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ON THE POINCARÉ AND DE SITTER GAUGE THEORIES OF GRAVITY
WITH PROPAGATING TORSION

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ABSTRACT

We consider a gauge approach to the gravitational theory based on the local Poincare P_{10} or De Sitter S_{10} groups. P_{10} gauge rotations and translations take place in the tangent spaces to the space-time manifold. The independence of matter fields on the tangent vectors is interpreted as the necessity to consider the non-linear realization of P_{10} or S_{10} thus effectively breaking the full symmetry to the Lorentz group. The lagrangian we choose is the S_{10} Yang-Mills invariant with the space-time metric being expressed in terms of the translational part of the S_{10} non-linear gauge field. Various consequences of the theory are discussed, including the correspondence with the general relativity, the propagating spin-connection interactions, the analogy with the chiral Higgs mechanism, the instanton-like solutions, the possibility of the gravitational repulsion due to non-compactness of the Lorentz group and etc. We also discuss quantization of the theories with torsion with the special emphasis on the presence of the non-linear realization. We stress the possibility to obtain the renormalizable theory if the metric is not quantized but is expressed in terms of a mean value of the quantized S_{10} -non-linear gauge field.

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1. Introduction

Recently there is a great interest in the attempts to extend the Einstein theory of gravity or the general relativity (GR) using the gauge approach (for reviews see [1-8]). In the most natural of these modifications (generalizing in that or another way the Einstein-Cartan theory (ECT) [8]) one considers the localization of the Poincare group $P_{10} = T_4 \rtimes L_6$ (or the semi-direct product of translation $T_4 = R^{4,1}$ and Lorentz $L_6 = SO(1,3)$ subgroups) thus obtaining the tetrad e^a_μ and the Lorentz connection $\omega^a_{\mu b}$ as the gauge potentials and the torsion $\Theta^a_{\mu\nu} = 2e^a_\mu \partial_\nu e^a_\nu - 2e^a_\nu \partial_\mu e^a_\mu$ and the curvature $R^a_{\mu\nu} = (2\omega_{\mu\nu} - 2\omega_{\nu\mu} + [\omega_{\mu\nu}, \omega_{\nu\mu}])$ as the gauge strengths. As a consequence, the Einstein interaction of e^a_μ (or the metric) with the energy-momentum tensor $t^{\mu\nu}$ is supplemented by the interaction of $\omega^a_{\mu b}$ and the spin density tensor $S^{\mu\nu}_a$ (" ω - S interaction"). The lagrangian of the theory contains the "translational" part (linear in R and quadratic in Θ), providing the correspondence with GR and also the "rotational" part (quadratic in R) giving a dynamical (or non-contact, cf. with ECT) ω - S interaction. The possibility of two independent coupling constants. (dimensional and dimensionless) follows from the structure of P_{10} itself.

The discussion of these theories is based on the following reasons: (i) the Poincare group and the concept of spin are fundamental in particle physics (cf. [1-3, 8]); (ii) the gauge principle is very successful as a guide in constructing the theories of fundamental interactions; (iii) one may hope that a gauge theory of gravity with torsion (or its

supersymmetric generalization) will improve the quantum behaviour of GR [9-12]. At the same time, there are a number of problems in the formulation of "kinematics" as well dynamics of the P_{10} gauge theory. The proposed approaches (see e.g. [8, 13, 14] and references in [1, 4, 7]) use in fact as a gauge group not P_{10} but the direct product $T_4 \times L_6$, or some deformed algebra with different from P_{10} commutation relations [8, 1]. As a result, e_μ^a cannot be consistently identified with an ordinary gauge field and the lagrangian is invariant under the independent T_4 and L_6 groups. In this paper (sect. 1) we consider T_4 translations acting in the t -tangent affine spaces (and not as the infinitesimal coordinate transformations) and show that in order to obtain the gravitational theory (and not the "internal" P_{10} -theory) one must use the non-linear realization of the T_4 -symmetry (the spontaneous breakdown of T_4 in our approach corresponds to the trivial invariance of the ordinary matter fields under the "internal" T_4 -transformations). Gravity is thus described by the nonlinear P_{10} gauge field (for the definition of nonlinear gauge fields see [15]) with $\kappa_0^{-1} e_\mu^a (\kappa_0 J = cm)$ and ω_μ^a being the T_4 and L_6 parts of it (and so it is L_6 -part of gravity that is the "usual" or linear gauge field).

One more problem is the ambiguity in the choice of the P_{10} -lagrangian: the independent T_4 and L_6 symmetries do not restrict the lagrangian in sufficient way (one can use various combinations of Θ^2 and R^2 terms with arbitrary

coefficients [1-3, 16] and so need some additional assumptions to select a particular one [17]. Our interpretation of this ambiguity is the following: the use of the (e, ω) -variables corresponds to the use of the non-linear realization of P_{10} and so the fixation of the dynamics is just the fixation of a "non-linear lagrangian" [15]. The only distinguished lagrangian is the Yang-Mills invariant of the linear gauge theory. However, for the Poincare group the latter choice is not a satisfactory one, because P_{10} is not semisimple (sect.3). For this reason we consider in sect.4 the nonlinear gauge theory of the De Sitter group

$S_{10} = SO(1,4)$ (or the nearest semi-simple extension of P_{10}) and choose the S_{10} -Yang-Mills lagrangian as the basic one. This in turn fixes the particular P_{10} non-linear lagrangian, possessing a number of attractive properties, presented in sect 5. They include the correspondence with GR, the spin-spin interactions, the instanton solutions, the possibility of indefiniteness of the energy for ω (due to the non-compactness of L_6) etc. In the last sect.6 we discuss the problems of quantization of the (e, ω) -theories with torsion and show that the fact of the non-linear realization and the Yang-Mills structure of S_{10} -lagrangian give the possibility to obtain the renormalizable theory where metric is not quantized but is equal to the mean value of a quantized field.

Some of the results of this paper were already summarized in [18].

2. Nonlinear gauge theory of the Poincare group

Let us consider first the construction of the linear P_{10} gauge theory, in the analogy with the "internal" gauge theories. In order to realize P_{10} on the linear vector space and not on the affine space we use the map $P_{10} \hookrightarrow GL(5, \mathbb{R})$

$$P_{10} = \left\{ A = \begin{bmatrix} L & \theta \\ 0 & 1 \end{bmatrix}, L \in SO(4,3), \theta \in \mathbb{R}^{4,3} \right\}, \quad (2.1)$$

$$A \begin{pmatrix} q \\ 1 \end{pmatrix} = \begin{pmatrix} Lq + \theta \\ 1 \end{pmatrix}, \quad q \in \mathbb{R}^{4,3}.$$

Defining the $GL(5, \mathbb{R})$ connection on the vector bundle over space-time M^4 with the fiber $\mathbb{R}^5$ and restricting it on P_{10} , we get the following P_{10} -connection (1-form on M^4 with values in the Lie algebra of P_{10})

$$\hat{\Omega} = \begin{bmatrix} \hat{\omega} & \hat{\theta} \\ 0 & 0 \end{bmatrix}, \quad \hat{\omega} \in \mathfrak{so}(4,3), \hat{\theta} \in \mathbb{R}^{4,3}, \quad (2.2)$$

$$\hat{\omega}' = L \hat{\omega} L^{-1} + L dL^{-1}, \quad \hat{\theta}' = L \hat{\theta} - \hat{\omega}' \theta,$$

$$\hat{\omega}' = d + \hat{\omega}'.$$

The corresponding curvature 2-form is

$$\hat{R} = d\hat{\Omega} + \hat{\Omega} \wedge \hat{\Omega} = \begin{bmatrix} \hat{R} & \hat{\Theta} \\ 0 & 0 \end{bmatrix}, \quad (2.3)$$

$$\hat{R} = d\hat{\omega} + \hat{\omega} \wedge \hat{\omega}, \quad \hat{\Theta} = \hat{\omega}' \hat{\theta}.$$

It is evident from (2.2), that the splitting of $\hat{\Omega}$ on $\hat{\omega}$ and $\hat{\theta}$ depends on the splitting of P_{10} on L_6 and T_4 . This freedom can be naturally separated by the introduction of the vector field $\vec{z}(x) \in \mathbb{R}_x^{4,3}$, fixing in each tangent space $T_x(M)$ a stability point for L_6 (here $\begin{pmatrix} a \\ 1 \end{pmatrix} \in \mathbb{R}_x^5$ is identified with $T_x(M)$) so that the corresponding bundle is soldered to M^4 with $\hat{\theta}$ playing the role of the genera-

lized soldering form [19-21]). Under the action of P_{10} :

$\hat{z}' = L\hat{z} + \theta$, $\hat{\theta}' = L\hat{\theta}$, where $\hat{\theta} = \hat{\theta} + \mathcal{D}\hat{z}$. As a result, one can identify $\hat{\omega}$ with the Lorentz connection ω on $T(M)$, $\kappa_0 \hat{\theta} ([\kappa_0] = cm)$ with the canonical 1-form $\theta (= e^a_\mu dx^\mu)$ and $\hat{\Omega}$ with the generalized affine connection on the tangent affine bundle over M [19], i.e.

$$\hat{\Omega} = \left[\begin{array}{c|c} \omega & \kappa_0^{-1}\hat{\theta} - \mathcal{D}\hat{z} \\ \hline 0 & 0 \end{array} \right], \quad \hat{R} = \left[\begin{array}{c|c} R & \kappa_0^{-1}\hat{\omega} - R\hat{z} \\ \hline 0 & 0 \end{array} \right] \quad (2.4)$$

where R and $\hat{\omega} = \mathcal{D}\hat{\theta}$ are the curvature and torsion of ω and θ .

