Prospects for the precision determination of the $W$ boson mass with the CMS detector at the LHC.

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Executive summary

The Standard Model of electroweak interactions has found excellent confirmations from experimental data in past and current experiments. The $e^+e^-$ colliders LEP and SLC have collected an enormous amount of electroweak precision data. High accuracy has been reached in measurements at the $Z$ pole of the $Z$ boson partial and total widths, polarization and forward-backward asymmetries. The $W$ boson properties have also been measured with increasing accuracy at LEP2, with a center of mass energy of 209 GeV, and at the Tevatron $p\bar{p}$ collider up to a center of mass energy of 1.96 TeV. The discovery of the top quark at Fermilab in 1995 by the CDF and D0 collaborations gave the direct experimental confirmation of the three-generation structure of the Standard Model and opened up the new field of top quark physics.

High precision electroweak measurements, compared to accurate theoretical predictions of electroweak observables, provide a precision test of the Standard Model. Moreover, they allow to put indirect constraints on the still experimentally unprobed scalar sector of the electroweak theory, that postulates the existence of a scalar particle, the Higgs boson, originating the masses of both the gauge bosons and the fermions. Improved measurement of electroweak parameters, in particular of the top quark mass, the $W$ boson mass and the electroweak mixing angle, are therefore of fundamental importance because they fix stronger constraints on the Higgs boson mass, which is an unknown parameter of the Model, and, after the discovery of the Higgs boson, will allow a strong consistency check of the Standard Model.

On the other hand, the discovery of deviations of the experimental results from theoretical predictions of the Standard Model constitutes a probe of New Physics beyond the Standard Model.

The direct search of the Higgs particle and the study of its properties will be one of the main goals of the Large Hadron Collider (LHC). In a com-
The LHC will be also the ideal place where the $W$ and $Z$ gauge bosons and top quark properties can be accurately studied, leading to a substantial improvement in the determination of electroweak parameters. In fact, thanks to the LHC center of mass energy and to the high collider luminosity, $W$, $Z$ and top events will be produced at very high rates, allowing to perform measurements with nearly negligible statistical errors and providing large control samples to check systematic effects.

In this thesis, a precision measurement of the $W$ boson mass through the leptonic $W \rightarrow e\nu$ decay is discussed. The LHC aimed precision on this parameter is about 10 MeV (to be compared to the current world average $M_W = 80.398 \pm 0.025$ GeV). In order to achieve this goal, new strategies of measurement can be envisaged exploiting the large $W$ and $Z$ statistics available. This work investigates, by means of detailed simulations of the CMS detector, the potentiality of the so-called “Scaled Observables Method”. This approach is based on the idea to predict the $W$ distributions sensitive to the boson mass using the corresponding measured distribution from the $Z$ boson decay, along with the theoretical ratio between $W$ and $Z$ distributions opportune scaled by the vector boson masses.

The main advantage of this approach is that the systematic uncertainties both of theoretical origin, related to the $W$ production and decay model, and of experimental origin tend to cancel in the ratio. In particular, from the theoretical side, the radiative corrections due to the soft gluon emission entering the prediction of the boson transverse momentum are reduced, making the ratio computable using fixed order perturbative QCD. A larger statistical uncertainty, due to the fact that the leptonic $Z$ cross section is an order of magnitude smaller than the leptonic $W$ cross section, is forseen with respect to the traditional approach for the $M_W$ measurement based on the comparison between $W$ data and Monte Carlo templates. This aspect, which made this approach unaffordable at the Tevatron, isn’t indeed a limiting factor at the LHC because of the extremely large statistics available. Moreover, just because most of the instrumental effects are cancelled by this approach, it seems to provide the best way to get an early measurement of the $W$ mass from the first 1 fb$^{-1}$ of data.

The discussion is organized as follows. The first Chapter gives an introductory outlook on the precision measurement of the $W$ boson mass, clarifying its motivations and its most relevant aspects. The current status of the $W$
mass measurement and the aimed precision at the LHC are presented. The conceptual design of the “Scaled Observables Method” is outlined. After an overview of the CMS detector given in Chapter 2, the reconstruction of the variables relevant to the measurement from $W \to e\nu$ decays will be addressed in Chapter 3. In Chapter 4 the selection of $W \to e\nu$ and $Z \to ee$ events and the main sources of backgrounds are discussed. Details on the analysis procedure and prospects on the statistical precision achievable using the “Scaled Observables Method” are given. The theoretical and experimental systematic uncertainties forseen on the measurement with the electron transverse energy spectrum are examined in Chapter 5 and Chapter 6 respectively, where the strategies to control these systematic effects directly from data, whenever possible, are also described. Finally, the uncertainties expected for 1 fb$^{-1}$ and 10 fb$^{-1}$ on the $M_W$ measurement are summarized in Chapter 7.
Chapter 1

Electroweak physics at the LHC

1.1 Introduction

Experimental measurements and theoretical developments in the last decades have brought to the definition of the Standard Model (SM) of electroweak interactions. Proposed by Glashow, Salam and Weinberg in the middle sixties [1], it has been later extensively tested over a wide range of energies. The discovery of the neutral current interactions and the production of intermediate vector bosons ($W^\pm$ and $Z$) with the expected properties increased the confidence in the model. Electroweak parameters have been measured with high precision, contributing to successfully test the model with sensitivity to genuine electroweak radiative corrections. However, the core of the theory, namely the spontaneous electroweak symmetry breaking mechanism that requires the existence of a neutral scalar field, the Higgs boson, originating all the masses and guaranteeing the renormalizability of the theory, has not been experimentally proved yet. The search of this particle and the study of its properties will be one of the main goals of the next-generation collider machines, such as the Large Hadron Collider (LHC), which, colliding protons at a center of mass energy of 14 TeV, will have direct access to unexplored range of energies, up to the TeV scale. Among the specific objectives are the direct search for the SM Higgs boson or for deviations from the SM, as predicted by several models, such as or including Supersymmetry [6]. In a complementary way, SM electroweak measurements are a very important part of the LHC physics program, because they represent "standard candles" to understand the detectors and the SM at the LHC.
energies. Moreover, precision measurements of electroweak parameters will be affordable and will allow to constrain the Higgs boson mass, which is an unknown parameter of the theory.

1.2 The Standard Model and beyond

The electroweak theory describing electromagnetic and weak interactions between quarks and leptons, is a gauge field theory based on the gauge symmetry group $SU(2)_L \times U(1)_Y$ of weak left-handed isospin and hypercharge. Combined with Quantum Chromo-Dynamics (QCD) \cite{2}, the theory of the strong interactions between colored quarks based on the $SU(3)_C$ group, the model provides a unified framework to describe these three forces.

The SM has two kinds of fields. The fundamental matter fields are quarks and leptons. They have spin 1/2 and appear in three generations (Table 1.1). Gauge fields correspond to the spin-1 bosons that mediate the interactions. In the electroweak sector, four gauge fields correspond to the $SU(2)_L \times U(1)_Y$ group, three related to the $SU(2)_L$ symmetry ($W^i$, i=1,2,3) and one to the $U(1)_Y$. The two charged $W^\pm$ bosons are obtained by a linear combination of two of them, $W^\pm = \frac{1}{\sqrt{2}}(W_1^\pm \mp W_2^\pm)$; the photon $A$ and the $Z^0$ neutral boson come from the combinations $B_\mu = A_\mu \cos \theta_W - Z_\mu \sin \theta_W$ and $W_3^\mu = A_\mu \sin \theta_W + Z_\mu \cos \theta_W$, where $\theta_W$ is the Weinberg angle. In the strong sector, there is an octet of gluon fields $G_\mu^{1,...,8}$ which corresponds to the generators of the $SU(3)_C$ group.

The gauge theory based on the $SU(2)_L \times U(1)_Y$ group is not compatible with explicit mass terms in the lagrangian as they would break the gauge invariance. The spontaneous symmetry breaking mechanism, first introduced by Higgs \cite{3}, gives masses to both bosons and fermions without violating the

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<tr>
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<td>+2/3</td>
<td>electron neutrino ($\nu_e$)</td>
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<tr>
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<td>III</td>
<td>top (t)</td>
<td>+2/3</td>
<td>tau neutrino ($\nu_\tau$)</td>
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<td>bottom (b)</td>
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<td>tau ($\tau^-$)</td>
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Table 1.1: Fermions of the Standard Model.
gauge invariance and in a way that is minimal and respects the renormalizability [4] and unitarity [5] requirements. In its simplest formulation, which is implemented in the Standard Model, an SU(2) doublet of complex scalar fields is introduced and its neutral component acquires a non-zero vacuum expectation value. The SU(2)\(_L\) \(\times U(1)\)\(_Y\) symmetry is spontaneously broken to the electromagnetic U(1)\(_Q\) symmetry. Three of the four degrees of freedom associated to the doublet of scalar fields are absorbed by the \(W^\pm\) and Z vector bosons to form their longitudinal polarizations and to provide masses. The remaining degree of freedom corresponds to a scalar particle, the Higgs boson. A Yukawa interaction of fermion fields with the scalar field and its conjugate generates the fermion masses.

In Minimal Supersymmetric Standard Model (MSSM) [6], the simplest extension of the SM to supersymmetric theories, the minimal choice to ensure a theory free of anomalies and to give mass to all the fermions requires the introduction of two doublets of complex scalar fields. Assuming CP conservation, the Higgs particles spectrum in the MSSM consists of two CP-odd eigenstates, \(h^0\) and \(H^0\), one CP-even eigenstate, \(A^0\) and two charged Higgs bosons, \(H^+\) and \(H^-\). Again, three of the eight degrees of freedom provide masses to the weak vector bosons.

One of the main goals of the Large Hadron Collider will be the search of an experimental evidence of the Higgs particle(s).

The only unknown parameter in the SM Higgs sector is the mass of the Higgs boson. Direct searches at LEP ruled out a Higgs boson with mass below 114.4 GeV with 95% confidence level as indicated by the excluded area drawn in yellow in Fig. 1.1. Indirect constraints are obtained from electroweak precision measurements, because the Higgs mass enters the radiative corrections to the theoretical predictions of electroweak observables, as it will be discussed in the following Section.

### 1.3 Electroweak parameters and precision data

The electroweak theory describes the relations between experimentally measurable quantities. Any observable, like the masses of the vector bosons, can be predicted in terms of a finite number of parameters, which have to be determined in experiments. The discussion of this section follows [9]. A useful standard set of reference measurements, used as input parameters for the model, is based on three quantities:
Figure 1.1: $\Delta \chi^2$ curve derived from high-\(Q^2\) precision electroweak measurements, performed at LEP and by SLD, CDF, and D0, as a function of the Higgs boson mass, assuming the Standard Model to be the correct theory. The yellow band corresponds to the mass region excluded by direct searches; the blue band represents the theoretical uncertainty. [8]
1.3 Electroweak parameters and precision data

- $\alpha$, the fine structure constant, characterizing the strength of the electromagnetic interaction; its value, at $q^2=0$, is $\alpha=1/137.03599911(46)$ [10];

- the Fermi constant, which determines the strength of the charged current; the measure of $G_F$ from the muon lifetime is $G_F = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$ [10];

- the mass of the $Z$ boson, $M_Z = 91.1876 \pm 0.0021$ GeV, determined from the $Z$ lineshape scan at LEP1 [33].

These three parameters reflect the three degrees of freedom of the gauge sector: the two coupling constants of the gauge groups $SU(2)_L$ and $U(1)_Y$ and the vacuum expectation value of the Higgs field. Within the framework of the SM, any process mediated by the weak or electromagnetic current can be computed at tree level from these three quantities, neglecting the small phase-space effects due to the non-zero mass of the final state fermions. In the perturbative expansion, radiative corrections, however, modify the tree level relations and other parameters need to be introduced in order to predict electroweak observables. They are

- the fermion masses, $m_f$. They give a contribution to the photon self-energy. They also appear in the weak boson self-energy corrections in the form $(m_i^2 - m_j^2)/M_Z^2$ (where $i$ and $j$ indicate two fermions in the same weak isospin doublet) and in the vertex corrections in the form $(m_f/M_Z)^2$; the most important contribution comes from the heavy top quark, $f = t$;

- $M_H$, the mass of the Higgs boson, which gives logarithmic contributions in the form $\log\left(\frac{M_H}{M_Z}\right)$ and has therefore small effects;

- $\alpha_s = \alpha_s(q^2 = M_Z^2)$, the strong coupling constant.

Each electroweak observable ($O_i$) depends in principle on the full set of input parameters [11]:

$$O_i = O_i\left(\alpha(M_Z^2), G_F, M_Z, m_f, M_H, \alpha_s\right) \quad (1.1)$$

and the theoretical prediction is mainly limited by the precision with which the least known parameters of the model are known, and not by missing high orders in the radiative corrections. Conversely (as is done at present), the comparison between electroweak observables measurement and theoretical
prediction allows to get further informations on the least known parameters of the model, such as the Higgs boson mass.

Within the SM, the theoretical prediction of $M_W$ is obtained in terms of $\alpha$, $G_F$ and of the $Z$ boson mass from the relation [12, 13]:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \alpha}{\sqrt{2} G_F} \frac{1}{1 - \Delta r}.$$  \hspace{1cm} (1.2)

At one loop order, the quantity $\Delta r = \Delta r(\alpha, M_W, M_Z, m_{t\bar{t}}, M_H)$ can be written as the sum of different terms:

$$\Delta r = \Delta \alpha - \frac{c_{W}^2}{s_{W}^2} \Delta \rho + (\Delta r)_{rem}$$  \hspace{1cm} (1.3)

where $c_W^2 = M_W^2/M_Z^2$ and $s_W^2 = 1 - c_W^2$. $\Delta \alpha$ is the photon vacuum polarization and contains the large logarithmic contributions from the light fermions, $\Delta \alpha \propto \log(m_f)$ . The second term $\Delta \rho$ is the leading contribution to the $\rho$ parameter [15], defined by the ratio of neutral-current to charged-current amplitude at $q^2 = 0$ and quadratically dependent on the top quark mass. At the tree level in SM, $\rho \equiv \rho_0 = \frac{M_W^2}{c_W^2 M_Z^2} = 1$. Adding the correction one order higher, it becomes $\rho = 1 + \Delta \rho$ with

$$\Delta \rho = \frac{\sqrt{2} G_F}{16 \pi^2} \sum_f N_f^i \Delta m_f^2$$  \hspace{1cm} (1.4)

where the sum is extended over all the fermions isodoublets and $\Delta m_f^2 = \left|m_f^2 - m_f^2\right|$ is the isodoublet mass splitting. The leading contribution to $\Delta \rho$ comes from the heavy quark doublet $(t, b)$ because of the large mass difference between the two quarks:

$$\Delta \rho \sim \Delta \rho_{tb} \sim \frac{3 G_F m_{t\bar{t}}}{8 \sqrt{2} \pi^2}$$  \hspace{1cm} (1.5)

where the quadratic dependence on $m_{t\bar{t}}$ is noteworthy. The higher order QCD corrections are known at two-loop [16] and three-loop [17] orders. Full fermionic contribution to $\Delta \rho$ have been recently derived at the two-loop level [18] and other higher orders corrections are also available [19].

At the one-loop level, the Higgs boson contributes also to the $\rho$ parameter [20]:

$$\Delta \rho_{\text{Higgs}} = \frac{3 \sqrt{2} G_F M_W^2}{16 \pi^2} f\left(\frac{M_H^2}{M_Z^2}\right),$$  \hspace{1cm} (1.6)

$$f(x) = x \left[ \ln c_W^2 - \ln x + \frac{\ln x}{c_W^2} \right] + \frac{x}{c_W^2 (1 - x)}.$$  \hspace{1cm} (1.7)
For a heavy Higgs boson, $M_H^2 > M_W^2$, this contribution gives approximately
\[
\Delta \rho^{1-Higgs} \sim \frac{3\sqrt{2} G_F M_W^2 s_W^2}{16\pi^2 c_W^2} \ln \frac{M_H^2}{M_W^2}.
\] (1.8)

The third term in Eq. 1.3 accounts for the remaining non-leading corrections. Among these, there are non-quadratic but still sizeable effects due to the top quark, additional light fermions contributions and some vertex and box corrections involved in the muon decay. At one-loop, the Higgs contribution to $\Delta r$ is still logarithmic, as it is for $\Delta \rho$ [12, 13]. Figure 1.2 shows how the Standard Model constraints between the top quark and the $W$ masses depend on the Higgs mass, which enter the loop corrections.

Besides the $W$ boson mass, the improvement on $m_{top}$ measurement will also have effects on the predictions of the $Z$ pole observables. The neutral-current at the $Z$ resonance is expressed by
\[
J^{NC}_\mu = (\sqrt{2} G_F M_Z^2)^{1/2}(g_V^f \gamma_\mu - g_A^f \gamma_\mu \gamma_5)
\] (1.9)
The couplings $g_V^f$ and $g_A^f$ are defined in the Standard Model by

\[ g_V^f = \sqrt{\rho_f^f} (I_3^f - 2Q_f^f \sin^2 \theta_{\text{eff}}^f) \quad g_A^f = \sqrt{\rho_f^f} I_3^f \]  

where $I_3^f$ are $Q_f^f$ the third component of the weak isospin and the fermion charge, while $\rho_f^f = 1 + \Delta \rho + ...$ is a normalization factor. The weak mixing angle is usually chosen as the on-resonance mixing angle for the leptons $f = e, \mu, \tau$ and denoted as $\sin^2 \theta_{\text{eff}}^f$. Also for this parameter, the radiative corrections depend on the top quark mass and on the Higgs mass, mainly through the $\Delta \rho$ term.

The Higgs mass enters, therefore, logarithmically in the radiative corrections to electroweak observables. Precise measurements of $m_{\text{top}}$, $M_W$ and $\sin^2 \theta_{\text{eff}}$ constrain the Higgs mass of SM or of $h$ in the MSSM. Currently, indirect measurements favour a rather light Higgs boson giving an upper limit of 144 GeV with 95% confidence level (Fig. 1.1).

At the LHC, improved measurements of $m_{\text{top}}$ and $M_W$ are forseen and will allow to put tighter limits on $M_H$. In particular, the top quark mass and the $W$ mass are expected to be measured with an accuracy at the level of 1 GeV and 15 MeV respectively, constraining the Higgs mass to 18% [14].

Besides the internal consistency checks of the Standard Model, the electroweak precision observables may be useful to distinguish between different models as candidates for the electroweak theory. New measurements, in fact, are to be compared with predictions. If these agree within their errors, the measurement can be used to further constrain the input parameters. Non-agreement indicates inconsistency of the theoretical framework and New Physics. At the LHC, a substantial improvement on the electroweak parameters will be achievable because the high center of mass energy and the high collider luminosity will make new phase space regions accessible and provide an enormous statistics.

In the following Section, details will be given about the precision measurement addressed in this thesis, namely the $W$ boson mass measurement at the LHC. In particular, the main aspects of the measurement at hadron colliders are outlined and the strategy adopted in this work is introduced.
A number of experiments have measured the $W$ boson mass since 1983 when the boson was first discovered by UA1 at CERN $p\bar{p}$ collider [32]. The current world average value for the $W$ mass is $80.398 \pm 0.025$ GeV. Figure 1.3 shows the present status of $W$ boson mass measurements from LEP and Tevatron. At LEP two methods have been used to obtain the $W$ mass. In the first method the measured $W$ pair production cross section, $\sigma(e^+e^- \rightarrow W^+W^-)$, has been used to determine the $W$ mass using the predicted dependence of this cross section on $M_W$. At 161 GeV, which is just above the $W$ pair production threshold, this dependence is a much more sensitive function of the $W$ mass than at the higher energies (172 to 209 GeV) at which LEP has run during 1996 - 2000. In the second method, used at the higher energies, the $W$ mass has been measured by directly reconstructing the $W$ from its decay products. At $p\bar{p}$ colliders and in general at hadron colliders, instead, the measurement is based on $W$ leptonic decays and, since the longitudinal momentum of the neutrino cannot be measured,
the $W$ mass has to be inferred by transverse quantities.

As discussed in Section 1.3, an improved measurement of the $W$ mass at the LHC, combined with other electroweak measurements will lead to fix strong indirect constraints on the mass of the Standard Model Higgs boson.

To ensure that the experimental errors on $m_{top}$ and $M_W$ equally contribute to the uncertainty on $M_H$ (the other electroweak parameters being far better measured), the precision on $m_{top}$ and $M_W$ has to satisfy the following relation [7]:

$$\Delta M_W \sim 0.7 \times 10^{-2} \Delta m_{top}$$

(1.11)

The mass of the top quark is foreseen to be measured at the LHC with an accuracy of the order of 1 GeV [34]. It follows that a global precision on $M_W$ at the level of 10 MeV has to be achieved, to avoid it to be the dominant error in the estimate of $M_H$. Such a precision measurement of the $W$ mass at the LHC becomes feasible because a huge sample of data available at the LHC. The production cross sections for heavy gauge bosons $W$ and $Z$ at the LHC are in fact significantly larger than at Tevatron, because of the higher center of mass energy. Already during the low luminosity phase ($\mathcal{L} = 2 \times 10^{33}$ cm$^{-2}$ s$^{-1}$), this results in large production rates (see Fig. 1.4). Approximately 200 $W$ and 50 $Z$ bosons will be produced every second. Considering only the leptonic decays, about 200 millions of $W \rightarrow l\nu$ will be produced with an integrated luminosity of 1 fb$^{-1}$ for each lepton species. The $Z$ cross section is about 1/10 of the corresponding $W$ cross section, then about 20 millions of $Z$ in each leptonic channel are expected. This large number of events will guarantee a small statistical error and a good control of the systematic effects.

1.4.2 Observables sensitive to the $W$ boson mass

At hadron colliders, the $W$ mass measurement is performed through the study of the two-body leptonic decays $W \rightarrow l \nu$ ($l = e, \mu$).

Since the longitudinal component of the missing momentum cannot be measured, there is not sufficient information to reconstruct the mass. The analysis is restricted to the plane transverse with respect to the beam axis, where observables sensitive to $M_W$ are the lepton transverse momentum and the transverse mass of the lepton-neutrino pair. The latter is defined as

$$M_T = \sqrt{2p_T^l p_T^\nu (1 - \cos(p_T^l, p_T^\nu))}$$

(1.12)
Figure 1.4: The production cross sections for different Standard Model processes in pp collisions at the LHC and p̅p collisions at the Tevatron.
where $p_T^l$, $p_T^\nu$ and $\cos(p_T^l, p_T^\nu)$ are the transverse momenta of the charged lepton, of the neutrino and the cosine of their angle, respectively. The observable $p_T^l$ is directly measured in the experiment, while the neutrino transverse momentum is reconstructed, exploiting the momentum balance in the transverse plane, from the transverse momenta of the lepton and of the hadronic system recoiling against the boson.

Considering, at first, the case in which the $W$ is produced at rest, the cross section can be expressed as follows:

$$\frac{d\sigma}{d(cos\hat{\theta})} = \sigma_0(\hat{s})(1 + \cos\hat{\theta})$$

where $\sqrt{s}$ is the center of mass energy of the colliding quarks and $\hat{\theta}$ is the polar angle between the electron with respect to the beamline. The function $\sigma_0(\hat{s})$ is proportional to a Breit-Wigner distribution.

Neglecting the electron mass, in the $W$ rest frame the lepton energy $\hat{E} = \hat{\rho}$ is $\hat{E} = \sqrt{\hat{s}}/2$ and the transverse energy is $\hat{E}^T = (\sqrt{\hat{s}}/2)\sin\hat{\theta}$. The cross section differential in $\hat{E}^T$ can thus be written as

$$\frac{d\sigma}{d\hat{E}^T} = \frac{2}{\sqrt{\hat{s}}} \frac{d\sigma}{d(sin\hat{\theta})}$$

$$= \frac{2}{\sqrt{\hat{s}}} \frac{d\sigma}{d(cos\hat{\theta})} \left| \frac{d(cos\hat{\theta})}{d(sin\hat{\theta})} \right|$$

$$= \frac{2}{\sqrt{\hat{s}}} \sigma_0(\hat{s})(1 + \cos\hat{\theta})|\tan\hat{\theta}|$$

$$= \sigma_0(\hat{s}) \frac{4(\hat{E}^T)^2}{\hat{s}} \left( 2 - 4(\hat{E}^T)^2/\hat{s} \right) \frac{1}{\sqrt{1 - 4(\hat{E}^T)^2/\hat{s}}}$$

Equation 1.17 is independent of the longitudinal momentum of the $W$ boson as both $\hat{E}^T$ and $\sqrt{\hat{s}}$ are invariant under longitudinal boosts. It shows a singularity at $\hat{E}^T = \sqrt{\hat{s}}/2$, which is also the maximum value for a given value of $\sqrt{\hat{s}}$. Hence, at a fixed $\sqrt{\hat{s}}$, the the transverse energy distribution has an infinitely sharp edge at $\sqrt{\hat{s}}$. This is usually referred to as “Jacobian edge”, because it derives from the jacobian $d(cos\hat{\theta})/d(sin\hat{\theta})$. The sharpness of the Jacobian edge is primarily broadened by the $W$ finite width: if the $\sqrt{\hat{s}}$ is distributed according to a Breit-Wigner distribution instead of a fixed value, the singularity in the lepton transverse energy distribution is made finite, but there is still a trailing edge at half the $W$ mass.

