $1/r$ -Coulomb potential close to the nuclear origin ($r \rightarrow 0$) are given by²⁰⁾

$$\tilde{f} = \sqrt{E+1} N (1 - e^{-2i\phi}) r^{s-1}, \quad (3.9)$$

$$\tilde{g} = i\sqrt{E-1} N (1 + e^{-2i\phi}) r^{s-1} \quad (3.10)$$

for energies $\epsilon = -E < -1$ and by

$$\tilde{f}' = i\sqrt{E'-1} N' (1 - e^{-2i\phi'}) r^{s'-1}, \quad (3.11)$$

$$\tilde{g}' = \sqrt{E'+1} N' (1 + e^{-2i\phi'}) r^{s'-1} \quad (3.12)$$

for $\epsilon = E' > 1$ with the abbreviations

$$r = [\kappa^2 - (Z\alpha)^2]^{1/2}, \quad r' = [\kappa'^2 - (Z\alpha')^2]^{1/2}, \quad (3.13)$$

$$e^{-2i\phi} = \frac{s + iB}{-\kappa + iB/E}, \quad e^{-2i\phi'} = \frac{s' - iB'}{-\kappa' + iB'/E'}, \quad (3.14)$$

$$B = \frac{Z\alpha E}{P}, \quad B' = \frac{Z\alpha' E'}{P'}, \quad (3.15)$$

$$P = [E^2 - 1]^{1/2}, \quad P' = [E'^2 - 1]^{1/2}, \quad (3.16)$$

$$N = \frac{(2P)^s e^{-\pi B/2} |\Gamma(\alpha+1)|}{2\sqrt{\pi P} \Gamma(b)}, \quad (3.17)$$

$$N' = \frac{(2P')^{s'} e^{+\pi B'/2} |\Gamma(\alpha'+1)|}{2\sqrt{\pi P'} \Gamma(b')}, \quad (3.18)$$

$$a = \gamma + i\beta, \quad a' = \gamma' - i\beta', \quad (3.19)$$

$$b = 2\gamma + 1, \quad b' = 2\gamma' + 1. \quad (3.20)$$

The functions $\tilde{f}$, $\tilde{g}$, $\tilde{f}'$, and $\tilde{g}'$ are complex. In order to obtain real functions we multiply with $(-i e^{i\phi})$ and $(-e^{-i\phi'})$, respectively.

$$f = 2\sqrt{E+1} N \sin \phi e^{-\gamma-1}, \quad (3.21)$$

$$g = 2\sqrt{E-1} N \cos \phi e^{-\gamma-1}, \quad (3.22)$$

$$f' = 2\sqrt{E'-1} N' \sin \phi' e^{-\gamma'-1}, \quad (3.23)$$

$$g' = -2\sqrt{E'+1} N' \cos \phi' e^{-\gamma'-1}. \quad (3.24)$$

After some lengthy but elementary transformations this yields for $k=k'$ and $\gamma=\gamma'$

$$\begin{aligned} |D e^{\epsilon} e^{\epsilon} (k=+1)|^2 &= \left| \lim_{\epsilon \rightarrow 0} \frac{g g'}{\epsilon^2 \gamma^2} \right|^2 \\ &= (1+\gamma)^2 \frac{(2NN'PP')^2}{(E-\gamma^2)(E'-\gamma'^2)} \left\{ (E+1)(E-1) + (1-\gamma)(2+E-E') - (1-\gamma)^2 \right\}, \end{aligned} \quad (3.25)$$

$$\begin{aligned} |D e^{\epsilon} e^{\epsilon} (k=-1)|^2 &= \left| \lim_{\epsilon \rightarrow 0} \frac{g g'}{\epsilon^2 \gamma^2} \right|^2 \\ &= (1+\gamma)^2 \frac{(2NN'PP')^2}{(E^2-\gamma^2)(E'^2-\gamma'^2)} \left\{ (E-1)(E'+1) + (1-\gamma)(2-E+E') - (1-\gamma)^2 \right\} \end{aligned} \quad (3.26)$$

and consequently

$$\sum_{k=\pm 1} |D e^{\epsilon} e^{\epsilon}|^2 = 8(1+\gamma)^2 (NN'PP')^2 \frac{EE'-\gamma^2}{(E^2-\gamma^2)(E'^2-\gamma'^2)}. \quad (3.27)$$