The above construction of the P_{10} -kinematics is natural and gives all necessary results by the restriction of various GL_5 formulas to the P_{10} case. For example, from the Bianchi identity $\hat{D}\hat{R} = d\hat{R} + \hat{\Omega} \wedge \hat{R} - \hat{R} \wedge \hat{\Omega} = 0$ we have

$$\mathcal{D}R = 0, \quad \mathcal{D}\hat{\omega} - R \wedge \theta = 0, \quad (2.5)$$

or the wellknown Bianchi and Ricci identities [19].

The introduction of $\hat{z}$ in (2.2) can also be treated in the following remarkable way: $\hat{z}$ is the "chiral" field providing the passage from the linear P_{10} -gauge field $\hat{\Omega}$ to the non-linear P_{10} -gauge field $\Omega = \left[\begin{array}{c|c} \omega & \kappa_0^{-1}\theta \\ \hline 0 & 0 \end{array} \right]$. The notion of a non-linear gauge field [15] was used in the theories with "phenomenological lagrangians" for the description of interactions of gauge and adjoint chiral fields and formally coincides with that of the connection on the bundle with a homogeneous space as a fiber. Let $P(M, G)$ be the principal bundle and $\mathcal{F}(M, F, G, P)$ be the associated bundle with

the fiber $F = G/K$, i.e. the homogeneous space (the space of a non-linear realization of G). Here K is a subgroup of G , i.e. for $S \in G$ there is a unique decomposition

$$S = S_F S_K, \quad S_F \in F, \quad S_K \in K$$

For the Lie algebra $\mathfrak{g}$ of G we have

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{f}, \quad [\mathfrak{k}, \mathfrak{k}] \subseteq \mathfrak{k}, \quad [\mathfrak{k}, \mathfrak{f}] \subseteq \mathfrak{f}, \quad (2.6)$$

where $\mathfrak{k}$ is the Lie algebra of K , $\mathfrak{f}$ is the vector space, corresponding to F and F is assumed to be a weakly reductive homogeneous space. Let $\hat{A} \in \mathfrak{g}$ be the 1-form on M^4 corresponding to the connection on $P(M, \mathfrak{g})$ for the fixed section of the bundle, i.e. the linear gauge field. Under the change of the section $\hat{A}' = S \hat{A} S^{-1} + S dS^{-1}$, $S \in G$. Now let us define the connection A on the bundle $\mathcal{F}$, or the non-linear gauge field

$$A = \phi^{-1} \hat{A} \phi + \phi^{-1} d\phi, \quad A \in \mathfrak{g}, \quad \phi \in F. \quad (2.7)$$

Here A is a 1-form on M' depending on $\hat{A}$ and on the fixed point $\phi(x) \equiv S_{oF} \in F_x$ in each copy of the homogeneous space, i.e. on the adjoint chiral field. Under the action of

$$S \phi = (S_F S_K) S_{oF} = S'_{oF} S'_K, \quad S'_{oF} = \phi'. \quad (2.8)$$

Decomposing A according to (2.6)

$$A = A_k + A_f, \quad A_k \in \mathfrak{k}, \quad A_f \in \mathfrak{f}, \quad (2.9)$$

we get the following transformation laws for the parts of A under (2.8)

$$A'_k = S'_k A_k S_k^{-1} + S'_k d S_k^{-1}, \quad A'_f = S'_k A_f S_k^{-1} \quad (2.11)$$

As a result, A_k transforms as an ordinary connection (or a linear gauge field) but A_f transforms homogeneously (this is the characteristic feature of a non-linear gauge field). The nontriviality of the passage from $\hat{A}$ to A is due to the fact that S'_k in general is a complicated function of S_k , S_f and ϕ and so the transformed fields in (2.10) depend on S in a non-linear way.

The Higgs mechanism is a remarkable example of the use of the non-linear gauge fields [22]. Consider a multiplet of scalar fields $\mathcal{Y}$ linearly transforming under the action of G . If there is a spontaneous symmetry breaking $G \rightarrow K$, then

$$\mathcal{Y} = \phi^{-1} \varphi, \quad \phi = \exp(\alpha^a t_a) \in F, \quad \{t_a\} \in \mathfrak{f}, \quad (2.12)$$

where ϕ corresponds to the set of goldstone fields and transforms as a chiral field according to (2.8) (as a consequence, $\mathcal{Y}$ transforms as a non-linear multiplet, $\mathcal{Y}' = S'_k \mathcal{Y}$). Introducing the vacuum $\mathcal{Y}_0$ (invariant under K) we get the Higgs field $\chi = \mathcal{Y} - \mathcal{Y}_0$, transforming according to the non-linear realization of G . As a result of (2.13), $\langle \hat{\mathcal{Y}} \hat{\mathcal{Y}} \rangle = \langle \partial \mathcal{Y} \partial \mathcal{Y} \rangle$, $\partial \mathcal{Y} = \partial \chi + A_f \mathcal{Y}_0$, i.e. $\hat{A}$ transforms into A by absorbing ϕ and $\mathcal{Y}_0 \neq 0$ leads to a non-zero mass for A_f . The final Higgs lagrangian is invariant under the non-linear realization of G and so the mass term for A_f is admissible by the transformation law (2.10). The field strength for A is

$$F = \phi^{-1} \hat{F} \phi = dA + A \wedge A \quad (2.12)$$

$$\mathcal{N} = \mathcal{N}_F \mathcal{N}_K, \quad \mathcal{N}_F \in \mathcal{F}, \quad \mathcal{N}_K \in \mathcal{K}$$

For the Lie algebra $\mathfrak{g}$ of G we have

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{f}, \quad [\mathfrak{k}, \mathfrak{k}] \subseteq \mathfrak{k}, \quad [\mathfrak{k}, \mathfrak{f}] \subseteq \mathfrak{f}, \quad (2.6)$$

where $\mathfrak{k}$ is the Lie algebra of K , $\mathfrak{f}$ is the vector space, corresponding to F and F is assumed to be a weakly reductive homogeneous space. Let $\hat{A} \in \mathfrak{g}$ be the 1-form on M^4 corresponding to the connection on $P(M, \mathfrak{g})$ for the fixed section of the bundle, i.e. the linear gauge field. Under the change of the section $\hat{A}' = S \hat{A} S^{-1} + S dS^{-1}$, $S \in G$. Now let us define the connection A on the bundle $\mathcal{F}$, or the non-linear gauge field

$$A = \phi^{-1} \hat{A} \phi + \phi^{-1} d\phi, \quad A \in \mathfrak{g}, \quad \phi \in F. \quad (2.7)$$

Here A is a 1-form on M' depending on $\hat{A}$ and on the fixed point $\phi(x) \equiv S_{o_F} \in F_x$ in each copy of the homogeneous space, i.e. on the adjoint chiral field. Under the action of

$$S' \phi = (S'_F S'_K) S_{o_F} = S'_{o_F} S'_K, \quad S'_{o_F} = \phi'. \quad (2.8)$$

Decomposing A according to (2.6)

$$A = A_k + A_f, \quad A_k \in \mathfrak{k}, \quad A_f \in \mathfrak{f}, \quad (2.9)$$

we get the following transformation laws for the parts of A under (2.8)

has no relation to coordinate transformations (see also [20-21], c.f. [4-5]). The ordinary physical fields $\varphi(x)$, depending only on the point $x \in M$, of the base space are trivially invariant under T_y . Thus P_{10} can be realized (as a semidirect product) only on the ring of smooth functions on the tangent bundle $C^\infty(T(M)) = \{\varphi(x, y)\}$ and not on $C^\infty(M) = \{\varphi(x)\}$, $\varphi'(x, y) = O(L)\varphi(x, L^{-1}(y-\theta))$ ($O(L)$ defines the representation of L_G), i.e. on the functions depending not only on x , but also on the tangent vector

$y(x) \in T_x(M)$ (similar fields are discussed in [20, 21, 23]).^{2/} One can consider this fact as a manifestation of a spontaneous T_y -symmetry breaking ($P_{10} \rightarrow L_G$) by the physical vacuum (for the fields depending only on x but not on y). In analogy with (2.11) we can define (here φ stands for p and ψ for φ in (2.11))

$$\varphi(x) = \exp(-i \int^y t_a) \varphi(x, y), \quad t_a = \frac{\partial}{\partial y^a} \quad (2.16)$$

and choosing $\xi = \eta$ (as a gauge) we get $\varphi(x) = \varphi(x, 0)$.