A further smearing of the Jacobian edge is due to the $W$ boson transverse
motion. The theoretical prediction of the boson transverse momentum is indeed affected by large uncertainties related to the soft gluons emission. Following the discussion in [35], it is possible to show that at the first order the $W$ transverse mass is insensitive to the $W$ boson transverse momentum while the lepton transverse energy is not.

In the laboratory frame the $W$ is not produced at rest and, given the lepton transverse energy $E_{\text{rest}}^T$ in the boson rest frame and the boson transverse momentum $p_W^T$, the charged lepton transverse energy $E^T$ can be written as

$$E^T \approx E_{\text{rest}}^T + \frac{1}{2} p^T_W$$  \hspace{1cm} (1.18)

which is valid to the first order in the ratio between the boson transverse momentum and the boson energy. The module $|\vec{E}^T|$ is given by

$$E^T \approx E_{\text{rest}}^T + \frac{1}{2} p_{W//}$$  \hspace{1cm} (1.19)

where $p_{W//}$ is the component of the $W$ transverse momentum parallel to the lepton direction. The falling edge in the laboratory frame is thus significantly wider than in $W$ rest frame.

The transverse mass has instead only a second order dependence on the boson transverse momentum. Starting from the relation

$$p_W^T = \vec{E}_\nu^T$$  \hspace{1cm} (1.20)

$\vec{E}_\nu^T$ can be replaced with $\vec{p}_W^T - \vec{E}_\nu^T$ everywhere in the definition of $M^T$:

$$M^T = \sqrt{(E^T + E_\nu^T)^2 - (\vec{E}_\nu^T + \vec{E}_\nu^T)^2}.$$  \hspace{1cm} (1.21)

The $M^T$ expression can be expanded to the first order in $p_W^T/E^T$ leading to the following approximation:

$$\frac{1}{2} M^T = E^T - \frac{1}{2} p_{W//}.$$  \hspace{1cm} (1.22)

By comparing this with Eq. 1.19, which is also approximate to first order in $p_W^T/E^T$, the transverse mass is expressed as

$$\frac{1}{2} M^T = E_{\text{rest}}^T.$$  \hspace{1cm} (1.23)

Correction to Eq. 1.23 are present only at the second order in $p_W^T/E^T$ and thus the distribution of this observable is far less sensitive to the $W$ boson $p^T$ modelling than the lepton transverse energy. On the other hand,
the transverse mass distribution depends largely on the neutrino transverse momentum reconstruction and is therefore plagued by larger experimental uncertainties. The complementarity between the two observables is well represented in Fig. 1.5, which shows the distribution of $M^T$ and $E^T$ as expected in CMS for $W \rightarrow e\nu$ decays and compared to the case of perfect detector resolution and of zero $p^T$ of the $W$ boson. The reconstruction of the transverse mass is dominantly affected by the experimental resolution on the missing transverse energy (neutrino transverse momentum), which is also dependent on the machine parameters, through pile-up events. The $E^T$ spectrum of the lepton, instead, is mainly affected by the $p^T$ distribution of the $W$ boson production, while the additional effect of the resolution on the lepton $p_T$, which in case of electrons is best estimated from the energy deposited in the calorimeter, is marginal at all the luminosities.

The $W$ transverse distributions must be predicted in a very precise way, taking accurately into account both the experimental and theoretical effects that shapes the spectra. These predictions are then compared to $W$ data in order to extract the boson mass. Two are the possible approaches. The first one fits $W$ data with Monte Carlo templates. The second one, referred to as “Scaled Observables Method”, is based on the direct comparison of $W$ transverse distributions with the distributions measured in $Z \rightarrow ll$ events. The latter represents a new measurement strategy with respect to the Monte Carlo template method traditionally applied at hadron colliders, and it is the method analyzed in this work.
Figure 1.5: *Boson transverse mass (top) and electron transverse momentum (bottom) distributions in W → ℓν decays. The distributions at the generator level with $p_T(W) = 0$ (black line), with finite W boson $p_T$ (blue dots) and including the experimental resolution in the low luminosity phase (red dashed line) are shown.*
1.4.3 The Monte Carlo template method

According to this approach, in practice, $M_W$ is extracted from the comparison between the $W$ experimental transverse distributions and Monte Carlo templates generated at different values of $M_W$. This method requires an excellent modelling both of the detector and of the theoretical description of the $W$ boson production and decay processes. For these reasons, the measure has been traditionally carried out using the transverse mass spectrum rather than the lepton transverse energy distribution, because the theoretical prediction of the latter is plagued by the large radiative corrections associated to the soft gluon emission.

In general, the most relevant contributions to the systematic uncertainties on the measurement of the $W$ mass come from the lepton energy/momentum scale, the lepton energy/momentum resolution, the modelling of the system recoiling against the $W$ boson, the parton distribution functions, the $W$ boson transverse momentum, the $W$ intrinsic width, the radiative decays and the background. This method has been largely studied at the Tevatron in the past years. The most recent results in the electron channel from the CDF collaboration are reported in Table 1.2 for the $M^T$ and $E^T$ fits with $\approx 200 \text{ pb}^{-1}$ of data [36].

At the LHC, each contribution to the $M_W$ uncertainty should be kept below about 10 MeV to achieve the aimed precision of 15 MeV on $M_W$.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Transverse mass</th>
<th>Electron transverse energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton scale</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Lepton resolution</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Lepton efficiency</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Recoil scale</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>Recoil resolution</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>$p_T^W$</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>PDF</td>
<td>11</td>
<td>20</td>
</tr>
<tr>
<td>Radiative decays</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Total systematics</td>
<td>39</td>
<td>45</td>
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<td>Statistical</td>
<td>48</td>
<td>58</td>
</tr>
<tr>
<td>Total</td>
<td>62</td>
<td>73</td>
</tr>
</tbody>
</table>

Table 1.2: Uncertainties on the $W$ mass measurement by CDF for the transverse mass fit and the lepton transverse energy fit in the electron channel with $\approx 200 \text{ pb}^{-1}$. The quoted errors are expressed in MeV [36].
1.4 W mass measurement at hadron colliders

combining channels and experiments. To accomplish this, new strategies for the W mass measurements must be considered.

1.4.4 A new analysis strategy: the “Scaled Observables Method”

The approach discussed in this thesis has been originally proposed in [37] and consists in predicting the distribution of experimental observables sensitive to the W mass, such as the transverse momentum or energy of the charged lepton ($p_T^l$) and/or the transverse mass of the boson ($M_T$), from the corresponding distribution measured in Z boson decays into two charged leptons. The concept of transverse mass measurement, and in general the selection of Z events to mimic W events, can be applied to Z boson events regarding one of the reconstructed leptons as missing energy. The theoretical description of both decays is similar and the resulting distributions in transverse mass are comparable for a wide range in kinematics. The advantage of this approach is that most of the experimental and theoretical uncertainties, being common between W and Z, cancel in the comparison, leading to a global reduction of the systematic uncertainty. The drawback is a larger statistical uncertainty due to the smaller production rate of Z bosons decaying to charged leptons, being the cross section for the process $pp \rightarrow Z$ with $Z \rightarrow e^+e^-$ or $\mu^+\mu^-$ about 1/10 of the corresponding W cross section. This analysis strategy is based on the prediction of the experimental distribution for W-boson observables scaled to the boson mass from the corresponding distribution measured for Z-bosons decaying into two charged leptons, along with the theoretical ratio between the W and Z cross-sections, calculated at a fixed perturbative order.

The main difference between W and Z boson production is due to the difference between the masses. For this reason, a useful choice to minimize this difference is to introduce the dimensionless ratio between the observable $O_V$ ($V = W, Z$) and the vector boson mass $M_V$:

$$X = \frac{O_V}{M_V}.$$  \hspace{1cm} (1.24)

It is straightforward to define the ratio between the W and Z differential cross sections in terms of the scaled variable $X$:

$$R(X) = \frac{d\sigma_W}{dX_W} \frac{d\sigma_Z}{dX_Z}.$$  \hspace{1cm} (1.25)
Hence, the differential cross section for the $W$ boson as a function of a given observable $O^V (V = W, Z)$ can be ideally predicted from the one measured for $Z$ boson as

$$\left. \frac{d\sigma_W}{dO_W} \right|_{\text{pred}} = \frac{M_Z}{M_W} R(X) \left. \frac{d\sigma_Z}{dO_Z} \right|_{\text{meas}} = \frac{M_Z}{M_W} O_{ZW}$$

where $R(X)$ has to be deduced from theoretical calculations. The parameter $M_W$ can be extracted by comparing this prediction to the distribution for $W$ events observed in the experiment. In practice, additional corrections to $R(X)$ are needed to account for the acceptance to $Z$ and $W$ events and for the experimental resolution. This calls for a detailed understanding of the detector response by means of Monte Carlo simulations compared to control samples. Clearly, the definition of $R(X)$ is the most critical aspect and must include both detector effects and theoretical predictions. This will be further addressed along with the discussion of the systematic limitations.

The precise knowledge of the $Z$ boson mass (about 2 MeV) implies no intrinsic limitations to the accuracy of this approach. Moreover, it has been demonstrated [37] that the ratio between $W$- and $Z$-boson observables can be reliably calculated using perturbative QCD, even when the individual $W$- and $Z$-boson observables are not. This analysis strategy is particularly relevant to the measurement of the $W$ mass through the measurement of the lepton transverse energy distribution, otherwise limited by the large radiative corrections affecting the prediction of the $p_T(W)$ spectrum. The major attention has therefore been payed to the measurement through the electron transverse energy spectrum because, in this case, the method appears more interesting: infact, the $W/Z$ ratio reduces the theoretical uncertainty due to the boson $p_T$ prediction allowing in principle a treatment based on fixed order perturbative calculations. Moreover, this distribution is well under control from experimental point of view because the missing energy due to the neutrino has only an indirect impact through the selection of events.

### 1.5 $W$ and $Z$ boson physics at the LHC

The abundant production of $W$ and $Z$ bosons at the LHC will be exploited, beyond a $W$ mass precision measurement, in a rich physics program since the initial phases of data taking. Infact, even when taking into account the branching ratios into clean leptonic final states (electron and muons) and
reasonable values for reconstruction and selection efficiencies, it is evident that large data samples will be acquired very shortly after the collider start up. This large number of events and the clean experimental signature of the leptonic decays make \( W \) and \( Z \) bosons a very useful tool for many purposes: on the technical side, \( W \) and \( Z \) events will be excellent means for the detector calibration and alignment and essential to tune the generators; moreover, they will allow to complement and extend the tests of the electroweak theory performed so far at the \( e^+e^- \) and \( p\bar{p} \) colliders.

For completeness, some details about \( W \) and \( Z \) bosons production are here reported. A brief overview of the physics measurements with \( W \) and \( Z \) is also given.

In proton-proton collisions, \( W \) and \( Z \) are predominantly produced via the processes illustrated in Fig. 1.6.

The production cross section \( \sigma_{W(Z)} \) for the \( pp \to W + X \) or \( pp \to Z + X \)
processes can be written as

$$\sigma_{W(Z)} = \sum_{ab} \sigma_{ab \rightarrow W(Z)} \otimes PDF(x_a, x_b, Q^2)$$  \hspace{1cm} (1.27)

It is expressed as the convolution of the partonic cross section $\sigma_{ab \rightarrow W(Z)}$ for the process $ab \rightarrow W(Z) + X$ with the Parton Distribution Functions $PDF(x_a, x_b, Q^2)$ of the partons $a$ and $b$ inside the proton, summed over all the partons; $x_a$ and $x_b$ represent the fraction of proton momentum carried by the partons $a$ and $b$.

The theoretical prediction can be compared to the cross section measured from data, in particular from the leptonic $W$ and $Z$ decays. The observed cross sections times the branching ratios is expressed as follows:

$$\sigma_W \cdot BR(W \rightarrow l\nu) = \frac{N_W - N_{bkg}^W}{A_W \cdot \epsilon_W \cdot \mathcal{L}}$$  \hspace{1cm} (1.28)

$$\sigma_Z \cdot BR(Z \rightarrow ll) = \frac{N_Z - N_{bkg}^Z}{A_Z \cdot \epsilon_Z \cdot \mathcal{L}}$$  \hspace{1cm} (1.29)

where $N_{W(Z)}$ and $N_{bkg}^{W(Z)}$ are the observed number of signal ($W$ or $Z$) events and the number of expected background events respectively; $A_{W(Z)}$ is the acceptance of the $W(Z)$ decays, defined as the fraction of these events satisfying the geometric constraints of the detector and the kinematic constraints of the experimental selection criteria; $\epsilon$ is the efficiency for identifying $W(Z)$ falling within the acceptance; finally, $\mathcal{L}$ is the integrated luminosity.

Rate and, hence, cross section measurements will be among the first measurements to be done at the LHC. It has been shown that an accuracy of a few percent will be reachable in the $W$ and $Z$ measured rates, with an integrated luminosity of 1 fb$^{-1}$ which translates in an error on the cross section measurements of a few percent plus the luminosity error (between 5-10% in the initial phase) [21]. As the statistical error is nearly negligible thanks to the high $W$ and $Z$ production rates, the measurement is dominated by systematic errors. An overall theoretical uncertainty at the 2% level has to be considered in the determination of the $W$ and $Z$ acceptances [22]. Another important component of the systematic error is related to the knowledge of the lepton reconstruction efficiency. This is an issue of more general interest and is crucial also for a precision measurement of the $W$ mass. Indeed, the lepton reconstruction efficiencies will be determined accurately from the data themselves, as already done in other collider experiments [23].

The comparison between the theoretical and experimental determination of
the $W$ ($Z$) cross sections can be solved for different variables. It can be used to extract parameters such as $M_W$ and $\Gamma_W$ or, conversely, if the $M_W$ and $\Gamma_W$ values are fixed to the world averages, it is possible to use the $W$ ($Z$) production as luminosity monitors.

At the LHC the study of the production rate of $W$ and $Z$ bosons will be a valuable mean to constrain the Parton Distribution Functions in kinematic regions that are still uncovered by low $Q^2$ experiments: the region corresponding to $W$ and $Z$ production ($Q^2 \sim 10^4$ GeV$^2$, $y < 2.5$) will give direct access at the $x$-range between $10^{-4}$ and $10^{-1}$. The kinematic plane for LHC parton kinematics is shown in Fig. 1.7. The measurement of the $Q^2$ and of the rapidity of the outgoing particles allows to determine the momentum fractions $x_a$, $x_b$ of the incoming partons as follows:

$$x_{a,b} = \sqrt{\frac{Q^2}{s}} e^{\pm y},$$  \hspace{1cm} (1.30)

where $Q$ is the centre of mass energy of the subprocess, $\sqrt{s}$ is the center of mass energy of the reaction ($\sqrt{s} = 14$ TeV at the LHC) and $y$ gives the parton rapidity, and where leading order kinematics is assumed. For the production of $Z$ or $W$ bosons $Q$ is represented by the vector boson mass, therefore the study of the differential rate of events at different boson rapidities explores different regions in $x$ (see Fig. 1.7). Consequently, the observable pseudorapidity distributions of the charged leptons from $W^\pm$ and $Z$ decays are also related to the $x$ distributions of quarks and antiquarks. The shape and the rate as a function of the lepton rapidity or the ratio of these distributions (e.g.: $\sigma_{W^+}/\sigma_{W^-}$) are expected to give strong constraints to the PDF parameters already with few hundred inverse picobarn of data [24, 25].

Improvements in the determination of the PDF are very important in many physics measurements and also in the $W$ mass measurement, since the PDF uncertainty is propagated to an uncertainty on $M_W$.

After the initial phase in which $W$ and $Z$ will be used both for the tuning of the generators (PDFs) and for detector calibration purposes, they will be available for many physics measurements. High precision measurements will be accessible, such as the already discussed measurement of the $W$ boson mass and, with large integrated luminosity ($O(100$ fb$^{-1}$)), the measurement of $\sin^2 \theta_W$ through the forward-backward asymmetry $A_{FB}$ in the Drell-Yan lepton pair production [26]. The LHC will allow to study Drell-Yan lepton pairs in a wide range of invariant mass. This is a benchmark process because
Figure 1.7: Graphical representation of the relationship between parton variables ($x, Q^2$) and the kinematic variables corresponding to final states of mass $M$ produced with rapidity $y$ at the LHC.
any deviation from the Standard Model can be reinterpreted as an effect of new physics that would manifests, for example, as new resonance above the Drell-Yan continuum. The high energy makes also the LHC the ideal place to study di-boson production: gauge boson pairs can be observed already in the initial phase with $1 \text{ fb}^{-1}$ [27], while with higher integrated luminosities ($\geq 30 \text{ fb}^{-1}$) an improvement on the limits on the anomalous triple gauge boson couplings is forseen [28, 29, 30, 31], providing another powerful test of the Standard Model.
Chapter 2

Experimental apparatus

2.1 The Large Hadron Collider

The Large Hadron Collider (LHC) [40] is the proton-proton \((p-p)\) collider under construction at CERN. It will collide protons with a center of mass energy \(\sqrt{s}= 14\) TeV with a design luminosity \(L=10^{34}\) cm\(^{-2}\)s\(^{-1}\). During the first 3 years of data taking, the luminosity is expected to be \(2\times10^{33}\) cm\(^{-2}\)s\(^{-1}\) (the so called “low luminosity” phase), while at the start up LHC is forseen to run at \(L=10^{32}\) cm\(^{-2}\)s\(^{-1}\).

LHC is being installed in the already existent LEP tunnel and the available CERN accelerators will be employed in the injection chain: the proton beam exiting a small linear accelerator at 50 MeV, will be injected in the PS at 1.4 GeV, then in the SPS at 25 GeV, and finally in the LHC ring at 450 GeV (Fig. 2.1).

One of the critical aspects in accelerating the protons up to an energy of 7 TeV is the required bending magnetic field which, for the LHC bending radius \((R \sim 2780\) m) is about 8.4 T. This magnetic field will be provided by the 1232 LHC superconducting 14.2 m long dipole magnets, placed in the eight curved sections which connect the straight sections of the LHC ring. The super-conducting magnets use a Ni-Ti conductor, cooled down to 1.9 K, by means of super-fluid Helium. The choice of a \(p-p\) collider obliges to install two separate magnetic chambers which, for economical reasons, will lay in the same mechanical structure and cryostat.

The high luminosity of the LHC is obtained by a high frequency of bunch crossing and by a high number of protons per bunch: two beams of protons with an energy of 7 TeV, circulating in two different vacuum chambers, will
Chapter 2 – Experimental apparatus

Figure 2.1: Scheme of the LHC injection chain.

contain each 2808 bunches filled with about $1.15 \times 10^{11}$ protons. The beams will cross at the rate of 40 MHz, at the interaction point, with a spread of 7.5 cm along the beam axis and 15 μm in the transverse directions. The main machine parameters are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Beam parameters</th>
<th>Technical parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>7 TeV</td>
</tr>
<tr>
<td>Maximum luminosity</td>
<td>$10^{34}$cm$^{-2}$s$^{-1}$</td>
</tr>
<tr>
<td>Time between collisions</td>
<td>25 ns</td>
</tr>
<tr>
<td>Bunch length</td>
<td>7.7 cm</td>
</tr>
<tr>
<td>RMS beam radius at the interaction point</td>
<td>16.7 μm</td>
</tr>
<tr>
<td>Ring length</td>
<td>26658.9 m</td>
</tr>
<tr>
<td>Radiofrequency</td>
<td>400.8 MHz</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>2808</td>
</tr>
<tr>
<td>Number of dipoles</td>
<td>1232</td>
</tr>
<tr>
<td>Dipole magnetic field</td>
<td>8.33 T</td>
</tr>
</tbody>
</table>

Table 2.1: The relevant LHC parameters for $p - p$ collisions.

The operating conditions at the LHC are extremely challenging for the experiments. The $p - p$ total inelastic cross section at $\sqrt{s} = 14$ TeV is about 80 mb, several orders of magnitude larger than the typical cross section for events with large momentum transfer (see Fig. 1.4). Most of the inelastic events consist of soft $p - p$ interactions characterized by outgoing particles
with a low transverse momentum. These events are referred to as minimum bias. It is expected that each bunch crossing will produce about 20 minimum bias events in the high luminosity phase and 5 minimum bias events in the low luminosity phase. Hence, each interesting events will be readout entangled with a large number of minimum bias events, which constitute the pile-up. The high interaction rate ($\sim 10^9$ events/s) and the high bunch crossing frequency impose stringent requirements on the data acquisition and trigger systems and on the detectors. The trigger has to provide an high rejection factor, maintaining at the same time a high efficiency in selecting the interesting events. The detectors has a fast response time (25-50 ns) and a fine granularity (and therefore a large number of readout channels) in order to minimize the effect of the pile-up. Furthermore the high flux of particles coming from the $p-p$ interactions implies that each component of the detector, including the read-out electronics, has to be radiation resistant.

2.2 The CMS detector

The CMS (Compact Muon Solenoid) detector is one of the two general purpose experiment that will take data at the LHC. The CMS structure is a typical one for experiments at colliders: a cylindrical central section (the barrel) closed at its end by two caps (the endcaps), as sketched in Fig. 2.2. The coordinates system in CMS are chosen with the $z$ axis along the beam direction, the $x$ axis directed toward the center of the LHC ring and the $y$ axis directed upward, orthogonally to the $z$ and $x$ axes. Given the cylindrical structure of CMS, a convenient and commonly used coordinate system is $r, \phi, \eta$, where $r$ is the distance from the $z$ axis, $\phi$ is the azimuthal angle in the $xy$ plane and $\eta$ is the pseudorapidity defined as

$$\eta = -\ln \left( \tan \left( \frac{\theta}{2} \right) \right),$$

where $\theta$ is the angle with respect to the beam axis. The use of pseudorapidity instead of the polar angle is motivated by the fact that the difference in pseudorapidity between two particles is invariant under Lorentz boosts along the beam axis.

CMS is characterized by high hermeticity with a full coverage in $\phi$ and up to $\eta = 5$ in pseudorapidity. The detector is constituted by different sub-detectors with different tasks. Starting from the innermost region to the
outermost they are: the inner tracking system; the electromagnetic calorimeter (ECAL); the hadron calorimeter (HCAL) and the muon chambers. A characteristic feature of CMS is that it is embedded in a 4 T axial solenoidal magnetic field, which leaded to a compact design for the detector.

In the following, the main characteristics of the CMS subdetectors are reported, with major emphasis on those aspects that are involved in the reconstruction of $W \rightarrow e\nu$ events, such as the calorimetry, the tracking and the trigger system. Among them, more details are given about the electromagnetic calorimeter and its calibration and about the electron trigger, as they are of direct interest in the measurement of the $W$ mass.

### 2.2.1 The tracking system

CMS tracker [41] is the subdetector closer to the interaction point, placed in the 4 T magnetic field of the superconductive solenoid.

It is designed to determine the interaction vertex, measure with good accuracy the momentum of the charged particles, identify the presence of secondary vertices.

The tracker must be able to operate without degrading its performances in the hard radiation environment of LHC and it has to comply with se-
vere material budget (see Fig. 2.3) constraints, in order not to degrade the excellent energy resolution of the electromagnetic calorimeter. The CMS collaboration has adopted silicon technology for the whole tracker. Three regions can be delineated, considering the charged particle flux at different radii at high luminosity:

- Closest to the interaction vertex where the particle flux is highest (\( \sim 10^7 \text{s}^{-1} \) at \( r \sim 10 \text{ cm} \)) pixel detectors are placed. The size of the pixels is \( \sim 100 \times 150 \mu \text{m}^2 \), leading to an occupancy of \( 10^{-4} \) per pixel per LHC crossing.

