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A NOTE ON PHASE TRANSITIONS AND SHOCK WAVES IN NUCLEI

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A Note on Phase Transitions and Shock Waves in Nuclei ⁺

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Abstract

The effect of density isomers on heavy ion collisions is studied in a one-dimensional shock wave model. A family of phenomenological equations of state, which include density isomers, is constructed. The thermodynamic properties resulting from these exotic equations of state are calculated, and some of the shortcomings of this conventional approach are pointed out. The nature of the phase transitions occurring in heavy-ion reactions is discussed in terms of the shock model.

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1. Introduction

It is exciting to imagine that high-energy heavy ion reactions will reveal exotic new features of the nuclear equation of state -- the existence of a limiting temperature, density isomers, quark matter, etc. However, such speculations present us with a serious problem. How will we recognise an exotic feature when it appears in the data? This is the subject of the present paper.

We seek not to give a general answer to this question; we limit ourselves to the subject of density isomers and the phase transitions which should occur in heavy ion collisions as results of density isomers. This is hardly a new topic; Galitski and Nishustin 1978; Hofmann et al 1979) however, earlier studies are not all in agreement, and they certainly have not provided a clear picture of the collision dynamics or the experimental consequences of a phase transition.

In what follows we first discuss the thermodynamic properties of exotic nuclear matter. The treatment is completely heuristic and phenomenological. A family of binding energy curves is introduced, with parameters which can be adjusted to produce secondary minima of desired depth and equilibrium density. The resulting first order phase transitions are discussed, as are the limitations of this approach.

Next, the consequences of using these equations of state in hydrodynamical calculations are explored. Ideally, one should make three dimensional hydrodynamic calculations of nuclear collisions at many energies, and with several different equations of state, in order to determine the consequences of the assumption that density isomers exist. However, this is not feasible because of the large numerical effort required to perform these calculations. Instead we present results based on a one-dimensional shock model (Scheid et al 1974; Baumgardt et al 1975; Abul-Magd 1975, Hofmann et al 1976; Stöcker et al 1980). This model is very limited but it should result in temperatures and densities in rough agreement with hydrodynamic calculations.

Equations of State and Phase Transitions

Examples of equations of state used in the present investigation are plotted in figure 1. The quantity plotted is the "compressional energy", i.e. the binding energy per nucleon, $W(\rho, 0)$, of nuclear matter at zero temperature. The symbol ρ represents the number density. Although the idea of density isomers originated in work by Migdal (Migdal 1978) (pion condensation) and Lee and Wick (Lee and Wick 1974), our approach is completely phenomenological, i.e. not rooted in microscopic theory.

The concern of the present work is not the physical nature of the condensate, but only the physical consequences of its existence. Accordingly we introduce the following simple parametrisation of $W(\rho, \sigma)$, (σ =entropy/nucleon):

$$W(\rho, \sigma) = W_A(\rho) + W_C(\rho) + W_I(\rho, \sigma) \quad (1)$$

The first term corresponds to "normal" nuclear matter and has the form (Brueckner et al 1969; Bethe 1971; Wong and Welton 1974),

$$W_A = b_0 \rho^{2/3} + b_1 \rho + b_2 \rho^{4/3} + b_3 \rho^{5/3} \quad (2)$$

The coefficients b_i are listed in table 1. The first term is the kinetic energy per particle of a Fermi gas at zero temperature, $E_F(\rho, 0)$. The b_i , $i=1, 2, 3$, are adjusted so that $W_A(\rho)$ has a minimum at $\rho_0 = 0.17 \text{ 1/fm}^3$ with $W_A(\rho_0) = -15.7 \text{ MeV}$ and a compressibility, $K = 180 \text{ MeV}$. In figure 1 the $W_A(\rho)$ curve is labeled A.

