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ENERGY LOSS MECHANISMS AND THE ANNIHILATION OF
CONFINED MONOPOLES

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A B S T R A C T

The annihilation process for a superheavy monopole-anti-monopole pair connected by a relatively light magnetic flux tube (which is possibly relevant to the cosmological monopole problem) is studied. Three energy loss mechanisms are considered in some detail. The first two are the radiation of photons and weak bosons due to the linear and Coulomb parts of the potential, respectively. Then the energy loss due to the frictional drag of monopoles moving through a relativistic plasma is analyzed. We argue that for the parameters relevant to standard theories this last mechanism dominates by many orders of magnitude and leads to an almost instantaneous annihilation at the time the confinement takes place. The implications of this work for the dilution of the baryon to entropy ratio are described.

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1. - INTRODUCTION

In recent years the importance of the interplay between grand unification and cosmology has become apparent ¹⁾. The possibility of explaining the baryon asymmetry of the present Universe is certainly one of the most interesting results. A more troublesome aspect, however, seems to be a dramatic abundance of superheavy monopoles (typically $m \approx 10^{16}$ GeV), which is expected to arise in virtually any grand unified scheme ²⁾.

For example, if the Universe underwent a second order transformation from a symmetric phase to a phase in which the grand unified symmetry is spontaneously broken at a temperature $T \approx M_X$ (the unification mass), then simple causality arguments ²⁾ have been given which suggest that monopoles were produced with an initial density

$$r \equiv \frac{n(T)}{T^3} \sim 10^{-6} - 10^{-10} \quad (1)$$

(the validity of these arguments has been questioned, however ³⁾). Because of the rapid expansion of the Universe, subsequent conventional annihilations could reduce a large initial r only to a value

$$r_i \leq 10^{-10} \quad (2)$$

This value of r would remain constant for an adiabatically expanding Universe (except for small changes due to the change in the number of effectively massless helicity states - these effects are unimportant for our purposes). However, the present observational limit on r from the limits on the deceleration of the Universe is ²⁾ $r_{\text{obs}} < 10^{-24}$ for a 10^{16} GeV monopole. To avoid any reference to the extremely late Universe it suffices to quote the limit $r_{\text{obs}} (T=1 \text{ MeV}) < 10^{-19}$ needed to avoid upsetting the usual successful calculation of the helium abundance. In either case, $r_{\text{obs}} \ll r_i$. This is the cosmological monopole problem.

Various conceivable cures to this problem have been suggested. Guth and others ⁴⁾ have studied the possibility of a strongly first order grand unified phase transition with a large supercooling as a solution to the monopole and horizon problems. The results of this work indicate ⁴⁾, however, that if the supercooling is large enough to solve the monopole problem, then the phase transition would probably never be completed.

Within the framework of a weakly first or second-order phase transition, Bais and Rudaz ³⁾ have pointed out that thermal fluctuations may play an important rôle in the reduction of the initial monopole density. However, it is not clear how this mechanism can reduce the density of monopoles below one per causal volume. Langacker and Pi ⁵⁾ have demonstrated that it is possible to construct acceptable models in which an unbroken $U(1)$ subgroup only appears at a much lower temperature ($T \approx 1$ TeV), thereby avoiding the formation of monopoles (which would still be superheavy). Finally, we mention the mechanism of monopole confinement ⁶⁾, in which it is envisaged that the superheavy monopoles whose topological charge is associated with the $U(1)_Y$ of weak hypercharge rather than with the $U(1)_Y$ of electromagnetism, will be confined by a Z_0 flux tube after the Weinberg-Salam phase transition at $T \approx 100$ GeV.