- In the intermediate region with \( 20 \text{ cm} < r < 55 \text{ cm} \) the particle flux becomes low enough to allow the use of silicon microstrip detectors, with a minimum cell size of \( \sim 10 \text{ cm} \times 80 \mu \text{m} \), giving an occupancy of 2-3% per LHC crossing.

- The outermost region is characterized by sufficiently low fluxes that enable to adopt larger-pitch silicon microstrips with a maximum cell size of \( \sim 25 \text{ cm} \times 80 \mu \text{m} \), keeping the occupancy to \( \sim 1\% \).
The pixel detector consists of three barrel layers and two endcap disks at each side (Fig. 2.4). The barrel layers are located at 4.4 cm, 7.3 cm and 10.2 cm and are 53 cm long. The two end disks, extending from 6 to 15 cm in radius, are placed on each side at $|z| = 34.5$ cm and 46.5 cm. This design allows to obtain at least two points per track in the $|\eta| < 2.2$ region, for tracks originating within $2\sigma_z$ from the central interaction point. The total number of channels is about 44 millions, organized in about 16000 modules of 52 columns and 80 rows. The total active area is close to 0.92 m$^2$. The presence of high magnetic field causes a noticeable drift of the electrons (and a smaller drift for the holes) from the ionization point along the track with a Lorentz angle of about $32^\circ$. This leads to a charge sharing between pixels which, using an analog readout, can be exploited to considerably improve the resolution, down to about 10 $\mu$m. In the endcap the modules of the detector are arranged in a turbine-like shape with a $20^\circ$ tilt, again in order to enhance the charge sharing.

The inner and outer tracker detector are based on silicon strips. They are $p^+$ strips on a $n$-type bulk whose thickness is close to 300 and 500 $\mu$m respectively in the inner and outer tracker. In the barrel the strips are parallel to the beam axis while for the endcaps they have a radial orientation. The inner tracker is made of 4 barrel layers, the two innermost are double
sided, and the endcaps count 3 disks each. The outer tracker consists of 6 layers in the barrel (the two innermost are double sided) while the endcaps are made of 9 layers (the first, the second and the fifth are double sided). On the whole the silicon trackers is made of about 10 millions of channels for an active area close to 198 m$^2$.

2.2.2 The electromagnetic calorimeter

The electromagnetic calorimeter (ECAL) measures the energy of the electrons and photons. The design of the CMS ECAL [42] was driven by the requirements imposed by the search of the Higgs boson in the channel $H \rightarrow \gamma \gamma$, where a peak in the di-photon invariant mass placed at the Higgs mass, has to be distinguished from a continuous background. A good resolution and a fine granularity are therefore required: both of them improve the invariant mass resolution on the di-photon system by improving respectively the energy and angle measurement of the two $\gamma$s. The fine granularity also helps to obtain a good $\pi^0/\gamma$ separation.

In order not to deteriorate the energy resolution the ECAL is placed inside the solenoid, hence compact calorimeter is required. ECAL is a hermetic, homogeneous calorimeter made of lead tungstate (PbWO$_4$) crystals, 61200 crystals mounted in the central barrel part, and 7324 crystals in each endcap (Fig. 2.5). The choice of lead tungstate scintillating crystals was driven by the characteristics of these crystals: they have a short radiation length ($X_0 = 0.89$ cm) and a small Moliere radius ($R_M = 2.2$ cm); they are fast, as the 80% of the scintillation light is emitted within 25 ns and radiation hard. The use of PbWO$_4$ crystals has thus allowed the design of a compact calorimeter to be placed inside the solenoid, and that is fast, with fine granularity and radiation resistant. However the relative low light yield (30 $\gamma$/MeV) requires the use of photodetectors with intrinsic gain that can operate in a magnetic field. In the barrel, silicon avalanche photodiodes (APDs) are used as photodetectors, while vacuum phototriodes (VPTs) have been chosen for the endcaps. In addition, the sensitivity of both the crystals and the APDs response to temperature changes requires temperature stability. A water cooling system guarantees a long term stability at the 0.1$^\circ$C [43] in order to preserve the ECAL energy resolution performances.

The barrel region has a pseudorapidity coverage up to $|\eta| < 1.479$. It has an inner radius of 129 cm and is structured in 36 supermodules, each containing
1700 crystals, covering half the barrel length and subtending a $20^\circ$ angle in $\phi$. Each supermodule is divided along $\eta$ into four modules which in their turn are made of submodules, the basic assembling alveolar units, containing $5 \times 2$ crystals each. The barrel crystals have a front face cross-section of $\sim 22 \times 22 \text{ mm}^2$ and have a length of 230 mm, corresponding to $25.8X_0$. The crystal axes are oriented with a $3^\circ$ tilt with respect to the pointing geometry, as can be seen in Fig. 2.6 to avoid that the particles can directly escape into the dead regions between the crystals. The granularity of the barrel is $\Delta \Phi \times \Delta \eta = 0.0175 \times 0.0175$ and the crystals are grouped, from the readout point of view, into $5 \times 5$ arrays corresponding to the trigger towers.

The endcaps cover the pseudorapidity region $1.48 < |\eta| < 3.0$, ensuring precision measurements up to $\eta < 2.5$. The endcap crystals have dimensions of $28.6 \times 28.6 \times 220 \text{ mm}^2$ [38]. Each endcap is structured in two “Dees” consisting of semi-circular aluminum plates from which are cantilevered structural units of $5 \times 5$ crystals, known as “supercrystals”.

A preshower device, whose principal aim is to identify neutral pions in the endcaps within $1.653 < |\eta| < 2.6$, is placed in front of the crystal calorimeter. The active elements are two planes of silicon strip detectors which lie behind disks of lead adsorber at depths of $2X_0$ and $3X_0$. 

Figure 2.5: Scheme of the barrel and of the endcaps of the CMS ECAL.
2.2 The CMS detector

ECAL calibration

Calibration is a severe technical challenge for the operation of the CMS electromagnetic calorimeter. It can be seen as composed of a global component, giving the absolute energy scale, and a channel-to-channel relative component, which is referred to as intercalibration.

The final goal of the calibration strategy is to achieve the most accurate energy measurement for electrons and photons. Schematically, the reconstructed energy can be factorized into three terms:

\[ E_{e,\gamma} = G \times F \times \sum_i c_i \times A_i \]  \hspace{1cm} (2.2)

where \( G \) is the global absolute scale; the function \( F \) is a correction function depending on the type of particle, its position, its momentum and on the clustering algorithm used. The \( c_i \) are the intercalibration coefficients and \( A_i \) are the signal amplitudes, which are summed over all the clustered crystals.

The intercalibration at the start-up relies on laboratory measurements of the crystal light yield, on test beam precalibration of some supermodules and on the commissioning of the supermodules with cosmic rays.

After the assembly of the detector, the *in situ* calibration with physics events will be performed, exploiting different tools:
• The $\phi$-symmetry [44, 45] of the deposited energy allows to rapidly improve the intercalibration within rings at constant $\eta$: with few tens of millions of events, equivalent to about 10 hours of data taking under the assumption of 1 kHz of Level-1 trigger bandwidth, it is possible to approach the limit precision of 1.5% in the barrel region and 1-3% in the endcap region. The main limit on precision comes from non-uniformities and inhomogeneities in the tracker material.

• The measurement of the momentum of isolated electrons from $W \rightarrow e\nu$ will provide a useful tool to intercalibrate crystals, once the tracker is fully operational and well aligned. The calibration precision dependence as a function of $\eta$ follows the tracker material budget distribution and it depends strongly on the available statistics; a precision better than 1.5% can be achieved in the barrel region with 5 fb$^{-1}$ [46].

• The $Z$ mass constrain in $Z \rightarrow ee$ decays is a further powerful mean for ECAL calibration. Many uses are envisaged, from the tuning of algorithmic corrections, to the intercalibration of ECAL regions (as a complement to the $\phi$-symmetry method at the start-up), to the absolute scale calibration [47]. A statistical precision on the absolute energy scale at the level of 0.05% is forseen with an integrated luminosity of 2 fb$^{-1}$.

• The calibration using mass reconstruction of $\pi^0$, $\eta \rightarrow \gamma\gamma$ and $Z \rightarrow \mu\mu\gamma$ is also being investigated.

**ECAL targets and performaces**

One of the relevant issue in evaluating the performances of the electromagnetic calorimeter is its energy resolution.

In the relevant energy range between 25 GeV and 500 GeV, the energy resolution is usually parametrized as the sum in quadrature of three different terms:

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c,$$

where $a, b$ and $c$ are named respectively stochastic, noise and constant term, and $E$ is the energy expressed in GeV.

The target values for CMS are 2.7% for $a$, 200 MeV when adding the signal of 5×5 crystals for $b$, and 0.5% for $c$. Their relative contributions are reported
in the plot of Fig. 2.7. It can be noticed that above 50 GeV the resolution is dominated by the constant term. Different effects contribute to the different terms in equation 2.3:

- the stochastic term $a$ receives a contribution from the fluctuations in the number of electrons which reaches the preamplifier ($n_e$). These fluctuations are proportional to $\sqrt{n_e}$ and therefore proportional to the square root of the deposited energy. Contributions to this term come from the light yield of the crystals, from the efficiency in collecting light onto the photodetector surface and from the quantum efficiency of the photodetector. The fluctuations in the electrons multiplication process also contribute to this term, as an excess noise term $F$.

- The noise term $b$ accounts for all the effects that can alter the measurements of the energy deposit independently of the energy itself. This term receives contributions from the electronic noise and from

Figure 2.7: The expected ECAL energy resolution versus the energy of the impacting electron. The different contributions are superimposed separately. The term “Intrinsic” includes the shower containment and a constant term (0.55%).
the pile-up events, whose contribution are different in the barrel and in the endcaps and can vary with the luminosity of LHC. The target values for the barrel (at $\eta = 0$) and the endcaps (at $\eta = 2$) in the low luminosity running are respectively 155 MeV and 205 MeV while in the high luminosity running they are 210 MeV and 245 MeV.

- The constant term determines the energy resolution at high energy. Many different effects contribute to this term: the stability of the operating conditions such as the temperature and the high voltage of the photodetectors; the rear and lateral leakage of the electromagnetic shower and the presence of the dead material of the supporting structure between the crystals; the light collection uniformity along the crystal axis; the intercalibration between the channels which contributes almost directly to the overall energy resolution since the most of the energy is contained into few crystals; the radiation damage of the PbWO$_4$ crystals $^1$.

### 2.2.3 The hadron calorimeter

The hadron calorimeter (HCAL) $[48]$, placed just outside the electromagnetic calorimeter, plays a major role in the reconstruction of jets and missing energy. Its resolution must guarantee a good reconstruction of the di-jets invariant mass and an efficient measurements of the missing energy which represent an effective signature in many channels of physics beyond the Standard Model. Similarly to the other subdetectors, HCAL has to provide a good hermeticity, which is critical for determining the missing energy, and a quite fine granularity to allow a clear separation of di-jets from resonance decays and improve the resolution in the invariant mass of the di-jets. Moreover it has to provide a number of interaction lengths sufficient to contain the energetic particles from high transverse momentum jets. The dynamic range has to be large to to detect signals ranging from the signal of a single minimum ionizing muon up to an energy of 3 TeV.

The pseudorapidity region $|\eta| < 3$ is covered by the barrel (up to $|\eta| < 1.74$) and the two endcaps. The HCAL is composed by brass layers as absorbers

$^1$Changes in the crystal transparency caused by irradiation and subsequent annealing lead to a variation in the crystal response to a given deposited energy; this effect develops on a short term scale ($\sim$ hours) and need to be tracked and corrected properly using a laser monitoring system.
2.2 The CMS detector

interleaved by plastic scintillator layers, 4 mm thick, used as active medium. The absorber layers thickness is between 60 mm thick in the barrel and 80 mm in the endcaps, while the scintillators layers are 4 mm thick. In terms of interaction lengths $\lambda$, the barrel ranges from $5.46\lambda$ at $|\eta|=0$ up to $10.82\lambda$ at $|\eta|=1.3$; the barrel corresponds on average to $11\lambda$. The scintillator in each layer is divided into tiles with a granularity matching the granularity of the ECAL trigger towers ($\Delta\eta \times \Delta\phi = 0.0875 \times 0.0875$) and the light is collected by wavelength shifters.

The two hadronic forward calorimeters improve the HCAL hermeticity, covering the pseudorapidity region $3<|\eta|<5$. It is placed at 11.15 m from the interaction point outside the magnetic field. Due to the extremely harsh radiation environment a different detection technique is used: a grid of quartz (radiation hard) fibers is embedded in a iron absorber.

2.2.4 The magnetic field

An important aspect of the CMS experiment is its axial high magnetic field. The magnet system of CMS [49] is composed of three main parts: the superconducting solenoid, the barrel return yoke and the endcap return yoke. The 4 T magnetic field allows to measure efficiently the muon momentum up to a pseudorapidity of 2.4. The return yoke is made of iron and contains the muon detectors. It is a 12-sided cylindrical structure, with a total length is about 11 m and it is divided into five rings of about 2.5 m each. It has an outer diameter of 14 m and a total weight of about 7000 tons. Each rings is divided into three iron layers where the muon detectors are inserted. The thickness of the border layers is 630 mm and the middle layer is 295 mm thick. Each endcap is composed by three independent disks, the outermost is 300 mm thick and the others are 600 mm thick.

The superconductive coil is housed into a vacuum tank and kept at the temperature of the liquid helium. The vacuum tank is supported only by the central barrel ring of the yoke and in its turn supports the calorimeter system (ECAL and HCAL) and the tracker.

2.2.5 The muon system

In CMS the muon detectors are placed beyond the calorimeters and the solenoid. The muon system [50] consist of four active stations interleaved by the iron absorber layers which constitute the return yoke for the magnetic
field. Three different typology of detectors are employed: drift tubes (DT) in the barrel region; cathod strip chambers (CSC) in the endcaps and resistive plate chambers (RPC) in both the regions in addition to DT and CSC. The DT and CSC are used to reconstruct the position and, hence, the muon momentum; while RPCs have an excellent time resolution (better than 2ns) and are therefore suitable for trigger purpose.

2.2.6 The trigger system

At the nominal LHC luminosity, the expected event rate is about $10^9$ Hz. Given the typical size of a raw event ($\sim 1$ MB) it is not possible to record all the informations for all the events. Indeed, the event rate is largely dominated by soft $p-p$ interactions with particles of low transverse momentum. The triggering system must have a large reduction factor and maintain at the same time an high efficiency on the potential interesting events, reducing the rate down to 100 Hz, which is the maximum sustainable rate for storing events. The trigger system consists of two main steps: a Level 1 Trigger and a High Level Trigger. The basic concepts will be described in the following with particular attention to the electron and photon trigger. A complete description can be found in [51].

The Level 1 trigger

The Level 1 trigger (L1) reduces the rate of selected events down to 50 (100) kHz for the low (high) luminosity running. The full data are stored in pipelines of processing elements, while waiting for the trigger decision. The L1 decision about taking or discarding data from a particular bunch crossing has to be taken in 3.2 $\mu$s. If the L1 accepts the event, the data are moved to be processed by the High Level Trigger. To deal with the 25 ns bunch crossing rate, the L1 trigger has to take a decision in a time too short to read data from the whole detector, therefore it employs the calorimetric and muons informations only, since the tracker algorithms are too slow for this purpose. The Level-1 trigger is organized into a Calorimeter Trigger and a Muon trigger whose informations are transferred to the Global Trigger which takes the accept-reject decision. The Calorimeter Trigger is based on trigger towers, arrays of 5 crystals in ECAL, which match the granularity of the HCAL towers. The trigger towers
are grouped in calorimetric region of $4 \times 4$ trigger towers. The Calorimeter Trigger identifies, from the calorimetric region information, the best four candidates of each of the following classes: electrons and photons, central jets, forward jets and $\tau$-jets identified from the shape of the deposited energy. The information of these objects is passed to the Global Trigger, together with the measured missing $E_T$. The Muon trigger is performed separately for each muon detector (see Sec. 2.2.5). The information is then merged and the best four muon candidates are transferred to the Global Trigger. The Global Triggers takes the accept-reject decision exploiting both the characteristic of the single objects and of combination of them.

The High Level Trigger

The High Level Trigger reduces the output rate down to 100 Hz. The idea of the HLT trigger software is the regional reconstruction on demand, that is only those objects in the useful regions are reconstructed and the uninteresting events are rejected as soon as possible. This leads to the development of three “virtual trigger” levels: at the first level only the full information of the muon system and of the calorimeters is used, in the second level the information of the tracker pixels is added and in the third and final level the full event information is available.

Electron and photon trigger

The L1 electromagnetic trigger is based on ECAL trigger towers. No distinction between electrons and photons are made at this stage. Energy deposits in trigger towers are classified as isolated or non-isolated according to the requirements described in [51]. The relaxed triggers accept both isolated and non-isolated trigger deposits. To identify electrons and photons the HLT can exploit the full granularity of the ECAL, and more refined algorithms that group the ECAL crystals into super-clusters [51]. The separation of electrons and photons is obtained looking for hits in the pixel detector compatible with the ECAL super-cluster. The matched super-clustered are considered electrons while the unmatched are classified as photons. For the start-up luminosity of $10^{32}\text{cm}^{-2}\text{s}^{-1}$, ten HLT trigger paths are foreseen for the electromagnetic triggers; they are listed in Table 2.2 with the corresponding transverse energy thresholds.
The electron HLT sequences are based essentially on the following [38]:

<table>
<thead>
<tr>
<th>HLT path</th>
<th>$E^T$ threshold (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single isolated electron</td>
<td>15</td>
</tr>
<tr>
<td>Single relaxed electron</td>
<td>17</td>
</tr>
<tr>
<td>Double isolated electron</td>
<td>10</td>
</tr>
<tr>
<td>Double relaxed electron</td>
<td>12</td>
</tr>
<tr>
<td>Single isolated photon</td>
<td>30</td>
</tr>
<tr>
<td>Single relaxed photon</td>
<td>40</td>
</tr>
<tr>
<td>Double isolated photon</td>
<td>20</td>
</tr>
<tr>
<td>Double relaxed photon</td>
<td>20</td>
</tr>
<tr>
<td>Single high energy EM</td>
<td>80</td>
</tr>
<tr>
<td>Single very high energy EM</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 2.2: Transverse energy thresholds for the HLT electromagnetic trigger paths.

- ECAL superclusters with a transverse energy above a certain threshold (dependent on the HLT path) are reconstructed in the regions corresponding to the L1 triggered events.

- The hadronic energy deposition in a cone around the supercluster direction is required to be lower than a certain threshold.

- The supercluster is matched to the pixel hits: the energy and position of the supercluster are propagated back through the magnetic field to search for compatible hits in the first layer of the pixel detector within a window of 40 mrad in $\phi$; this search window is about 1/5 narrower than the one used for in the offline electron tracking and allows to reject a large fraction of QCD events.

- The track is reconstructed using the a combinatorial Kalman Filter method [38] and, for the single-electron trigger only, the $E/p$ matching between the energy reconstructed in ECAL and the corresponding track momentum measured in the tracker is required ($E/p < 1.5$ for electrons in the barrel region, $E/p < 2.45$ in the endcaps).

- The electron track is finally requested to be isolated.

The isolation in the hadron calorimeter and isolation in the tracker are the most effective isolation variables against QCD background. In the hadron
calorimeter, the energy collected in a cone around the supercluster is required to be below a certain threshold (3 GeV for single-electron trigger and 9 GeV for double-electrons). The track isolation is defined by the sum of the $|p_T|$ of the tracks in the region between two cones around the electron candidate track direction. An inner cone of radius 0.02 to exclude the electron track and an outer cone of 0.2 are chosen. Only tracks with $|p_T| > 1.5$ GeV and with a z-impact parameter consistent with the electron candidate z-impact parameter, namely $|z_e - z_{track}| < 0.1$ cm, are considered. The quantity $|p_T|/p_{electrons}^T$ is required to be below 0.06 for single-electron and below 0.4 for double-electron.

According to the thresholds in table 2.2, the total rate for the isolated single electron stream is about 16.9 Hz: 0.4 Hz from $W \to e\nu$, 16.4 Hz from jets and 0.1 Hz from $Z \to ee$ events. The background rate is strongly dependent on the $E_T$ threshold and can be reduced by increasing it. Studies for the $2 \times 10^{33}$ cm$^{-2}$s$^{-1}$ have shown that with an $E_T$ threshold for the single electron HLT at 26 GeV, the rate of jet events is brought in a ratio about 1:1 with respect to the $W \to e\nu$ rate (9.8 Hz for $W \to e\nu$ and 9.4 Hz for the QCD background) [38]. This threshold value at 26 GeV has been used for the selection of the $W \to e\nu$ events for the $W$ mass measurement.
Chapter 3

Simulation and reconstruction

The feasibility of a precision $W$ mass measurement with the “Scaled Observables Method” has been studied by means of a detailed simulation of the CMS detector. In this Chapter, the main features of the simulation are presented and the reconstruction of the physics objects used to identify $W \rightarrow e\nu$ and $Z \rightarrow ee$ events (in particular the electron and the missing energy related to the presence of the neutrino) is described.

3.1 Detector simulation

The number of $W$ and $Z$ events to be simulated in order to perform a study on statistical samples corresponding to 1 fb$^{-1}$ is very large ($\approx$20 millions of $W \rightarrow e\nu$ and $\approx$2 millions $Z \rightarrow ee$) and makes necessary the use of a fast simulation of the CMS detector.

The analysis of the $W$ mass measurement has, therefore, been performed in the CMS fast simulation framework, referred to as FAMOS [56]. The reliability of the fast simulation has been evaluated by comparisons to smaller data samples produced through the full simulation and reconstruction chain known as ORCA [55].

3.1.1 Full simulation in the ORCA framework

The detailed simulation, referred as OSCAR [53], is based on the GEANT4 [54] simulation toolkit. It provides a rich set of physics processes describing electromagnetic and hadronic interactions and tools for modelling the full CMS
detector geometry and the interfaces for retrieving informations from particle tracking through the detectors and the magnetic field. The full simulation program implements the sensitive detector behaviour, track selection mechanism, hit collection and digitization, which constitutes the simulation of the electronic readout used to acquire data and the DAQ system. The reconstruction step, following the digitization one, is the operation of constructing physics quantities from the raw collected data. The collection of the full detector simulation and reconstruction code developed in this framework is known as ORCA (Object-oriented Reconstruction for CMS Analysis).

3.1.2 FAMOS: the FAst MOntecarlo Simulation

The framework for the fast simulation of particle interactions in the CMS detector is known as FAMOS, FAst MOntecarlo Simulation [56]. The interactions simulated in FAMOS are (1) electron bremsstrahlung; (2) photon conversion; (3) charged particle loss by ionization; (4) charged particle multiple scattering; (5) electron, photon and hadron showering. The first 4 are applied to particles passing through the tracker layers, while the latter is parametrized in ECAL and HCAL. The fast simulation of the muon interaction is also based on a parametrization of resolutions and efficiencies.

As output, FAMOS delivers a series of “high-level objects”, such as the reconstructed hits for charged particles in the tracker layers, energy deposits in the calorimeters cells, which can be immediately be used as inputs of the same “higher-level algorithms” (track fitting, calorimeter clustering, electron identification, jet reconstruction and calibration, etc.) as in the full reconstruction and analysis package. This parallelism between fast and complete reconstruction allows for comparisons between the fast and full simulations and subsequent tuning in a straightforward manner.

The CPU time needed to simulate an event in FAMOS is about 3 orders of magnitude smaller than that needed in the full chain, for a level of agreement at the percent level or below.

3.1.3 The CMSSW framework

Recently, a comprehensive set of changes have been made in the underlying software framework, the services it provides and the model of data storage in preparation to the LHC data-taking. These changes address additional requirements on the software to implement calibration and aligne-
ment strategies, ensure tracking and reproducibility of the reconstruction results, simplify and standardize the way physicists develop reconstruction algorithms and facilitate the interactive analysis. The overall collection of software is referred to as CMSSW [52] and is built around a Framework, an Event Data Model and Services needed by the simulation, calibration, alignment and reconstruction modules that process the event data.