Secondary minima in $W(\rho, 0)$ at $\rho = \rho_1$ occur because of the addition of the second term, the "condensation energy". It has the form,

$$W_C(\rho) = \frac{E_C}{1 + \exp(\frac{\rho_0 - \rho}{\delta})} \quad (3)$$

The values of the parameters E_C , ρ_C , and δ corresponding to cases B, C, and D in figure 1 are also listed in table 1. The final term in

equation (1) is the thermal energy. It is taken for simplicity to be the internal energy per nucleon of a non-relativistic Fermi gas, i.e.

$$W_I(\rho, \sigma) = E_F(\rho, \sigma) - E_F(\rho, 0) \quad (4)$$

In hydrodynamics, this is a very convenient choice, since it results in a thermal pressure contribution given by

$$P_I = \frac{2}{3} \rho W_I(\rho, \sigma) \quad (5)$$

making it possible to solve the hydrodynamic equations without introducing a temperature or entropy variable explicitly. Other quantities of interest can be calculated from the thermodynamic relations,

$$P = \rho^2 \left(\frac{\partial W}{\partial \rho} \right)_\sigma, \quad T = \left(\frac{\partial W}{\partial \sigma} \right)_\rho \quad (6)$$

A first order phase transition comes about if the condensation term is sufficiently strong to produce a region of ρ values for which the P - ρ isotherms have mechanically unstable (spinodal) regions with $(\partial P / \partial \rho)_T < 0$, similar to the unstable regions of the Van der Waals equation of state. A density isomer, i.e. a secondary minimum in $W(\rho, 0)$ is not necessary for this to occur. For example, parameter set B does not result in a secondary minimum but does result in a phase transition.

Some isotherms corresponding to case C in figure 1 and table 1 are plotted in figure 2. The standard interpretation of equations showing such behaviour is that there exists a density region, $\rho_a \leq \rho \leq \rho_b$, for which the homogeneous fluid state is unstable against separation into a mixture of fluids of densities ρ_a and ρ_b . The boundary points, ρ_a and ρ_b , are determined by the Maxwell construction procedure, which satisfies the equilibrium conditions,

$$\begin{aligned} P(\rho_a, T) &= P(\rho_b, T) \\ g(\rho_a, T) &= g(\rho_b, T) \end{aligned} \quad (7)$$

in which g is the Gibbs free energy per nucleon, $g = W(p)/p - T\sigma$. In figure 2 the dashed curve is the locus of points p_a, p_b . In the interior of this curve only phase mixtures are stable. When the critical temperature is reached, $p_a = p_b$. The critical temperatures of the cases shown in figure 1 are also listed in table 1. Note that these critical temperatures are extremely high, greater than 200 MeV in most cases studied. The critical temperature for the nucleon gas - normal fluid phase transition, by contrast, is only 17 MeV.

This discussion of the thermodynamic properties of the hypothetical matter defined by equations (1) through (4) is helpful in understanding the shortcomings and limitations of the present approach. The critical temperatures in table 1 are clearly unphysical. Above 100 MeV temperature, relativistic effects become important, and the degrees of freedom of meson and baryon resonances must be accounted for. (The temperature values given in parentheses in table 1 are stated only for comparison. The model is clearly invalid for those temperature regions.) An improved W_t is essential if one is to put any faith in model predictions at higher T .

In addition, these extremely high critical temperatures may be an indication that the idea of simply adding on the thermal energy of a gas (even a relativistic gas) should be carefully examined. This treatment of thermal effects is consistent with mean field theory. In mean field theory nuclear interactions affect thermal properties only by adding a constant, density dependent, one-body potential energy to the free particle energies. As a result, the entropy, for example, is that of a gas, completely independent of nuclear forces.

A simple example of a system which cannot be described by mean field theory is the Van der Waals gas. Its entropy at high densities depends strongly on the molecular repulsion at short distances. In the nuclear case mean field theory is also very suspect at high temperatures. Fortunately, our shock wave results predict relatively low temperatures for most bombarding energies which are currently of interest.

Clearly the present model, which has been used widely, in which $W(p,0)$ has a secondary minimum and thermal effects are added as a Fermi gas term, hardly exhausts the possibilities. For example one

might make the condensation term in equation (1) a function of σ as well as p , so that its strength decreases with temperature. The result would be a substance with the same $W(p,0)$ as in the present model but with completely different thermal properties. A model of this sort is described in the appendix. In figure 2, the dots indicate isotherms, at $T=0$ MeV, 50 MeV, and 100 MeV of such a substance, constructed from the $W(p,0)$ of parameter set C. The critical temperature drops from roughly 300 MeV to 100 MeV, and the thermodynamic properties at all but the lowest temperatures are markedly changed.

