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A LIMIT ON THE NEUTRINO MASSES FROM DOUBLE β DECAY

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ABSTRACT

Limits on the masses of the light and heavy Majorana neutrinos of $m_\nu \leq 3$ eV and $m_\sigma \geq 10^4$ GeV have been obtained. Also a limit on the neutrino-anti-neutrino Higgs coupling $|g| \leq 10^{-4}$ has been set. These results follow from the experimental limits on the double β decay $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ and a new calculation of the nuclear matrix elements.

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Neutrinoless double β decay is the oldest lepton charge (number) violating process studied both experimentally and theoretically¹⁾⁻³⁾. Recent developments in unified gauge theories⁴⁾⁻⁸⁾, which, in order to provide a satisfactory understanding of the neutrino mass, predict that lepton number must be broken at some level, have rekindled interest in this process. In fact, with lifetimes not much longer than the present limits of $(T_{1/2})_{0\nu} \gtrsim 10^{21}$ yr it appears that the reaction

$$(A, Z) \longrightarrow (A, Z+2) + e^- + e^- \quad (1)$$

is the most likely to be first observed. In fact, from such experimentally established limits on the lifetimes of ^{76}Ge and ^{82}Se a limit $\langle m_\nu \rangle \approx 15 \text{ eV}/c^2$ has recently⁹⁾ been set for the neutrino mass. A somewhat longer value $\langle m_\nu \rangle \approx 30 \text{ eV}/c^2$ has been obtained by Doi et al.¹⁰⁾ from the available data and the available nuclear matrix elements.

In the present paper we will extract the lepton violating parameters and set a limit on the neutrino mass, focusing our attention on the theoretically and experimentally desirable $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ decay. In addition we will show that a limit can be put on the neutrino-antineutrino massless Higgs (χ_0) coupling g recently discussed by Georgi, Glashow and Mussinov¹¹⁾.

The $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ double β decay was initially favoured¹²⁾ from an experimental point of view because of the large energy release involved ($E_0 \approx 4.2 \text{ MeV}$). It later became disfavoured when it was shown¹³⁾ that the relevant nuclear matrix element was exceedingly small. Our results indicate that the smallness of the nuclear matrix element is related to the approximate effective transition operator employed in the earlier treatment. The matrix element need not be small in a more exact treatment.

From a theoretical point of view, the reaction (1) can proceed in the gauge theory model provided

- i) there exists a Majorana neutrino ν_m :

$$\nu_m^c = \nu_m \quad \nu_m^c \equiv C(\bar{\nu}_m)^T \quad (2)$$

where C is the charge conjugation operator. ν_m can be either the electron neutrino ν_e or any neutrino to which ν_e is coupled to (ν_m , of course, enters as a virtual particle and does not appear in the final state).

- ii) One (or both) of the following must happen:

a) there exists a small right-handed component in the usual left-handed current given by a parameter η .

b) The neutrino ν_m has a mass which is not much smaller than a few eV.

Process a), which is not favoured nowadays, has been extensively studied in the past^{1),3),10),13),14)}. For the reader's convenience we briefly summarize the results. The transition probability for $0^+ \rightarrow 0^+$ decays takes the form

$$W_{0\nu}(i \rightarrow f e^- e^-) = K_2 \bar{A}^{2/3} |\chi(\alpha(z+z))|^2 \tilde{f}_2(\epsilon_0) |\langle f | \eta \Omega | i \rangle|^2 \quad (3)$$

where

$$\Omega = \sum_{i \neq j} \tau_+(i) \tau_+(j) \frac{R_0}{r_{ij}} \left[3 \frac{\vec{s}_i \cdot \vec{s}_j}{s_A^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j + \frac{2\sqrt{6}}{\sqrt{5}} \sqrt{4\pi} Y_{22}(\hat{r}_{ij}) (\vec{\sigma}_i \otimes \vec{\sigma}_j)^2 \right] \quad (4a)$$

$$\chi(\alpha z) = 2\pi\alpha z / (1 - \exp(-2\pi\alpha z)), \quad \vec{r}_{ij} = \vec{r}_i - \vec{r}_j, \quad \hat{r} = \frac{\vec{r}_{ij}}{r_{ij}} \quad (4b)$$