Before starting a more detailed discussion of this confinement mechanism we would like to describe our main motivation for studying the dynamics of this process. Obviously the annihilation of the large number of heavy monopoles will lead to a rather substantial entropy generation. Consequently the baryon to entropy ratio which was generated just after the grand unified phase transition will be reduced accordingly. Within the context of the presently known scenario it is only admissible to dilute the baryon to entropy ratio by (at most) a few orders of magnitude. To get an idea, let us calculate the reheating due to the instantaneous annihilation of monopole-antimonopole pairs with a relative density $r = n/T^3 = 10^{-10}$ at $T = M_W$. The monopoles dominate the initial energy density, so the annihilation reheats the plasma to a final temperature

$$T' \simeq \left(\frac{n m}{30} \right)^{1/4} \quad (3)$$

The ratio of entropy before and after annihilation becomes

$$\frac{S'}{S} = \left(\frac{T'}{T} \right)^3 = \left(3 \times 10^{-12} \frac{m}{T} \right)^{3/4} \underset{T \sim M_W}{\simeq} 10^2, \quad (4)$$

where we have assumed that the number of helicity states of interacting relativistic particles has not changed in the phase transition. We note that the ratio as calculated in (4) is still acceptable. Clearly if the annihilation did not take place instantaneously then the annihilation temperature T would be lower, thereby increasing the ratio T'/T . It is therefore interesting to investigate the annihilation process in some detail to determine whether or not it is effectively instantaneous.

Let us now return to the question of under what conditions monopole confinement is expected to take place ⁷⁾. The simplest example of a situation in which it would occur is the simple $O(3)$ Georgi-Glashow model with the original 't Hooft-Polyakov monopole. One could imagine breaking the residual $U(1)$ symmetry completely at a lower scale. The situation is then completely analogous to the Abelian Higgs model where all the flux emanating from the monopole would be squeezed into a topologically stable magnetic flux tube of the Nielssen-Olesen type. The monopoles would indeed be confined. The standard $SU(5)$ model is less attractive in this respect. The $SU(3)_C \times SU(2)_L \times U(1)_Y$ monopoles of mass M_X/α_{GUT} are not expected to be confined at $T < M_W$. Rather, they will be converted into pure electromagnetic monopoles. This is so in spite of the fact that the Z_O component of the weak hypercharge acquires a mass because there is no topologically stable flux tube possible. On the other hand it is topologically allowed for the $U(1)_Y$ monopole to decay into a $U(1)_Y$ monopole. In fact, this problem seems to be generic to most unified models because of the way the electromagnetic gauge group is usually embedded.

In spite of the fact that confinement is not likely to occur in currently popular models we nevertheless consider it worthwhile to study the process by which confined monopoles will annihilate or form multiply charged magnetic hadrons.

The basic problem is that subsequent to the confining phase transition the monopole-antimonopole pair will typically be found in highly excited vibrational states of the linear potential associated with the magnetic flux tube. The size of each monopole ($\approx 1/M_X$) is very small compared to both the transverse size of the flux tube ($\approx 1/M_W$) and the initial monopole separation ($\approx n^{-1/3}$). Hence annihilation can only occur after the system has lost most of its initial energy. We have identified what we think are the three dominant energy loss mechanisms, and will discuss them subsequently in this paper. The first is the radiation of photons and weak bosons due to the oscillations in the linear potential [photons could be radiated if the monopole charge had an unconfined $U(1)_Y$ component]. Then we consider the additional energy loss that occurs because the oscillating pairs occasionally come close enough together to feel the relatively strong Coulomb part of the potential. Finally, assuming that the monopoles are immersed in a thermal plasma of charged particles, there is the possibility of energy dissipation due to friction. It is our conclusion that with the standard values for the input parameters, i.e., $r \approx 10^{-10}$, $M_W/M_X \approx 10^{-14}$ and $T_C \approx M_W$, this drag effect is the dominant mechanism and that it will lead to a practically instantaneous annihilation.

Throughout, we ignore the oscillations or other motions of the flux tube itself. This is because the massive monopoles move too slowly to significantly excite the string modes (i.e., an adiabatic approximation is justified), and because thermal excitations are unimportant.