It should be pointed out that the hydrodynamic approach encounters fundamental difficulties in dealing with density isomers. In classical hydrodynamics, fluid cells are macroscopically small but large enough and sufficiently uniform in properties that each can be assumed at any given instant to be in thermodynamic equilibrium. In a phase transition the fluid separates into macroscopic regions of different density. It is not clear that it makes sense to talk about a fluid element containing a phase mixture, since the different phases do not interpenetrate, but rather one phase exists in the form of small but macroscopic bubbles or droplets in a fluid of the other.

In nuclear hydrodynamic calculations made to date, the Maxwell construction has not been made, and the mechanically unstable regions have been treated as physically meaningful. This seems reasonable if one assumes that nucleation and the growth of droplets or bubbles are processes which proceed slowly compared to a collision time. Then the thermodynamic functions in the unstable regions can be thought of as properties of matter existing momentarily in unstable states. There is some justification for this viewpoint in the classical theory of nonhomogeneous fluids. In the present paper, because of the oversimplified shock model, we do not have to resolve these problems. The compressed region is "macroscopic" with sharp boundaries, and it may make sense to talk about a phase mixture. Shock solutions either don't exist, or are unstable in the thermodynamically unstable region.

Shock Waves and the Collision Model

Heavy ion collisions are here simulated in one dimension by the collision of semi-infinite slabs of nuclear matter. Such a collision will result in shock waves as shown in figure 3. The frame of reference chosen is that in which the velocities of the slabs are equal in magnitude and opposite in direction.

The hot, compressed central region is stationary in this system, except for its boundaries, which move outward with the shock velocity. The thermodynamic properties of the central region, and the velocity of the shock, can be determined from the relativistic equations of continuity for mass, momentum, and energy at the boundary surface between compressed and cold matter. These lead to the Rankine-Hugoniot equation (Landau and Lifschitz 1959),

$$h_1^2 - h_0^2 - p_1 \left(\frac{h_0}{s_0} + \frac{h_1}{s_1} \right) = 0 \quad (8)$$

The quantity h is the enthalpy per nucleon, $h = mc^2 + W + p/\rho$. Equation (8) provides an implicit relation between p_1 and T_1 , which is most easily made explicit by choosing a value for p_1 and solving for T_1 numerically.

We do this in two ways. First, we will use the equation of state of the last section without the Maxwell construction, thereby ignoring the question of phase separation, and instability. Because of equation (5), equation (8) can be cast in the form of a quadratic equation in $W_c(p, c)$, with p -dependent coefficients. For a chosen value of p_1 , the coefficients are evaluated and the quadratic equation solved for W_t . For physically meaningful solutions we must require $W_t \geq 0$; the temperature can then be found by numerically inverting the Fermi-gas expression.

The second computation assumes a mixed-phase system in the central compressed region. A parameter λ is defined to be the fraction of particles in phase a. The extensive variables of the system can be written in terms of λ , as, for example

$$h(\rho, T) = \lambda h(\rho_a, T) + (1 - \lambda) h(\rho_b, T), \quad (9)$$

$$\frac{1}{s} = \frac{\lambda}{s_a} + \frac{1 - \lambda}{s_b} \quad (10)$$

Substituting these into equation (8), with $p(\rho, T) = p(\rho_a, T) = p(\rho_b, T)$, one obtains a quadratic equation in λ with coefficients depending on p_a , p_b , and T . The latter variables are in turn connected through equations (7), so that equation (8) can be solved for $\lambda(T)$. The physically meaningful solutions must fulfill $0 \leq \lambda(T) \leq 1$. For such solutions, equation (10) can be used to obtain solutions $T(\rho)$, as before.

The equations of continuity determine the velocity, β_0 ($= v_0/c$), of the cold matter relative to the shock as

$$\beta_0^2 = \frac{p_1^2 (h_1^2 - h_0^2)}{h_1^2 (p_1^2 - p_0^2)}; \quad (11)$$

the compressed matter has a velocity, β_1 , relative to the shock, which is given by

$$\beta_1^2 = \left[1 + \frac{p_1^2}{\beta_0^2 s_0^2 p_0^2} \right]^{-1}, \quad (12)$$

$$\gamma = (1 - \beta^2)^{-1/2}$$

The velocities shown in figure 3 are then given by

$$\beta_s' = -\beta_1,$$

$$\beta_f = \frac{\beta_o - \beta_1}{1 - \beta_o \beta_1}$$

In the laboratory frame, i.e. the frame of reference in which one slab is at rest, the velocity, β_o , of the "incoming" slab is given by a velocity transformation,

$$\beta_o = \frac{2\beta_f}{1 + \beta_f^2}$$

If this is assumed to be the speed of a nucleon in the projectile, then the kinetic energy per projectile particle in the laboratory frame is

$$E_{kin} = (\gamma_o - 1) m c^2 = \frac{2\beta_f^2}{1 - \beta_f^2} m c^2 \quad (13)$$

By means of equation (13) one can directly relate ρ_1 , T , and the shock velocity, $\beta_s = \beta_o$, to the bombarding energy.