^{2/}It is worth noting that in earlier approaches, dating back to Sciama and Kibble [14] P_{10} was realized as a direct rather than a semidirect product. Really, as compared to the case of $\mathbb{R}^4$, $\varphi'(x) = O(L)\varphi(L^{-1}(x-\theta))$, on M^4 the rotation of the point $L^{-1}x$ is undefined, though the infinitesimal translation $x \rightarrow x - \theta$ can be imitated by the infinitesimal coordinate transformation. The partial resolution of this difficulty (which gives the theory with a conserved total angular momentum) is to consider the following simultaneous transformations $\varphi'(x) = O(L)\varphi(x-\theta)$ and $e'_a = e_a(L^{-1})$, ($\{e_a\}$ is a lorentz basis in $T_x(M)$ what, however, effectively unlinks T_4 and L_G transformations (see also [7, 13]).

Thus on $\{\varphi(x)\}$ P_{10} is realized non-linearly (in the mentioned trivial sense, cf. (2.14)) and we come to the necessity of the use of the non-linear gauge field (2.15). The "chiral field" θ plays the role of (four) goldstone fields, all physical fields, are of the "Higgs type" for gravity and gravity itself is described by the non-linear P_{10} -gauge field. In that final interpretation our approach differs considerably from the earlier suggestions of treating gravity as a non-linear gauge field [24-27]. It is interesting to note that translations play a rather peculiar role (cf. [28]): they do not act on the physical fields (and thus the P_{10} -covariant derivative coincides with the L_6 -one) but their localization is necessary for the introduction of the tetrad 1-form (and thus metric) as the part of the non-linear gauge field (note that this interpretation of θ is unrelated to the attempts to consider metric as a goldstone field [29]).

We come to the conclusion that the L_6 -symmetry is unbroken (and so is realized linearly) and the corresponding L_6 -range field ω is the part of the gravitational field that is really a (linear) gauge field. The tetrad field (or metric), i.e. the Einstein gravitational field, is the "non-linear" T_4 -part of the non-linear P_{10} -gauge field, which (due to the dimensional character of the T_4 -coupling constant) provides the clear explanation of the possibility of the non-polynomial (in $e^{\mu\nu}$ or $g_{\mu\nu}$) interactions (present in the Einstein theory).

3. The choice of the dynamics for the Poincare gauge theory

Now let us consider two possible ways of constructing the dynamics of the P_{10} -theory. First, one can start from the non-linear gauge field (2.15) and use its parts ω and θ separately in the lagrangian, invariant under the non-linear realization of P_{10} (or actually under L_6) and of course under the general coordinate transformations. Hence the lagrangian is not fixed uniquely (note that the possibility of the linear in curvature $R(\omega)$ term, providing the correspondence with GR is due to the homogeneous transformation law for θ cf. [30]). Second, one can try to construct the dynamics in analogy with internal gauge theories, i.e. by considering the lagrangian depending on Ω (or R) as a whole object. The simplest nontrivial choice is the Yang-Mills one

$$L_P = \frac{1}{4} t_2 (R \wedge * R) \quad (3.1)$$

However, here is necessary to assume that $g_{\mu\nu}$ in ∇^2 operation in (2.17) is expressed in terms of θ . That is why Ω as a whole is not enough and this is a well known obstacle on the way of description of gravity by the lagrangian polynomial in the linear gauge field (cf. [5, 30-32]). In view of the non-semisimple nature of P_{10} the Killing-Cartan form t_2 in (3.1) is degenerate, what leads to the lack of equivalence of two procedures: (i) to vary (3.1) as a GL_5 -Yang-Mills lagrangian and restrict on P_{10} only on the level of the field equations; (ii) to treat (3.1) as the P_{10} lagrangian from the very beginning, i.e. substituting (2.15) and varying the independent

P_{10} -potentials. The latter procedure gives the unsatisfactory result: $L_P = L_L = \frac{1}{4\lambda} t_2(R \wedge R)$ or only L_6 -lagrangian [6,33] without T_γ -part (necessary to establish the correct Einstein limit). The first procedure gives the following field equations

$$D * R = -\lambda * J, \quad J = \left[\begin{array}{c|c} S & -\kappa_0 t \\ \hline 0 & 0 \end{array} \right], \text{ or} \quad (3.2)$$

$$D * R = -\lambda * S, \quad D * \Theta = * R \wedge \theta = \frac{\kappa^2}{2} * t, \quad (\kappa^2 = 2\lambda \kappa_0^2) \quad (3.3)$$

These equations provide the dynamical ωS interaction and correspondence with GR (when $\Theta = 0$ and $J = 0$) (3.3) is equivalent to the vacuum Einstein equations, see also [34]. However, eqs (3.3) are non-variational in two-fold sense. Firstly, they can not be obtained from some action by varying ω and θ . Secondly, the dependence of $g_{\mu\nu}$ (in $*$ in (3.1)) on the θ -part of the dynamical variable Ω is not reflected in (3.2) (otherwise t in (3.3) would be supplemented by the energy-momentum tensor of R). As a result, one cannot identify $t_a^{\mu\nu}$ with the energy-momentum tensor of matter ($= e^{-1} \frac{\partial(e\mathcal{L}_m)}{\partial e^{\mu\nu}}$, $e = \det e_\mu^a$) though it is possible to identify $S_{ab}^{\mu\nu}$ with the spin density ($= \frac{\partial \mathcal{L}_m}{\partial \omega^{\mu\nu}}$).

We conclude that the approach based on (3.1) does not give the acceptable theory like the case of P_{10} as a gauge group. It seems natural now to pass to the De Sitter group, or the nearest semi-simple extension of P_{10} (with the non-degenerate trace "tr") and to consider again a unique Yang-Mills invariant (3.1). This gives the theory with the variational field

equations, analogous in their structure to eqs. (3.3).

4. Non-linear gauge theory of the De Sitter group

The following parametrization of the De Sitter group

$S_{10} = SO(1,4)$ is useful in order to separate the $L_6 = SO(1,3)$ subgroup

$$S = \left[\begin{array}{c|c} (1 - \epsilon^2 \theta \theta^T) L & \theta \\ \hline \theta^T L & \epsilon \end{array} \right] = S_\theta S_L, \quad L \in L_6, \theta \in \mathbb{R}^{1,3} \quad (4.1)$$

$$S_L = \left[\begin{array}{c|c} L & 0 \\ \hline 0 & 1 \end{array} \right], \quad S_\theta = \exp \left[\begin{array}{c|c} 0 & \beta \\ \hline \beta^T & 0 \end{array} \right], \quad \theta = \frac{\sinh \mu}{\mu} \beta, \quad \mu^2 = -\theta^2 \quad (4.2)$$

$$\epsilon = (1 + \theta^2)^{1/2}, \quad (\beta^T)^T = \beta$$

Let $E(M^4, \mathbb{R}^{11}, S_{10}, P)$ be the vector bundle, associated with the principal bundle $P(M^4, S_{10})$ and $\hat{\Omega}$ be the connection on E , or the linear S_{10} -gauge field (1-form on M^4 with values in the $SO(1,4)$ -algebra)

$$\hat{\Omega} = \left[\begin{array}{c|c} \hat{\omega} & \hat{\theta} \\ \hline \hat{\theta}^T & 0 \end{array} \right], \quad \hat{\omega} \in SO(1,3), \quad \hat{\theta} \in \mathbb{R}^{1,3} \quad (4.3)$$

The corresponding curvature is given by

$$\hat{R} = \left[\begin{array}{c|c} \hat{R} + \hat{\pi} & \hat{\odot} \\ \hline \hat{\odot}^T & 0 \end{array} \right], \quad \hat{R} = d\hat{\omega} + \hat{\omega} \wedge \hat{\omega}, \quad \hat{\pi} = \hat{\theta} \wedge \hat{\theta}, \quad \hat{\odot} = 2\hat{\theta} \quad (4.4)$$

The splitting of $\hat{\Omega}$ (4.3) is invariant only under the L_6 -transformations ($\hat{\theta} = 0$). In order to have a possibility to establish the relation of $\hat{\omega}$ and $\hat{\theta}$ to the Lorentz connection and the canonical 1-form on $T(M)$ it is necessary to consider the non-linear realisation of S_{10} . This effectively provides us with the space-time (non-internal) gauge theory and corresponds to the "breaking" of the symmetry $S_{10} \rightarrow L_6$. We can now construct the non-linear S_{10} -gauge field by considering the connection on the associated bundle $F(M^4, \Sigma^4, S_{10}, P)$ with the De Sitter spaces $\Sigma^4 = S_{10}/(L_6)_0$ as fibers