2. - FORMULATION OF THE PROBLEM

The system we consider consists of a monopole-antimonopole pair each of mass m ($m \approx 10^{16}$ GeV), connected by a flux tube of length R with energy per unit length $\alpha_g M_W^2$, where $\alpha_g = g^2/4\pi = 1/\alpha \approx 10^2$. Close to the monopole (at distances smaller than $1/M_W$) the magnetic field is radial. It is convenient to introduce the following dimensionless variables: time $\tau = M_W t$, separation $x = M_W R$, energy $\epsilon = E/M_W$, and the inverse monopole mass

$$K = \frac{M_W}{m} \approx 10^{-14}.$$

We work in units for which $\hbar = c = k = \pi = 10 = 1$. The potential energy in units of M_W is thus a sum of a linear and a Coulomb part

$$V(x) = \alpha_g x - \frac{\alpha'_g}{x}. \quad (5)$$

The difference between α_g and α'_g should account for the fact that not all the flux from the monopole is assumed to be squeezed in the tube (i.e., the magnetic charge may have an unconfined component). In all calculations we will set $\alpha_g \approx \alpha'_g \approx 10^2$, however. Let us assume that the formation of the flux tubes is completed at $T = T_c \approx M_W$, and furthermore that the relative density at T_c is $r \approx 10^{-10}$, as would occur ²⁾ if a large initial density were reduced to this value by conventional annihilation processes at $T \lesssim M_X$. This allows us to estimate the average initial separation as

$$x_0 \approx n^{-1/3} M_W \approx 10^3 \quad (6)$$

and the average initial energy as

$$\epsilon_0 = V(x_0) \approx \alpha_g x_0 \approx 10^5. \quad (7)$$

ϵ_0 , which is obviously dominated by the linear part of the potential, is large compared to the thermal energy $\epsilon_{Th} \approx T/M_W \approx 1$ and therefore to the rotational energy $\epsilon_{Rot} \approx \epsilon_{Th}$. The motion of the monopoles is non-relativistic because $\epsilon_0 K \approx 10^{-9}$. Let us also remark that for large quantum numbers n the levels of the linear potential behave ⁸⁾ as

$$\mathcal{E}(n) \simeq n^{2/3} K^{1/3}, \quad (8)$$

which means that the system starts out in the highly excited vibrational state $n_0 \simeq 10^{15}$.

3. - RADIATION DUE TO THE LINEAR PART OF THE POTENTIAL

The classical formula for the power radiated in photons from an accelerated charged particle is

$$P = - \frac{dE}{dt} = \frac{2}{3} \alpha a^2, \quad (9)$$

where a is the acceleration. The same formula applies to a monopole with a $U(1)_Y$ magnetic charge provided we replace α by α_g . To make a comparison with the quantum mechanical formula it is instructive to write x in terms of its Fourier components x_j :

$$x = \text{Re} \sum_j x_j e^{ij\omega_0 t}. \quad (10)$$

Equation (9) then takes the form of a sum over modes

$$\left\langle \frac{dE}{dt} \right\rangle = - \frac{1}{3} \alpha \omega_0^4 \sum_j j^4 |x_j|^2. \quad (11)$$

This expression strongly resembles the quantum mechanical formula

$$\frac{dE}{dt} = - \frac{4}{3} \alpha \omega_{fi}^4 |Z_{fi}|^2. \quad (12)$$

In fact it can be shown⁹⁾ that for large values of the quantum number n , $Z_{n-j,n}$ becomes proportional to x_j if $E_n - E_{n-j} \equiv \omega_{n-j,n} \simeq j\omega_0$, where the proportionality constant is such that the classical result is obtained. Since in our case $n \simeq 10^{15}$, the use of classical formulae is justified. In particular, Eq. (11) allows us to say something about the energy loss per mode, which will be useful later when we estimate the radiation into heavy vector bosons.