Results

A qualitative feeling for the results of the calculations can be obtained from the sketches in figure 4. The curves in figure 4 do not yet represent results from an actual calculation, they are used to illustrate certain points which are very difficult to see in the actual results, this will become clear later.

In figure 4, curves of $T(\rho)$ are shown for cases of weak and strong condensation. The solid lines labeled RH represent Rankine-Hugoniot adiabatics for which the compressed matter is assumed to consist of a single phase obeying the equation of state. The negative branches represent solutions for which the thermal energy W_t is negative, i.e. if $T = T(\rho, W_t)$, then for solutions of the quadratic equation with $W_t \leq 0$, we plot $-T(\rho, -W_t)$.

The boundary of the mixed phase region, shown as a dashed line, intersects RH at the two points I and II. The dot-dashed line connecting these points represent the mixed phase solutions of the Rankine-Hugoniot equation. In the case of strong condensation two distinct cases emerge, depending on whether point II occurs for positive or negative thermal energy. If the intersection occurs for negative temperatures, no mixed phase solution exists between points III and II'.

The sketches strongly exaggerate the angle between RH and the phase mixture boundary curve at the point II; in reality both curves are so steep that it is extremely difficult to plot them in the neighborhood of their intersection. In figure 5 the results of an actual calculation for equation of state B are shown. An exponential horizontal scale has been used to spread out the figure in the region of the intersection. Even with this device the angle of intersection between the two curves appears very small.

As a result of the steepness of these curves the question of whether there is a Rankine-Hugoniot solution at all densities is an extremely sensitive function of the parameters of the equation of state. We know of no simple criteria for ensuring a continuous shock adiabat. If the condensate is sufficiently weak there are solutions for all ρ even without the Maxwell construction, as indicated in the

right hand part of figure 4. For stronger condensation a region of p exists with no single phase solutions. If the minimum in the p - p diagram is not too deep, or if it occurs at sufficiently high density, then the shock adiabat becomes continuous if the formation of a phase mixture is included. The equation of state B, figure 5, is an example for this behaviour. If the minimum is too deep, or if it occurs too close to p_0 , then the shock adiabat will always be discontinuous.

For example, if the condensation strength parameter, E_c , in equation of state B is made stronger by 1 MeV, the mixed phase shock adiabat will change its behaviour from being continuous to being discontinuous. If this extreme sensitivity to the parameters is overlooked the existence of a continuous shock adiabat could easily be missed entirely.

In figure 6 the variables p_1 , T , and the shock velocity in the laboratory system, β_s , are plotted against laboratory energy for equation of state B. The dashed lines indicate the mixed phase solution. The curves are multiple valued and for a given bombarding energy one must decide which is the physical branch. The branches marked UU' on the curves are in the completely unstable spinodal region. In this region the slightest perturbation will grow and result in phase separation. A further indication for this behaviour is the fact that no sound speed exists in the region UU' indicated in figure 5. The attempt to propagate a small sound-like perturbation in this region will result in exponential growth of the amplitudes.

In the region beyond I, on the low density branch, phase separation occurs as a result of the growth of droplets of dense material. Nucleation occurs as a result of thermal fluctuations. The probability of nucleation and the speed of growth of instabilities increases as one approaches U.

Another instability occurs between I and U; the sound speed passes through a maximum as a function of density, so that a low density perturbation travels with higher speed than a high density perturbation. Beyond such a maximum in the sound speed there is a broadening of the shock front. In attempting to predict what will happen one must keep in mind that the shock wave equations assume

steady flow conditions, whereas shock-like compressions in real nuclear collisions are transient phenomena. Whether an instability results in a jump from one branch to another depends on how much it grows during a collision time. Without further study one can only conclude that at some energy, lying between I and U, one should see a jump in properties -- an increase in density and a decrease in temperature and shock velocity.