($\mathcal{O}$ is a fixed point in $\mathbb{R}^{44} \supset \Sigma'$). Using the prescription (2.7) with ϕ equal to S_e (4.2) we get (compare with (2.14), (2.15)). (The correspondence with the P_{10} -theory can be obtained by the Inönü-Wigner contraction $S \xrightarrow{10} P_{10}$, i.e. formally putting all quantities with subscript "T" equal to zero).

$$\Omega = \left[\begin{array}{c|c} \omega & \bar{\theta} \\ \hline \bar{\theta}^T & 0 \end{array} \right], \quad \omega \in \mathfrak{so}(4,3), \quad \bar{\theta} \in \mathbb{R}^{43}, \quad (4.5)$$

$$\omega = \hat{\omega} - e(\theta \theta^T \hat{\omega} + \hat{\omega} \theta \theta^T), \quad \bar{\theta} = \gamma \hat{\theta} + d\theta + \hat{\omega} \theta + e(\hat{\theta} \theta^T) \theta. \quad (4.6)$$

Under the action of S_{10}

$$\omega' = L' \omega L'^{-1} + L' d L'^{-1}, \quad \bar{\theta}' = L' \bar{\theta}, \quad (4.7)$$

where L' is defined from $S S_e' = S_e' S_e'$, $S = S_e' S_e'$.

We finally are able to establish necessary identifications of

ω and $\theta = \kappa_0 \bar{\theta}$ (coinciding with those of P_{10} -case). Two remarks are now in order. First, one can observe that the goldstone fields here are represented by 5-vector $\begin{pmatrix} \theta_{(a)} \\ \gamma_{(a)} \end{pmatrix} \in \Sigma'_x$ or the section of the bundle $\mathcal{F}$ (see also [27, 35]). The second remark is that the bundle $\mathcal{F}$ is soldered to M^4 with $\bar{\theta}$ being the soldering form (for detailed discussion of the De Sitter bundle see ref. [20]). The curvature of (4.5) can be written in the form

$$R = \left[\begin{array}{c|c} R + \bar{\pi} & \bar{\Theta} \\ \hline \bar{\Theta}^T & 0 \end{array} \right], \quad \bar{\pi} = \kappa_0^{-2} \pi, \quad \bar{\Theta} = \theta \wedge \theta^T \quad (4.8)$$

where R and $\bar{\Theta}$ coincide with the same quantities in (2.15).

Having defined the metric on M^4 in terms of $e_\mu^a = \kappa_0 \bar{\theta}_\mu^a$ we choose as a lagrangian the S_{10} -Yang-Mills term (cf. (2.13), (3.1))

$$L_s = \frac{1}{4\lambda} t_2 (\hat{R}_1 * \hat{R}) = \frac{1}{4\lambda} t_2 (R_1 * R), \quad (4.9)$$

invariant under the non-linear realization of S_{10} (or even under the linear realization of S_{10} if $g_{\mu\nu}$ is considered as an invariant object).

It is important to recognize, that only the L_6 -symmetry is physical for the ordinary matter fields (this fact is a manifestation of a non-linear realization of S_{10} (or P_{10})). Thus various earlier attempts [32, 35-39] of treating gravity directly as the De Sitter gauge field seem to be inappropriate. The correct attitude to the problem (i.e. basing on a non-linear realization) has been presented in [25-27], but from the different point of view on the reason for the non-linear realization (see also sect.2).

As far as the dynamics of the S_{10} -theory is concerned, there have been proposed different variants of the S_{10} -lagrangian, but only (4.9) corresponds to the natural physical requirements ((4.9) was also discussed in [36, 38] but with the different emphasise and in the different context). For example, the lagrangians of refs. [27, 32, 37, 40] are simply reparametrizations of the ECT lagrangian [8]. The approach of ref. [39] (though similar to ours in the requirement of non-contactness of ω - S -interaction) is unsatisfactory: the lagrangian here is the total derivative while all physical effects were obtained from the gauge fixing term.

Returning to our lagrangian (4.9) we can rewrite it (using (4.8) as follows, $L_s = L_s \sqrt{g} d^4x$ ($g = -\det g_{\mu\nu}$))

$$\mathcal{L}_s = -\frac{1}{\kappa^2} [R(\omega) - 2\Lambda] + \frac{1}{2\kappa^2} \Theta^a_{\mu\nu} \Theta^a{}^{\mu\nu} + \frac{1}{8\lambda} R^a_{\mu\nu} R^a{}^{\mu\nu}, \quad (4.10)$$

$$\Lambda = -\frac{6\lambda}{\kappa^2}, \quad \kappa^2 = 2\kappa_0^2 \lambda.$$

Varying (4.9) plus the matter part

$$\mathcal{L}_m = -\frac{1}{2} t_2 (*J \wedge \Omega), \quad J = \left[\begin{array}{c|c} S & -\kappa_0 t \\ \hline -\kappa_0 t & 0 \end{array} \right],$$

$$\mathcal{L}_m = \frac{1}{2} \omega^a{}_\mu S_{ab}{}^\mu - e^a{}_\mu t^a_\nu \quad (4.11)$$

with respect to ω and e we get equations

$$2 * R + 2\lambda \kappa^{-2} (2 * \pi + \theta \wedge * \Theta^T - * \Theta \wedge \theta^T) = -\lambda * S, \quad (4.12)$$

$$2 * \Theta - * R \wedge \theta - 2\lambda \kappa^{-2} * \pi \wedge \theta = \frac{\kappa^2}{2} (*t + \lambda^{-1} * T_0), \quad (4.13)$$

differing from the S_{10} -Yang-Mills equations only by the presence of T_0 , or the energy-momentum tensor of Ω , arising

from the dependence of $\mathcal{L}_m$ in (4.10) on θ . Eqs. (4.12),

(4.13) are variational as compared to the P_{10} -equations (3.3).

The lagrangian (4.10) can be given the following useful interpretation (from the point of view of internal gauge theories): there is the spontaneous symmetry breaking $S_{10} \rightarrow L_6$ in the free S_{10} -gauge theory itself, leading to the "mass term" for the L_6 -gauge field ω . The point is (explaining the difference from the internal non-linear gauge theory) that in our case the "exterior" metric in (4.9) is constructed from the part $\bar{\theta}$ of the gauge potential and thus it can "eat up" (in contraction of indices) $\bar{\theta}$ -multipliers in the different terms of (4.10). As a simple analog of our case let us consider the chiral Higgs mechanism. Let A_μ be SO_3 -Yang-Mills field, $\varphi \in S^2 = S^3/S^1$ - adjoint chiral field ($\varphi^2 = m^2$, $\{\varphi^i\} \in \mathbb{R}^3$), and

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \quad (4.14)$$

Due to the local SO_3 -invariance of (4.14) we can simply put

φ equal to $\varphi_0 = \begin{pmatrix} 0 \\ m \end{pmatrix}$, thus obtaining

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{m^2}{2} [(A_\mu^1)^2 + (A_\mu^2)^2] \quad (4.15)$$

with the reduced symmetry $SO_3 \rightarrow SO_2$ (the drawback of this "Higgs" mechanism is its perturbative non-renormalizability see however [41]). The analogy with our theory is quite transparent

$$\begin{aligned} \varphi &\rightarrow e_\mu^a, \quad \varphi^2 = m^2 \rightarrow e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu} \\ \partial \varphi &\rightarrow \Theta_{\mu\nu}^a = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a. \end{aligned} \quad (4.16)$$

As a result, e_μ^a being (from the geometrical point of view) the "covariant part" of the non-linear S_{10} -gauge field, plays the role of the (vector) "chiral" Higgs field from the dynamical point of view (i.e. in the lagrangian (4.10)).

(The possible interpretation of Θ as a vector Higgs field was also mentioned in [42]). Quantizing ω on the flat background $e_\mu^a \rightarrow \delta_\mu^a$ we really do obtain the mass terms for ω (see sects 5, 6).