Let us now compute the energy loss due to the linear part of the potential using Eq. (9). We are dealing with a constant acceleration

$$|a| = \alpha_g K M_W \simeq 10^{-12} M_W, \quad (13)$$

which is very small because of the enormous mass of the monopole. Therefore

$$\frac{d\varepsilon}{d\tau} = -\frac{2}{3} \alpha_g^3 K^2 \simeq 10^{-22} \quad (14)$$

yielding the linear time dependence

$$\varepsilon(\tau) = \varepsilon_0 \left[1 - \frac{2}{3} \alpha_g^3 K^2 \varepsilon_0^{-1} \tau \right] \quad (15)$$

The time needed to radiate away all of the initial energy is

$$\tau_{\text{tot}} = \left[\frac{2}{3} \alpha_g^3 \frac{K^2}{\varepsilon_0} \right]^{-1} \simeq 10^{27} \quad (16)$$

corresponding to $t_{\text{tot}} \approx 10$ seconds. This is a very ineffective mechanism indeed ! The temperature T (in GeV) of the Universe at time t (in seconds) during the radiation dominated era is given by

$$T^2 = 2.4 \times 10^{-6} g_t^{-1/2} t^{-1}, \quad (17)$$

where

$$g_t = g_B + \frac{7}{8} g_f \simeq 100, \quad (18)$$

and $g_B(g_f)$ is the number of light boson (fermion) helicity states at T . Hence, if this were the only mechanism by which the system could lose energy, it would correspond to an effective annihilation temperature of $T \approx 10^{-4}$ GeV. Therefore the increase of entropy would be $s'/s \approx 10^7$, which would be unacceptably large. To calculate the total number of cycles it takes, we first calculate the cycle time as a function of $\varepsilon(\tau)$

$$\tau_c(\varepsilon) = \frac{1}{\omega_0(\varepsilon)} \simeq \frac{1}{\alpha_g} \sqrt{\frac{\varepsilon}{K}}, \quad (19)$$

so that initially

$$\tau_c(\varepsilon_0) \simeq 10^8. \quad (20)$$

The total number of cycles N_{lin} is calculated as

$$N_{\text{lin}} = \int_0^{\tau_{\text{tot}}} \frac{1}{\tau_c} d\tau \simeq 10^{20}. \quad (21)$$

The average energy of the emitted photon is typically

$$\mathcal{E}_\gamma \simeq \omega_0 \simeq 10^{-8} \quad (22)$$

and consequently the total number of photons is approximately $N_\gamma \simeq 10^{13}$, i.e., one photon per 10^6 cycles ! Note also that the photon energy roughly equals the average level splitting as one would expect.

We have so far considered the radiation of photons from monopoles with a $U(1)_Y$ magnetic charge. Similar considerations should apply to the radiation of any massless quanta associated with the monopole. For example, the power radiated in gravitons should be suppressed by $\alpha_g^{-1} \kappa \epsilon (m/m_p)^2 \simeq 10^{-17}$ compared to photons¹⁰⁾. The next question to be considered is the emission of massive quanta, which would be the only significant radiation process that could occur for monopoles that are not associated with long range fields. We will discuss the emission of W and Z bosons for definiteness. Similar considerations should apply to the radiation of gluons which for high temperatures acquire an effective mass $\sim g^2 T$, presumably due to magnetic screening caused by the topologically unstable Wu-Yang type $SU(3)_C$ monopoles (our view differs from the one expressed by Linde⁶⁾). As the mass of the W and Z bosons corresponds to roughly 10^8 energy levels a proper treatment should involve the quantized boson field. On the other hand, it is clear that because of the large boson mass, the emission of massive bosons should be smaller than that of photons by many orders of magnitude. A crude estimate can be made based on the following picture. We can calculate the radiation spectrum assuming massless quanta and then make the restriction that only those quanta which have an energy larger than M_W can get away, i.e., materialize as real W or Z bosons. In other words we want to calculate $\langle dE/dt \rangle$ from Eq. (11) but only summing the contributions with $j > j_{\min}$ with

$$j_{\min} \omega_0 \simeq M_W \quad (23)$$

In order to determine this rate we calculate the Fourier coefficients x_j for motion in a linear potential. They are

$$x_j = \frac{4 |a| (-1)^{\frac{j-1}{2}}}{j^3 \omega_0^2 \pi} \quad (24)$$

for j odd and $x_j = 0$ for j even. a is the acceleration in (13) and ω_0 is the frequency (19). Substituting (24) back into (11) we obtain