This conclusion is relatively independent of the equation of state and it is also independent of whether or not the branches of the curve are joined by a continuous phase transition. Note that there is a stable solution at every bombarding energy, contrary to earlier ideas that the signature of a density isomer might be the vanishing of shock behaviour in some density region.

The bombarding energy at which these phenomena occur depends on the equation of state; this is illustrated in figure 7. The compression, p_1/p_0 , for a single phase system, is plotted as a function of E_{lab} , for a series of different equations of state which are shown in the inserts. (Equation F_2 has not been plotted since it cannot be distinguished from F_1 on this scale.) The parameters used are listed in table 1.

In the sequence F_1 , F_2 , F_3 , shown in the lower half of figure 7, only the condensation strength, E_c , has been varied. The compression curves for F_1 and F_2 are both continuous, however, with F_2 we obtain a multiple valued curve and a jump will occur. Equation of state F_3 results in a behaviour similar to B. (The high density branch has been omitted in figure 7.)

In the sequence E_1 , E_2 , E_3 a secondary minimum in $W(p,0)$ has been created at different densities, the depth of the minimum has been kept fixed. The results are shown in the upper part of figure 7; a shift of the minimum from $p/p_0 = 4$ to $p/p_0 = 5$ causes the approximate location of the jump to shift from 100 to 400 MeV/nucleon. (The high density branches have again not been plotted.)

In summation it should be pointed out that the collision model used here is very schematic; it can at best give a qualitative indication of the effects to be expected in a heavy ion collision. However, if interpreted properly, the model can be very useful in surveying

the effects of changes in the equation of state on the outcome of a collision.

Keeping the limitations of the model in mind we conclude that a phase transition of the type considered here could cause jumps in the excitation function of measurable quantities, for example the temperature. The question whether a phase mixture was actually formed, however, cannot be answered from the behaviour of either compression, temperature, or shock velocity.

Appendix

We present here a more detailed discussion of the thermodynamics of the hypothetical matter treated in the text of the paper. Particular attention is directed to the temperature dependence of the condensation term in equation (1).

The thermodynamic functions of a Fermi gas play an important role in the models used. For a non-relativistic fermi gas, these functions, or simple combinations of them, can be written in terms of a single variable,

$$\tau = \frac{T}{\epsilon_0} \quad , \quad \epsilon_0 = \frac{\hbar^2}{2m} \left(\frac{3N}{2} \rho \right)^{2/3} \quad (A-1)$$

The temperature is given in energy units; ϵ_0 is the top of the Fermi sea at zero temperature. For example, in terms of the variable τ , the energy per particle E_F of equation (4) is of the form,

$$\frac{E_F(\beta T)}{\epsilon_0(\beta)} = f(\tau) \sim \frac{3}{5} + \frac{\pi^2}{4} \tau^2 + \dots \quad (A-2)$$

The low temperature form on the right is valid at normal density for $T \ll 20$ MeV and at higher densities for somewhat higher temperatures. All the thermodynamic functions of interest can be obtained from the tabulation of Beer et al (Beer et al 1955). In numerical calculations one can interpolate between the tabulated values or fit these values accurately with simple analytic forms.

We construct an ad hoc model of temperature dependent condensation as follows: We write

$$W(\beta, \epsilon) = W_0(\beta) G(\epsilon) \quad , \quad (A-3)$$

in which W_0 is the function defined in equation (3) and $G(\epsilon)$ is a decreasing function of ϵ with the property, $G(0) = 1$. The resulting expression for the internal energy per particle is

$$W(p, \epsilon) = W_A(p) + W_4(p, \epsilon) + W_c(p, \epsilon) . \quad (A-4)$$

In equation (A-4) we take W_{th} to the same as before, i.e., it depends on p and σ in the same way as the Fermi gas function, equation (4). We emphasize again that our models are not rooted in microscopic considerations. It seems hardly likely that a microscopic calculation would result in an equation exactly like (A-4), in which the energy neatly separates into a "normal nuclear matter" term, an excitation term and a condensation term. However, equation (A-4) has the virtue of being as reasonable (or unreasonable) as equation (3), but embodying different physics -- $W_c(p, \sigma)$ becomes less effective with increasing temperature, presumably as the result of disorder resulting from thermal agitation.