$$K_2 = c \frac{m_e c^2}{\hbar c} \left(\frac{G_F m_p^2}{(2\pi)^3} \right)^4 \left(\frac{\hbar c}{m_p c^2 r_0} \right)^2 \left(\frac{m_e}{m_p} \right)^6 f_A^4 = 2.6 \times 10^{-15} \text{ yr}^{-1} \quad (4c)$$

$$\tilde{f}_2(\epsilon_0) = \frac{1}{81} \frac{1}{210} \epsilon_0^4 (\epsilon_0^3 + 14 \epsilon_0^2 + 77 \epsilon_0 + 77) \quad (4d)$$

$R_0 = r_0 A^{1/3}$ = nuclear radius ($r_0 = 1.1$ fm) and ϵ_0 is the available energy in units of the electronic rest energy ($\epsilon_0 = 8.4$ for $A = 48$).

The transition operator has been multiplied^{*)} by three so that Ω reduces to the well-known Gamow-Teller operator $-\vec{Y} \cdot \vec{Y}$, $\vec{Y} = \sum_i \tau^+(i) \vec{\sigma}(i)$ if the radial dependence, as well as the tensor component, are neglected.

We remind the reader that this mechanism also allows, to leading order, transitions of the type $0^+ \rightarrow J \neq 0^+$. Mechanism b) only allows $0^+ \rightarrow 0^+$ transitions.

*) A compensating factor of $1/9$ was put into the energy function $\tilde{f}_2$ [the other factor of $1/9$ comes from a recoupling of the leading (dipole) term in the expansion of the electron plane wave].

The most interesting situation arises, however, when the Majorana neutrino has non-vanishing mass. Then the relevant Feynman diagrams at the nucleon level are presented in Fig. 1. They are associated with a charged current of the form

$$\begin{pmatrix} \nu_e + \beta N_0 \\ e^- \end{pmatrix} \quad (5)$$

where ν_e and N_0 are the light and heavy components of the neutrino given by

$$\nu_e = \sum_j X_{ej}^{(1)} \nu_j \quad N_0 = \sum_j X_{ej}^{(2)} N_j \quad (6)$$

where ν_j (N_j) are the (Majorana) mass eigenstates in the light (heavy) mass sector and β is a convenient parametrization of the mixing between the two groups.

Now the rate for process (1) becomes:

$$W_{0\nu}(l \rightarrow f e e^-) = K_2 \bar{A}^{2/3} |\chi(\alpha(z+z))|^2 f_2(\epsilon_0) |\eta_0 \langle f | \Omega_0 | i \rangle + \eta_\sigma \langle f | \Omega_\sigma | i \rangle|^2 \quad (7)$$

where

$$\eta_0 = \langle m_\nu \rangle / m_e, \quad \langle m_\nu \rangle = \sum_j (X_{ej}^{(1)})^2 m_j \quad (8a)$$

(m_j are the light mass eigenvalues)

$$f_2(\epsilon_0) = \frac{2}{15} \epsilon_0 [\epsilon_0^4 + 10 \epsilon_0^3 + 40 \epsilon_0^2 + 60 \epsilon_0 + 30] \quad (8b)$$

$$\Omega_0 = \sum_{i \neq j} \tau_+(i) \tau_+(j) \left[\frac{f_v^2}{f_A^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right] \frac{R_0}{r_{ij}} \quad (8c)$$

The parameter η_σ in general, depends in a complicated fashion on the nuclear properties. However, if the mass of N_j 's is large ($\gg 1$ GeV) we get

$$\eta_\sigma = \beta^2 \frac{m_p}{m_\sigma}, \quad \frac{1}{m_\sigma} = \sum_j (X_{ej}^{(1)})^2 \frac{1}{M_j} \quad (9)$$

(M_j are the heavy mass eigenvalues).

