Ref.TH.3155-CERN

ORDER α_s CORRECTIONS TO INTERMEDIATE BOSON MASSES

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A B S T R A C T

We present the order α_s corrections to the masses of the Z and W bosons in the standard model. For the low mass quarks u, d, c and s the corrections are small and positive. Due to the Coulomb singularity which appears in the vector boson self-energy graph with one gluon exchange, we find a negative contribution from the t, b doublets.

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Recently, several groups have calculated one-loop corrections to the masses of the Z and W bosons¹⁾⁻⁵⁾. The result of these calculations was an increase of the zeroth order mass values of these bosons. The zeroth order masses are given by :

$$M_W^{(0)} = \frac{M^{(0)}}{\sin \theta_W} ; \quad M_Z^{(0)} = \frac{M_W^{(0)}}{\cos \theta_W} ; \quad M^{(0)} = \left(\frac{\pi \alpha}{G_F \sqrt{2}} \right)^{1/2} . \quad (1)$$

The calculated mass shift originates from boson loops (Higgs bosons and intermediate bosons) and from fermion loops (quarks and leptons). It turns out that the dominant correction comes from fermion loops.

Since quarks do not only participate in weak but also in strong interactions, it is clear that higher order QCD contributions must further change the masses of the W and Z bosons. There are several reasons why one wants to study these shifts :

- i) it is an effect where QCD changes the parameters of the electroweak theory ;
- ii) since the QCD coupling constant is not very small, one expects the contributions to be noticeable.;
- iii) since the mass of the top quarks is not yet known, there may be considerable threshold effects, if $m_t + m_b \approx M_W$ or if $2m_t \approx M_Z$.

These threshold effects may be enhanced by QCD contributions. In particular, in order α_s , there is a Coulomb singularity at the threshold.

In this paper we give the results of the $O(\alpha_s)$ correction to the one-quark loop graph of Fig. 1a, which consists of all possible one-gluon exchanges between the quarks (Fig. 1b). We start with a few general comments on the quark contribution to the mass shift.

From the one-loop calculation, it appears that the mass shift is mainly due to low mass (u,d) quarks¹⁾⁻⁵⁾. This can be understood if one looks at the origin of the radiative corrections. The corrections consist of two parts. On the tree level the masses of the W and Z are expressed in terms of three (phenomenological) parameters α , G_F and $\sin \theta_W$ as in Eq. (1). The first part of the mass correction can be traced back to the renormalization of these parameters. The second part is the self-mass of the vector bosons (Fig. 1).

The renormalization of α , G_F and $\sin\theta_W$ due to fermions only gets contributions from self-energy graphs of γ , Z and W taken at the point $q^2 = 0$, where q is the external momentum of graph 1. It is known that this renormalization procedure leads to renormalized parameters containing mass singularities ^{6),7)}. These singularities reveal themselves if the fermion masses go to zero. For this reason the most important contribution comes from the u and d quarks. For the one-loop correction it appears that the heavy quark (t) contribution is small since the self-energy at threshold is not significant.

For the $O(\alpha_s)$ correction the mass-shift also gets important contributions from the light quarks like in the lowest order case. Contrary to the one-loop situation, here the heavy quark (t) also gives a noticeable contribution to the mass-shift. Its origin can be attributed to the logarithmic enhancement at the threshold due to the one-gluon exchange graph (Fig. 1b). Therefore, if the top-quark mass approaches $M_W - m_b$ or $M_Z/2$ we can expect important corrections to M_W or M_Z respectively. The origin of this Coulomb singularity is explained in Ref. 6).

For the calculation of the $O(\alpha_s)$ correction we will use the method of dispersion relations as outlined in Ref. 8). Sometimes this is a more convenient method of computing two-point functions than the usual Feynman integral technique.

Before presenting our results we express the formulas for the mass-shifts due to fermions ¹⁾ into dispersion integrals ⁹⁾. The vacuum polarization tensor which stands for the self-energy (Fig. 1) is denoted by

$$\Pi_{ij}^{\mu\nu} = g^{\mu\nu} \Pi_{1,ij} + g^\mu g^\nu \Pi_{2,ij} \quad (2)$$

where $i, j = \gamma, Z, W$. The couplings of Z , W and γ to the quarks are given by the general vertex

$$\Gamma^\mu = \lambda_i \gamma^\mu (a_i - b_i \gamma^5) \quad (3)$$

with

$$\lambda_\gamma = g \sin\theta_W ; \lambda_W = \frac{g}{2\sqrt{2}} ; \lambda_Z = \frac{g}{4 \cos\theta_W} \quad (4a)$$