$$\left\langle \frac{dE}{dt} \right\rangle = - \frac{16}{3\pi^2} \alpha_g |a|^2 \sum_{j=\text{odd}} \frac{1}{j^2} , \quad (25)$$

hence the energy radiated in the j^{th} mode falls like $1/j^2$. This implies that roughly

$$\left\langle \frac{dE}{dt} \right\rangle_{W,Z_0} = \sum_{j=j_{\min}} \frac{dE_j}{dt} \simeq \frac{1}{j_{\min}} \left\langle \frac{dE}{dt} \right\rangle_{\gamma} . \quad (26)$$

From earlier remarks it follows that $\omega_0 \simeq 10^{-8}$, so that the energy radiated away in the form W and Z bosons is indeed down by some eight orders of magnitude !

4. - COULOMB SCATTERING

We have seen that the radiation energy loss for monopoles oscillating in a purely linear potential is negligible. However, as the monopoles oscillate back and forth they may occasionally come close enough together to feel the Coulomb part of the potential. For small enough impact parameters the monopoles may be strongly deflected and therefore radiate a significant amount of energy. The estimate for the energy loss due to the Coulomb part of the potential will be made using the classical formalism. First we estimate a distribution of impact parameters b and then consider the energy loss as a function of b .

We assume that the average impact parameter b_{av} is determined by the thermal transverse velocity

$$V_{\perp} \simeq \sqrt{\frac{2T}{m}} \simeq \sqrt{K} \quad (27)$$

at the start of each cycle. (This is equivalent to assuming a thermal distribution of angular momenta for the monopole-antimonopole system.) This implies

$$b_{\text{av}} = b_{\text{av}} M_W \simeq \tau_c \sqrt{K} = \frac{1}{\alpha_g} \sqrt{\epsilon} \simeq 3 (1 - 10^{-27} \tau)^{1/2} . \quad (28)$$

Note that for $\tau < 10^{27}$ the average impact parameter is larger than the radius of the flux tube. In what follows we assume that for each oscillation cycle there is a uniform distribution of impact parameters over a disc of radius β_{av} . We assume that the distribution of impact parameters is uncorrelated from cycle to cycle (e.g., because of radiation reaction and drag effects). To the extent that

these effects are small the impact parameters will be the same from cycle to cycle (angular momentum conservation) implying less radiation on the average. Hence, our approach can be considered an upper bound on the radiated power. (In fact, the estimates of drag effects to be presented below support this randomization assumption.)

To estimate the loss due to Coulomb scattering we use the classical energy loss formula ¹⁰⁾. Let χ be the centre-of-mass scattering angle. χ is given by

$$\cot \chi/2 = \frac{\epsilon \beta}{\alpha_g} . \quad (29)$$

Then, the energy loss is

$$\Delta \epsilon \simeq -\epsilon^{5/2} K^{3/2} \tan^3 \frac{\chi}{2} \left\{ (\pi + \chi) \left(1 + 3 \tan^2 \frac{\chi}{2} \right) + 6 \tan \frac{\chi}{2} \right\} . \quad (30)$$

First we determine the impact parameter β^* for which $\Delta \epsilon_{\text{Coul}} \simeq \Delta \epsilon_{\text{Lin}}$. Recall that the energy loss per cycle for the linear part is $-\Delta \epsilon \simeq 10^{-14}$. With $\epsilon_0 = 10^5$ this corresponds to $\cos \chi/2 = 10^3 \beta$ in Eq. (30). Thus $\beta^* \simeq 10^{-2} \simeq 10^{-2} \beta_{\text{av}}$, so that the Coulomb energy loss is only important in about one cycle in 10^4 .