Some specific studies (in particular on electron efficiency and QCD background rejection) in this work have been performed using samples simulated and reconstructed with CMSSW, after the validation against the ORCA full simulation.

### 3.2 Simulated samples

The study of the $W$ mass measurement have been performed using large data samples produced with FAMOS. Signal and background events were generated with PHYTEA 6.227 [57] using the CMS software package CMKIN [58] to interface the generator to the detector simulation. Table 3.1 summarizes the samples generated the for the signal and the main backgrounds. $W \to e\nu$ events have been generated with $M_W = 80.45$ GeV; $Z$ events have been generated with $M_Z = 91.187$ GeV and with an invariant mass $M_{ee}$ greater than 50 GeV.

<table>
<thead>
<tr>
<th>Event sample</th>
<th>Cross section</th>
<th>Number of simulated events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W \to e\nu$</td>
<td>17.4 nb</td>
<td>$27 \times 10^6$</td>
</tr>
<tr>
<td>$Z \to ee$</td>
<td>1.75 nb</td>
<td>$12.5 \times 10^6$</td>
</tr>
<tr>
<td>$Z \to \tau\tau$ (all $\tau$ decays)</td>
<td>1.75 nb</td>
<td>$1.75 \times 10^6$</td>
</tr>
<tr>
<td>$W \to \tau\nu$ ($\tau \to e\nu\nu_\tau$)</td>
<td>3.2 nb</td>
<td>$3.2 \times 10^6$</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>488 pb</td>
<td>$488 \times 10^3$</td>
</tr>
</tbody>
</table>

Table 3.1: Cross sections and number of events generated with PYTHIA and simulated with FAMOS for signal ($W \to e\nu$, $Z \to ee$) and backgrounds ($Z \to \tau\tau$, $W \to \tau\nu$, $t\bar{t}$).

Smaller samples of fully simulated events are used for cross checks. About 200 000 events belonging to a fully simulated $W \to e\nu$ dataset have been studied for such purpose. This dataset was generated with preselection cuts ($p_T^{e} > 25$ GeV and $|\eta| < 2.7$) and the digitisation was performed within the ORCA framework including the effects of pile-up at low luminosity ($2 \times 10^{33}$ cm$^{-2}$s$^{-1}$). Both the fast and full simulation frameworks allow to overlay pile-up.
up events on top of real signal events, according to a Poisson distribution with average value $\mu$ equal to the expected average number of inelastic non-diffractive interactions per bunch crossing. At low luminosity, $\mu=5$. These events are sampled from a data base of minimum bias events, generated with the parameters reported in [34].

The fully simulated CMSSW samples used for the QCD background analysis are $W \rightarrow e\nu$ events and QCD di-jet events generated in different $p_T$ ranges \(^1\) (see Table 3.2).

<table>
<thead>
<tr>
<th>Event sample</th>
<th>Cross section</th>
<th>Number of simulated events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W \rightarrow e\nu$</td>
<td>17.4 nb</td>
<td>$7 \times 10^5$</td>
</tr>
<tr>
<td>QCD 20-30</td>
<td>0.63 mb</td>
<td>$0.97 \times 10^5$</td>
</tr>
<tr>
<td>QCD 30-50</td>
<td>0.15 mb</td>
<td>$1.93 \times 10^5$</td>
</tr>
<tr>
<td>QCD 50-80</td>
<td>0.02 mb</td>
<td>$13.8 \times 10^5$</td>
</tr>
<tr>
<td>QCD 80-120</td>
<td>0.003 mb</td>
<td>$5.95 \times 10^5$</td>
</tr>
</tbody>
</table>

Table 3.2: Cross section and number of events simulated with CMSSW for $W \rightarrow e\nu$ and QCD di-jets.

### 3.3 Reconstruction

In this Section, the electron reconstruction and the missing transverse energy reconstruction (related to the presence of a neutrino in the final state) are treated. Moreover, jet reconstruction is briefly discussed, as a jet veto is imposed in the selection of events (see Sec. 4.1).

#### 3.3.1 Electron reconstruction and identification

An electron is characterized by an energy deposit in the electromagnetic calorimeter and a track pointing towards it. Electromagnetic showers deposit their energy in several crystals in the ECAL. The presence in CMS of material in front of the calorimeter results in bremsstrahlung and photon conversions. Because of the strong 4 T magnetic field the energy reaching the calorimeter is spread in the $\phi$ direction. The spread energy is clustered by building a cluster of clusters, called "supercluster", which is extended in $\phi$. Details about superclustering algorithms can be found in [51].

Since electrons are affected by non-gaussian fluctuations due to bremsstrahlung, \(p_T\) is the parton transverse momentum of the 2$\rightarrow$2 process in the center of mass frame.
haloing emission along their trajectory, dedicated track reconstruction strategies are used. Differently from the default CMS track reconstruction method based on the Kalman Filter, a non linear filter approach such as the Gaussian Sum Filter (GSF), along with a specific Bethe-Heitler modeling of the electron energy losses during track building [59], has been found to better describe the propagation of electrons.

Compatibility between the supercluster and track angular position and energy estimate are required to select electron candidates in an event. The list of electron candidates, that is superclusters with a matched track, is obtained from dedicated algorithms described in [38].

For this specific study on $W \rightarrow e\nu$ events, electrons are selected from a sample of events passing the single-electron HLT (see Section 2.2.6). The HLT efficiency is about 38% for an $E_T$ threshold fixed at 26 GeV and 51% for the $E_T$ threshold fixed at 15 GeV. These efficiencies are to be considered relative to a sample of $W \rightarrow e\nu$ events preselected at generation by requiring the electron pseudorapidity $|\eta| < 2.5$ and the electron transverse momentum $p_T > 5$ GeV. The efficiencies for each selection of the single electron HLT sequence are reported in Table 3.3. Beyond the $E_T$ cut, the step that mostly affects the resulting efficiency is the matching between the supercluster and the track and the filter based on the quantity $E/P$.

The energy deposition and the shower shape in the calorimeters are then re-

<table>
<thead>
<tr>
<th>Selection</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 match</td>
<td>75.9%</td>
</tr>
<tr>
<td>$E_T$ filter ($E_T &gt;15$ GeV)</td>
<td>95.5%</td>
</tr>
<tr>
<td>Hcal isolation</td>
<td>99%</td>
</tr>
<tr>
<td>Supercluster-pixel match</td>
<td>88.2%</td>
</tr>
<tr>
<td>$E/P$ filter</td>
<td>83.3%</td>
</tr>
<tr>
<td>Track isolation</td>
<td>97%</td>
</tr>
</tbody>
</table>

Table 3.3: Efficiencies of the different selections in the isolated single-electron HLT sequence. The efficiencies are computed for a sample of simulated $W \rightarrow e\nu$ events with preselections at the generator level ($|\eta| < 2.5$ and $p_T > 5$ GeV).

required to be consistent to the one characteristic of an electron. An isolation criterion is also applied, as electrons from $W$ and $Z$ decays are expected to be well isolated from possible hadron jets in the event. This is accomplished through the following set of selections:
• the difference in pseudorapidity between the track and the supercluster in the electromagnetic calorimeter is required to satisfy $\Delta \eta = |\eta_{\text{trk}} - \eta_{\text{SC}}| < 0.005$;

• the difference in $\phi$ between the track and the supercluster in the electromagnetic calorimeter is required to satisfy $\Delta \phi = |\phi_{\text{trk}} - \phi_{\text{SC}}| < 0.05$;

• the variable $|1/E - 1/P|$, where $E$ is the energy of the supercluster and $P$ the momentum of the matching track is required to be lower than 0.02 (for $E, P$ measured in GeV);

• the ratio $E_{\text{HC}}$ between the energy deposited in the hadronic calorimeter and the energy deposited in the electromagnetic calorimeter is required to be lower than 0.05;

• the ratio $E_{\text{HC}}/P_{\text{HC}}$ between the energy of the supercluster and the momentum at vertex of the associated track is required to exceed 0.8;

• the quantity $\sigma_{\eta\eta}^2$, defined as

$$\sigma_{\eta\eta}^2 = \sum (\eta_i - \eta_{\text{seed}})^2 \times \frac{E_i}{E_{\text{seed}}}$$

is required to be below a certain threshold. The sum in Eq. 3.1 runs over all the crystal in the seed cluster, $\eta_i$ and $E_i$ are respectively the pseudorapidity of each crystal in the cluster and its energy, while $\eta_{\text{seed}}$ and $E_{\text{seed}}$ are the pseudorapidity and the energy of the seed cluster. This variable represents the spread of the electromagnetic shower in $\eta$ with respect to the electron $\eta$ direction. It is rather insensitive to bremsstrahlung, allowing for separation of signal and background shower shapes. $\sigma_{\eta\eta}$ is required to be lower than 0.01 for electron candidates in the barrel region and lower than 0.03 in the endcaps.

• the track isolation, defined as the sum over all the tracks (except the electron track) in a cone of radius $R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.35$ around the electron track direction, divided by the transverse energy of the supercluster:

$$I_{\text{so}} = \frac{1}{E_{\text{SC}}^T} \left( \sum_{r<R} p_{\text{track}}^T - p_e^T \right)$$

is required to be lower than 0.2.
3.3 Reconstruction

Figure 3.1: Distributions of the electron variables $\Delta \phi$ and $\Delta \eta$ giving the geometrical matching between track and supercluster. Electrons from $W \rightarrow ev$ events (full line) are compared to “fake-electrons” found in the QCD di-jet sample (dotted line); all the distributions are obtained with electron candidates with $E_T > 26$ GeV and with $|\eta| < 2.4$.

For illustration, figures 3.1, 3.2 and 3.3 represent the distributions of the electron identification variables and the track isolation, described above, for electrons coming from $W \rightarrow ev$ decays and for jets faking electrons from a QCD di-jet sample.

The efficiencies of the HLT, of the GSF tracking and of the electron identification criteria are shown in Fig. 3.4 as a function of the generator level electron transverse momentum and pseudorapidity, respectively. The efficiency of the electron identification selections is about 90% on electrons from $W \rightarrow ev$ decays that have passed the single electron HLT (see Fig. 3.5) and are intended to select electrons with good purity against QCD background due to jets faking electrons. The QCD background contamination is reduced both by the HLT and by the electron identification selections: more precisely, the former provides a rejection factor at the $10^5$ level, while an additional rejection factor of $\approx 3$ is given by the offline electron identification. A procedure to measure from data the electron efficiency and the probability that a jet is misidentified as an electron will be discussed later, in Chapter 6.

The electron transverse energy spectrum in $W \rightarrow ev$ events predicted by the fast simulation FAMOS has been compared to the spectrum obtained from the full simulation after the application of the electron selection criteria.
The distributions are in good agreement, as shown in Fig. 3.6. The RMS of the $(E_{T,\text{reco}} - E_{T,\text{true}})$ distribution is about 1.4 GeV both in ORCA and FAMOS samples. The mean of the distribution is also consistent in the two simulation programs.

### 3.3.2 Missing energy and hadronic recoil reconstruction

The missing transverse energy (often referred to as MET) is used to get an estimate of the transverse momentum of the neutrino. It is defined as the vector sum with opposite sign of the transverse energies deposited in the detector calorimeters (ECAL and HCAL):

$$\vec{E}_{\text{missing}} = - \sum_i \vec{E}_{i,\text{measured}}$$

(3.3)

Within the ORCA framework, different algorithms were implemented to reconstruct this variable:

- \textit{METfromCaloRecHit}: the missing transverse energy is obtained from the sum of the energy deposits in all the cells in the calorimeters; hits are taken in the sum if their energy exceeds the noise thresholds of 500 MeV for HCAL cells, 90 MeV for ECAL barrel and 450 MeV for ECAL endcap crystals [60];
Figure 3.3: Distributions of the $\sigma_\eta$ for barrel and endcaps (top) and of the track isolation (bottom). Electrons from $W \rightarrow e\nu$ events (full line) are compared to “fake-electrons” found in the QCD di-jet sample (dotted line); electron candidates with $E_T > 26$ GeV and with $|\eta| < 2.4$ are considered.
Figure 3.4: The efficiency in $W \rightarrow e\nu$ events of the HLT, of the electron GSF tracking reconstruction and of the electron identification as a function of the generated electron transverse momentum (left) and pseudorapidity (right).

Figure 3.5: The electron identification efficiency in $W \rightarrow e\nu$ events relative to the single electron HLT, as function of the reconstructed electron transverse energy (left) and pseudorapidity (right).
Figure 3.6: The transverse energy spectrum of the electron in $W \rightarrow e\nu$ events (left) and the resolution (right). The distributions obtained with FAMOS (solid line) and ORCA (open dots) are shown.

- **METfromCaloTowers**: the missing transverse energy is measured from the transverse energy deposited in all the calorimeters towers with a transverse energy above a threshold of 0.5 GeV; in forming towers, individual calorimeter cells must pass the energy threshold cuts quoted above;

- **METfromJets**: the missing energy is reconstructed from the transverse energy of all the jets in the event;

For $W \rightarrow e\nu$ events, the best reconstruction method has found to be the one based on the reconstructed hits. The other three methods give a bias on the missing energy, represented by the non-zero mean value of the distributions, and a worse resolution given by the larger width. In particular, reconstruction using jets, whatever is the jet algorithm used, is not fully appropriate because, in this kind of events where low $p_T$ are involved, the most of the energy is not clustered in jets. Figure 3.7, representing the MET resolution obtained with four different algorithms, well illustrates the differences between the four mentioned algorithms.

The effect of pile-up on the modelling of the transverse energy is quite important, as it contributes and adds fluctuations to the energy deposition in the calorimeters. For illustration, the distributions of the missing transverse energy and its resolution are presented in Fig. 3.8, where the fast simulation with pile-up and for the full simulation with ORCA. Without the inclusion
Figure 3.7: The missing transverse energy resolution in $W \rightarrow e\nu$ events using four different algorithms for MET reconstruction in the ORCA framework: on top, MET is obtained from the calorimetric towers (left, green histogram) and from calorimeter hits (right, blue histogram); on bottom, MET is reconstructed starting from jets, using the iterative cone algorithm (left, yellow histogram) or a $K_T$ algorithm (right, purple histogram).
Figure 3.8: The missing transverse energy spectrum in $W \rightarrow e\nu$ events (left) and the missing transverse energy resolution (right). The distributions obtained with FAMOS with pile-up (solid line) and ORCA (open dots) are shown.

of pile-up in the fast simulation, the MET resolution is about 9 GeV, to be compared to 12 GeV from the full simulation with pile-up. For comparison, the Tevatron experiments have shown a better resolution, about 5 GeV, on this variable.

The missing transverse energy is related to another kinematic variable, the hadronic recoil. All the particles recoiling against the $W$ boson are collectively referred to as the hadronic recoil. The missing transverse energy can be expressed as

$$E_{\text{missing}}^T = -\vec{u} - \vec{p}_T$$

(3.4)

where $\vec{u}$ is the transverse momentum of the hadronic system recoiling against the boson and $\vec{p}_T$ is the lepton transverse momentum. The recoil vector $\vec{u}$ is defined as the vector sum of transverse energy over all the calorimeter hits (or towers) in the detector but those corresponding to leptons; hence hits belonging to electron superclusters are explicitly removed from the recoil calculation. In order to avoid noise contributions, only hits with energy above a certain threshold are included in the sum. Different thresholds values are used for each subdetector: 500 MeV in HCAL, 90 MeV in the ECAL barrel and 450 MeV in the ECAL endcaps. These thresholds corresponds to roughly 2-3 standard deviations above the expected root-mean-square electronic noise. In Fig. 3.9 the recoil distribution and its resolution for $W \rightarrow e\nu$
events are shown.

Further issues concerning the recoil reconstruction and modelling will be discussed in Section 6.4 with details specifically related to the $W$ mass measurement study.

### 3.3.3 Jet reconstruction

Different jet reconstruction algorithms are available. They are the iterative cone algorithm [51, 61], the midpoint cone algorithm [62] and the inclusive $k_T$ algorithm [63, 64, 65]. In this analysis, the iterative cone algorithm, simpler and faster with respect to the others, has been used with a cone size $R=0.5$ in the $\eta, \phi$ space. The jet calibration is checked using the $p_T$ balance in $\gamma$+jet events to calibrate the absolute energy scale [66].
Chapter 4

Analysis procedure

The basic idea of the “Scaled Observables Method” has been introduced in Chapter 1. Here, details about the analysis procedure are given: the ratio of cross sections, $R(X)$, used in the analysis to correct the $Z$ distributions to fit those of the $W$ boson is defined and the statistical method to compare scaled $Z$ spectra to the corresponding ones for $W$ is described. In the whole procedure, an important aspect is related to the definition of criteria for the selection of events. They are, in fact, to be chosen with two purposes: primarily, they are designed to select $W$ and $Z$ samples with low backgrounds; moreover, they are intended to minimize the differences between $W$ and $Z$ in order to distort $R(X)$ with respect to the theoretical prediction in a minimal way.

4.1 Events selection and backgrounds

4.1.1 $W \rightarrow e\nu$ events selections

$W \rightarrow e\nu$ candidates are selected among the events that passed the HLT for isolated single electrons by requiring:

- one isolated electron with $E^T > 29$ GeV within the pseudo-rapidity region $|\eta| < 2.4$;
- missing transverse energy MET $> 25$ GeV;
- transverse momentum $|\vec{u}|$ of the system recoiling against $W$ has to be lower than 20 GeV;
- no jets in the event with $p_{T,jet} > 30$ GeV.
The minimum accepted $E^T$ is somewhat higher than the HLT single lepton threshold (26 GeV), to reduce the impact of a non uniform trigger efficiency near threshold, which is relevant when $W$ events are compared to $Z$ events (see below). The last two selections are intended to select $W$ bosons produced with a small transverse momentum and to suppress background from QCD processes. An additional selection on the electron pseudorapidity is introduced to avoid dead regions between the calorimeter modules where the energy reconstruction is not optimal (see Fig. 4.1). The fiducial volume is identified by the following pseudorapidity regions: $0.0 < |\eta| < 0.41, 0.47 < |\eta| < 0.76, 0.83 < |\eta| < 1.12$ and $1.68 < |\eta| < 2.4$. The requirement on the electron to be in the fiducial volume leads to a loss of efficiency, but it is acceptable considering the high statistics available.

The selection efficiency is about 12.8% and the background contribution is at the 2% level, dominated by $Z$ decays to leptons (see Fig. 4.2). The background in the $W$ sample mainly consists of single electron events coming from $Z \rightarrow e^+e^-$ events with one electron escaping detection (1.3%) and from $W$ and $Z$ decays to $\tau$’s followed by a $\tau$ decay to an electron (1.2% and 0.2% respectively). The background events with $\tau(s)$ have typically electrons with lower transverse energy, on average, with respect to the transverse energy.

![Figure 4.1: The ratio between the reconstructed transverse energy and the true transverse energy of the electrons in the $W$ sample in function of the supercluster pseudorapidity.](image-url)
of electrons from $W$ decays. Then, the most of $W \rightarrow \tau \nu$ and $Z \rightarrow \tau \tau$ events are rejected by the lepton transverse energy cut. The stability of these predictions against the assumptions in the Monte Carlo generator has been tested. A harder $W$ boson $p_T$ spectrum would imply a higher efficiency of the $E_T^{miss}$ cut to electrons coming from $\tau$ decays. This is however balanced by a lower efficiency of the requirement that the system recoiling against the boson be lower than 20 GeV. The final effect is a relative 1.2\% variation in the $W \rightarrow \tau \nu$ contribution to the background for a $p_T$ spectrum twice as hard as in the Monte Carlo generator, which is indeed incompatible with existing data.

The contribution from $b\bar{b}$ and $t\bar{t}$ production with either true or misidentified electrons in the final state are expected to be negligible, even in spite of large production cross sections. The former channel mainly provides electrons with small $E_T^{miss}$ close to the beam axis, the latter is characterized by $W$ bosons with high transverse momentum coming from the top quark decay and by jets of large $p_T$; the high rejection factor on the $t\bar{t}$ background is mainly due to the requirements that the recoil transverse momentum be less than 20 GeV and that no jets with $p_T > 30$ GeV be present.

The efficiencies of the selection criteria for $W$ signal and backgrounds are shown in Table 4.1 and Table 4.2 respectively.

<table>
<thead>
<tr>
<th>Selection</th>
<th>number of events</th>
<th>% remaining events</th>
</tr>
</thead>
<tbody>
<tr>
<td>no selections</td>
<td>$1.75 \times 10^7$</td>
<td>100%</td>
</tr>
<tr>
<td>1 isolated HLT electron</td>
<td>6238091</td>
<td>35.6%</td>
</tr>
<tr>
<td>1 electron, $E_T^{miss} &gt; 29$ GeV</td>
<td>5475592</td>
<td>31.3%</td>
</tr>
<tr>
<td>MET $&gt; 25$ GeV</td>
<td>4564244</td>
<td>26.1%</td>
</tr>
<tr>
<td>$</td>
<td>\vec{q}</td>
<td>&lt; 20$ GeV</td>
</tr>
<tr>
<td>$p_T^{jet,max} &lt; 30$ GeV</td>
<td>3117226</td>
<td>17.8%</td>
</tr>
<tr>
<td>electron in fiducial volume</td>
<td>2241893</td>
<td>12.8%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Selection</th>
<th>number of events</th>
<th>% remaining events</th>
</tr>
</thead>
<tbody>
<tr>
<td>no selections</td>
<td>$1.75 \times 10^6$</td>
<td>100%</td>
</tr>
<tr>
<td>2 isolated HLT electrons</td>
<td>424430</td>
<td>24.2%</td>
</tr>
<tr>
<td>1 electron, $E_T^{miss} &gt; 29$ GeV $\cdot M_Z/M_W$</td>
<td>301664</td>
<td>17.2%</td>
</tr>
<tr>
<td>MET $&gt; 25$ GeV $\cdot M_Z/M_W$</td>
<td>267501</td>
<td>15.3%</td>
</tr>
<tr>
<td>$</td>
<td>\vec{q}</td>
<td>&lt; 20$ GeV $\cdot M_Z/M_W$</td>
</tr>
<tr>
<td>$p_T^{jet,max} &lt; 30$ GeV</td>
<td>184117</td>
<td>10.5%</td>
</tr>
<tr>
<td>electron in fiducial volume</td>
<td>129273</td>
<td>7.4%</td>
</tr>
</tbody>
</table>

Table 4.1: Selection efficiencies for $W \rightarrow e\nu$ and $Z \rightarrow e^+e^-$. 

Figure 4.2: The electron transverse momentum distribution (top) and the boson transverse mass distribution (bottom) in $W \rightarrow e\nu$ decays and the backgrounds from $Z \rightarrow e^+e^-$ (dashed line), from $Z \rightarrow \tau^+\tau^-$ (dotted line) and from $W \rightarrow \tau\nu$ (dot-dashed line) for 1 fb$^{-1}$.
### 4.1 Events selection and backgrounds

#### 4.1.2 QCD background to $W \rightarrow e\nu$ events

A contribution to the background can also be expected from di-jet events in which one jet mimics an electron and the other is mismeasured, creating missing transverse energy. Such events, referred to as QCD background, are contaminating the $W$ samples collected at the Tevatron at the percent level or below [39].

At the LHC the production ratio of $W$ to di-jet events of large $p_T$ is somewhat less favourable than at the Tevatron. The total production cross section for QCD di-jets at the LHC is about 58 mb. A precise figure for the level of contamination due to QCD events could not be obtained with full detail, as a rejection power larger than $10^8$ has to be established, which is beyond the statistical reach even of the fast simulation. However, the single-electron HLT selections, the electron identification criteria and the topological selections used to select $W \rightarrow e\nu$ events are expected to strongly reduce the QCD background to a level below 1%.

Moreover, the QCD background needs to be controlled carefully, as the uncertainty associated to it is larger than for the other backgrounds mentioned above. With respect to this, it is important to stress that methods to model the QCD background directly from the data can be envisaged (see also [39]).