5. Some consequences of the S_{10} -theory

5.1 Correspondence with general relativity

Putting $S, t, \Theta = 0$, in (4.12), (4.13) we get

$$\tilde{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (\tilde{R} - 2\Lambda) = \frac{\kappa^2}{2\lambda} \tilde{e}_{\mu\nu}, \quad (5.1)$$

$$\tilde{e}_{\mu\nu} = \frac{1}{2} (\tilde{R}^a_{\mu\lambda} \tilde{R}^{\lambdaa}_{\nu} - \frac{1}{2} g_{\mu\nu} \tilde{R}^a_{\lambda\lambda} \tilde{R}^{\lambdaa}_{\nu})$$

$$\tilde{R}^{\lambda\mu\nu}_{\mu\nu} = 0, \text{ or } \tilde{R}_{\mu[\nu\lambda]} = 0. \quad (5.2)$$

(here $\sim$ denotes the curvature tensor of the riemannian connection; $;$ denotes the riemannian covariant derivative). Using the definition of the Weyl tensor $C_{\lambda\mu\nu\rho}$, we have from (5.1)

$$-\frac{1}{12} \tilde{R} E_{\mu\nu} = C_{\mu\lambda\nu\rho} E^{\lambda\rho}, \quad \tilde{R} = \gamma\Lambda, \quad E_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \tilde{R} \quad (5.3)$$

With the help of the method of ref. [43] one can show that the only solution of (5.3) is $\tilde{R}_{\mu\nu} = \Lambda g_{\mu\nu}$ (satisfying also (5.2)). Thus the Einstein spaces are the only vacuum solutions with zero torsion of the system (4.12), (4.13).

Moreover, one can try to prove for (4.12), (4.13) some sort of the Birkhoff theorem: the unique solution of (4.12),

(4.13) for $S, t = 0$ and SO_3 -spherically symmetric ansatz for e and ω is $\Theta = 0$ and $g_{\mu\nu} =$ Schwarzschild(-De Sitter) metric. This is really true for some classes of the $(R + \Theta^2 + R^2)$ lagrangians [44-46]. At the same time, for the $(\Theta^2 + R^2)$ lagrangian of Hehl et al [2,17] there is the SO_3 -symmetric solution of Baekler [47] with $\Theta \neq 0$. It is probable that in our case (4.10) torsion will be non-zero but will swiftly fall down away from the origin with $\tau \rightarrow \infty$ (thus representing the "torsion monopole", see also [9,46,48]).

The apparent problem is the relation of magnitudes of Λ and the ω - S interaction constant λ (note that $\Lambda \sim \lambda^2$ also for the Baekler solution [47]). One can solve the problem of great Λ for $\lambda \neq 0$ "by hands" merely subtracting Λ -term from (4.10). However, one should keep in mind that the great Λ -term arise also in a number of different cases [49] and also take into account the

possibility of some kind of "confinement" on the microlevel due to the ω - S -interaction (hadrons as the Schwarzschild-De Sitter "universes"). Finally, let us note that even for spinless matter ($S=0$) the condition of $\Theta=0$ is too stringent one, because according to [50] nearly all solutions of (5.2) are the vacuum Einstein spaces (cf. [2], see also the discussion of the Newtonian limit for $(R+R^2)$ -theories in [45]).

5.2. Spin-spin interactions

Taking $e^a_\mu = \delta^a_\mu$, $\Lambda=0$, one can write the $(R+\Theta^2)$ - part of (4.10) in the form $(\hat{\omega}_d = \frac{1}{3!} \epsilon_{abcd} \omega^{abc})$

$$\mathcal{L} = \frac{1}{\kappa^2} (\omega_a \omega^a + 18 \hat{\omega}_a \hat{\omega}^a), \quad \omega_a = \omega_{\alpha\beta\gamma} e^\alpha_\mu e^\beta_\nu e^\gamma_\rho \quad (5.4)$$

Using the fact that $S_{abc} = \epsilon_{abcd} \hat{S}^d$, $\hat{S}^d = -\frac{1}{2} \gamma^d_{\alpha\beta} \hat{S}^{\alpha\beta}$ for spin 1/2 fermions and assuming that $\omega_{abc} = \epsilon_{abcd} \hat{\omega}^d$ we get the lagrangian

$$\mathcal{L} = \frac{3}{2} \{ (\partial_a \hat{\omega}_b)^2 - \frac{1}{2} (\partial_a \hat{\omega}_a)^2 + \mu^2 \hat{\omega}_a^2 - 3 (\hat{\omega}_a \hat{\omega}^a)^2 - 2 \hat{\omega}_a \hat{S}^a \} \quad (5.5)$$

This gives the following static non-relativistic potential [51] for the interaction of two fermions with spins $\vec{\sigma}_1$

$$\text{and } \vec{\sigma}_2 \text{ in the linear approximation and massless limit } (\mu \rightarrow 0)$$

$$V_E \sim \lambda^2 \left\{ A \frac{\vec{\sigma}_1 \vec{\sigma}_2}{r^2} + B \frac{(\vec{\sigma}_1 \vec{r})(\vec{\sigma}_2 \vec{r})}{r^3} \right\} + \{\delta(r) - \text{contact terms}\} \quad (5.6)$$

Another approximation is obtained if we neglect the transferred momentum in $\hat{\omega}$ -propagator ($\mu \neq 0$), giving the contact potential as in the ECT case [8]. Thus the presence of the L_6 Yang-Mills term R^2 in (4.10) modifies ECT just like the theory with intermediate bosons do it with the Fermi theory. The potential (5.6) will lead to some new physical effects, depending on the value of the L_6 -dimensionless

constant λ . In ref. [39] λ was taken to be 10^{-38} (or m the mass of the proton), but the absence of contradiction with known atomic effects allows λ to be sufficiently greater [52] (e.g., 10^{-17} [10]). One can observe that the spin-spin interaction is already present in the Einstein theory ($V_E \sim -\kappa^2 \frac{\vec{S}_1 \cdot \vec{S}_2}{2}$, or the analog of the Breit potential [51]). The point, however, is that the gauge-like ω - δ interaction provides the $\frac{1}{2}$ spin-spin potential analogous to the fundamental $\frac{1}{2}$ -potentials in electrodynamics and gravitation.

5.3. Instanton solutions

The field equations (4.12), (4.13) in the vacuum possess the remarkable solutions, intimately connected with the Yang-Mills structure of (4.9) or the self-dual solutions (usually considered in the euclidean case $SO(4) \rightarrow SO(5)$, $\gamma_{ab} \rightarrow -\delta_{ab}$ or $\kappa_a^2 \rightarrow -\kappa_a^2$ in (4.10))

$$R = *R, \text{ or } *(R - \bar{R}) = R - \bar{R}, \quad \Theta = *\Theta. \quad (5.7)$$

Solutions of (5.7) realize the local minima of the euclidean action $I_S = \frac{1}{8\lambda} \int d^4x \sqrt{g} \{ (R - \bar{R})^2 + 2\bar{\Theta}^2 \}$ bounded from below by the topological invariant $P_{1/2} = -\frac{1}{8\pi^2} \int \text{tr} (R \wedge R) = P_{1/2}$. It is important to stress that here we do not confront the difficulty of the indefiniteness of the euclidean action, present in the Einstein theory [53] (and also in ECT and supergravity). Note that though the action of ref. [32] is also non-negative, it does not lead to the convergence of the path integral (as compared to I_S). The absolute minimum $I_S = 0$ is realized on the De Sitter space S^4 , $R = 0$, or

$\Theta = 0$, $R^{ab}_{\mu\nu} = \frac{2}{k_0^2} e^a_\mu e^b_\nu$. Then go the minima with $\Theta = 0$ and selfdual $(\tilde{R} - \tilde{T})$, which coincide with the solutions of the Einstein equations with selfdual Weyl tensor (e.g. CP^2 and its generalizations considered in [54]). Instantons with $\Theta \neq 0$ have greater action (or topological number).

One can a priori hope that the theory (4.10) may provide the realization of the idea of Hanson and Regge [9] about the analogy of gravity with torsion $\Theta \neq 0$ and the theory of superconductivity (with the Meissner effect corresponding to the phase of zero torsion, thus dynamically explaining why $\Theta = 0$ in GR and localized regions with $\Theta \neq 0$ being the analogs of the Abrikosov vortices). In [3] this analogy was proposed to hold in the euclidean case with e and ω being spherically symmetric in 4-dimensional sense ("torsion instantons"). However, (5.7) does not admit non-trivial solutions of this type: all solutions for the most general SO_4 ansatz ($\rho = x^a x_a$)

$$\omega^{ab}_\mu = 2A x^{[a} \delta^{b]}_\mu + B \epsilon^{ab}_{\mu\nu} x^\nu, e^a_\mu = f \delta^a_\mu + g x^a x_\mu \quad (5.8)$$

are exhausted by the trivial S^1 instanton: $A = \frac{2}{\rho^2 + q^2}$, $f = \frac{\sqrt{2} q k_0}{\rho^2 + q^2}$, $B = 0$, $g = 0$ (ω corresponds to the SO_4 connection for the SU_2 one-instanton solution [55]).