For the conversion of bound state electrons we restricted our investigations to the $1s_{1/2}$ -state only. The K-shell wavefunction for $r \rightarrow 0$ can be approximated by²⁰⁾

$$f = -\sqrt{1-\gamma} K e^{-\gamma-1}, \quad (3.28)$$

$$g = \sqrt{1+\gamma} K e^{-\gamma-1} \quad (3.29)$$

with the normalization factor

$$K = \frac{(2Za)^{\gamma+\frac{1}{2}}}{\sqrt{2\Gamma(2\gamma+1)}}. \quad (3.30)$$

The final state again is described by the electron continuum wavefunction, where the energy is substituted by $E' = \bar{E} = \gamma + \omega$. The same manipulations as for the pair creation lead to

$$\begin{aligned} |D e^{\epsilon}|^2 &= \left| \lim_{\epsilon \rightarrow 0} \frac{g g'}{\epsilon^2 \gamma^2} \right|^2 \\ &= (2\bar{N}K)^2 (1+\gamma)(\bar{E}+1) \cos^2 \bar{\phi} = (1+\gamma)^2 \frac{2(\bar{N}K\bar{P})^2}{\omega}. \end{aligned} \quad (3.31)$$

In summary we explicitly obtain

$$\frac{dP e^{\epsilon} e^{\epsilon}}{dE} = \frac{\omega^2}{3} |M|^2 R^{\gamma+\gamma'} \left(\frac{\gamma+1}{2}\right)^2 \frac{(4Za\bar{P})^{2\gamma} e^{-2\bar{\gamma}} |\Gamma(\gamma+i\bar{\gamma})|^2 2Za(\omega+2\gamma)}{\pi P P' (\Gamma(2\gamma+1))^4}, \quad (3.32)$$

$$P e^{\epsilon} = \frac{\omega^2}{3} |M|^2 R^{\gamma+\gamma'} \left(\frac{\gamma+1}{2}\right)^2 \frac{(4Za\bar{P})^{2\gamma} e^{-2\bar{\gamma}} |\Gamma(\gamma+i\bar{\gamma})|^2 2Za(\omega+2\gamma)}{\bar{P} (\Gamma(\gamma+\frac{1}{2}))^3} \quad (3.33)$$

and for the ratio $\frac{dn}{dE}$

$$\frac{dn}{dE} = \frac{(E'E') e^{\pi(B'-B-E)}}{\pi (2\lambda)^2 (\omega + 2\lambda) \Gamma(2\lambda+1)} \left(\frac{PP'}{2\lambda P} \right)^{2\lambda-1} \left| \frac{\Gamma(\lambda+iB) \Gamma(\lambda+iB')}{\Gamma(\lambda+i\bar{B})} \right|^2 \quad (3.34)$$

In the limiting case $E=1$ we find immediately

$$\frac{dn}{dE} (E=1) = 0 \quad (3.35)$$

whereas the opposite limit $E = \omega-1$ yields a finite value.

Our analytical results are now compared with the differential pair creation probability $\frac{dw}{dE}$ and the K-shell conversion probability w_K of R. Thomas⁸⁾.

With

$$G = \left(\frac{\lambda+1}{2} \right)^2 \quad (3.36)$$

and

$$H = \frac{12}{[1-\lambda + \sqrt{1+\lambda}]^2} \quad (3.37)$$

it follows

$$\frac{dP_{e^+e^-}}{dE} = G \frac{dW_P}{dE}, \quad (3.38)$$

$$P_e = G H W_K, \quad (3.39)$$

$$\frac{dn}{dE} = H^{-1} \frac{dW_P}{dE} \cdot \frac{1}{W_K}. \quad (3.40)$$

Therefore the difference depends only on $Z\alpha$ in a weak manner.

Complete agreement is achieved in the limit $Z \rightarrow 0$. Our result concerning the K-shell conversion probability P_e^- for nuclear EC-transitions may also be compared with the equivalent quantity $\Omega_K \rho^2$ with $\rho^2 = M^2/R^4$ derived by Church and Weneser⁹⁾. We find total agreement

$$P_e^- = \Omega_K \rho^2. \quad (3.41)$$

Finally we mention that eqs. (3.32) and (3.33) have to be multiplied by G^{-1} (eq. (3.36)) if one takes $S(r) = f(r) f'(r) + g(r) g'(r)$ instead of eq. (3.2). However it is important to note that dn/dE remains unchanged.