A procedure to measure it from data will be discussed in details in Sec. 6.3. For completeness, in this Section, the efficiencies of the HLT, electron identification and of the topological selections are summarized (Table 4.3) for each analyzed sample in the different $p_T$ bins. The relative contribution to the $W \rightarrow e\nu$ background, together with the statistical error corresponding to the available Monte Carlo statistics is reported. At the moment, the simulated QCD samples have still a too small number of events and the estimate of the QCD background is limited by the large statistical uncer-
tainty. In particular, the $p_T$ bin 30-50 GeV, no events out of the $\sim195000$ analyzed survive the HLT plus electron identification selections. The topological efficiency, in this bin and, for consistency, in the others, has thus been computed using “quasi-electron objects” from the QCD di-jets sample. This “quasi-electrons” are electron candidates passing the single electron HLT but failing at least one of the other electron identification cuts. The assumption that the efficiency of the topological selections obtained on electrons, quasi-electrons and electron candidates is the same has been validated on events in the $p_T$ bin 50-80 GeV, where enough events are available to perform such a comparison. These efficiencies have been found to be compatible within the statistical errors.

<table>
<thead>
<tr>
<th>$p_T$ (GeV)</th>
<th>HLT efficiency</th>
<th>Electron ID efficiency</th>
<th>Topological selection efficiency</th>
<th>Background to $W \rightarrow e\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-50</td>
<td>$(1.95\pm1)\times10^{-3}$</td>
<td>$50%\pm25%$</td>
<td>$5.7%\pm3%$</td>
<td>$3.3% \pm 2.9%$</td>
</tr>
<tr>
<td>50-80</td>
<td>$(1.15\pm0.09)\times10^{-4}$</td>
<td>$29.6%\pm3.6%$</td>
<td>$2%\pm1.1%$</td>
<td>$0.57% \pm 0.31%$</td>
</tr>
<tr>
<td>80-120</td>
<td>$(1.4\pm0.15)\times10^{-4}$</td>
<td>$31%\pm5.1%$</td>
<td>$1.29%\pm1.28%$</td>
<td>$0.06% \pm 0.06%$</td>
</tr>
</tbody>
</table>

Table 4.3: The efficiencies of the HLT, electron identification selections and of the topological selections on fake-electrons with $E_T > 26$ GeV from the QCD di-jets events are shown in the first three columns for the $p_T$ range between 30 and 120 GeV. In the last column the contribution to the to the $W \rightarrow e\nu$ are reported. The errors are due to the limited statistical samples.

The efficiency of each topological selection on the QCD di-jets is shown in Table 4.4. The cut on the electron transverse energy rejects more events as the $p_T$ range decreases; the impact of the MET cut is quite imporant in all the $p_T$ bins since di-jets are expected to be balanced in the transverse plane; finally, the jet veto has a slightly larger effect in rejecting events with high $p_T$.

### 4.1.3 $Z \rightarrow ee$ events selections

$Z$ events used to predict the $W$ spectrum are selected from the sample of events accepted by single lepton HLT. A $Z/\gamma^*$ candidate event is tagged by requiring a pair of identified electrons in the region $|\eta| < 2.4$, with opposite charge and an invariant mass larger that 50 GeV. One of the two electrons, randomly chosen, is then removed from the event to mimic a $W$ decay. The same selections discussed above are then applied, with the difference that
4.2 Effects of events selections on $R(X)$

<table>
<thead>
<tr>
<th>Selection</th>
<th>$p_T^e = 30 - 50$ GeV</th>
<th>$p_T^e = 50 - 80$ GeV</th>
<th>$p_T^e = 80 - 120$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_T^e &gt; 29$ GeV</td>
<td>55%</td>
<td>72.8%</td>
<td>84.4%</td>
</tr>
<tr>
<td>MET $&gt; 25$ GeV</td>
<td>10.7%</td>
<td>4.6%</td>
<td>24.7%</td>
</tr>
<tr>
<td>$</td>
<td>\vec{p}_T^\beta</td>
<td>&lt; 20$ GeV</td>
<td>6.7%</td>
</tr>
<tr>
<td>$p_T^{\mu_T} &lt; 30$ GeV</td>
<td>5.7%</td>
<td>2%</td>
<td>1.3%</td>
</tr>
</tbody>
</table>

Table 4.4: Efficiency of the topological selections on fake-electrons with $E_T^e > 26$ GeV from the QCD di-jets sample. The efficiencies have been computed using “quasi-electron” objects in order to have a significant statistical sample.

The study of the expected precision on the $M_W$ measurement has been fully addressed for the analysis of the electron transverse energy spectrum and of the vector boson transverse mass spectrum with a statistical sample corresponding to 1 fb$^{-1}$ of integrated luminosity.

The analysis procedure is based on a $\chi^2$ test of the compatibility of the scaled observable spectra from $Z$- and $W$-boson decays. A statistical sam-

---

1A similar restriction is not applied to the other electron, which is not directly used to predict the electron $E_T^e$ spectrum from $W$ decays, but is only needed to tag a $Z$ event.
Figure 4.3: The electron transverse momentum distribution in $Z \rightarrow e^+e^-$ events and the backgrounds from $Z \rightarrow \tau^+\tau^-$ (dashed line).

The understanding of the effect of each selection on $R(X)$ is in order, as it was considered in the optimisation of the selection criteria. The selection on the invariant mass of the electron-positron pair is kept loose ($M_{e^+e^-} > 50$ GeV) on purpose as a corresponding selection cannot be applied to the $W$ sample. The selection on the pseudo-rapidity of the electron chosen to
predict the $E_T$ spectrum in $W$ decays ($|\eta_1| < 2.4$) slightly distort $R(X)$ in the region with $X < 1$, because of the different rapidity distributions of the electrons in $W$ and $Z$ events. Being $W^+$ and $W^-$ rapidity distributions different, the study of $W^+$ and $W^-$ samples separately will provide a useful tool to understand and estimate possible systematic effects related to this aspect. More relevant is, instead, the cut on the pseudorapidity of the electron chosen to mimic the neutrino ($|\eta_2| < 2.4$) in $Z$ events, which implies an unmatched selection between $W$ and $Z$ events. Low $E_T$ electrons from the $Z$ will have on the average an higher rapidity and therefore the cut on the rapidity of the second lepton mainly distort $R(X)$ in the low $E_T$ region (that means low $X$). Finally, the requirement that $|\vec{u}|$, the transverse momentum of the system recoiling against the boson, be lower than a certain threshold has an effect on the tails of the $X$ distribution, due to differences in the boson $p_T$ spectra. The largest variations are anyway limited to the high $X$ region and their impact can be reduced by properly restricting the fit range around the Jacobian peak.
4.3 Statistical precision

In the fitting procedure, the scaled observables \( X = 2E^T/M_V \) and \( X = M^T/M_V \) have been considered. In \( W \to e\nu \) events the observables are scaled with several different values of \( M_W \). These spectra are then compared at each step (i.e. for each value of \( M_W \) considered), by means of a \( \chi^2 \) test, to the corresponding \( Z \) distributions scaled with \( M_Z \) and weighted with \( R(X) \). The \( \chi^2 \) test is restricted to the region about the Jacobian peak where the sensitivity to the \( W \) mass is larger: \( 0.75 < X < 1.4 \), corresponding to \( 30 \text{ GeV} < E^T < 56 \text{ GeV} \), for the scaled transverse energy fit and \( 0.8 < X < 1.25 \), corresponding to \( 64 \text{ GeV} < M^T < 100 \text{ GeV} \), for the scaled transverse mass fit. \( W \)- and \( Z \)-boson spectra are normalised in the fit range window. The \( \chi^2 \) statistics reads:

\[
\chi^2 = \sum_i \left( \frac{N_i^W (M_W) \frac{(N^Z R)_{tot}}{N_{tot}^W} - N_i^Z R_i}{N_i^Z R(X_i)^2} \right)^2,
\]

where the sum is over the number of bins in the fit range window, \( N_i^W \) is the number of electrons from \( W \) decays in the \( i \)-th bin, \( N_i^Z R_i \) is the number of electrons from \( Z \) decays after proper rescaling through \( R(X) \), \( (N^Z R)_{tot}/N_{tot}^W \) is the relative normalisation of the spectra within the range considered in the fit. The term in the denominator only accounts for the statistical fluctuation on the number of \( Z \) events, as the relative contribution from \( W \) events is ten times lower and the statistical inaccuracy of \( R(X) \) is also marginal. Events have been divided in approximately 1 GeV wide bins in the \( E^T \) spectra and 2 GeV wide bins in the \( M^T \) spectra.

Figure 4.5-left shows the comparison of the electron scaled transverse energy spectra for \( W \)- and \( Z \)-bosons at the best-fit point for a simulated statistics corresponding to 1 fb\(^{-1}\) of integrated luminosity. The \( \chi^2 \) dependence on \( M_W \) is displayed in Fig. 4.5-right. The \( \chi^2 \) was computed for a discrete set of \( M_W \) values at steps of 10 MeV and interpolated around the minimum with the quadratic form

\[
\chi^2 = \chi^2_{min} + \frac{(M_W - M_{\text{best}})^2}{\Delta M_W^2}.
\]

A statistical precision of about 40 MeV, limited by the quantity of \( Z \) events, is obtained in the analysis using the scaled transverse energy of the lepton. The smearing of the Jacobian peak in the lepton \( E^T \) spectrum is dominated
by the finite boson transverse momentum. The statistical precision would worsen of about 5%, if the boson $p^T$ spectrum was about 20% harder than predicted by the current Monte Carlo. Extrapolation to 10 fb$^{-1}$ of integrated luminosity readily results in a statistical precision of less than about 15 MeV.

The statistical precision from the fit of the transverse mass spectrum is comparable to the one obtained from the scaled $E^T$ (Fig. 4.6-right). In this case, however, the sharpness of the $M^T$ jacobian edge is badly smeared by the detector resolution, and not by the boson transverse motion.

In the following Chapters the theoretical and experimental systematic uncertainties of the $M_W$ measurement will be discussed relatively to the measurement through the scaled electron $E^T$ spectrum. A complete analysis using the transverse mass has also been performed, but it resulted in experimental uncertainties related to the limited capability of reconstructing the neutrino transverse momentum much larger than the corresponding ones in the measurement with the lepton $E^T$. A better understanding of the missing energy reconstruction and, eventually, the study of techniques to improve its determination are mandatory for a precision $M_W$ measurement using the $M^T$ spectrum. Since at the current status of the studies the transverse mass analysis does not seem very promising, it has not been reported in this work,
that, hereafter, will treat the measurement using the electron transverse energy spectrum.

The evaluation of the systematic uncertainties of theoretical origin affecting the measurements is performed by determining the distortion to $R(X)$ implied by different effects affecting the theoretical prediction (PDF, $\Gamma_W$, perturbative expansion, ...) and by fitting the $W$ event sample to $Z$ events. The effects of instrumental origin have been studied by fixing $R(X)$ to the theoretical prediction exactly describing the samples of generated events (i.e. an exact knowledge of the theory is assumed) and by introducing distortions and biases in the detector response. The resulting shift in $M_W$, determined from a fitting procedure, is assumed as the systematic uncertainty associated to the effect. In this way systematics effects of instrumental origin and of theoretical origin result factorised.
Chapter 5

Theoretical uncertainties

5.1 Parton distribution functions

The incomplete rapidity coverage of the detectors introduces a dependence of the measured $E_T$ distributions on the longitudinal momentum distribution of the produced $W$ boson determined by the Parton Distribution Functions (PDF). PDF uncertainties can arise both from the experimental side, as PDF are obtained from a global fit to deep inelastic scattering and other data, and from the theoretical models. The limited knowledge of the PDFs involved in the $W$ and $Z$ production is then determining an uncertainty on the acceptance and on the differential distributions predicted for $W$- and $Z$-boson events. In the context of the method discussed here, these effects are best represented by looking at the distortion induced on $R(X)$ by the uncertainty on the PDF parameters. Examples of these distortions are shown in Fig. 5.1 for some sets of PDF of the CTEQ6.1 [67] and MRST [68, 69, 70] families. These were computed by means of the LHAPDF libraries [71] to re-weight the simulated data samples.

The impact on the determination of $M_W$ can then be determined quantitatively by repeating the fitting procedure for several re-weighting functions $R(X)$, corresponding to different PDF parameters’ sets and by determining the effect on $M_W$. In this procedure the PDF set CTEQ6.1 [67] has been mainly considered. The PDF uncertainties of this set are estimated by CTEQ using the Hessian Matrix Method [72]. In the PST approach, the Hessian Method both constructs a $N$ ($N = 20$ for CTEQ6.1) eigenvector basis of PDFs and provides a method from which uncertainties on observables can be calculated. The first step is to perform a fit with $N$ free parameters and
obtain the central or best fit parameter set by minimizing the global $\chi^2$. The $N \times N$ hessian error matrix is then diagonalized yielding an orthonormal basis of $N$ eigenvectors, that provides a representation of the parameter space around the minimum. Each eigenvector probes a direction in the parameter space that is a combination of the $N$ free parameters used in the global fit.

According to this approach, the CTEQ6.1 PDF family contains one central PDF set ($i = 0$) and 40 PDF members, $F^\pm = F(x, Q; s_i^\pm)$ ($i = 1, 20$), representing the PDF variations induced by a change within a certain tolerance of each independent parameter describing the PDF set. The tolerance chosen by the CTEQ collaboration corresponds to the 90\% C. L. limit [73].

The 'Master Equation' proposed by Nadolsky and Sullivan [74] is used to determine the 1\(\sigma\) effect on $M_W$:

$$\Delta M_W^+ = \frac{1}{1.6} \sqrt{\sum_{i=1}^{20} \left[ \max (M_W(s_i^+), M_W(s_i^-) - M_W(0), 0) \right]^2}$$  \hspace{1cm} (5.1)

$$\Delta M_W^- = \frac{1}{1.6} \sqrt{\sum_{i=1}^{20} \left[ \max (M_W(0) - M_W(s_i^+), M_W(0) - M_W(s_i^-), 0) \right]^2}$$  \hspace{1cm} (5.2)
where \( M_W(s_i^\pm) \) represent the best-fit value for the \( W \) mass corresponding to the PDF set under consideration. Following the recipe adopted by CDF [36], the uncertainty is scaled from the 90\% C.L. PDF error to the 1\( \sigma \) error by dividing by a factor 1.6.

Equation 5.2, which considers maximal positive and negative variations of the physical observable separately, is known as “modified tolerance method” and, among many versions of ‘Master Equations’ that can be found in the literature [75], it is the preferred one [74, 76]. Figure 5.2 shows the quanti-

\[ \Delta M_W(s_i^\pm) = M_W(s_i^\pm) - M_W(0) \]

Figure 5.2: \( \Delta M_W(s_i^\pm) = M_W(s_i^\pm) - M_W(0) \) as determined by the fitting procedure using the 40 different members of the CTEQ6.1 family to determine \( R(X) \) for \( X = 2E_T/M_V \). The quoted shifts \( \Delta M_W(s_i^\pm) \) correspond to 90\% error.

The largest variations are of order 10 MeV and correspond to CTEQ6.1 members, in particular eigenvectors 10 and 11, implying a large distortion of \( R(X) \) in the low \( X \) region (\( X<1 \)), where most of the events lie. In most cases, it is not easy to tie the eigenvector behaviour to a particular PDF in a certain kinematic region, because each PDF parameter has, in general, components along each eigenvector direction. Only in some cases, the de-
dependence is remarkable, such as for eigenvector 15 which is very sensitive to the high-\(x\) gluon PDF. In general, the large eigenvalues, or low eigenvector numbers, correspond to well determined directions and are primarily determined by valence quark distributions which are well constrained at moderate \(x\), while larger eigenvectors are primarily determined by sea quark and gluon PDFs. For the \(W\) and \(Z\) production at the LHC, both valence quarks and sea anti-quarks are involved, hence the dependence from specific eigenvectors is not so direct.

The resulting uncertainty on \(M_W\) from the fit of the electron scaled transverse energy is:

\[
\Delta M_W^+ = 14.6 \text{ MeV} \tag{5.3}
\]
\[
\Delta M_W^- = 10.6 \text{ MeV} \tag{5.4}
\]

These figures can be quoted as the expected initial systematic error coming from PDF uncertainties, as the PDF sets belonging to the MSRT family at NLO and NNLO give results consistent to the CTEQ6.1 central value within the errors reported in equation (5.4). The largest differences on \(R(X)\) between the CTEQ6.1 and the MRST sets are indeed limited to the region of large \(X\) (see Fig. 5.1), and have a small impact on the determination of \(M_W\).

It must be noted that early LHC results will definitely imply an improved determination of the PDF sets relevant to this analysis, in particular studying the pseudorapidity distributions of leptons from \(Z\) or \(W\) decays that are very sensitive to the PDFs.

For example, Fig. 5.3 shows the ratio \(N_{W^+}/N_{W^-}\) between the pseudorapidity distributions of positrons and electrons from \(W \rightarrow e\nu\) decays obtained for different PDF sets. In these distributions, detector effects by means of the FAMOS simulation and events selections are accounted for. It can be observed that the statistical precision is enough, already with a few inverse picobarns of integrated luminosity, to distinguish not only between different PDF families (e.g., CTEQ with respect to MRST), but also between different members in the same PDF family. A comparison based on a \(\chi^2\) test has been performed between the differential distributions \(N_{W^+}/N_{W^-}\) as a function of the pseudorapidity obtained with each PDF set belonging to the
5.1 Parton distribution functions

Figure 5.3: The ratio $N_{W+}/N_{W-}$ between the rapidity distributions of leptons from $W \to e\nu$ decays determined with different PDF sets. The error bars indicate the statistical uncertainty with an integrated luminosity of 370 pb$^{-1}$.

CTEQ6.1 family and the one corresponding to CTEQ6.1 central member:

$$\chi^2 = \sum_{i=1}^{N} \frac{(n_i - n_0)^2}{\sigma_i^2 + \sigma_0^2} \quad (5.5)$$

where $n_0$ is the number of events in each bin of the reference histogram (corresponding to the central PDF set), $n_i$ is the number of events in each bin for each PDF set considered, $\sigma_0^2$ and $\sigma_i^2$ are the errors on $n_0$ and $n_i$ respectively. Only 1/3 out of the 40 PDF sets in the CTEQ6.1 family has $\chi^2$ variation $\leq 1$ (fig.5.4), i.e. is one sigma within the prediction of the CTEQ6.1 central value. The PDF members that give the largest $\Delta \chi^2$ for the $N_{W+}/N_{W-}$ correspond to eigenvector 8 and are not the same that give the largest $\Delta M_W$, probably because the two observables are sensitive to different PDF features.

By removing the PDF sets with $\Delta \chi^2 > 1$ from the master equation 5.2, the uncertainties on $M_W$ become $\Delta M_W^+ = 8.2$ MeV and $\Delta M_W^- = 4.1$ MeV.

This figures are indicative as they account only for statistical errors. The final effects on $M_W$ has to be studied including backgrounds and possible systematic errors (e.g. the charge misidentification probability). Anyway, the sensitivity of the $W$ leptons pseudorapidity distributions is evident and the procedure suggests that an improvement of the PDF knowledge is achievable.
Figure 5.4: The $\Delta \chi^2$ between the distribution $N_{W^+}/N_{W^-}$ obtained for each CTEQ6.1 PDF set with respect to the one obtained from the CTEQ6.1 central set.

For example, a quantitative study of the LHC potential in constraining the PDF is reported in [25], where the possibility of reducing the PDF uncertainty is investigated taking into account realistic expectations for measurement accuracy, kinematic cuts and backgrounds. In the cited work, a sample of ATLAS [77] fully simulated $W \to ev$ events, generated using the CTEQ6.1 PDF set, has been used as pseudo-data including the pseudorapidity distributions of electrons and positrons from $W$ decay in the ZEUS-S global fit [78]. Assuming a 4% measurement accuracy, a reduction at the level of 35-40% of the error on the ZEUS parameter $\lambda_g$, which describes the gluon shape at low-$x$, is predicted with an integrated luminosity corresponding to only a few days of LHC running at low luminosity.

A full analysis of the improvement on the PDFs knowledge would indeed require a complete PDF fit, that goes beyond the scope of this thesis, considering all the observables sensitive to the PDFs. Hence, the figures quoted above in Equation 5.4 have to be regarded as a conservative estimate of the uncertainty induced on $M_W$ for an integrated luminosity of 1 fb$^{-1}$, while a reduction of this error below 5 MeV is expected at increased luminosities.
5.2  $W$ boson transverse momentum

The measurement of the $W$ mass through the analysis of the lepton transverse energy distribution is directly affected by the uncertain knowledge of the $p_T(W)$ distribution, while the measurement using the transverse mass spectrum is far less sensitive, as previously discussed in Sec. 1.4.4. Measurements of the $W$ mass by the CDF and D0 collaborations rely on the modelization of the $W$ boson transverse momentum spectrum using RESBOS [79, 80], which computes the quintuple differential cross section

$$\frac{d^5\sigma}{dp_T^W dq^2_W dq^2_W d\phi}$$

for the process $p\bar{p} \rightarrow W^\pm$. RESBOS models the $W$ boson $p_T$ spectrum at low $p_T$ via multiple-soft-gluon-resummation techniques. The non-perturbative part is parametrized by three parameters, commonly indicated as $g_1, g_2, g_3$, that have to be determined experimentally. For a fixed beam energy and a fixed $Q$ value, the boson $p_T$ spectrum can be described by a single parameter, namely $g_2$, which indeed determines the mean boson $p_T$. $g_2$ is fitted to the $p_T^Z$ data. The use of $Z$ boson data to predict the $W$ transverse momentum spectrum is based on the fact that QCD effects are the same for $W$ and $Z$; the differences between $W$ and $Z$ arising from the phase space and the electroweak couplings are accurately accounted for in the Monte Carlo. Currently, the systematic uncertainties with CDF Run II deriving from the $g_2$ uncertainty are 3 MeV, 9 MeV and 5 MeV for the $W$ mass measurement using respectively the $M_T$, $E_T$ or missing transverse energy spectra [36].

In the context of the “Scaled Observables Method”, the $p_T^W$ uncertainty is better quantified as the uncertainty associated to the theoretical prediction of $R(X)$ due to soft gluon emission. As the $W$ boson transverse momentum becomes smaller, the QCD radiation becomes more and more independent of the hard process and factorizes. By taking the ratio, therefore, the radiative corrections tend to cancel and a reliable calculation at fixed perturbative order is reliable also in phase space regions where individual observables are not.

A comparison of the available calculations at the NLO to the LO in $\alpha_S$ shows a difference of up to 10% in the ratio in the region above the resonance for $X = 2E_T/M_V > 1$, while differences within a few percent are found in the $R(X)$ theoretical prediction for the scaled transverse mass, $X = M^T/M_V$ [37].

In this section the impact of the boson $p_T$ uncertainty on the scaled lepton
transverse spectrum will be discussed in details, being this uncertainty more severe for this observable.

A preliminary estimate of the error associated to the missing orders in the perturbative expansion has been addressed by studying the dependence of the NLO prediction on the choice of the renormalisation and factorisation scales. The computation has been performed with the DYRAD program [81], which also allows for the introduction of experimental selections (lepton transverse energy and rapidity, missing energy and leading jet energy). A reference point has been computed with the renormalisation ($R$) and factorisation ($F$) scales both set to the boson mass ($M_V; V = W, Z$): a natural scale of the process under study. Test points have been calculated by changing both the scales to $0.5M_V$ and $2M_V$, with $R = F$, a commonly adopted choice. The distortions induced on $R(X)$ are shown in Fig. 5.5 where the ratio of $R(X)$ as a function of $X = 2E_T/M_V$ for the two choices to the reference $R(X)$ is shown. These variations, parametrized by means of third order polynomials, have been propagated to the fitting procedure in order to quantify the uncertainty on $M_W$. Shifts on the $W$ mass value not greater than -15 MeV are observed.

![Figure 5.5](image)

Figure 5.5: Ratio of $R(X)$ as a function of $X = 2E_T/M_V$ for the two choices $R = F = 2M_V$ and $R = F = 0.5M_V$ of the renormalisation ($R$) and factorisation ($F$) scales to the reference $R(X)$ computed with $R = M_V$ and $F = M_V$ with a third polynomial fit superimposed.