This result has a simple explanation: looking for solutions with $\Theta \neq 0$ we are looking for "multiinstanton" configurations, which are not spherically symmetric. The correct point of view probably is that the Hanson-Regge analogy takes place in the Minkowski-signature space-

time and what we really need is the SO_3 -symmetric static solution, corresponding to the torsion "monopole" (similar solutions were already found in a number of theories [45-48]). This solution is analogous to the SU_2 -t'Hooft-Polyakov monopole with e^a_μ playing the role of the Higgs field and $\omega^a_{\mu\nu}$ - of the Yang-Mills potential. In the case of M^4 being the Minkowski space one is able to prove the topological stability of such kind of solutions. Putting $e^0_0 = 1$, $e^0_i = e^i_0 = 0$, $e^i_j = \frac{\partial x^i}{\partial x^j} \in SO_3$ and choosing $e^a_\mu \rightarrow \delta^a_\mu$ we get the map $e^a_\mu : R^3 U(1) \rightarrow SO_3$ classified by $\pi_3(SO_3) = \mathbb{Z}$. The necessary degeneracy of the e^a_μ -vacuum is established by the condition $e^a_\mu e^b_\nu = \delta^a_b \delta^0_0$. The discussion of the analogy with the chiral Higgs mechanism in sect. 4).

5.4. Model lagrangian and physical consequences of non-compactness of the Lorentz group.

Dropping the torsion "mass terms" (and Λ -term) in (4.10) in $(R + \Theta^2)$ we obtain the following approximation for (4.10)

$$\mathcal{L}_0 = -\frac{1}{k_2} \tilde{R}(g) + \frac{1}{8\lambda} R^a_{\mu\nu}(\omega) R^b_{\mu\nu}(\omega) \quad (5.9)$$

The $\mathcal{L}_0$ -theory may be considered as a model of the P_{10} -dynamics, corresponding in some sense to the $T_4 \times L_6$ gauge theory (the analogous lagrangians were discussed in [56-58], [7]). One can treat $\mathcal{L}_0$ also as the lagrangian for the Einstein gravity, interacting with the $SO(4,3)$ gauge field. (Note that ω interacts only with spinors). Field equations have the form (we use matrix notations in $a, b, \dots$)

$$\tilde{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \tilde{R} = \frac{\kappa^2}{2} (t_{\mu\nu} + \frac{1}{2} \tilde{\epsilon}_{\mu\nu}^{23}), \quad t_{\mu\nu} = 0, \\ \tilde{\epsilon}_{\mu\nu} = \frac{1}{2} t_{\lambda} (R_{\mu\lambda} R_{\nu}{}^{\lambda} - \frac{1}{2} g_{\mu\nu} R_{\lambda\rho} R^{\lambda\rho}), \quad (5.10)$$

$$\bar{\mathcal{D}}_{\mu} R^{\mu\nu} = \lambda S^{\nu}, \quad \bar{\mathcal{D}} \equiv \mathcal{D} + \{ \} \quad (5.11)$$

Introducing $E_i = R_{0i}$, $H_i = \frac{1}{2} \epsilon_{ijk} R^{jk}$, $\vec{E}_e = \vec{E}^{i0}$, $\vec{E}_h = \vec{H}^{i0}$,

$$\vec{H}_e = \frac{1}{2} \epsilon^{ikj} \vec{E}_{kj}, \quad \vec{H}_h = \frac{1}{2} \epsilon^{ikj} \vec{H}_{kj} \quad \text{we get}$$

$$\gamma_{00} = \frac{1}{2} (\mathcal{H}_e^2 + \mathcal{H}_h^2 - \vec{E}_e^2 - \vec{E}_h^2), \quad \mathcal{H} = \frac{1}{2} (\vec{E}_h^2 + \mathcal{H}_e^2 - \vec{E}_e^2 - \mathcal{H}_h^2), \quad \text{i.e.}$$

the non-compactness of $L_6 = SO(4, 2)$ (the indefiniteness

of the group metric) imply the indefiniteness of the energy for ω . The lack of the positive definiteness of the energy

is the necessary consequence of the localization of L_6 ,

drastically influencing physical predictions of the theory

(e.g. violating the conditions for singularities, providing the possibility for the "gravitational repulsion" etc.). We

conclude that the Poincare gauge theory, taking into ac-

count the ω - S -interaction of the L_6 Yang-Mills type,

may provide the possible answer to the question of Wu and

Yang [59] about the physical applications of gauge theories

with non-compact gauge groups.

There exists the situation when the system of spins

creates the field ω with negative energy (the following

example is also interesting from the point of view of pos-

sible experimental observation of the ω - S interaction).

Namely, let us consider the cylindrical ferromagnetic specimen

with all spins polarized in Z -direction so that inside the

specimen $\Psi = \sqrt{n} e^{-im\vec{r} \cdot \left(\frac{1}{\hbar}\right)} \vec{S} = \{0, 0, 0, \pm \hbar\}$ (cf. sect.

5.2) (n is the concentration of electrons). Supposing $g_{\mu\nu} = \eta_{\mu\nu}$

we have from (5.11) in linear approximation the following

interior solution

$$R^{i0} = \frac{1}{3} x^i S^0, \quad R^{ik} = \frac{1}{3} x^i S^k, \\ \Delta M = \int_{\tau=0}^{\tau} d\tau = -\frac{1}{2} h^2 \pi r^2 \ell \left(\frac{1}{3} \ell^2 \frac{B}{A} \tau^2 \right), \quad (5.12) \\ A = 14/288, \quad B = 5/288$$

(τ is the radius and ℓ is the length of the specimen).

As a result, spins produce the L_6 -gauge field with the negative energy and thus decrease the total mass of the system "sources + field" and hence, effectively, of the specimen. For example, when $\ell \sim 10$ cm, $\tau \sim 1$ cm, $h \sim 10^{24} \text{ cm}^{-3}$, then $\Delta M \sim 10^{12} \text{ g}$ ($\hbar = c = 1$) and therefore in the world with $\lambda \sim 10^{-5}$ one will witness "antigravity" after the instantaneous polarization of the specimen (cf. with the recent proposal of antigravity in supergravity [60]).^{2/}

Another consequence of the non compactness of L_6 is the possibility of static regular solutions of the L_6 Yang-Mills equations in vacuum, absent for compact gauge groups. For example, let us consider the following complex solution

for SU_2 gauge theory found in [59] (in the flat space)

$$A_i^k = \epsilon_{ikj} f(\tau) n^j, \quad A_0^k = h^k g(\tau), \quad \vec{n} = \frac{\vec{x}}{r}, \quad (i, j, k = 1, 2, 3) \quad (5.13)$$

$$1 + \tau f = \beta \tau / \sinh(\beta \tau), \quad 1 + \tau g = \beta \tau \coth(\beta \tau), \quad \beta = a + i b \in \mathbb{C}, \quad a > 0$$

The corresponding $SL(2, \mathbb{C})$ or $SO(4, 1)$ solution is obtained

$$\text{according to the rule } A_\mu^k = W_\mu^k + i V_\mu^k, \quad \omega_\mu^{0k} = V_\mu^k, \quad \omega_\mu^{ij} = \epsilon^{ijk} W_\mu^k.$$

The field strength for (5.13) is regular in $\tau = 0$ and goes as τ^{-2} when $\tau \rightarrow \infty$. The energy of the solution is zero,

i.e. it is possible to generalize it to the approximate solution of (5.10), (5.11) for conformally flat metrics. We

remark that the existence of regular solutions like (5.13) is

^{2/}The corresponding effect in the EGT is negligible $|\Delta M| \sim 10^{-35} \text{ g}$.

probably the manifestation of the long-range character of "non-compact" gauge fields (analogous to that of the abelian fields). Thus the $L_6(\omega)$ gravity has a long-range nature just like the Einstein $T_4(e)$ -gravity (so the idea of "confinement" of ω proposed in [1-3] seems to be inappropriate and based on an incorrect choice of variables).

Treating the vacuum $\mathcal{L}_0$ -theory as $GR + SO(4,3)$ gauge field it is easy to obtain the general spherically symmetric solution of (5.10), (5.11), cf. [62] (for analogous solution for an arbitrary gauge group see [61])

$$\begin{aligned} \theta^0 &= a dt, \quad \theta^1 = b dr, \quad \theta^2 = r d\theta, \quad \theta^3 = r \sin\theta d\varphi, \\ \omega &= \frac{u}{2} dt - b \cos\theta d\varphi, \quad R = E \theta^0 \theta^1 + H \theta^2 \theta^3, \quad (5.14) \\ E &= \frac{u}{2^2}, \quad H = \frac{b}{2^2}, \quad a^2 = b^2 = 1 - \frac{2GM}{r} + \frac{4\pi G}{2^2} g^2, \quad g^2 = -\frac{1}{4} \text{tr}(u^2 + b^2) \end{aligned}$$

(u and b are commuting matrices from the algebra of L_6)
Note that $g^2 \neq 0$ due to the non-compactness of L_6 . Here u and b are connected with the "charge" (i.e. with the spin) of the central body and hence there is a possibility to prevent the formation of a horizon by a non-zero total spin $\vec{S}$ of the star ($g^2 = \frac{1}{6\pi^2} \vec{S}^2$). It is interesting to note that torsion is non-zero for (5.14) (and thus the Birkhoff theorem of ref. [44] is not valid for (5.9)) but goes too slowly ($\sim \frac{1}{r}$) to provide the interpretation of a "torsion monopole" to the abelian solution (5.14).