4. Results and conclusions

According to eqs. (2.26) and (2.18) we computed the differential conversion ratio dn/dE with respect to the positron energy E for EO-transitions. As bound state only the atomic K-shell has been taken into account. The conversion probability of higher bound states is at least an order of magnitude smaller. For the nucleus ^{92}U the energy distribution of emitted positrons is depicted in fig. 2. Nuclear transition energies of 1323 keV, 1423 keV, 1523 keV and 1623 keV are considered, which correspond to maximum positron energies of $E_{\text{kin}}^{\text{max}} = 300$ keV, 400 keV, 500 keV and 600 keV. The spectra are centered around $E_{\text{kin}}^{\text{max}}$, reflecting the strong Coulomb repulsion of low energetic positrons. For comparison of the spectral shape we show in fig. 3 the equivalent differential conversion coefficient dB/dE for nuclear E1 and E2 transitions, which are defined as

$$\frac{dB}{dE} = \frac{dP_{e^+e^-}}{dE} \quad (4.1)$$

and

$$\beta = \int_1^{\omega-1} \frac{dB}{dE} dE = \frac{P_{e^+e^-}}{P_\gamma} \quad (4.2)$$

Obviously the ratio β determines the number of created electron-positron pairs divided by the number of created photons. For details concerning the theoretical formalism and for the numerical numbers we refer to refs 4-6). Similar positron distributions as in fig. 2 are found. Fig. 4 displays the change of dn/dE with the nuclear charge Z for the transition energies $\omega=1423$ keV and

$\omega=2023$ keV, respectively. With decreasing Z and increasing ω the energy distribution of positrons becomes more and more symmetric and therefore equal to the electron spectra. On the other hand, the Coulomb repulsion of positrons is best visible in high Z elements and transition energies close to the pair production threshold. In fig. 5 we compare our numerical results for dn/dE (solid lines) using Dirac wavefunction for finite size nuclei with the analytical formula (3.34) (dashed lines). Good agreement is achieved for low and medium Z nuclei. Even for $Z=92$ the agreement is still remarkable. In conclusion we can state that nuclear EO-transitions with $\omega > 2m^{-1}$ MeV may be identified by measuring the positron to K-shell vacancy production rate. The differential ratio $\frac{dn}{dE}$ as well as η are typical quantities for EO-transitions. For the purpose of nuclear spectroscopy formula (3.34) can be applied as a good approximation.

The numerical integration of eq. (2.26) yields the dimensionless ratio η . For $Z=92$ η is presented in fig. 6 as function of the nuclear transition energy.

For $\omega=1423$ keV we find that the probability for K-shell conversion is 556 times larger than the e^+e^- pair creation probability. In the considered range of transition energies η varies from 10^{-4} to $2 \cdot 10^{-2}$. The dashed line represents results obtained by numerical integration of our analytical expressions for the differential positron production probability (eq.(3.34)). Both calculations are in fair agreement. Hence there is practically no need for sophisticated numerical computations. In fig. 7 η is presented versus the nuclear charge Z for the transition energies $\omega=1423$ keV,

2000 and 3000 keV. These calculations are also extended to the region of superheavy nuclei ($Z > 100$). η decreases monotonically until one reaches almost critical systems ($Z \sim 170$), where the binding energy of the $1s_{1/2}$ -state exceeds twice the electron rest mass and the bound state gets imbedded as a resonance into the negative energy continuum. The onset of this behaviour is reflected in a slight increase of η around $Z \sim 160$. However, the numerator of eq. (2.26) and therefore the pair production probability increases by almost 4 orders of magnitude if the nuclear charge is extended from $Z=92$ to $Z=169$. In consequence the nuclear transition time due to internal pair creation decreases also by 4 orders of magnitude in the considered Z range. Our calculated values for the inverse ratio $1/\eta$ are compared in table 1 with some available experimental data (see ref. 13, 22, 23) and with the calculations of Lombard¹³. η decreases by 7 orders of magnitude from oxygen ($Z=8$) to Polonium ($Z=84$) mainly reflecting the change in the K-shell density distribution inside the nucleus. Fair agreement with the experimental data is achieved. As a further possibility we discuss now the creation of monoenergetic positrons in nuclear EO-transitions (process c of fig.1). Especially we calculate the ratio

$$\{ (nlj, n'l'j') \} = \frac{P_{e^+ - nlj}}{P_{e^- - n'l'j'}} \quad (4.3)$$

Here $P_{e^+ - nlj}$ denotes the probability of monoenergetic positron production, where an electron of the negative energy continuum is excited to a vacant bound state described by the quantum numbers n, l and j .