The possibility to narrow the fit window has been investigated in order to avoid $X$ regions where the distortions are large. The results are summarized in table 5.1. A slight improvement is obtained for $R = F = 2M_V$ as the major distortion on $R(X)$ happens in the low $X$ region which is avoided by restricting the fit window. In the other case, $R = F = 0.5M_V$, $R(X)$ is dis-
5.2 $W$ boson transverse momentum

orted just around the jacobian peak (i.e. around $X=1$) where the sensitivity to $M_W$ is greater; hence, the bias on $M_W$ remains almost unchanged even when varying the width of the fit window.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$F$</th>
<th>$\Delta M_W$ (MeV)</th>
<th>$\Delta M_W$ (MeV)</th>
<th>$\Delta M_W$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2M_V$</td>
<td>$2M_V$</td>
<td>+6</td>
<td>+3</td>
<td>-1</td>
</tr>
<tr>
<td>$\frac{1}{2}M_V$</td>
<td>$\frac{1}{2}M_V$</td>
<td>-15</td>
<td>-15</td>
<td>-14</td>
</tr>
</tbody>
</table>

Table 5.1: Shifts on $W$ mass obtained for different choices of the fit window. The functions shown in Fig. 5.5 have been used to weight events in the fitting procedure.

The DYRAD computation has an uncertainty due to the numerical precision of the Monte Carlo integration (shown by the error bars in Fig. 5.5). It results in a finite precision on the $M_W$ biases obtained for the different values of $R$ and $F$. The evaluation of this precision has been accomplished as follows. The ratio $R(X)$ has been computed introducing, in each $X$ bin, a factor from a gaussian distribution with mean equal to the $R(X)$ value in that bin and with width determined by the numerical error on $R(X)$ in that bin. This exercise has been repeated 100 times for each choice of the renormalisation and factorisations scales and allows to verify that the bias collected on $M_W$ is gaussian with a width of about 6 MeV, determined by the numerical precision of the NLO $R(X)$ computation (see Fig. 5.6). The

Figure 5.6: Distributions of the biases on $W$ mass for the two test points $R = F = 2M_V$ and $R = F = 0.5M_V$ from the DYRAD calculation.
resulting shifts on $M_W$, given by the mean value of these distributions, are $+0.3$ MeV for $R = F = 2M_V$ and $-15$ MeV for $R = F = 0.5M_V$.

A further check has been performed using different combinations of $R$, $F$. In particular, the choices of $R$, $F$ with one of the two scales fixed to $M_V$ and the other moved to $2M_V$ and $0.5M_V$ have also been studied. The results are summarized in Fig. 5.7, showing the ratio between $R(X)$ for different choices of the scales and the reference $R(X)$. Shape variations at the 1% level are observed in the region relevant to the fitting procedure ($0.75 < X < 1.4$).

![Figure 5.7](image)

**Figure 5.7:** Ratio of $R(X)$ as a function of $X = 2E_T/M_V$ for different choices of the renormalisation ($R$) and factorisation ($F$) scales to the reference $R(X)$ computed with $R = M_V$ and $F = M_V$. In the left panel, all the curves are normalised at $X = 1$ to put in evidence shape variations. The error bars indicate the numerical precision of the computation.

However, since the DYRAD computation is quite time-consuming, the new points have been calculated with lower numerical precision with respect to the previous ones. In this case, the numerical uncertainty, determined with the procedure explained above, is about $13$ MeV. The results are reported in Fig. 5.8.

This statistical precision is also comparable with the RMS of the biases
5.2 \( W \) boson transverse momentum

Figure 5.8: Distributions of the biases on \( W \) mass for different choices of the factorisation and renormalisation scales using the DYRAD program. The mean value of each distribution is the expected shift on \( W \) mass due to the scale variation and the width (about 13 MeV) corresponds to the numerical precision of the DYRAD computation.

obtained from the different combinations of \( R, F \). For some points (for ex-
ample, $R = 2M_V$ and $F = M_V$), the shift is somewhat larger than the ones estimated in the previous calculation with high numerical precision, but still compatible within errors.

For each point defined by equal values of the renormalisation and factorisation scales ($R = F = 2M_V$ and $R = F = 0.5M_V$) two independent results have been obtained, one with high numerical accuracy and one with lower numerical accuracy. Combining the two measurements, we obtain the following values for the biases on $M_W$: $+1.3$ MeV for $R = F = 2M_V$ and $-13.2$ MeV for $R = F = 2M_V$. Hence, a bias on $M_W$ of the order of 10 MeV is deduced.

The same study has been repeated using the MC@NLO program [82], which merges NLO calculations to a parton shower approach. Two points have been tested, with both the scales set to twice the vector boson mass and with both the scales set to half the vector boson mass, because currently MC@NLO only allows to chose equal values for the renormalisation and factorisation scales. About 15 millions of $Z/\gamma^* \rightarrow ee$ events, 15 millions of $W^+ \rightarrow e^+\nu$ and 15 millions of $W^+ \rightarrow e^-\bar{\nu}$ have been generated to study

![Figure 5.9: Ratio of $R(X)$ as a function of $X = 2E^T/M_V$ for different choices of the renormalisation ($R$) and factorisation ($F$) scales to the reference $R(X)$ computed with $R = M_V$ and $F = M_V$. The curves, obtained using MC@NLO, are normalised at $X = 1$ and the error bars indicate the statistical precision. A third order polynomial fit is superimposed.](image-url)
the variations on $R(X)$ depending on scale modifications. The distortions on $R(X)$ are shown in Fig. 5.9.

In this case, the biases on $M_W$ are greater than those obtained using the NLO DYRAD computation: +32 MeV and -69 MeV for $R = F = 2M_V$ and $R = F = 0.5M_V$ respectively. However, the statistical uncertainty related to the limited number of events used to evaluate $R(X)$ is quite large, resulting of about 30 MeV (see Fig. 5.10).

Figure 5.10: Distributions of the biases on $W$ mass for different choices of the factorisation and renormalisation scales using the MC@NLO program. The mean value of each distribution is the expected shift on $W$ mass due to the scale variation and the width (about 30 MeV) corresponds to the statistical precision on it.

Figure 5.11, summarizes the bias (and its numerical/statistical error) on $M_W$ due to the uncertainty related to the boson $p_T$ prediction, for the study with DYRAD and MC@NLO. It is evident that the current statistical precision from the MC@NLO generated samples is not enough to quantify the uncertainty on $M_W$. In order to obtain a statistical precision on these biases of about 5 MeV, a sample of MC@NLO generated events 36 times larger than the one used in this study would be needed.

Summarizing the results, a bias on $M_W$ at the level of 10 MeV is obtained from the DYRAD calculation tested moving both the scales to $2M_V$ or $0.5M_V$. Additional checks for other choices of the renormalisation and factorisation scales or using MC@NLO would suggest somewhat larger biases, but they are to be considered very preliminary because of their high numerical uncertainty (especially in the MC@NLO test). A preliminary estimate of
the systematic uncertainty on the $W$ mass implied by the uncertain knowledge of QCD radiative corrections is hence conservatively quoted to about 20 MeV. A more detailed analysis, with improved numerical precision, is required in order to quantify the ultimate precision of this method at 10 fb$^{-1}$ of integrated luminosity.

Should the precision turn out to be insufficient, the extension of the calculation one order higher in $\alpha_S$ is technically feasible and has been recently become available [87].

5.3 $W$ boson width

The present uncertainty on $\Gamma_W$ is of 41 MeV [10] from direct measurements. Such a variation of $\Gamma_W$ would directly affect the shape of the Jacobian peak and modify the high tail both of the lepton transverse energy distribution and of the boson transverse mass. This can be exploited in a two-parameter fit of the observed distribution, which has not been attempted yet. As discussed in [37], a change in $\Gamma_W$ results in a variation of the $Z$ to $W$ normalisation (the $R(X)$ shape) above the resonance ($X > 1$), while leaving unaffected the normalisation below the resonance.

The uncertainty induced on the $W$ mass has been tested assuming an uncertainty of about 40 MeV on $\Gamma_W$, which can be accounted for in the procedure by modifying $R(X)$ accordingly. In order to perform this study, the MC@NLO program, which allows to chose the value of $M_W$ and $\Gamma_W$
independently, has been used. Samples of \( W^+ \) and \( W^- \) events have been generated with \( M_W = 80.45 \text{ GeV} \) for three different values of \( \Gamma_W \): 2.141 \text{ GeV} (the actual world average value [10]), 2.100 \text{ GeV} and 2.182 \text{ GeV}. The last two values correspond to about \( \pm 1 \) sigma from the central value. For each \( \Gamma_W \) value, about 15 million of \( W^+ \) and 15 million of \( W^- \) events and have been generated. In order to compute \( R(X) \) without reprocessing all the events through the simulation, a simple reweighting procedure has been attempted. Events have been weighted according to the ratio \( \frac{dN}{dQ}^{|\Gamma_\text{ref}|} \) of the distributions of the invariant mass \( Q \), at the generator level, of the lepton-neutrino pair obtained with a certain value \( \Gamma_W \) with respect to one obtained with the central \( \Gamma_W = \Gamma_\text{ref} \) reference value (see Fig.5.12).

The ratio of \( R(X) \) obtained with a choice of the \( W \) width with respect to the one obtained with the reference \( \Gamma_W \) value is shown in Fig. 5.13.

An uncertainty of about 40 MeV in \( \Gamma_W \) translates in a normalisation uncertainty above the resonance lower than about 0.2\% in the region \( X > 1 \) and greater than about 0.5\% only in the region with high \( X \) values (\( X = \)

Figure 5.12: Ratio of the distribution \( \frac{dN}{dQ}^{|\Gamma_\text{ref}|}/\frac{dN}{dQ}^{|\Gamma|=\Gamma_\text{ref}} \) as a function of the invariant mass \( Q \) of the electron-neutrino pair for two different choices of the \( W \) width value corresponding to \( \pm 40 \) MeV with respect to the \( \Gamma_W \) reference value. A fit with a function corresponding to the ratio of two Breit-Wigner distributions characterized by different widths, \( \frac{2\pi}{\sigma_{\Gamma_\text{ref}}} \cdot (Q-M_W)^2+(\Gamma_\text{ref}/2)^2 \), is superimposed.
Figure 5.13: Ratio \( R(X)/R_{\text{ref}}(X) \) for two different values of the \( W \) boson width with respect to a reference value, as a function of the scaled electron transverse energy. The full line corresponds to \( \Delta \Gamma_W = -40 \text{ MeV} \) and the dashed line to \( \Delta \Gamma_W = +40 \text{ MeV} \).

\( 2\sqrt{E_T}/M_V > 1.4 \), outside the range used by the fitting procedure. The shape distortions result in about 10 MeV uncertainty on the \( W \) boson mass. This figure is quoted as uncertainty for 1 fb\(^{-1}\) of integrated luminosity, while an improvement is expected for a measurement with 10 fb\(^{-1}\) when \( \Gamma_W \) will be measured from LHC data. Two methods are usually applied at hadron colliders to extract \( \Gamma_W \). The first is the indirect one from the measurement of the ratio \( R = \frac{\sigma_W}{\sigma_Z} \cdot \frac{\text{BR}(W \rightarrow l\nu)}{\text{BR}(Z \rightarrow ll)} \). Using the very precise measurement of the \( Z \) branching ratio (BR) from LEP [83] and NNLO calculations of the ratio between \( W \) and \( Z \) total cross sections, together with the Standard Model prediction of \( \Gamma(W \rightarrow l\nu) \), one can extract \( \Gamma_W \). The second method is the direct measurement from a fit of the high mass tails of the \( W \) transverse mass spectrum (100 GeV < \( M_T \) < 200 GeV), which are sensitive to the \( W \) width. Using this method, a combined uncertainty \( \Delta \Gamma_W \sim 35 \text{ MeV} \) is expected with 2 fb\(^{-1}\) from Tevatron Run II [84], where the main sources of uncertainty can be identified in the knowledge of the lepton energy scale and in the recoil modelling. A further improvement in the \( W \) width measurement can thus be expected at the LHC because of the availability of larger \( Z \) control samples to be used for calibration purposes.
5.4 Radiative decays

The size of the NNLO QCD corrections and the improved stability of the results against variations of the renormalisation and factorisation scales [85, 86, 87] impose to worry about electroweak corrections to $W$ and $Z$ production, being $O(\alpha_{em}) \sim O(\alpha_s^2)$. The understanding of electroweak radiative corrections is then crucial for a precision $W$ mass measurement.

The dominant process is the final state photon radiation (FSR) from the charged lepton, the effect of which strongly depends on the lepton identification criteria and the energy or momentum measurement methods employed. In particular, calorimetric energy measurements, such as those adopted in the electron channel, are more inclusive than track based momentum measurements used, for example, in the muon channel and the effect of FSR is consequently reduced. Since the final state radiation has a significant impact on $W$ mass, multi-photon radiation has to be taken into account.

In the context of the “Scaled Observables method”, unlike other uncertainties, the effect of photon radiation is not cancelled in the ratio of $W$ distributions to $Z$ distributions, because in $Z \rightarrow ll$ events both final state leptons couple to photons, in constrast to $W \rightarrow l\nu$ where only one of the final state particles carries electric charge. Electroweak radiative corrections are therefore mandatory using the ratio method.

In the last years, complete $O(\alpha)$ calculations for $W$ production [88, 89, 90, 91, 92] and $Z$ production [93] have been carried out. Tools to include higher order electroweak radiative corrections are indeed available [92, 94]. Extrapolating from Tevatron results, an uncertainty on $M_W$ of about 10 MeV related to radiative decays can be assumed.
Chapter 6

Experimental uncertainties

The final precision on $M_W$ will be affected by several systematic uncertainties of instrumental origin. In this Chapter the following experimental effects are considered: the electron energy scale and its linearity, the electron energy resolution, the electron identification efficiency, the background modelling and the recoil reconstruction. Emphasis is put on the definition of the analysis strategies aimed at controlling these effects with data, whenever possible.

In the context of the “Scaled Observables Method”, the uncertainties that are common on $W$ and $Z$ are naturally reduced by taking the ratio between $W$ and $Z$ distributions. This is remarkable for the absolute energy scale uncertainty. Other effects, such as the non-linearity of the electron energy scale, the lepton efficiency and the background modelling, are instead not canceled in the ratio, since their impact on the scaled electron $E_T$ spectrum in $W$ events is different than in $Z$ events, and must be controlled carefully.

The precision expected from the first sample of 1 fb$^{-1}$ of data collected during the low luminosity phase are quoted.

6.1 Electron energy scale and resolution

The lepton energy scale uncertainty is the dominant source of uncertainty of instrumental origin in the $W$ mass measurement performed with the Monte-Carlo templates method traditionally used at hadron colliders. The energy scale can be modified in two ways: by an absolute scale factor and by a non linearity term. The effect of the former miscalibration is expected to be largely reduced by the adopted analysis strategy, as a common miscalibration in $Z$ and $W$ events will cancel in the ratio. The latter effect is more
nasty because, by definition, non-linearities are not canceled in the $W/Z$ ratio.

### 6.1.1 Absolute energy scale calibration

The impact of the uncertainty on the electron energy scale has been estimated introducing a miscalibration factor $\delta E_T/E_T$ on the reconstructed transverse energy of the leptons, both in the $W$ and $Z$ samples.

In Fig. 6.1, the shift on the $W$ mass extracted from the fitting procedures applied to the electron scaled transverse energy spectrum is shown as a function of the injected miscalibration factor. A shift $\Delta M_W$ of 4 MeV for $\delta E_T/E_T \sim 0.1\%$ is found. This is about 20 times smaller than the impact of the scale uncertainty in the traditional approach. In fact, ideally, in the method adopted, the effect of a scale uncertainty should be exactly zero, the residual effect is due to the slope of the re-weighting function: a wrong scale results in a wrong weight.

![Figure 6.1: The shift $\Delta M_W$ as a function of the relative mis-calibration factor $\delta E_T/E_T$ in the transverse energy scale.](image)

The statistical precision on the overall energy scale with $2 \text{ fb}^{-1}$ is expected to about 0.05% from the calibration of the electromagnetic calorimeter using $Z \rightarrow e^+e^-$ events [47]. Non-uniformities in the response to electrons as a function of $\eta$ are expected, as a result of the variation of the tracker material distribution in front of the calorimeter. These would spoil the resolution and
not the overall scale. Nevertheless, we have conservatively assumed that the energy scale be known with a precision of 0.25% in the initial phase. This figure, comparable to the precision of the calorimeter inter-calibration [47], would lead to an uncertainty on $W$ mass measurement from the lepton scaled transverse energy distribution of about 10 MeV, that is anyway marginal with respect to the statistical precision of the method.

The estimate of the transverse energy of the electron is based on the calorimetric measurement of the energy and on the measurement of the $\theta$ direction from the tracker. The major impact on the $W$ mass precision is expected to come from the energy scale measured in the electromagnetic calorimeter. The measurement of the $\theta$ direction is not critical and the precision available in the first run will suffice. The direct measurement of the electron impact position in the calorimeter would already give a precision of 0.1%. The uncertainties on the magnetic field mapping, relevant in the bending plane, are not of direct importance in this measurement. They might only affect the precision of the energy scale calibration through the measurement of the electron directions, needed in the computation of the invariant transverse mass of the electron-positron pair from the $Z$.

### 6.1.2 Energy scale non-linearity

The effect of non-linearities in the estimate of the transverse energy, which are expected to be present, are definitely more critical. The prediction of the transverse spectra for the observables sensitive to the $W$ mass is based on a scaling procedure from a corresponding variable in the $Z$ events. The 10% mass difference between the two vector bosons implies a 10% difference in the relevant range of $E_T$ spectra in the two cases. A non-linearity in the energy scale about the Jacobian peak in $Z$ events can be parameterised as

$$E_T' = E_T \left( 1 + \epsilon \left( E_T - \frac{M_Z}{2} \right) \right) \quad (6.1)$$

While this results, on the average, in a correct transverse energy estimate for $Z$ events, a shift $\delta E_T$ in the transverse energy estimate for events around the Jacobian peak of the $W$ boson is expected:

$$\delta E_T \sim \frac{1}{4} M_W \epsilon (M_W - M_Z) \quad (6.2)$$

This implies a bias in the determination of the $W$ mass of about $2\delta E_T$, which is of order of 10 MeV for a non-linearity of $2 \times 10^{-5}$ GeV$^{-1}$. This rough
estimate, well consistent with the required precision quoted by the CDF experiment [39], has been confirmed by changing the non-linearity in the sample simulating the experiment, while leaving unaffected the computation of \( R(X) \). Any differential non-linearity between the Monte Carlo and the final experiment will therefore need to be controlled at this level. This is a serious challenge, since the methods used at the Tevatron to control non-linearities will not be easily available. In particular, the \( \Upsilon \) and \( J/\psi \) resonances will be largely filtered by the single- and double-electron HLT and in any case will provide mostly events of low transverse energy. On the other hand, the control of the linearity based on a comparison of the momentum to the energy scale, also attempted at Tevatron, is difficult and would bring magnet and tracking uncertainties into the game. This is not desirable in particular with the first sample of collected data. To cope with these difficulties and limitations, a different procedure is envisaged, based on the use of \( Z \rightarrow e^+e^- \) events alone. The invariant mass, \( Q \), of an \( e^+e^- \) pair is indeed modified (twice) by the same non-linearity factor entering equation (6.1). This can be measured by looking at the distribution of the average value of \( \log(Q^2/M_2^2) \) as a function of the sum of the transverse energies \( E_{T1} + E_{T2} \) of the two leptons of the Drell-Yan pair. Indeed, a mis-calibrated and non-linear energy response would result in a squared invariant mass \( Q'^2 \) given by

\[
< \log(Q^2/M^2) > = < \log(Q^2/M_2^2) > + \log(1 + s_E) + \\
+ \log(1 + \epsilon \Delta E_{T1}^T) + \log(1 + \epsilon \Delta E_{T2}^T) \quad (6.3)
\]

\[
\sim < \log(Q'^2/M_2^2) > + s_E + \epsilon \Delta E_{T1}^T + \epsilon \Delta E_{T2}^T \quad (6.4)
\]

where \( \Delta E_{T1}^T = E_{T1}^T - M_Z/2 \), \( \Delta E_{T2}^T = E_{T2}^T - M_Z/2 \). The second equality holds upon the condition that the absolute scale miscalibration \( (s_E) \) and the non-linearity terms be small.

Differential non-linearities \( \xi = \Delta \epsilon \) between the data and the Monte Carlo simulation used to define the re-weighting function could be observed by comparing the distribution of \( < \log(Q^2/M^2) > \) as a function of \( \Delta E_{T1}^T + \Delta E_{T2}^T \) obtained from the the data to the same distribution obtained from the simulation. The difference between the two \( < \log(Q^2/M^2) > \) distributions as a function of \( \Delta E_{T1}^T + \Delta E_{T2}^T \) shows the differential non-linearity \( \xi = \Delta \epsilon \):

\[
\Delta(< \log(Q^2/M^2) >) \sim \xi(\Delta E_{T1}^T + \Delta E_{T2}^T) \quad (6.5)
\]
A method could then be designed to extract the differential non-linearity \( \xi \) from a two dimensional analysis of the difference \( \Delta(< \log(Q^2/M^2>) \), between the data and the simulation, as a function of \( \Delta E_T^1 + \Delta E_T^2 \).

Figure 6.2 shows the distribution of \( \log(Q^2/M_Z^2) \) as a function of sum \( \Delta E_T^1 + \Delta E_T^2 \).

![Figure 6.2: The distribution of \( \log(Q^2/M_Z^2) \) as a function of \( \Delta E_T^1 + \Delta E_T^2 \).](image)

In events characterized by an invariant mass value far from the \( Z \) peak, the leptons are allowed to acquire a transverse energy, roughly \( E_T^\ell \sim Q/2 \), which can be far from values around the Jacobian peak. In these cases, the \( (Q^2/M_Z^2) \) distribution is not flat with respect \( E_T^1 + E_T^2 \) but increases with \( E_T^1 + E_T^2 \), i.e. \( \log(Q^2/M_Z^2) \sim \log(E_T^1 + E_T^2) \), giving an explanation of the structure observed in the \( \log(Q^2/M_Z^2) \) distribution of Fig. 6.2. A cut on the invariant mass of the lepton pair coming from the \( Z \) decay is then applied to select events in a narrow window, \( |Q - M_Z| < 10 \text{ GeV} \), around the \( Z \) peak. The additional requirements that the two electrons, passing the double electron HLT, lie within the rapidity acceptance of the electromagnetic calorimeter and that \( |E_T^1 - E_T^2| < 30 \text{ GeV} \) are imposed. The latter corresponds to the requirement of \( Z \) events with small values of the boson transverse momentum. These events well describe the kinematic region of events considered in the \( W \) mass analysis.

To test the method, differential non-linearities up to \( \pm 0.0005 \text{ GeV}^{-1} \) have been artificially introduced on a sample of simulated \( Z \to ee \) events, while
another sample has been left unaffected. According to equation (6.5), a linear fit of $\Delta < \log(Q^2/M_Z^2) >$ as a function of $\Delta E_T^1 + \Delta E_T^2$ allows to extract the differential non-linearity between the two samples. The extraction of the non-linearity requires an iterative procedure. Instead, the bare application of this method returns a value of the non-linearity biased but strongly correlated to the true one, as shown in Fig. 6.3. The existence of a bias is due to the invariant mass selection, that, in the sample with the non-linearity added, is actually applied on $Q' = Q\sqrt{(1 + \xi_{true}\Delta E_T^1)(1 + \xi_{true}\Delta E_T^2)}$ which is slightly different from $Q$, implying that the selection is applied to events in slightly different invariant mass regions. The bias is removed by iterating the procedure. This consists in adjusting the invariant mass cut at the $n$-step according to the non-linearity value $\xi_{n-1}$ reconstructed from the previous step. More precisely, the invariant mass cuts are rearranged as follows:

\[
Q' = Q\sqrt{1 + \xi_n \Delta E_T^1} \sqrt{1 + \xi_n \Delta E_T^2} \quad (6.6)
\]

\[
< M^+ (1 + \frac{1}{2}\xi_n \Delta E_T^1)(1 + \frac{1}{2}\xi_n \Delta E_T^2) \quad (6.7)
\]

\[
Q' = Q\sqrt{1 + \xi_n \Delta E_T^1} \sqrt{1 + \xi_n \Delta E_T^2} \quad (6.8)
\]
6.1 Electron energy scale and resolution

\[ > M^-(1 + \frac{1}{2}\xi_{n-1}\Delta E_1^T)(1 + \frac{1}{2}\xi_{n-1}\Delta E_2^T) \]  
\[ (6.9) \]

where \( M^+ = M_Z + 10 \text{ GeV} \) and \( M^- = M_Z - 10 \text{ GeV} \).