As far as cosmological solutions of (5.10), (5.11) are concerned ω can be considered as some background field (due to a probable absence of its sources). Here, in

general $\Theta \neq 0$. It is important to realize, however, that torsion is not a directly observable quantity. Namely, from (5.10), (5.11) one gets the following conservation laws (or equations of motion)

$$\bar{D}_\mu S^\mu = 0, \quad T^{\mu\nu}_{;\nu} = \frac{1}{2} t_2 (R^{\mu\nu} S_\nu) \quad (5.15)$$

and so spinless particles move along the riemannian geodesics but in the presence of spin they are influenced by ω via the generalized Lorentz force (see also [8]). As a result, one can in principle detect the existence of the ω - S -interaction observing the differences in the trajectories of particles with and without spin near a probable source of ω (e.g. ferromagnetic specimen or rotating neutron star).

6. Quantization of gauge theories of gravity with torsion

6.1. General remarks and propagator

The non-compactness of $\angle_6$ (or the absence of positive definiteness of the energy) leads to the problems in quantization of the theory, including $\mathcal{L}_6$ part in the lagrangian (as in (5.9)) the hamiltonian $H \geq 0$ is not bounded from below, there is no ground state and unitarity. The correct quantization scheme is absent for the "non-compact" gauge fields. From the point of view of path integral quantization the indefiniteness of H apparently implies the divergence of the euclidean path integral. However, our case of $\angle_6$ is a remarkable exception: here the Wick rotation takes place not only on the base space ($g_{\mu\nu} \rightarrow g_{\mu\nu}^{(E)}$) but also "in the

fibers" $(SO(4,3) \rightarrow SO(4))$, because the fiber is here the tangent space to the base. As a result, the euclidean action is positive (cf. again with GR [53]), signifying that inspite of ghosts in the Minkowski sector the corresponding quantum theory may be meaningful.

Let us now consider the propagator in the S_0 -theory or the bilinear approximation in (4.10) in terms of ω and B , $e_\mu^a \approx \delta_\mu^a + \kappa B_\mu^a$ (we disregard A -term). The analysis of the spectrum of particles in the theory allows the following ambiguity: (i) one can treat (e_μ^a, ω_μ^a) as the multiplet of ten vector gauge fields, i.e. $\chi(e, \omega) = 10 \times (1 \oplus 0^1)$;

(ii) in the bilinear approximation there is formally no difference the latin and greek indices and so one can consider as the basic variables $B_{ab} = \bar{B}_{ab} + \frac{1}{4} \delta_{ab} B + B_{ab}$ =
 $= \{2^+, 1^-, 0^+, 0^+; 1^-, 1^+\}$; $\omega_{abc} = \bar{\omega}_{abc} +$
 $+ \frac{2}{3} \delta_{c[a} \hat{\omega}_{b]} + \epsilon_{abcd} \hat{\omega}^d = \{2^+, 2^-, 1^-, 1^+, 0^+, 1^-, 0^+, 1^+\}$. The

second approach is preferable when dealing with the propagator and is in correspondence with GR (graviton as 2^+), while the first one is essential in the exact theory. Let us note that the appearance of the 2^+ -particles from ω_{abc} (besides the graviton from B) provides the interpretation of the (e, ω) theory as a variant of "strong gravity" [2, 57, 10]. The propagators in (e, ω) -theories with quadratic lagrangians recently were discussed in detail in refs [10, 11, 16] so we shall only emphasize the essential points of our approach. From the above discussion it is evident that there is the $(e-\omega)$ mixing

in $2^+, 1^-, 1^+, 0^+$ -sectors of the propagator. However, for the aim of revealing the pole structure it is advantageous to analyze the propagator in terms of the $SO(4)$ and not of the $SO(3)$ -spin. Here again the mixing occurs and the propagator P has the following sectors: "trace" $\check{P}$, "pseudo-trace" $\hat{P}$ and "irreducible" $\bar{P}$. It turns out that P has no ^{massive} poles, but $\check{P}$ and $\hat{P}$ have the following tachyonic poles: $\check{P}^2 = -12\kappa^{-2}\lambda$, $\hat{P}^2 = -6\kappa^{-2}\lambda$, $\lambda > 0$ (c.f. (5.5)). The passage to the euclidean theory implies $S_0 \rightarrow SO_4$ or $\kappa^2 \rightarrow -\kappa^2$ and the change of the sign of the square of momentum (our signature is $(+---)$). As a result, the euclidean propagator has no real poles just as in the case of a "respectable" euclidean theory (we hope that this fact makes the objection of the absence of unitarity raised against (4.10) in [1] harmless). Thus one may suspect (as one does in the case of the Einstein theory [53]) that it is the euclidean approach that possibly provides the correct quantization of the theory.

Analysing the propagator one can formally act in terms of the non-linear S_0 potential Ω (4.5). However, it is necessary to take into consideration the spontaneous symmetry breaking $\langle e^q_\mu \rangle = \delta^q_\mu$. It is remarkable that in our approach this condition (extremely essential also in GR) is the direct consequence of the use of the non-linear realization of S_0 (or P_{10}). Really, assuming that for the linear gauge field $\hat{\Omega}$ (4.3) $\langle \hat{\Omega} \rangle = 0$, or $\langle \hat{\omega} \rangle = 0$, $\langle \hat{\theta} \rangle = 0$, we get from the definitions (4.5), (4.6) that $\langle \omega \rangle = 0$, $\langle \bar{\theta} \rangle = d\beta \neq 0$ (goldstone parameter β in (4.6) is consider-

ed as a classical field). Dealing for simplicity with the case of P_{10} -theory (2.4), (2.15) we have $\langle \theta_\mu^a \rangle = \langle \xi_\mu^a \rangle = \delta_\mu^a$ (where $\xi_\mu^a = \chi^a$ when M^4 is flat, i.e. coincides with the tangent space). Thus the problem is reduced to the question about the values of goldstone parameters in the vacuum state and clearly corresponds to the spontaneous T_4 -symmetry breaking.

One could notice that the non-zero $\langle \theta \rangle$ had led to the mass terms for ω (more accurately, to the poles of the combined (e, ω) -propagator). This fact prompts the possibility to obtain the spontaneous symmetry breaking in a sourceless internal (e.g. SO_n) gauge theory by the use of a non-linear realization ($SO_n \rightarrow SO_{n-1}$) under some natural assumptions about the vacuum values of goldstone parameters. As a consequence, the SO_{n-1} gauge field will get some mass and so this mechanism may be an interesting alternative to the ordinary Higgs mechanism, being based only on gauge fields.

One more remark is about the Λ -term in (4.10). Even if $2\Lambda \kappa^{-2}$ is subtracted from (4.10), the minimum ($= -2\Lambda \kappa^{-2}$) of the euclidean action is again realized on the De Sitter space. Thus the value of Λ is a serious problem (note, however, that in GR even the notion of the euclidean vacuum is senseless in view of the indefiniteness of the action and hence the choice of the flat space as the vacuum state is an additional assumption).

6.2. The problem of renormalizability

We shall consider successfully a number of examples of the (e, ω) -theories to illustrate the main difficulties confronted in the case of the $\mathcal{S}_0$ -theory (4.10). Let us start with

$\mathcal{L}_0$ -theory (5.9). In the euclidean variant it is unitary but non-renormalizable (as GR plus SO_4 gauge theory). Taking into account the result of [63] one can get the 1-loop counterterms in the form $(\bar{E} = 8\pi^2(n-4))$, $z_6 = 6$, $c_{2L} = c_2(SO_4)$,

$$\Delta \mathcal{L}_0 = \frac{1}{\bar{E}} \left[\left(\frac{137}{60} + \frac{z_6-1}{10} \right) \tilde{R}_{\mu\nu}^2 + c_{2L} \frac{11}{12} \left(-\frac{1}{2} \right) \text{tr} (R_{\mu\nu}(\omega) R^{\mu\nu}(\omega)) \right] \quad (6.1)$$

Using the field equations (5.10), (5.11) we have $\tilde{R} = 0$,
 $\tilde{R}_{\mu\nu}^2 = \frac{1}{4} \tilde{R}_{\lambda\mu\nu\rho}^2 + \text{div}$, or $\Delta \mathcal{L}_0 = a_1 \tilde{R}_{\lambda\mu\nu\rho}^2 + a_2 (R_{\mu\nu}^2)$. Extraction of the torsion part from ω yields $(R_{\alpha\beta\mu\nu})^2 = (\tilde{R}_{\lambda\mu\nu\rho} + T_{\lambda\mu\nu\rho})^2$,
 $T = \nabla \tilde{T} + \tilde{T} \tilde{T}$, $\tilde{T} = \omega - \omega_0(e)$, and thus the renormalizability

takes place only after the additional assumption that the background torsion is zero (this condition may have physical sense if the idea about the Meissner effect for torsion is true [9]). This "conditional renormalizability" may be compared with the finiteness of supergravity beyond the one loop level when the background fields satisfy certain self-duality relations [64]. If one puts also the quantum torsion equal to zero, the $\mathcal{L}_0$ -theory reduces to the $\tilde{R} + \tilde{R}^2$ -theory, which is renormalizable [65] but apparently non-unitary (see also [66] and references therein). We conclude that torsion plays the role of the field, restoring the unitarity (see also [11]) but in general breaking the renormalizability of the theory with quadratic lagrangian.