Let us now calculate the critical impact parameter β_0 for which capture occurs. We simply demand $\Delta \epsilon = -\epsilon$ and obtain (with $\chi \simeq \pi$)

$$\beta_0 \simeq \alpha_g K^{3/10} \epsilon^{-7/10} , \quad (31)$$

yielding $\beta_0 \simeq 10^{-5.7}$ initially. Ignoring the Coulomb interaction except for the capture probability

$$\beta_0^2 / \beta_{\text{av}}^2 \simeq 10^{-13} , \quad (32)$$

one obtains an estimate of the total lifetime of

$$\tau_{\text{tot}} \simeq 10^{13} \tau_c \simeq 10^{20} \quad (33)$$

This indicates that the Coulomb scattering is certainly more effective than the radiation due to the linear vibrations alone. For an improved calculation one performs an average over all impact parameters

$$\left\langle \frac{\Delta \varepsilon}{\varepsilon} \right\rangle \simeq - \frac{2}{\beta_{av}^2} \int_0^{\beta_{av}} \rho(\beta) \beta d\beta \quad (34)$$

with

$$\rho(\beta) = \begin{cases} 1 & 0 < \beta < \beta_0 \\ (\beta_0/\beta)^5 & \beta_0 < \beta < \beta_{av} \end{cases} \quad (35)$$

This obviously leads to the result

$$\frac{\Delta \varepsilon}{\varepsilon} \simeq - \left(\frac{\beta_0}{\beta_{av}} \right)^2, \quad (36)$$

omitting constants of order unity. The formula (35) is approximate in the sense that it is strictly only valid in the domain where $\cot \chi/2 \ll 1$ or $\beta \ll 10^{-3}$; however, at that value of β the integrand has already dropped by some 15 orders of magnitude compared to its value at $\beta = \beta_0$, implying a negligible error. Using the expressions (31) and (28) for β_0 and β_{av} in terms of ε , we can write a differential equation for the energy loss $\Delta \varepsilon$ as a function of the (large) number of cycles n

$$\frac{d\varepsilon}{dn} \simeq - \alpha_g^4 K^{3/5} \varepsilon^{-7/5}. \quad (37)$$

Upon integration one finds that the total number of cycles N_{Coul} for which $\varepsilon(N_{Coul}) = 0$ is

$$N_{Coul.} \simeq \alpha_g^{-4} K^{-3/5} \varepsilon_0^{12/5} \simeq 10^{12.4} \quad (38)$$

which actually coincides with our naive estimate (33) up to irrelevant numerical factors. Let us finally note that the entropy generation due to this mechanism is easily computed from (4) and (17) to be

$$S'/S \Big|_{Coul} = \left(\frac{N_{Coul}}{N_{Lin}} \right)^{3/8} S'/S \Big|_{Lin} \simeq 10^4. \quad (39)$$

Therefore, if these were the only energy loss mechanisms a barely tolerable situation would arise.

5. - ENERGY DISSIPATION DUE TO THE FRICTION OF THE CHARGED PLASMA

We finally come to what in our opinion is the dominant energy loss mechanism in monopole confinement, namely the scattering of the monopoles with the plasma of relativistic charged particles. A reasonable approximation can be made as follows. We write the energy loss per unit time as

$$\frac{dE}{dt} \simeq \frac{\langle \Delta E \rangle}{\Delta t} , \quad (40)$$

where $\langle \Delta E \rangle$ denotes the average kinetic energy loss of a monopole moving with initial speed v_i as the result of a single collision with a charged relativistic particle with momentum $p_i \simeq T$. The quantity Δt is then the average collision time

$$\Delta t = \frac{1}{\sigma n_p} , \quad (41)$$

where σ is the charge-monopole large angle cross-section and n_p the number density of the particles in the plasma (per helicity state). A simple kinematic computation shows that

$$\Delta E \simeq -\epsilon K . \quad (42)$$

For the cross-section we take the classical capture cross-section

$$\sigma = B'/T^2 \quad (43)$$

with

$$B' = \sum_i \left\{ \frac{q_i g}{4\pi} \right\}^2 , \quad (44)$$

where the sum is over all helicity states of the species in the plasma. For $T \lesssim M_W$ we expect (43) to be a reasonable estimate of the capture cross-section even if the monopole charge does not have a $U(1)_Y$ component. Finally, using the fact that $n_p \simeq cT^3$ ($c \simeq 1/10$) we obtain