After four or five iterations the correct value of the non-linearity is recovered, as it is represented in Fig.6.4 for a value of the non-linearity equal to 0.0004 GeV\(^{-1}\).

The statistical precision expected with an integrated luminosity of 1 fb\(^{-1}\) is about \( 1.7 \times 10^{-5} \) GeV\(^{-1}\) (Fig. 6.5).

This figure can be contrasted to the systematic limitation of about \( 6 \times 10^{-5} \) GeV\(^{-1}\) quoted by the CDF experiment [39], based on the comparison of the energy and the momentum scales, and implies an estimated uncertainty of 10 MeV on the \( W \) mass. Should one keep the CDF value as the initial estimate of the differential non-linearities in the data and Monte Carlo, an uncertainty of about 30 MeV would be obtained. This number is conservatively quoted as uncertainty on \( W \) mass due to non-linearities for 1 fb\(^{-1}\) leaving room for possible systematic effects or possible rapidity dependences due to the variation of the tracker material, not yet accounted within the present analysis. At 10 fb\(^{-1}\) when larger statistical samples will be available and a better understanding of the overall detector response including tracking will be gained, the method proposed here can be cross-checked with
Figure 6.5: Dependence of $\Delta(< \log(Q^2/M_Z^2) >)$ as a function of the $E_T$ of one of the two electrons of a Drell-Yan pair. The default FAMOS simulation is taken as a reference and compared to simulated data samples where additional non-linearities of $\pm 0.0004$ GeV$^{-1}$ have been introduced. The effect of the differential non-linearity is evident in the slope of the distribution.

more traditional approaches to help reduce the uncertainty below 10 MeV.

6.1.3 Electron energy resolution

The transverse energy resolution modifies the shape of the Jacobian peak. This and the uncertainty in the knowledge of the resolution function need to be accounted for in the modelling of $R(X)$. For illustration, Figure 6.6 shows the ratio between $R(X = 2E_T^e/M_V)$ obtained with transverse energy of the electron at the generator level and $R(X = 2E_T^e/M_V)$ computed from the reconstructed electron transverse energy: the presence of a slope, though marginal, suggests that if the resolution is not accounted for in the computation of $R(X)$, $Z$ events are wrongly weighted leading to a shift in the $W$ mass.

The study of resolution effects has been performed modifying the width of the $E_T^e$ resolution in the $W$ and $Z$ samples, keeping its shape fixed. The fitting procedure was then repeated for different values of the resolution using the weighting function of Section 4.2. In Fig. 6.7 the variation of the $W$
mass value obtained from the fit of the electron scaled $E^T$ spectrum is plotted as a function of the relative change in the transverse energy resolution. The knowledge of the resolution with a relative accuracy of 1%, would give an uncertainty below 1 MeV on the $W$ mass.

The energy resolution can be extracted from the study of the width of the $Z$ line shape. The CDF experiment measured it with a precision of about 10%, limited by the $Z$ statistics, while a systematic limitation of 1.5%, related to photon radiation from the final state electrons, was claimed [39].

A study of the statistical precision achievable with CMS on the resolution from the $Z$ line shape has been performed in the following way. $Z \rightarrow ee$ samples, in which the electron energy resolution has been modified with respect to the one provided by the fast simulation, are used to obtain reference distributions for the lepton pair invariant mass spectrum. These reference distributions are then compared to that obtained from a sample where no modifications on the resolution have been introduced. This latter sample works like a sample of pseudo-data.

This study has been performed using a pseudo-data sample corresponding to $1 \text{ fb}^{-1}$ of integrated luminosity, while the reference samples corresponding to about $\sim 7 \text{ fb}^{-1}$ in order to limit the statistical fluctuations due to the fi-
nite statistics of the reference distributions. Moreover, the analysis has been performed in different electron/positron pseudorapidity regions, since a dependence of the resolution on the pseudorapidity is expected: in fact, given a certain $E_T$, different energies are sampled, corresponding to different values of the polar angle; moreover, the effects of tracker material depend on the pseudorapidity.

Events are selected with both the leptons in the same pseudorapidity bin. The ECAL pseudorapidity coverage has been divided into 6 $\eta$ regions: four regions in the barrel, defined by $0 < |\eta| < 0.4$, $0.4 < |\eta| < 0.8$, $0.8 < |\eta| < 1.2$, $1.2 < |\eta| < 1.4$; and two regions in the endcaps defined by $1.6 < |\eta| < 2.0$ and $2.0 < |\eta| < 2.4$. As usual, the gap region between barrel and endcap has not been considered. The invariant mass spectra for the different regions are shown in Fig. 6.8. An invariant mass peak slightly below the $Z$ mass value is observed for leptons in the pseudorapidity region $1.2 < |\eta| < 1.4$: in this region, the electron energy reconstruction is deteriorated by the presence of a large dead region between the barrel and the endcaps of the electromagnetic calorimeters, as it was shown in Fig. 4.1 of Section 4.1.

The resolution has been varied in different ways:

- a global factor $(1+\delta)$, energy independent, is introduced; $\delta$ represents...
6.1 Electron energy scale and resolution

Figure 6.8: The invariant mass spectrum of the electron-positron pair from $Z \rightarrow ee$ events obtained with leptons belonging to different pseudorapidity regions. Both electrons are intended to lie in the same $\eta$ bin.

the fractional difference in the resolution width with respect to the nominal one;

- an extra-smearing to the stochastic term or to the constant term in the energy resolution has been added.

In the former case, the energy is modified accordingly to

$$E' = E^{true} + (1 + \delta)(E^{meas} - E^{true}),$$

where $E'$ is the energy corresponding to the new resolution, $E^{meas}$ is the measured energy and $E^{true}$ is the true (i.e., at the generator level) energy. This approach suppose the knowledge of the shape of the resolution and has the effect to modify the width of the resolution, leaving the shape unaffected. A $\chi^2$ test between the reference distributions obtained for different values of the $\delta$ parameter and the pseudo-data distribution in the invariant mass range 85 - 97 GeV is performed for each $\eta$ region. The $\delta$ values to be tested have been chosen at step of 0.006 in a range between -0.5 and +0.5. The exercise has been repeated using as pseudo-data 1000 sub-samples randomly extracted from the global sample of the generated $Z \rightarrow ee$ events.
The distribution of the best fit parameter values is expected to be gaussian and centered in $\delta = 0$; its width corresponds to the statistical precision on the determination of the parameter (see Fig. 6.9).

Table 6.1 summarizes the statistical precision on the $\delta$ parameter for 1 fb$^{-1}$ in the different pseudorapidity bins. The collected uncertainties are about 3% and translate in an uncertainty on $M_W$ of about 3 MeV.

In the second case, the stochastic and constant term are allowed to vary in an independent way and the resolution is modified according to:

$$\frac{\sigma'(E)}{E} = \frac{\sigma(E)}{E} \oplus \frac{\alpha}{\sqrt{E}}$$

(6.10)

if we are considering the stochastic term, or

$$\frac{\sigma'(E)}{E} = \frac{\sigma(E)}{E} \oplus \beta$$

(6.11)

when considering the constant term.

The procedure to determine the best parameters’ value and the statistical
6.1 Electron energy scale and resolution

| $|\eta|$ bin | statistical precision with 1 fb$^{-1}$ [85 GeV $\leq M_{ee} < 97$ GeV] | number of events |
|-----------|---------------------------------|-----------------|
| 0.0 - 0.4 | 2.8% | 71210 |
| 0.4 - 0.8 | 3.7% | 64191 |
| 0.8 - 1.2 | 3.1% | 57165 |
| 1.2 - 1.4 | 3.5% | 16912 |
| 1.6 - 2.0 | 2.4% | 35675 |
| 2.0 - 2.4 | 3.4% | 30516 |

Table 6.1: The statistical precision on the electron energy resolution defined by the parameter $\delta$ (see text) in different pseudorapidity regions.

uncertainty is the same described above. Values for the parameter $\alpha$ up to 0.05 and $\beta$ up to 0.01 have been tested. The resulting statistical precisions for an integrated luminosity of 1 fb$^{-1}$ are summarized in Table 6.2.

| $|\eta|$ bin | statistical precision on $\alpha$ (1 fb$^{-1}$) | statistical precision on $\beta$ (1 fb$^{-1}$) |
|-----------|---------------------------------|---------------------------------|
| 0.0 - 0.4 | $1.4 \times 10^{-3}$ | $1.0 \times 10^{-4}$ |
| 0.4 - 0.8 | $1.8 \times 10^{-3}$ | $1.9 \times 10^{-4}$ |
| 0.8 - 1.2 | $1.6 \times 10^{-3}$ | $1.8 \times 10^{-4}$ |
| 1.2 - 1.4 | $1.5 \times 10^{-3}$ | $1.7 \times 10^{-4}$ |
| 1.6 - 2.0 | $2.4 \times 10^{-3}$ | $1.2 \times 10^{-4}$ |
| 2.0 - 2.4 | $2.1 \times 10^{-3}$ | $1.3 \times 10^{-4}$ |

Table 6.2: The statistical precision on the electron energy resolution defined by the parameters $\alpha$ and $\beta$ (see text) in different pseudorapidity regions. Both leptons from $Z \rightarrow ee$ are required to lie in the same $\eta$ bin.

A simultaneous fit of the two parameters has not been attempted yet. Considering that the nominal values for the stochastic term and the constant term are respectively 0.027 and $5 \times 10^{-3}$, the statistical errors obtained on the extra-smearings correspond to an uncertainty at the level of a few percent. These precisions would cause shifts of a few MeV on $M_W$: more precisely, adding an extra-smearing to the constant term, a bias of $-6$ MeV is found from the scaled $E_T$ fit; the extra-smearing on the stochastic term gives $\Delta M_W = -3$ MeV.

Still the precise knowledge of the resolution will be dominated by systematic effects such as the photon radiation mentioned above. Lacking at this very moment any full detailed study on the resolution modelling for the CMS detector through the analysis of the $Z$ line shape, we conservatively assume
an initial uncertainty of 5 MeV on the $W$ mass from a fit of the scaled $E_T$ spectrum, corresponding to a relative uncertainty of about 8% on the energy resolution.

### 6.2 Trigger and selection efficiency

Trigger and selection efficiency are relevant because they induce distortions to the $R(X)$ theoretical prediction that need to be corrected for. In particular if the electron efficiency shows a dependence on the electron transverse energy and pseudorapidity this is not compensated taking the ratio of $W$ to $Z$ distributions as leptons from $W$ and $Z$ cover slightly different regions in $E_T$, $\eta$.

At the trigger level a selection of single electron events with a transverse energy $E_T$ threshold of 26 GeV is applied. The observable relevant to this analysis is not directly $E_T$, but the scaled variable $2E_T/M_V$. A threshold at $X_T = 0.75$ is applied in the analysis, corresponding to about 30 (33) GeV for electrons coming from the $W$ ($Z$) decay. At these energies the trigger efficiency is almost independent of the transverse energy [38]. The residual dependence on the electron transverse energy, related to the effects of other HLT selections (such as the $E/P$ or isolation selections), can be established through the analysis of $Z \rightarrow ee$ events triggered by the double-electron HLT and compensated for in the definition of $R(X)$ the final effect on $M_W$ is expected to be small.

The efficiency of the offline electron identification criteria will also be measured from data. Two strategies have been considered to measure the efficiency and will be discussed in the following. One method relies on $Z \rightarrow ee$ events. The other approach exploits HLT electron candidates selected according specific topological selection, aimed to define a sample of pure electrons and to reject the most of the fake-electrons from the QCD background. An uncertainty on $M_W$ due to the lepton efficiency knowledge at the level of 10 MeV can be anticipated.

**Offline electron efficiency using “tag-and-probe”**

The efficiencies of the trigger selections and of the lepton identification selections can be determined directly from the data using $Z \rightarrow ee$ events. The procedure is known as “tag-and-probe” method. One of the two legs of the
6.2 Trigger and selection efficiency

\( Z \) is required to satisfy tight electron selections and is used as tag; the second electron leg in the events, referred to as probe, is reconstructed as an electron candidate with criteria independent of the ones for which the efficiency has to be measured; and finally, to further reduce the backgrounds, the invariant mass of the tag-probe pair is required to be in a narrow window around the \( Z \) peak. Assuming that the criteria for which the efficiency is going to be measured are included in the tag definition, the efficiency \( \epsilon \) is obtained as follows:

\[
\epsilon = \frac{2N_{TT} + N_{TA}}{2N_{TT} + N_{TA} + N_{TF}},
\]

where \( N_{TT} \) is the number of events with two tags, \( N_{TA} \) is the number of events with one tag and one probe surviving the selection criteria and \( N_{TF} \) is the number of events with the probe failing. The critical aspects are the study of possible correlations between the selections applied to the tag and to the probe and the background estimation and subtraction. The main backgrounds are represented by by QCD di-jets where the jets are misidentified as electrons or by \( W \to e\nu \) events with an associated hadronic jet faking an electron. Following the procedures adopted by CDF, the first background can be estimated using the number of same-sign events in the \( Z \to ee \) candidates samples, i.e. of events with two electron candidates with the same charge. This estimation relies on the assumption that no charge correlation is expected in hadronic jet events between the fake electrons and, consequently, the number of same-sign events is expected to be equal to the number of opposite-sign pairs. A correction for real \( Z \to ee \) reconstructed as same-sign events has to be included and can be taken from the Monte Carlo simulation. The contribution from \( W+\)jets can be accounted for from a combination of Monte Carlo informations and measured jet fake rates. More details can be found in [23].

The “tag-and-probe” method has been tested here to study its application in the determination of the offline electron identification efficiency. The tag is chosen among electron candidates passing the isolated single electron HLT requirements and all the electron identification criteria (see Sec. 3.3.1). A probe is identified as a supercluster in ECAL matched to a track \( (p_T > 10 \text{ GeV}) \) within a cone of radius \( R = \sqrt{\Delta \eta^2 + \Delta \phi^2} = 0.2 \) and it has to satisfy the isolated single electron HLT. The invariant mass of the tag-probe pair is required to be in the mass window 82-92 GeV. The offline efficiency is given by the number of probes passing the electron identification
selections over the total number of probes. The study has been performed in the CMSSW framework using the available sample of $Z/\gamma^* \to ee$ events, that corresponds to about 100 pb$^{-1}$ of integrated luminosity. The offline electron efficiency from “tag-and-probe” mapped as a function of $E_T, \eta$ is shown in Fig. 6.10.

![Figure 6.10: Efficiency of the offline electron identification selections from the “tag-and-probe” method in $E_T, \eta$ bins.](image)

The electron identification efficiency $\epsilon(E_T, \eta)$ obtained from “tag-and-probe” can be used to predict the efficiency in $W \to e\nu$ events: a good agreement with the Monte Carlo offline electron efficiency, obtained as the number of electron candidates in $W \to e\nu$ events satisfying the electron identification criteria divided by the total number of electron candidates passing the HLT, is found and the comparison is shown in Fig. 6.11.

**Offline electron efficiency using isolated HLT electrons**

Another possible approach to study the offline electron efficiency uses electron candidates filtered by the isolated single electron HLT. The isolated single electron HLT accepts fake electrons from di-jets in a ratio about 1:1 to electrons from $W \to e\nu$. It is possible to select among these events a sample consisting mainly of real electrons, by applying topological selections that reject the most of fake-electrons. In particular, the following variables have been found to be useful to discriminated between electrons from $W \to e\nu$ and fake-electrons:
6.2 Trigger and selection efficiency

Figure 6.11: The efficiency of the offline electron identification selections in the $W \rightarrow e\nu$ sample from the Monte Carlo simulation (line) is compared to the efficiency obtained from the “tag-and-probe” method (markers) as a function of the lepton transverse energy and pseudorapidity.

- $\Delta\phi$, the azimuthal separation between the HLT electron supercluster and the leading jet;

- $E_{SC}^T/E_{jet}^T$, the ratio between the supercluster transverse energy and the leading jet transverse energy.

Any reconstructed jet cluster with $E_{jet}^T > 5$ GeV has been considered as a jet.

The choice of these quantities to study the QCD background is motivated by the fact that it is preferable to use topological variables different, even if partially correlated, from those used in the $W$ mass analysis.

For fake-electrons from di-jets, the first variable is peaked at $\pi$ because the two jets (one of which is the fake-electron) are back-to-back in the transverse plane; the ratio $E_{SC}^T/E_{jet}^T$ peaks at 1 as the two jets are balanced. The scatter plot of these two variables is shown in figure 6.12.

A sample of pure electrons can be defined by requiring, for example, $\Delta\phi < 2$ and $E_{SC}^T/E_{jet}^T > 1.5$ and, in addition, that events have missing transverse energy (MET > 25 GeV). With this selections, the purity of the electron sample is at the 98% level (see Fig. 6.13).

The comparison between the Monte Carlo efficiency in $W \rightarrow e\nu$ and the efficiency obtained with the procedure described above using the sample tagged by the $\Delta\phi$, $E_{SC}^T/E_{jet}^T$ and MET requirements, is shown in Fig. 6.14.
Figure 6.12: Scatter plot of the ratio $E_{SC}^T/E_{jet}^T$ between the supercluster transverse energy of the electron candidate and the leading jet transverse energy as a function of the azimuthal separation $\Delta \phi$ between the electron candidate supercluster and the leading jet. $W \to ev$ and QCD events are shown with blue and red markers respectively.

Figure 6.13: The purity of the electron sample tagged by $\Delta \phi < 2$, $E_{SC}^T/E_{jet}^T > 1.5$ and MET > 25 GeV.

integrated efficiencies are compatible within the statistical errors obtained with a sample of events corresponding to about 10 pb$^{-1}$: the Monte Carlo efficiency in $W \to ev$ is $88.8 \pm 0.2\%$, the other is $88.9 \pm 1.1\%$. 
The described procedures will allow to compare the MC efficiency to the measured one and eventually to tune it. The finite accuracy with which the efficiency will be measured is unavoidably propagated to the $W$ mass measurement.

Modifying $R(X)$ according to the efficiency $\epsilon(E_T^*, \eta)$ obtained from “tag-and-probe”, a bias on $M_W$ of $10\pm 14$ MeV is found, where the error is the statistical precision corresponding to an efficiency measurement with $100 \text{ pb}^{-1}$. This precision is reduced to about 5 MeV for an integrated luminosity of $1 \text{ fb}^{-1}$. However the accuracy in the determination of efficiencies is limited by the knowledge of backgrounds, especially from hadronic jets faking electrons. The available simulated QCD samples have not enough events to perform a complete study of the background to $Z \rightarrow ee$ in the “tag-and-probe” method. Hence a conservative figure of 10 MeV is assumed as initial uncertainty on $M_W$ due to the lepton efficiency knowledge.

For comparison, the difference between the MC efficiency, in the $E_T^*, \eta$ plane, and the efficiency obtained using the single electrons sample, that includes the QCD background, has been taken as preliminary estimate of the uncertainty in the electron efficiency measurement. The impact on $M_W$ is a bias of about 5 MeV, still within the 10 MeV quoted above. The final uncertainty is expected to decrease with higher integrated luminosity as larger samples
and a better understanding of the systematic effects (mainly related to the background and to its subtraction) will be available.

## 6.3 Background modelling

In the method proposed there are two sources of backgrounds that need to be kept under control: the background in the \( W \rightarrow e\nu \) sample and the background in the \( Z \) sample. The former requires a precise modelling and needs to be added to the \( Z \) scaled sample in the fitting procedure. The latter is dangerous as it could spoils the reliability of the \( Z \) to \( W \) scaling procedure. Yet, as discussed in section 4.1, it is expected to be small.

Background samples corresponding to 1 fb\(^{-1}\) of integrated luminosity for the processes \( Z \rightarrow e^+e^- \), \( Z \rightarrow \tau^+\tau^- \), \( W \rightarrow \tau\nu \) have been generated and processed with the FAMOS simulation. Events surviving the selection criteria have been added to the \( W \rightarrow e\nu \) candidate sample. The same background model was also added to the sample of scaled \( Z \) events entering the fitting procedure. In the latter case, the relative normalisation of the background has been assumed to be uncertain and the best fit value for \( M_W \) has been extracted for several different background normalisations. The variation of \( M_W \) determined form the fitting procedure as a function of the background normalisation is shown in Fig. 6.15, indicating that a precision of about 1 MeV would require a background knowledge at the 1% level.

A small contribution from the QCD background is also expected, but it can be reduced below 1% applying tight electron identification selections and exploiting the rejection power of the topological selections of \( W \rightarrow e\nu \) events. Since the available simulated QCD sample provide a number of events too small to survive all these selections, it has not been possible to include its contribution in the study done for electroweak backgrounds to analyze the effect of the background normalization uncertainty on \( M_W \). Therefore, a conservative estimate of the uncertainty on \( M_W \) due to the background modelling is here given. Assuming a background knowledge at the level of 10% (corresponding to the overall background uncertainty quoted by the CDF experiment [36]), an uncertainty on \( M_W \) of about 10 MeV can be forseen. Indeed, the uncertainty related to the electroweak backgrounds is expected to be smaller than 10%, since their shapes and rates are accurately predictable, as it has been demonstrated by CDF studies [36]. On the other hand, even if the QCD background contamination is expected to
be reduced to an almost negligible level, its modelling is more difficult than for the electroweak backgrounds. For this reason, it is necessary to design a procedure to measure the QCD background directly from data. This aspect will be discussed below.

### 6.3.1 QCD background determination

A suppression of the QCD background greater than $10^8$ is required to keep the QCD background below 1%. As anticipated in Section 4.1, different tools are exploited in order to reduce the QCD contamination in $W \rightarrow e\nu$ events: the High Level Trigger, the electron identification selections and the topological selections defining the $W \rightarrow e\nu$ sample. The single-electron HLT accepts electron candidates with a transverse energy above a threshold of 26 GeV and satisfying the other requirements discussed in Section 2.2.6 (E/p matching, isolation, matching between pixels and supercluster). This provides a rejection factor of about $10^5$ on jets faking electrons. The electron identification exploits variables related to the geometrical matching between track and supercluster, to the energy and momentum matching and to the shower shape, in order to discriminate between electrons and fake-electrons. According to the Monte Carlo simulation, the probability that a jet is misidentified as an electron in events filtered by the isolated...
single electron HLT is about 0.3. The topological selections described in Section 4.1 further reduce the QCD background: the electron transverse energy cut is sufficiently high (29 GeV) in order to get rid of the most of the QCD events with low $p_T$; the selection of events with missing transverse energy removes a considerable part of the QCD di-jets as they are expected to be balanced in the transverse plane; and, finally, the jet veto and the requirement of low hadronic activity are more directly intended as anti-QCD cuts. The effect of these selections depends on the $p_T$ of the QCD events, giving rejection factors between 20 and 100 (see Section 4.1.2).

In order to model the QCD background from data, it is necessary to measure, on a sample of events passing the single electron HLT, the probability to misidentify a jet as an electron and the probability that an event with a fake-electron survives the topological selections. In both cases, samples of pure fake-electrons from di-jets have to be defined. The strategy to define such samples from the data is discussed in the next paragraphs.

**Measurement of electron misidentification probability**

In Sec. 6.2, the two variables $\Delta \phi$ and $E_{SC}^T/E_{jet}^T$ have been identified to distinguish regions of the phase-space mainly populated by electrons from $W \rightarrow e\nu$ or by fake-electrons from di-jets (see Fig. 6.12). The distributions of these two variables for electron candidates passing the HLT are shown in Fig. 6.16 and Fig. 6.17.

A sample consisting mainly of di-jets can be thus obtained imposing, for example, $\Delta \phi > 2.5$ and an upper cut on $E_{SC}^T/E_{jet}^T$. Figure 6.18 shows the effect of the $\Delta \phi$ cut on the $E_{SC}^T/E_{jet}^T$ distribution for HLT electron candidates in QCD events and in $W \rightarrow e\nu$ events.

Moreover the di-jets sample purity can be further improved by an “anti-MET” cut, i.e. by requiring events with low missing transverse energy: this helps to reject, by definition, $W$ events where the presence of a neutrino creates missing transverse energy, and to keep QCD di-jet events that, instead, are expected to be balanced. For example, a purity better than 98% is expected requiring $E_{SC}^T/E_{jet}^T < 1.$ and MET<20 GeV. The dependence of the di-jet sample purity on the $E_{SC}^T/E_{jet}^T$ cut is shown in figure 6.19 for different choices of the “anti-MET” cut.