$P_{e^- - n'l'j'}$ signifies the probability for the formation of a free electron due to excitation of a bound state electron with quantum numbers n', l' and j' . The ratio $\{$ can be calculated exactly like eq. (2.26) if one inserts the corresponding wavefunctions in eq. (2.18). Numerical values for $\{$ are presented in table 2 for various nuclear transition energies in ^{92}U and several bound states. The quantities in columns 3 and 4 are physically not well defined provided that the spin substates are not treated explicitly. Due to the Pauli principle the monoenergetic positron emission may only occur for a vacant final bound state. Hence it is meaningless to study at the same moment the excitation of this bound state electron. Nevertheless we can definitely conclude that the EO-conversion of a K-shell electron has a probability larger by about 3 orders of magnitude than the monoenergetic e^+ -production, where the electron occupies the vacant K-shell. This ratio even increases by one order of magnitude if one considers higher bound states in relation to the positron production process. Now we investigate a realistic heavy ion collision of ^{92}U on ^{92}U with a bombarding energy close to the Coulomb barrier. The number of created K-vacancies per collision with an impact parameter $b \lesssim 20$ fm is typically of the order of $10\%^{21, 24, 25}$. The transition time (γ -decay) of an electron from a higher shell to the K-shell is about 10^{-17} s²⁶. This has to be compared with nuclear transition times of about 10^{-13} s, after which conversion takes place. Hence the K-vacancy will be decayed before it could eventually be filled again due to pair conversion of a nuclear state. Therefore the monoenergetic positron formation is suppressed in contrast to ordinary K-shell conversion by at least 7 orders of

magnitude, and thus can be neglected completely.

First measurements concerning the energy distribution of positrons emitted in collisions of very heavy ions were reported by Backe et al^{27,28)}. In this experiment no specific structure has been found.

We finally discuss the possibility that the recently observed structure^{1,2)} in positron spectra measured in collisions of ${}^{92}\text{U}$ - ${}^{92}\text{U}$, ${}^{92}\text{U}$ - ${}^{90}\text{Th}$ and ${}^{92}\text{U}$ - ${}^{96}\text{Cm}$, originates from nuclear EO-transitions.

One convincing argument against this interpretation is related to the shape of the e^+ -energy distribution. According to fig. 2 the halfwidth of the spectra should be at least $\Delta E \geq 150$ keV. However, the observed structure is narrower by more than a factor 2. Our second argument is connected with the energy distribution of the emitted δ -electrons. The measured differential positron production probability $\frac{dP_e^+}{dE}$ with respect to the kinetic positron energy at $E \sim 400$ keV for ${}^{92}\text{U}$ - ${}^{92}\text{U}$ collisions with a bombarding energy of $E_{\text{lab}} = 5.9$ MeV/u and ion scattering angles of $\theta_{\text{lab}} = 45^\circ \pm 10^\circ$ amounts to $\frac{dP_e^+}{dE} \approx 10^{-7} \frac{1}{\text{keV}}$.

Assuming that these positrons are of pure nuclear origin one can easily deduce the probability P_N for nuclear excitation

$$P_N \approx \frac{dP_e^+}{dE} \bigg/ \frac{dn}{dE} \approx 10^{-2}.$$

In principle the competing K-shell conversion should lead to a sharp line in the electron spectra at $E_e \approx 1300$ keV.

However the experimental energy resolution for these high energy electrons is presently not better than $\Delta E_e = 25$ keV, which therefore yields a differential e^- -production probability of

$$\frac{dP_e^-}{dE_e} \approx \frac{P_N}{\Delta E_e} \approx 4 \cdot 10^{-4} \frac{1}{\text{keV}}.$$

This is still by a factor $F=40$ larger than the expected δ -electron production rate^{24,25)} of $\frac{dP_e^-}{dE_e} \approx 10^{-5} \frac{1}{\text{keV}}$ at a kinetic electron energy of $E_e \approx 1300$ keV, which results from dynamical electron excitations during the collision. The factor F can be increased by an order of magnitude if one measures the emitted electrons in coincidence with the decaying K-vacancy (e.g. the K_α -radiation). Hence we can conclude: If the observed structure in the positron spectra is caused by nuclear EO-transitions one should also observe a dominant peak in the δ -electron distribution at $E_e \approx 1300$ keV.

Furthermore we emphasize that sharp peaks in the energy distribution of emitted positrons with a halfwidth of less than 100 keV may not originate from nuclear EO-conversion processes.

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Figure caption

Fig. 1: Schematic representation of electron conversion processes accompanying nuclear E0-transitions. a) Electron-positron pair production leading to a continuous energy distribution of positrons. b) Conversions of K-shell electrons. c) Monoenergetic positron production.