The relation between temperature, density, and entropy is obtained from equation (6). The entropy of a Fermi gas depends only on the variable τ , equation (A-1). Because of this and the form of equation (A-2), it is possible to express W_{th} in the form,

$$W_{th}(p, \epsilon) = p^{2/3} F(\epsilon) \quad (A-5)$$

Substituting equations (A-3) and (A-5) into equation (6) one obtains

$$T = p^{2/3} F'(\epsilon) + W_c(p) G'(\epsilon) . \quad (A-6)$$

In order for $T = 0$ at $\sigma = 0$, we require that $G'(0) = 0$. A simple function with this property is a Gaussian,

$$G(\epsilon) = \exp(-\epsilon^2/\epsilon_0^2) . \quad (A-7)$$

In calculating the pressure at fixed T , equation (A-6) must be solved for $\sigma(p)$, which is then used in

$$P = p^2 \frac{dW_A}{ds} + \frac{2}{3} p W_{th}(p, \epsilon) + p^2 \frac{dW_c}{ds} G(\epsilon) . \quad (A-8)$$

The dots in figure 2 were calculated using parameter set C with $\sigma_0 = 3.5$. By making σ_0 smaller the phase transition can be made to vanish at even lower temperatures.

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Table 1

Parameters of the equations of state
investigated in this paper.

A. Normal nuclear matter (all quantities in MeV)

$b_0 \rho_0^{2/3}$	$b_1 \rho_0$	$b_2 \rho_0^{4/3}$	$b_3 \rho_0^{5/3}$	w_0	κ
23.04	-136.1	124.66	-27.22	-15.77	180

B. Condensation term

Parameter set	E_c [MeV]	ρ_c / ρ_0	δ / ρ_0	T_c [MeV]
B	-15.0	3.0	0.20	245
C	-35.0	3.0	0.30	282
D	-24.0	3.5	0.20	(635)
E ₄	-70.0	4.5	0.35	(1100)
E ₂	-56.0	4.0	0.35	
E ₃	-41.0	3.5	0.35	
F ₄	-6.5	3.5	0.20	
F ₂	-10.0	3.5	0.20	
F ₃	-24.0	3.5	0.20	(635)

$$\rho_0 = 0.17 \text{ fm}^{-3}; \quad \kappa = \rho_0^2 \left(\frac{\partial^2 w_A}{\partial \rho^2} \right)$$

$$\rho = \rho_0$$

Figure captions

Figure 1. Energy per particle, $W(\rho, T)$, for $T = 0$ and various parameterisations. The curves are labeled according to the parameter sets in table 1. The insert shows the "condensation" energy, $W_c(\rho)$, for curve B.

Figure 2. Isotherms in the p - p plane (solid lines) for equation of state C. The dashed line indicates the envelope of the mixed-phase region. The Maxwell construction for each temperature is shown as horizontal solid line. The dots indicate isotherms for $T = 0, 50, 100$ MeV, using the equation of state described in the appendix.

Figure 3. Illustration of the one-dimensional shock model. The compressed matter is at rest in this frame of reference.

Figure 4. Sketches of the possible types of shock solutions in the T - p plane. The solid lines represent solutions of the shock equation for a single-phase system, the dashed lines indicate assumed envelopes of the mixed phase region. The mixed phase shock solutions are represented by dot-dashed lines.

Figure 5. The mixed-phase region envelope (vertical dashed lines) and the solutions of equation (8) for equation of state B are shown. The solid line represents the single phase solutions, the mixed phase solutions are shown as dashed line connecting points I and II. Note that the p -axis has been stretched using an exponential scale.

Figure 6. The compression, ρ/ρ_0 , the temperature, T , and the shock velocity, β_s , are shown as functions of the laboratory energy per nucleon for shock solutions using equation of state B. The solid lines represent single-phase solutions, the dashed lines mixed-phase ones.

Figure 7. Single-phase shock solutions, ρ/ρ_0 as functions of E_{lab} , for various equations of state. The parameters of these equations of state are listed in table 1, the binding energies per nucleon for each one are shown in the inserts. (The $W(\rho)$ curves for F_1 and F_2 cannot be distinguished on this scale.)

Fig. 1

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UFfm TP 81- 212

Fig. 4

Fig. 5

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Fig. 6

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Fig. 7