The purity of the sample of fake-electrons obtained by requiring $\Delta \phi > 2.5$, $E_{SC}^T/E_{jet}^T < 1.$ is also shown in Fig. 6.20 as a function of the transverse
6.3 Background modelling

![Figure 6.16](image1)

**Figure 6.16:** Distribution of the azimuthal separation $\Delta \phi$ between the HLT electron and the leading jet. The blue line represents electron candidates from $W$ events, the black one refers to electron candidates from QCD di-jets; both of them have passed the single electron HLT selections.

![Figure 6.17](image2)

**Figure 6.17:** Distribution of the variable $E_{SC}^T/E_{jet}^T$, the ratio between the supercluster transverse energy and the leading jet transverse energy for electron candidates in events ($W$, blue line; QCD, black line) that passed the single electron HLT selection.
Figure 6.18: Distribution of the variable $E_{SC}^T/E_{jet}^T$, the ratio between the supercluster transverse energy and the leading jet transverse energy; the full histograms represent the distributions after the selection $\Delta \phi < 2.5$.

Figure 6.19: Purity of the di-jet sample obtained from events passing the single electron HLT and with $\Delta \phi > 2.5$ as a function of the $E_{SC}^T/E_{jet}^T$ cut, for different choices of the “anti-MET” cut.
6.3 Background modelling

Figure 6.20: Purity of the di-jet sample as a function of the transverse energy (left) and of the pseudorapidity (right) of the electron candidate. The di-jet sample purity is improved by requiring low missing transverse energy in the events. The two cases with no MET cut (open dots) and with a cut MET$<20$ GeV (full dots) are shown.

The procedure described in this paragraph defines a sample of di-jet events sufficiently pure to measure the misidentification probability ($\eta_{ID}$) on fake-electrons from QCD di-jet events. The purity of the di-jet test sample can be indeed controlled from the data. In fact, the contamination of real electrons could be estimated by studying the shape of the $E_{T}^{SC}/E_{T}^{jet}$ distribution in regions with low $\Delta\phi$ and extrapolating it to model the $E_{T}^{SC}/E_{T}^{jet}$ distribution in the region $\Delta\phi > 2.5$ that is used to define the test sample. Figure 6.21 shows the comparison between the electron misidentification probability obtained from the Monte Carlo and the one that can be measured from data using the di-jet test sample as defined above. The first is calculated as the ratio between the number of di-jet events after electron identification criteria has been applied and the total number of di-jets. The second case represents the measure that can be performed with real data: the misidentification probability has thus been computed as the number of events surviving HLT tagged as di-jets and satisfying the electron identification requirements, divided by the number of events surviving HLT tagged as di-jets.

The total misidentification probability on the QCD di-jets obtained from the Monte Carlo is 40\,$\pm\,$3\% and the one measured from the test sample is 29.6\,$\pm\,$9\%. They are comparable within the errors that are given by the limited number of simulated and reconstructed QCD events surviving all the
Figure 6.21: *Comparison between the Monte Carlo (red line) electron misidentification probability and the one that can be measured from test sample (green line).*

selections. This statistical limitation will not affect the real data, as this test was performed on a sample of events corresponding to about 10 pb$^{-1}$ of integrated luminosity. Moreover the large statistical sample of events available from data will enable to tune the selection criteria defining the test sample in order to maximize the purity.

Assuming the purity of the test sample to be known from the data as discussed above, the measured misidentification probability can be corrected in order to subtract the contribution due to the contamination of residual real electrons. More precisely, given the purity $P$ of the di-jets test sample, the electron identification efficiency $\epsilon_{ID}$ and the measured misidentification probability $\eta_{ID}^{meas}$, the true misidentification probability $\eta_{ID}^{true}$ is obtained as

$$\eta_{ID}^{true} = \frac{\eta_{ID}^{meas} - \epsilon_{ID}(1 - P)}{P}$$  \hspace{1cm} (6.13)

The precision on the misidentification probability that can be achieved using the described procedure is limited by the precision with which the contamination of real electrons in the di-jets test sample will be known.

**Measurement of the efficiency of topological selections.**

The efficiency of topological selections on QCD background and a modelization of it can be also measured from data.

Among events passing the single electron HLT requirements it is possible to identify a sample of electron candidates mainly composed of fake-electrons
and with a small contamination of true electrons. This sample will be referred to as “truly-fake-electrons” and can be used to test the effect of topological selections in the reduction of the QCD background. The assumption behind this procedure is that the topological distributions of the QCD events containing one “truly-fake-electron” is identical to the one of QCD events with fake electron candidates. This is true if only some of the criteria defining an electron is vetoed. “Truly-fake-electrons” are thus defined as those electron candidates that

- satisfy the single electron HLT;
- are not isolated;
- fail at least one of the other electron identification criteria.

The sample obtained in this way will be composed almost at all by fake-electrons, but can be partially contaminated by real electrons from $W \rightarrow e\nu$. From the simulation, the contamination of real electrons to the “truly-fake-electrons” sample has been found to be at the level of 1% (Fig. 6.22). However, this contamination from real electrons can also be measured from the data themselves using the “tag-and-probe” method or the test sample defined by $\Delta\phi < 2., E_{SC}^T/E_{jet}^T > 1.5$ used to study the electron identification efficiency in Sec. 6.2. The contribution of real electrons in the “truly-fake-electron” sample can thus be measured and subtracted from it, enabling to measure the efficiency of topological selections on sample of pure fake-electrons.

6.4 Recoil reconstruction

One of the advantages of the analysis of the electron transverse energy spectrum is that the knowledge of the hadronic recoil scale and resolution as well as the detailed modelling of the underlying event is not as critical as it is in the reconstruction of the transverse mass observable. In fact, for the electron $E_T$, these effects are only indirectly entering in the selection procedure and are not directly affecting the variable considered to extract $M_W$. Thus, their contribution to the total uncertainty on $M_W$ in the $E_T$ analysis is expected to be smaller than in the transverse mass analysis.

These contributions have been evaluated by fixing the $R(X)$ to be used in the scaling procedure to the one obtained from the standard set of selection
cuts, and by repeating the fitting procedure on simulated experiments with $W$ and $Z$ samples selected modifying the missing transverse energy and the recoil transverse momentum according to their uncertainties. The transverse momentum $\vec{u}$ of the system recoiling against the bosons was defined as the sum over all the energy deposited in the calorimetric towers but those corresponding to the electron(s). The missing transverse energy, as it was discussed in Section 3.3.2, is related to $\vec{u}$ and it is obtained summing over all the calorimetric towers, including the energy deposited by the electrons. The result is that, from the electron $E_T^T$ spectrum analysis, a $W$ mass uncertainty of about 10 MeV due to the uncertainty in the recoil modelling requires the hadronic scale to be known with a precision of 2% and the hadronic recoil resolution to be known with a precision better than 2.5%. The hadronic recoil scale and the hadronic recoil resolution can be studied, and possibly tuned, using $Z \rightarrow ee$ events. Also in this case, the effect on $W$ mass is not due to the absolute hadronic scale, but on the contrary it comes from the precision with which any difference between the scale in the MC model used to define $R(X)$ and the data is known. Drell-Yan $Z \rightarrow ee$ events, in which the boson transverse momentum can be measured from the two leptons with good accuracy, can be used to this purpose exploiting
the balance of the recoil momentum $\vec{u}$ and of the boson momentum in the transverse plane.
In order to disentangle the effect due the scale from that associated to the resolution, it is better to define the transverse momentum of the system recoiling against the boson, $\vec{u}$, in terms of the components $u_1$ and $u_2$, respectively, parallel and perpendicular to the boson transverse momentum.

In $Z \to ll$ events, the $Z$ boson transverse momentum is well measured from the two charged leptons, making these variables well defined and accessible.

The parallel component $u_1$ is the hard part of the hadronic recoil balancing the boson transverse momentum, $p_T^Z$, and hence is useful to test the hadronic scale. The other one, $u_2$, is the soft component, associated to pile-up and underlying events, and gives informations on the hadronic recoil resolution.

Figures 6.23 and 6.24 show the distributions of the variables $u_1$ and $u_2$ for a sample of $Z \to ee$ events FAMOS simulated.

The average value of $u_1$ is the average calorimeter response to the hadronic recoil, supposed to balance $p_T^Z$. Hence, the correlation between $u_1$ and $p_T^Z$ (Fig. 6.23) can be used to calibrate the hadronic response with respect to electromagnetic scale. The average value of $u_2$ is zero, as expected, and the resolution measured by the width of the $u_2$ distribution is about 8.5 GeV (Fig. 6.24). These distributions will be used to cross-check data and Monte Carlo.
Some restrictions are to be considered when transporting this calibration to \(W\) events, because of several reasons. Firstly, at variance with \(W\) decays, where only a single lepton and true missing energy are present, in \(Z \rightarrow ee\) events two leptons need to be suppressed and their hits completely removed in the computation of \(u\). Moreover, differences due to radiative decays can arise between \(W\) and \(Z\) when the transverse energy of the photon radiated by the lepton(s) is not completely collected in the electron supercluster, leading to an incorrect subtraction of the energy associated to the lepton(s). Finally, the underlying event in \(Z\) and \(W\) boson production is expected to be different. Thus, systematic limitations arise in the achievable precision of the hadronic recoil modelling.

An estimate of the impact of these systematic effects has been obtained from a comparison between \(W\) and \(Z\) of the distributions of the relevant variables \(u_1\) and \(u_2\). As the \(W\) boson \(p_T\) cannot be measured from the reconstructed leptons like in \(Z\) events, the \(p_T^W\) variable for this study is taken at the generator level.

Figure 6.25 shows the ratio between \(W\) and \(Z\) of the \(u_1\) distributions as a function of the vector boson transverse momentum. These distributions are obtained from events selected through the criteria developed for the \(W\) mass measurement and defined in Section 4.1. A difference at the level of 2\% is observed and is taken as an estimate of the systematic uncertainty on the
hadronic recoil scale.

The distributions of the recoil component parallel to the boson $p_T$ are shown in Fig. 6.26 for $Z \rightarrow ee$ and $W \rightarrow e\nu$ events: in both cases, the mean value is consistent with zero; the width of the $u_2(W)$ distribution is 8.43 GeV, while the width of the $u_2(Z)$ distribution is 8.49 GeV. The observed difference is at the 1% level and it is taken as an estimate of the systematic uncertainty on the hadronic recoil resolution. Assuming 2% as systematic error on the hadronic scale and 1% on the hadronic resolution in the initial phase, the corresponding biases expected on the $W$ mass are 10 MeV from the scale uncertainty and 5 MeV from the resolution uncertainty.

$W$ data, indeed, are more naturally described in term of the recoil components $u_||$ along the charged lepton direction and $u_\perp$ perpendicular to the charged lepton direction (Fig. 6.27). Hence, the $u_||$ and $u_\perp$ represents an additional tool to perform a cross-check between the data and the MonteCarlo. They obviously don’t allow to decouple the scale and resolution contributions like $u_1$ and $u_2$ in $Z \rightarrow ee$ events, but are helpful to test the calibration from $Z$ events and, eventually, to put further constraints to better model the hadronic response.

The modelling of the hadronic recoil requires also an accurate description of the underlying event (UE). This aspect has not been addressed in details in this analysis. However, it has been demonstrated elsewhere [95] that the underlying event at the LHC can be studied by examining the charged particle density in jet events and Drell-Yan muon production (after removing the Drell-Yan pair). With CMS under nominal conditions, it will be already possible to distinguish between different UE models with an integrated luminosity of about 100 pb$^{-1}$.

## 6.5 Summary of the experimental uncertainties

Table 6.3 summarizes the examined experimental contributions to the uncertainty on $M_W$. The precision required to keep the effect of each of them within 10 MeV is quoted, together with the expected impact on $M_W$ after an integrated luminosity of 1 fb$^{-1}$. The total instrumental error is about 36 MeV from the scaled $E_T^T$ fit and it is dominated by the electron energy scale non-linearity.

An extrapolation to 10 fb$^{-1}$, when the detector knowledge will be improved, has also been attempted.
Figure 6.25: The ratio $u_1(W)/u_1(Z)$ between the distributions of hadronic recoil component parallel to the boson transverse momentum as a function of the boson transverse momentum. A 2% difference between $W$ and $Z$ in the hadronic recoil reconstruction is deduced.
Figure 6.26: The $u_2(W)$ and $u_2(Z)$ distributions of the hadronic recoil. The width is 8.49 GeV in $Z$ events and 8.43 GeV in $W$ events.

Figure 6.27: Distributions of the variables $u_{||}$, the recoil component parallel to the charged lepton direction, and $u_{\perp}$, perpendicular to the lepton direction, in a $W \rightarrow e\nu$ sample.
Chapter 6 – Experimental uncertainties

Statistical and experimental uncertainties.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>10 MeV effect on $M_W$</th>
<th>$\Delta M_W$ [MeV] at 1 fb$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Statistics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron energy scale</td>
<td>0.25%</td>
<td>10</td>
</tr>
<tr>
<td>Scale linearity</td>
<td>$2 \times 10^{-5}$ GeV$^{-1}$</td>
<td>30</td>
</tr>
<tr>
<td>Electron energy resolution</td>
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<td>5</td>
</tr>
<tr>
<td>Electron efficiency</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Background</td>
<td>10%</td>
<td>10</td>
</tr>
<tr>
<td>Recoil scale</td>
<td>2%</td>
<td>10</td>
</tr>
<tr>
<td>Recoil resolution</td>
<td>1%</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total instrumental</strong></td>
<td></td>
<td>36</td>
</tr>
</tbody>
</table>

Table 6.3: The columns list the experimental effect considered, the required precision for 10 MeV shift in the W mass and the resulting error on $M_W$ with 1 fb$^{-1}$ from the lepton scaled transverse energy fit.

The statistical error scales down to 13 MeV. Since the higher integrated luminosity provides larger control samples useful to constrain systematic effects, a scaling of the systematic uncertainties with the collected statistics is also expected.

The electron energy scale and resolution after 10 fb$^{-1}$ will be known with better precision. The absolute energy scale has been conservatively assumed to be precise within 0.25% with 1 fb$^{-1}$; with 10 fb$^{-1}$, that uncertainty can be lowered to 0.05%; this would account both for the statistical precision (0.05% is the statistical precision achievable with just 2 fb$^{-1}$) and for other possible systematic errors in its determination. The non-linearity, that can be controlled to $6 \times 10^{-5}$ GeV$^{-1}$ with 1 fb$^{-1}$ using $Z \rightarrow ee$ events, is assumed to be known with a precision of $2 \times 10^{-5}$ GeV$^{-1}$, scaling with the available $Z$ statistics. Moreover, with 10 fb$^{-1}$, other techniques will be exploited to measure the non-linearity: the tracker will be well aligned and the measurement of the quantity $E/p$ of isolated electrons will also be a useful tool in addition to the procedure discussed in Section 6.1.2 that exploits only $Z$ events.

The electron energy resolution is also measured from the $Z$ peak. With 10 fb$^{-1}$ the statistical uncertainty on the resolution will be negligible and the error will be dominated by the systematic uncertainty related to the photon radiation, which ultimately limits the reachable precision. This systematic contribution, deduced from Tevatron studies, amounts to about 1.5%. For the measurement with 10 fb$^{-1}$ with CMS, a 3% uncertainty, twice
the number quoted by CDF, has been conservatively assumed as, at the moment, specific studies on this topic has not been performed.

The recoil reconstruction similarly relies on the measurement of the hadronic recoil in \( Z \rightarrow ee \) events and is also expected to be improved by the larger statistics in the control data sample.

As it can be seen from Table 6.4, almost all the contributions to the uncertainty on \( M_W \) can be kept at the level or below 10 MeV with an integrated luminosity of 10 fb\(^{-1}\), when the measurement through the electron transverse energy spectrum is considered. In this case, a total experimental uncertainty less than about 15 MeV can be envisaged, while the theoretical uncertainty, according to this preliminary study, is still dominated by the \( p_T \) uncertainty (20 MeV), that can, however, be reduced by higher order calculations.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Assumed uncertainty</th>
<th>( \Delta M_W ) [MeV]</th>
</tr>
</thead>
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<tr>
<td>Electron energy scale</td>
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<td>(&lt;10)</td>
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<td>Electron energy resolution</td>
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</tr>
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<td>2</td>
</tr>
<tr>
<td>Recoil scale</td>
<td>(&lt;1.5)%</td>
<td>(&lt;8)</td>
</tr>
<tr>
<td>Recoil resolution</td>
<td>(&lt;1)%</td>
<td>(&lt;5)</td>
</tr>
<tr>
<td><strong>Total instrumental</strong></td>
<td></td>
<td>(&lt;15)</td>
</tr>
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</table>

Table 6.4: The columns list the effect considered, the assumed uncertainty for each contribution for 10 fb\(^{-1}\) and the corresponding uncertainty on \( W \) mass.
Chapter 7

Conclusions

In this work, a method for the precision measurement of the $W$ boson mass at the LHC has been investigated for $W \rightarrow e\nu$ decays. Differently from the traditional Monte Carlo based approach, the “Scaled Observable Method” relies on the direct comparison between $W$ and $Z$ spectra scaled by the vector boson masses. $Z$ events used to predict the $W$ distributions are weighted in order to compensate for unavoidable differences between $W$ and $Z$. All the systematic effects, both from theory and from experiment, are then projected on the reweighting function, that is obtained from a detailed simulation.

The method has been fully studied for an integrated luminosity of 1 fb$^{-1}$ and strategies, exploiting the large number of $Z \rightarrow ll$ events, have been designed to control systematic effects from data. In Table 7.1, the main sources of error are summarized, together with the assumed uncertainty on each variable entering the measurement for 1 fb$^{-1}$ and its corresponding impact on $M_W$. The expected statistical uncertainty, limited by the number of $Z$ events, is about 40 MeV. A total instrumental error of 36 MeV is forseen on the electron transverse energy measurement. On the theoretical side, the dominant contribution (~20 MeV) is expected to come from the boson transverse momentum uncertainty. The measurement with 1 fb$^{-1}$ gives a global uncertainty comparable with current results obtained at Tevatron using the traditional method: for comparison, the most recent measurement by CDF with 200 pb$^{-1}$ has a statistical uncertainty of 58 MeV, an experimental uncertainty of 37 MeV and a theoretical uncertainty of 25 MeV (see Table 1.2) from the electron transverse energy fit.
Electron scaled transverse energy

<table>
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<th>Assumed uncertainty</th>
<th>$\Delta M_T [\text{MeV}]$</th>
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<td>Scale linearity</td>
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<td>$&lt;2 \times 10^{-5}\text{GeV}^{-1}$</td>
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<td>5</td>
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<td>2</td>
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<td>Background</td>
<td>10%</td>
<td>10</td>
<td>2%</td>
<td>2</td>
</tr>
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<td>$&lt;1%$</td>
<td>$&lt;5$</td>
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<tr>
<td><strong>Total instrumental</strong></td>
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<tr>
<td>$\Gamma _W$</td>
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<td>$&lt;40$ MeV</td>
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<tr>
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<td>$\sim 20$ (or NNLO)</td>
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<td>$\sim 10$</td>
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</tbody>
</table>

Table 7.1: The columns list the effect considered, the assumed uncertainty for each contribution for 1 fb$^{-1}$ and the corresponding uncertainty on $W$ mass. In the last two columns extrapolation to 10 fb$^{-1}$ are reported.

The “Scaled Observables Method” has the advantage that the uncertainties common between $W$ and $Z$ are cancelled in the ratio or, conversely, that the precision requirements on the quantities used in the measurement are allowed to be much looser than in the traditional approach: for example, as discussed in Section 6.1.1, the impact of the absolute lepton energy scale becomes marginal, since an effect on $M_W$ of about 10 MeV is reachable with a precision of only 0.25% on the scale, when in the MC template method the same effect requires a knowledge of the lepton energy scale at the level of $10^{-4}$. The theoretical uncertainties result also reduced. In particular, this study has shown that an uncertainty of about 20 MeV related to the boson $p_T$ prediction can be expected using fixed order perturbative QCD and it doesn’t require the resummed calculations needed by the Monte Carlo templates method.

Extrapolating the uncertainties to an integrated luminosity of 10 fb$^{-1}$, the “Scaled Observables Method” applied to the electron $E_T$ spectrum is expected to provide a $W$ mass measurement with a statistical uncertainty of about 13 MeV and an experimental systematic uncertainty lower than 15 MeV. The uncertainties of theoretical origin will be reduced, at least for those effects that can be controlled directly from the data: the $W$ boson width, which will be directly measured, and the Parton Distribution
Functions, that will be constrained by early LHC data. Both of them are expected to contribute to the $W$ boson mass uncertainty for less than 10 MeV. The uncertainty in the $p_T(W)$ prediction, that has been estimated by the dependence of the available NLO prediction on the choice of the renormalisation and factorisation scales, could be a limiting factor for the measurement with $10 \text{ fb}^{-1}$. NNLO calculations might be necessary to improve this error in order to match the expected experimental precision.

The uncertainties that have been summarized are relative to a measurement performed using one observable, the electron transverse energy, and one leptonic channel. The final measurement of the $W$ boson mass will combine the results obtained from the measurements of different distributions (the lepton $E_T$ and the boson transverse mass) and from different leptonic channels (electrons and muons).

A study of the uncertainties expected at CMS analysing the transverse mass spectrum in $W \to \mu \nu$ decays and, again, exploiting the ratio between $W$ and $Z$ transverse spectra can be found in [96].

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Assumed uncertainty</th>
<th>$\Delta M_W$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Momentum scale</td>
<td>$&lt;0.1%$</td>
<td>$&lt;10$</td>
</tr>
<tr>
<td>$1/p_T^2$ resolution</td>
<td>$&lt;3%$</td>
<td>$&lt;10$</td>
</tr>
<tr>
<td>Acceptance definition</td>
<td>$&lt;3 \times 10^{-3}$</td>
<td>$&lt;10$</td>
</tr>
<tr>
<td>Background</td>
<td>2%</td>
<td>Negligible</td>
</tr>
<tr>
<td>Recoil scale</td>
<td>$&lt;1%$</td>
<td>$&lt;20$</td>
</tr>
<tr>
<td>Recoil resolution</td>
<td>$&lt;3%$</td>
<td>$&lt;18$</td>
</tr>
<tr>
<td><strong>Total Instrumental</strong></td>
<td>$&lt;30$</td>
<td></td>
</tr>
<tr>
<td>PDF uncertainties</td>
<td>$&lt;5$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_W$</td>
<td>$&lt;40$ MeV</td>
<td>$&lt;10$</td>
</tr>
<tr>
<td>Other theory errors</td>
<td>not yet evaluated</td>
<td>for this experimental study</td>
</tr>
</tbody>
</table>

Table 7.2: The columns list the effect considered, the assumed uncertainty for each contribution for $10 \text{ fb}^{-1}$ and the corresponding uncertainty on $W$ mass for the analysis of the transverse mass spectrum in the muon channel [96].

Table 7.2 summarizes the main sources of uncertainty and their effect on $M_W$ for the $M_T$ measurement with muons. As it has been already discussed for the electron channel, a measurement through the transverse mass is largely limited by the experimental uncertainty related to the reconstruction of the neutrino transverse momentum. In this case, in fact, the missing transverse energy measurement enters directly in the determination of the observable
\(M^T\) from which \(M_W\) is extracted, while it enters only indirectly through the events selection in the lepton \(E^T\) study. A deeper understanding of the missing transverse energy reconstruction is mandatory in order to reduce the associated uncertainty and to perform a competitive \(M_W\) measurement. On the other side, the analysis based on the lepton \(E^T\) is more affected by the uncertainty related to the \(p^T(W)\) prediction, to which the \(M^T\) distribution is only slightly sensitive.

The strategy of the “Scaled Observables Method” can be applied to the muon \(p^T\) spectrum. In this case, a better statistical precision of 30 MeV (10 MeV) for 1 fb\(^{-1}\) (10 fb\(^{-1}\)), due to the higher acceptance for muons compared to electrons, is foreseen. A detail analysis is not yet available, but the size of the other uncertainties is expected to be comparable to those obtained in the measurement with electrons. Common uncertainties between electrons and muons are expected to be mainly related to theoretical effects, while from the experimental side the two analysis are expected to be largely independent, since measurements with muons rely on different part of the detector with respect the measurements with electrons. A global reduction of the uncertainty on \(M_W\) will be therefore achievable combining the two channels.
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