$$\frac{d\epsilon}{d\tau} \simeq -B K \epsilon , \quad (45)$$

where $B \equiv cB' \simeq 10$, leading to an exponential decrease of the energy

$$\epsilon(\tau) \simeq \epsilon_0 e^{-B K \tau} . \quad (46)$$

The time constant is

$$\tau_{\text{tot}} \simeq (BK)^{-1} \simeq 10^{13} \quad (47)$$

which implies that it will take only about $N_{\text{drag}} \approx 10^6$ cycles to annihilate. (Note, however, that the dissipation effect is very small during a single cycle, as we have implicitly assumed in treating the drag as a perturbation.) τ_{tot} in (47) corresponds to an annihilation time of $t_{\text{tot}} \approx 10^{-12}$ seconds, which is short compared to the age of the Universe ($\approx 10^{-11}$ seconds) at the time of the $SU(2) \times U(1)$ transition. Hence, the plasma drag effect causes the annihilation to occur almost instantaneously, so that the entropy is increased only by a factor of $\gtrsim 10^2$ [as in Eq. (4)].

6. - DISCUSSION

Having identified what we believe are the three major contributions to the energy loss of a confined monopole-antimonopole pair, it is interesting to compare their relative importance but now as a function of the two relevant parameters $\kappa = M_W/m$ and ϵ_0 , the initial potential energy.

The calculations we have described should be valid as long as $\epsilon_0 > \alpha_g$ (so that the linear part of the potential dominates the monopole motion), $\kappa < \alpha_g^{-1}$ (so there exists an unconfined phase between monopole formation and confinement - this assumption could be relaxed somewhat depending on the gauge and Higgs parameters of the theory), and $\epsilon_0 \kappa \ll 1$ (non-relativistic motion). Other conditions, such as $\epsilon_0 > 1$ (neglect of thermal modes), $\epsilon_0 \gg \alpha_g^2 \kappa$ (adiabatic motion), and $\epsilon_0 \gg \kappa^{\frac{1}{3}}$ (highly excited initial state) then follow automatically. As we have calculated the various rates, ignoring as a certain extent how they influence each other, due care should be taken in comparing the formulae (14), (37) and (45) with each other, especially for those values of the parameters where any two of them predict comparable rates. With this reservation in mind one may indicate the various areas of the κ, ϵ_0 plane where any one of the three mechanisms is dominant as we have done in the Figure. For ϵ_0 and κ both small, Coulomb scattering becomes dominant. There is a small region near $\kappa = 1$ for which the linear part of the potential dominates the energy loss. For ϵ large but $\kappa \ll \epsilon$ it is the drag which dominates. This is of course the situation of interest in most grand unified schemes. Besides the canonical set of parameters we have used throughout this paper there is another possibly interesting set, $\kappa \approx 10^{-6}$ and $\epsilon_0 \approx 10^6$, which corresponds to a unification mass of $\approx 10^6$ GeV ($m \approx 10^8$ GeV, $r_1 \approx 10^{-18}$). Because of the small value r_1 the monopole problem is not a serious difficulty to begin with in such schemes.

We have assumed throughout that confinement occurs at the time of the $SU(2) \times U(1)$ phase transition, when the temperature is $T \approx T_C \approx M_W$. We note, however, that the formulae for τ_C and for $dc/d\tau$ and N (for each of the energy loss

mechanisms) are independent of T_c (except for a very weak dependence on the number of light helicity states at T_c). They can therefore be applied to other U(1) breaking transitions provided of course that ϵ , τ and κ are appropriately redefined.

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FIGURE CAPTION

The dominant energy loss mechanism for a confined monopole-antimonopole pair plotted logarithmically as a function of the parameters $\kappa = M_W/m < 1$ and ϵ_0 , the initial potential energy. We have marked the point which corresponds to the standard values of the parameters which we have used throughout the paper.

FIG. 1