$$a_\gamma(e_q) = e_q ; b_\gamma = 0 \quad (4b)$$

$$a_W(e_q) = 1 ; b_W(e_q) = 1 \quad (4c)$$

$$a_z(e_q = +\frac{2}{3}) = 1 - \frac{8}{3} \sin^2 \theta_W ; \quad b_z(e_q = +\frac{2}{3}) = 1 \quad (4d)$$

$$a_z(e_q = -\frac{1}{3}) = -1 + \frac{4}{3} \sin^2 \theta_W ; \quad b_z(e_q = -\frac{1}{3}) = -1 \quad (4e)$$

where e_q is the charge of the quark. Interference terms between vector and axial vector parts of the vertices disappear in Eq. (2), and we can write :

$$\Pi_{m,ij} = a_i a_j \Pi_m^V + b_i b_j \Pi_m^A, \quad m=1,2 \quad (5)$$

Since the polarization tensor is contracted with the average sum of the vector boson polarizations, we can drop the last term in Eq. (2). The mass shifts for M_W and M_Z in lowest order in g are given by ¹⁾ :

$$\frac{\delta M_W^2}{M_W^2} = \left[\frac{\Pi_{WW}(q^2)}{M_W^2} - \frac{\cos \theta_W}{\sin \theta_W} \frac{\Pi_{\gamma Z}(q^2)}{q^2} - \frac{\Pi_{ZZ}(q^2)}{q^2} \right]_{q^2=0} - \text{Re} \frac{\Pi_{WW}(M_W^2)}{M_W^2} \quad (6)$$

$$\frac{\delta M_Z^2}{M_Z^2} = \left[\frac{\Pi_{WW}(q^2)}{M_Z^2} - \frac{1-2\cos^2 \theta_W}{\cos \theta_W \sin \theta_W} \frac{\Pi_{\gamma Z}(q^2)}{q^2} - \frac{\Pi_{ZZ}(q^2)}{q^2} \right]_{q^2=0} - \text{Re} \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} \quad (7)$$

In expressing the right-hand side of (6) and (7) in dispersion integrals one is not allowed to subtract them prematurely at $q^2 = 0$. In doing so, some properties of $\Pi^{V,A}$ are lost, especially the characteristic that $\Pi^V - \Pi^A \sim (1/n-4) \cdot (m^2/q^2)$ in the n dimensional regularization scheme.

In order to conform with the results for the one-loop corrections obtained in Ref. 1), one has to proceed in the following way. Introduce a cut-off Λ^2 and write unsubtracted dispersion relations for $\Pi^{V,A}$:

$$\begin{aligned} \pi \text{Re} \overline{\Pi}_1^{V,A}(q^2, m_1, m_2, \Lambda^2) &= \int_{(m_1+m_2)^2}^{\Lambda^2} ds \frac{\text{Im} \overline{\Pi}_1^{V,A}(s, m_1, m_2)}{s - q^2} = \int_{(m_1+m_2)^2}^{\Lambda^2} \frac{ds}{s} \text{Im} \overline{\Pi}_1^{V,A}(s, m_1, m_2) + \\ &+ q^2 \int_{(m_1+m_2)^2}^{\Lambda^2} \frac{ds}{s^2} \text{Im} \overline{\Pi}_1^{V,A}(s, m_1, m_2) + q^4 \int_{(m_1+m_2)^2}^{\Lambda^2} \frac{ds}{s^3} \frac{\text{Im} \overline{\Pi}_1^{V,A}(s, m_1, m_2)}{s - q^2} \end{aligned} \quad (8)$$

where (8) has to satisfy two conditions :

$$\text{Re } \Pi_1^V(q^2, m, m) = -q^2 \text{Re } \Pi_2^V(q^2, m, m) \quad (9a)$$

(gauge invariance)

and

$$\text{Re } \Pi_1^V(q^2, m_1, m_2) = \text{Re } \Pi_1^A(q^2, m_1, -m_2) \quad (9b)$$

(γ^5 reflexion)

We therefore modify Eq. (8) in the following way :

$$\begin{aligned} \pi \text{Re } \Pi_1^{V,A}(q^2, m_1, m_2, \Lambda^2) &= \int_{(m_1+m_2)^2}^{\Lambda^2} \frac{ds}{s} \Im_m \Pi_1^{V,A}(s, m_1, m_2) \\ &- \frac{1}{2} \int_{4m_1^2}^{\Lambda^2} \frac{ds}{s} \Im_m \Pi_1^V(s, m_1, m_1) - \frac{1}{2} \int_{4m_2^2}^{\Lambda^2} \frac{ds}{s} \Im_m \Pi_1^V(s, m_2, m_2) \\ &+ q^2 \int_{(m_1+m_2)^2}^{\Lambda^2} \frac{ds}{s^2} \Im_m \Pi_1^{V,A}(s, m_1, m_2) + q^4 \int_{(m_1+m_2)^2}^{\Lambda^2} \frac{ds}{s^2} \frac{\Im_m \Pi_1^{V,A}(s, m_1, m_2)}{s - q^2} \end{aligned} \quad (10)$$