The operator Ω_σ is also complicated. For point-like nucleons it behaves like $\vec{e}^{mr} i j$ where $m = \min(m_W, M_j)$. Thus, since m is very large, the matrix element of Ω_σ will be very much suppressed and will depend critically on aspects that are not so well understood (short-range correlations). It was for this reason that people looked for mechanisms other than the two nucleon mode. One such

possibility is to proceed via isobars in the nuclear medium¹⁵⁾. This of course does not, to leading order, contribute to $0^+ \rightarrow 0^+$ transitions. The other is the double charge exchange of pions in flight between two nucleons¹⁶⁾. This process will be examined elsewhere¹⁶⁾.

There is no reason, however, to assume that the nucleons are point-like particles. We know that they have a size which in fact is adequately described in terms of a dipole form factor

$$F(q^2) = \frac{1}{(1 - q^2/m_A^2)^2}, \quad m_A = .85 \text{ GeV}/c^2 \quad (10)$$

If we introduce such a form factor into the diagram (1b) we find that

$$\Omega_{\sigma} = \frac{m_p}{m_e} \sum_{i \neq j} \tau_+(i) \tau_+(j) \left[\frac{S_V^2}{f_A^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right] \frac{R_0}{r_{ij}} F(r_{ij}) \quad (11a)$$

where

$$F(x) = \frac{1}{48} x (x^2 + 3x + 3) e^{-x}, \quad x = \frac{m_A c^2}{\hbar c} r_{ij} \quad (11b)$$

Now the range of the transition operator is set by m_A and thus it does not suffer from the above-mentioned pathologies. Its matrix elements change only by 15% when short-range correlations are included. Furthermore, the contribution of $J = 0$ nucleon pairs now dominates and thus it may be more stable against nuclear structure variations. It depends though rather sensitively on the mass parameter m_A .

From the experimentally established limit of no neutrino double β decay^{2),17)} of the $A = 48$ system

$$(T_{1/2})_{\text{exp}} > 2 \times 10^{21} \text{ yr}$$

we obtain the constraint:

$$I(\epsilon_0) / |\eta \langle f | \Omega | i \rangle|^2 + |\eta_0 \langle f | \Omega_0 | i \rangle + \eta_c \langle f | \Omega_c | i \rangle|^2 \leq 0.4 \times 10^{-10} \quad (12)$$

where

$$I(\epsilon_0) = \tilde{f}_2(\epsilon_0) / f_2(\epsilon_0) \approx 0.05$$

In order to extract the interesting parameters η , η_0 and η_c , the nuclear matrix elements must be known. In the present work they have been evaluated within the simple $1(f_{7/2})$ shell using reasonable effective interactions. Without short-range correlations ($r_c=0$) we find^{*})

^{*}) The matrix element $\langle f | \Omega_0 | i \rangle$ is large because it has been scaled by m_p/m_e [see Eq. (11a)].

$$\langle f | \Omega | i \rangle = 4.0, \quad \langle f | \Omega_0 | i \rangle = 1.0, \quad \langle f | \Omega_\sigma | i \rangle = 72 \quad (13)$$

Using a hard core radius of $r_c = 0.4$ fm they become 3.6, 0.82 and 62 respectively. We stress that these matrix elements are much larger than the approximate matrix element

$$(ME)_4 = - \langle f | \vec{\tau} \cdot \vec{\tau} | i \rangle \quad (14)$$

used in the earlier work¹³⁾. Of course, since the matrix element $(ME)_4$ is very small, its precise value is uncertain. For purposes of comparison we will adopt the value $|ME|_4^2 = 1.7 \times 10^{-2}$ which was found previously¹³⁾. We emphasize, however, that it is not this matrix element which enters the neutrinoless double β decay.