Fig. 2: Differential conversion probability ratio $d\eta/dE$ with respect to the kinetic positron energy E for nuclear E0-transitions in ^{92}U . Nuclear transition energies $\omega=1623$ keV, 1523 keV, 1423 keV and 1323 keV are considered.

Fig. 3: Differential conversion coefficient dg/dE with respect to the kinetic positron energy E for nuclear E1- and E2-transitions in ^{92}U . The same transition energies as in fig. 2.

Fig. 4: Z-dependence of the differential conversion probability ratio $d\eta/dE$ versus the kinetic positron energy E for electric monopole transitions with $\omega=1423$ keV and $\omega=2023$ keV.

Fig. 5: Differential conversion probability ratio $d\eta/dE$ as function of the kinetic positron energy E . The nuclei $Z=50$ and $Z=92$ are considered. The nuclear transition energy was chosen to be $\omega=1623$ keV. The solid line represents a numerical calculation utilizing Dirac wavefunction for finite size nuclei. The dashed line follows from the analytical expression (3.34).

Fig. 6: The ratio η versus the nuclear transition energy ω for $Z=92$. The dashed line is obtained by numerical integration of the analytical expression of $d\eta/dE$ (see eq. (3.34)).

Fig. 7: Z-dependence of η for nuclear transition energies $\omega=1423$, 2000 and 3000 keV.

Table 1

Nucleus	Transition energy ω [MeV]	$\frac{1}{\eta}$ (exp)	$\frac{1}{\eta}$ (Lombard)	$\frac{1}{\eta}$ (this work)
$^{16}_8\text{O}$	6.06	$(4.0 \pm 0.46) \cdot 10^{-5}$	$3.92 \cdot 10^{-5}$	$3.8 \cdot 10^{-5}$
$^{40}_{20}\text{Ca}$	3.353	$6.9 \cdot 10^{-3}$	$6.0 \cdot 10^{-3}$	$7.16 \cdot 10^{-3}$
$^{42}_{20}\text{Ca}$	1.836	0.111 ± 0.02	0.072	0.139
$^{90}_{40}\text{Zr}$	1.752	2.08 ± 0.08	1.42	2.52
$^{140}_{58}\text{Ce}$	1.903	6.3	5.22	6.9
$^{214}_{84}\text{Po}$	1.416	440-625	168	388.6

Table caption: Experimental data and theoretical values of K-shell electron to internal pair conversion probability ratios for EO-transitions.

For the refs. concerning the experimental data (see ref. ¹³).

Table 2

ω [keV]	E [keV]	$\xi(1s_{1/2}, 1s_{1/2})$	$\xi(2s_{1/2}, 2s_{1/2})$	$\xi(2s_{1/2}, 1s_{1/2})$	$\xi(3s_{1/2}, 1s_{1/2})$	$\xi(2p_{1/2}, 1s_{1/2})$
1323	433	$1.57 \cdot 10^{-3}$	$8.72 \cdot 10^{-4}$	$1.77 \cdot 10^{-4}$	$4.68 \cdot 10^{-5}$	$5.60 \cdot 10^{-5}$
1423	533	$2.17 \cdot 10^{-3}$	$1.39 \cdot 10^{-3}$	$2.80 \cdot 10^{-4}$	$7.70 \cdot 10^{-5}$	$7.90 \cdot 10^{-5}$
1523	633	$2.77 \cdot 10^{-3}$	$1.92 \cdot 10^{-3}$	$3.88 \cdot 10^{-4}$	$1.09 \cdot 10^{-4}$	$1.00 \cdot 10^{-4}$
1623	733	$3.35 \cdot 10^{-3}$	$2.47 \cdot 10^{-3}$	$4.96 \cdot 10^{-4}$	$1.41 \cdot 10^{-4}$	$1.19 \cdot 10^{-4}$

Table caption: Ratio $\xi(nlj, n'l'j') = P_{e^+nlj}/P_{e^-n'l'j'}$ of probabilities for monoenergetic positron production in EO-transition for $Z=92$. P_{e^+nlj} denotes the probability for exciting a negativ energy electron to a vacant bound state with quantum numbers (nlj) . $P_{e^-n'l'j'}$ is the probability for conversion of a bound state electron with quantum numbers $(n'l'j')$. The first column indicates the considered nuclear transition energy. The second column specifies the positron energy for transitions to the vacant K-shell.

Fig.1

Fig.2

Fig. 4

Fig. 3

Fig. 7