Our next example is the theory of Fairchild [17]

$\mathcal{L}_F \sim R(\omega) + R(\omega)R(\omega)$ (coinciding with (4.10) up to the Λ - and Θ^2 -terms). Here the kinetic term for e_μ^a is absent making the quantization of e (or metric) somewhat senseless. If e is not quantized, $\mathcal{L}_F$ is simply the lagrangian of the S_0 -gauge field in the curved space-time, interacting in a gauge invariant manner with the external source $\pi^{ab} = e_\mu^a e_\nu^b$ by means of the Pauli-like term $R = R^{\mu\nu}{}_{\alpha\beta} \pi^{\alpha\beta}$. This theory is unitary (cf. with the opposite claim in [11] in the case of Minkowski signature and quantized e). Using the algorithm of ref. [6-7] one can obtain the 1-loop counterterms in the form

$$\Delta \mathcal{L}_F = \frac{1}{\epsilon} \left\{ \frac{1}{20} C_{\mu\nu\rho\sigma}^2 + C_{\mu\nu}^2 \left(-\frac{1}{2}\right) \text{tr} \left[\left(R_{\mu\nu} + \frac{21}{8} \pi_{\mu\nu} \right)^2 - \frac{1}{12} R_{\mu\nu}^2 \right] \right\} \quad (6.2)$$

where the Weyl tensor $C_{\mu\nu\rho\sigma}$ and π depend only on the external field and so do not disturb renormalizability [6b]. However, if e is quantized, the additional $\tilde{R}^2$ terms appear in (6.2) thus breaking renormalizability if the background torsion is non-zero.

Turning now to the case of the S_{10} -theory (4.10) we conclude, that it is unitary (in the euclidean region) but in general nonrenormalizable in view of unavoidable $\tilde{R}^2$ counterterms (arising from the coupling of metric to the kinetic $(\partial\omega)^2$ and $(\partial e)^2$ terms in (4.10)). Thus we have a strong feeling that lagrangians like (4.10) lead to non-renormalizable theories (this is especially obvious when non-supersymmetric matter is present, giving additional counterterms absent in the initial lagrangian). As a result, the S_{10} -theory has the analogous quantum behaviour as GR

(excluding the remarkable fact of the positivity of the euclidean action and interesting possibility to obtain the renormalizable theory when the classical torsion is equal to zero). There is, however, a chance to improve the situation by treating the metric as a classical object and quantizing only the linear S_0 -gauge field $\hat{\Omega}$ (4.3). Let us assume (in the context of the background field method [69])

that

$$\hat{\Omega} = \hat{\Omega}_{ce} + \hat{\Omega}_g, \quad \hat{\Omega}_{ce} = \Omega, \quad \text{or } \langle \hat{\omega} \rangle = \omega, \quad \langle \hat{\theta}_f^a \rangle = \kappa_0 e_f^a \quad (6.3)$$

i.e. that the breaking of the symmetry $S_0 \rightarrow L_6$ is established by the condition that the classical part of the quantized linear gauge field (4.3) is equal to the non-linear gauge field (4.5). If the condition (6.3) is absent and

$g_{\mu\nu}$ is some classical metric we have for (4.9)

$$\Delta \mathcal{L}_S = \frac{1}{\epsilon} \left\{ \frac{1}{20} (C_{\lambda\mu\nu\rho})^2 + \frac{11}{12} C_{\lambda\mu} \left(\frac{1}{2} \right) \text{tr}(\hat{R}_{\mu\nu} \hat{R}^{\mu\nu}) \right\}, \quad (6.4)$$

implying the renormalizability [68]. Taking now into consideration (6.3) and the equality (4.9) one can observe that renormalizability is still present (up to a subtle point of establishing the background gauge invariance in view of the dependence of $g_{\mu\nu}$ on the part of the background S_0 -gauge field). The latter problem does not arise in the proof of renormalizability (in the flat space-time) of the SQ_n gauge theory with spontaneous symmetry breaking $SQ_n \rightarrow SQ_{n-1}$ of the type discussed in sect.6.2. We emphasize that the obtained result is based essentially on: (a) the Yang-Mills (renormalizable) structure of the lagrangian (4.9); (b) on the use of the non-linear realization (necessary to establish

the interpretation of (4.9) as the gravity theory). The approach based on (6.3) is a compromise one: though the metric is not quantized, it is not purely classical, being expressed in terms of the mean value of the quantum field $\hat{\theta}$. The apparent drawback of this suggestion lies in the fact that matter interacts naturally with e and ω but not with $\hat{\theta}$ and $\hat{\omega}$. The radical point of view is that the quantized part of the gravitational field is the connection ω (or L_g - gauge field) [58], tetrad or metric being purely classical.

Our final comment is about the interaction of ω with spin $\frac{1}{2}$ - fermions. As is well known (see e.g. [8]), the corresponding term in the lagrangian is $\mathcal{L}_{int} \sim (\bar{\psi} \gamma^\mu \gamma_5 \psi) \hat{\omega}_\mu$, $\hat{\omega}_\mu = -\frac{1}{2} \epsilon_{\mu\lambda\rho\sigma} \omega^{\lambda\rho\sigma}$, i.e. only the axial part of connection (or torsion) contributes in it. As a result, in order to establish the renormalizability (even if the metric is classical) one needs the following bare term in the (gravity + matter) lagrangian

$$\mathcal{L} = \frac{1}{2\kappa^2} W_{\mu\nu} W^{\mu\nu}, \quad W_{\mu\nu} = \partial_\mu \hat{\omega}_\nu - \partial_\nu \hat{\omega}_\mu. \quad (6.5)$$

It is this term (and not $R^2(\omega)$ in (5.9)) that should be added to the Einstein lagrangian for the renormalizability of the ω - ψ interaction. This fact seems to be overlooked in earlier investigations (i.e. various lagrangians of refs. [1-3, 8, 16, 17, 36-40] are non-renormalizable in the presence of fermions).

7. Further questions

Let us recollect here a number of problems left in the theory: 1) The first question is about the value of λ in (4.10) and thus the magnitude of physical effects due to the interaction of spin and connection; 2) Are there any monopole-like solutions of (4.12), (4.13) with localized torsion and what is their impact on quantum theory (cf. [9, 48])?; 3) Does there exist the supersymmetric extension of the lagrangian (4.10) thus partially improving the quantum behaviour of the theory (see in this connection [70, 71])? 4) Is it possible to develop a senseful quantum theory for (4.10) by the continuation of the euclidean theory back to the lorentzian sector?; 5) What is the exact form of the 1-loop counterterms for (4.10) when both e and ω are quantized? (Let us stress that the 1-loop counterterms in the theories with non-zero torsion can be evaluated using the wellknown algorithm of refs. [67, 71] and do not require the modification proposed in [72]). The questions related to our theory in internal gauge theories are: (i) how to quantize the gauge theories with noncompact gauge groups; (ii) is it possible to establish the viable alternative to the Higgs mechanism by working only in terms of the pure gauge theory under the assumption of the non-linear realization and appropriate ^{conditions} on the "broken" part of the gauge potentials (cf. (6.3)) (see also [73] about the noncompact gauge theories). The real question is of course about the existence in nature the interaction of the Lorentz connection and spin. If the answer is negative, the Lorentz

group should not be gauged and one must abandon the attempts to interpret gravity as a gauge theory (analogous to the internal gauge theories). However, if the answer is positive, we believe that the ω -S interaction will be of the Yang-Mills type and thus the total lagrangian will look like (4.10) (probably plus (6.5)).

After the completion of this work several useful references appear to our view: similar to ours approaches to kinematics of the Poincare gauge theory are proposed in [74]; the analogous to (4.10) lagrangians are discussed in [75]; for instantons in R^2 -theory see ref. [76]; for supergravity with propagating ω see [77].

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