Since η , η_0 and η_σ both contribute to reaction (1), one can set limits on them by considering the contribution of each of them separately. Thus,

i) $\eta_0 = \eta_\sigma = 0$. Then

$$|\eta| \leq 0.7 \times 10^{-5} \quad (15)$$

which is consistent with the estimate of $|\eta| \leq 10^{-7}$ given by Mohapatra and Senjanovic⁸⁾.

ii) $\eta = 0$, $\eta_\sigma = 0$. Then

$$|\eta_0| \leq 0.6 \times 10^{-5} \quad \text{or} \quad \langle m_\nu \rangle \leq 3 \text{ eV} \quad (16)$$

iii) $\eta = 0$, $\eta_0 = 0$. Then

$$\beta^2 \frac{m_e}{m_\sigma} \leq 10^7 \quad (17)$$

Equation (17) sets a limit on m_σ provided that β^2 is not terribly small. For $\beta^2 \approx 10^{-3} - 10^{-2}$, as in the Mohapatra-Senjanovic⁸⁾ model, one finds that $m_\sigma \geq 10^4 - 10^5$ GeV. Even if $\beta^2 = m_e/m_\sigma$, as is the case in some models¹⁸⁾, one finds $m_\sigma \geq 1$ GeV. If, however, $\beta \approx m_\nu/m_\sigma$ with m_ν in the eV region m_σ is essentially unrestricted, i.e., in the MeV region.

If the old value of 1.7×10^{-2} for the nuclear matrix element is used, the above limits become $\eta \leq 2 \times 10^{-4}$, $\eta_0 \leq 0.5 \times 10^{-4}$ and $m_\chi \leq 24$ eV.

An interesting possibility, which may confuse the analysis of double β decay experiments, has recently been discussed by Georgi-Glashow-Nussinov¹¹⁾, i.e., a process like

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \chi^0 \quad (18)$$

where χ^0 is a scalar massless boson. This lepton violating process, with χ^0 escaping detection, will then be confused with the lepton conserving process

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \tilde{\nu}_e + \tilde{\nu}_e \quad (19)$$

which, of course, is always present. The transition rate for process (18) is given by

$$W_{0\nu}(i \rightarrow f e^- e^- \chi^0) = \frac{K_2}{(2\pi)^2} |\chi(\alpha(Z+2))|^2 A^{2/3} f_M(\xi) \left| \frac{g}{\sqrt{2}} \langle f | \Omega_0 | i \rangle \right|^2 \quad (20)$$

where $g/\sqrt{2}$ is the $\chi^0 \tilde{\nu}_e \nu_e$ coupling¹¹⁾ and

$$f_M(\xi) = \frac{1}{630} \xi^3 (6\xi^4 + 14\xi^3 + 84\xi^2 + 210\xi + 210) \quad (21)$$

The decay rate for the two neutrino process of Ca^{48} is^{10),13)}, $(T_{1/2})_{2\nu}^{\text{exp}} = 4.0 \times 10^{20}$ yr which is an order of magnitude longer than the present limit¹⁷⁾

$$(T_{1/2}^{\text{exp}})_{2\nu} \geq 3.6 \times 10^{19} \text{ yr}$$

The parameter g can be extracted from this limit using the relation

$$\frac{1}{(T_{1/2})_{0\nu, \chi^0}} = \frac{1}{(T_{1/2}^{\text{exp}})_{2\nu}} - \frac{1}{(T_{1/2}^{\text{th}})_{2\nu}} \approx \frac{1}{(T_{1/2}^{\text{exp}})_{2\nu}} \quad (22)$$

From this we deduce $|g| \leq 3 \times 10^{-4}$. Its value will become even smaller if the last equality of the previous equation does not hold. In any case, for values of $|g|$ not much smaller than 10^{-4} , process (18) will compete with the 2ν double β decay and in the case of ${}^{48}\text{Ca} \rightarrow {}^{48}\text{Ti}$ it may even dominate.

To summarize, we find that the radial dependence of the transition operator is important in the case of ^{48}Ca . It may be even more important in other transitions and in particular those in which neutrons and protons occupy different oscillator orbits.

Double β decay sets fairly stringent limits on the lepton violating parameters. The above limits may be somewhat modified if more complicated nuclear wave functions are used. We have, however, no reason to expect that the nuclear matrix elements given by Eq. (13) will be drastically reduced.

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FIGURE CAPTION

Lepton violating diagrams arising from mass insertion interactions for the light (1a) and heavy (1b) Majorana neutrinos. β is the mixing angle between the light and heavy components.