Here (10) satisfies the conditions (9a) and (9b). It can be shown that (10) produces the same result for the two-point Green functions as obtained by n dimensional regularization up to a finite renormalization. This finite renormalization disappears if (10) is substituted into (6) and (7), and we obtain :

$$\begin{aligned} \frac{\delta M_w^2}{M_w^2} &= \frac{3g^2}{8\pi} \left[-M_w^2 \int_{(m_1+m_2)^2}^{\infty} \frac{ds}{s^2} \frac{\Im_m \Pi_1^{V+A}(s, m_1, m_2)}{s - M_w^2} \right. \\ &+ \int_{4m_1^2}^{\infty} \frac{ds}{s^2} \left\{ \frac{4}{3} \Im_m \Pi_1^V(s, m_1) + \frac{2}{3} \Im_m \Pi_1^V(s, m_2) - \Im_m \Pi_1^{V+A}(s, m_1, m_2) \right\} \\ &\left. + \frac{2}{3} \int_{4m_2^2}^{4m_1^2} \frac{ds}{s^2} \Im_m \Pi_1^V(s, m_2) - \int_{(m_1+m_2)^2}^{4m_1^2} \frac{ds}{s^2} \Im_m \Pi_1^{V+A}(s, m_1, m_2) \right] \quad (11) \end{aligned}$$

$$\begin{aligned}
\frac{\delta M_2^2}{M_2^2} = & \frac{3g^2}{16\pi \cos^2 \theta_W} \left[-M_2^2 \int_{4m_1^2}^{\infty} \frac{ds}{s^2} \frac{1}{s-M_2^2} \left\{ a_2^2\left(\frac{2}{3}\right) \Im \Pi_1^V(s, m_1) + b_2^2\left(\frac{2}{3}\right) \Im \Pi_1^A(s, m_1) \right\} \right. \\
& - M_2^2 \int_{4m_2^2}^{\infty} \frac{ds}{s^2} \frac{1}{s-M_2^2} \left\{ a_2^2\left(-\frac{1}{3}\right) \Im \Pi_1^V(s, m_2) + b_2^2\left(-\frac{1}{3}\right) \Im \Pi_1^A(s, m_2) \right\} \\
& + \frac{1}{M_2^2} \int_{4m_1^2}^{\infty} \frac{ds}{s} \left\{ 2 \Im \Pi_1^{V+A}(s, m_1, m_2) - \Im \Pi_1^{V+A}(s, m_1) - \Im \Pi_1^{V+A}(s, m_2) \right\} \\
& - \frac{1}{M_2^2} \int_{4m_2^2}^{\infty} \frac{ds}{s} \Im \Pi_1^{V+A}(s, m_2) + \frac{2}{M_2^2} \int_{(m_1+m_2)^2}^{4m_1^2} \frac{ds}{s^2} \Im \Pi_1^{V+A}(s, m_1, m_2) \\
& + \int_{4m_2^2}^{\infty} \frac{ds}{s^2} \left\{ \frac{5}{3} \Im \Pi_1^V(s, m_1) + \frac{1}{3} \Im \Pi_1^V(s, m_2) - \Im \Pi_1^A(s, m_1) - \Im \Pi_1^A(s, m_2) \right\} \\
& \left. + \int_{4m_2^2}^{4m_1^2} \frac{ds}{s^2} \left\{ \frac{1}{3} \Im \Pi_1^V(s, m_2) - \Im \Pi_1^A(s, m_2) \right\} \right]
\end{aligned}
\tag{12}$$

Here m_1^* is the mass of the charge $+2/3$ quark and m_2 is the mass of the charge $-1/3$ quark. We have used the notation $\Pi^{V+A} = \Pi^V + \Pi^A$, the couplings a and b have been defined in (4) and we also have factored the coupling constant g . Expressions (11) and (12) are manifestly finite. If we insert the absorptive part of Fig. 1a into (11) and (12) we reproduce the results given in Eqs. (5) and (6) of Ref. 1). For the order α_s contribution we have evaluated the absorptive parts of diagrams 1b. All of these diagrams contain infra-red singularities which are cancelled in the sum according to the K.L.N. theorem ^{6),7)}. The result of this calculation is rather long and we will present it elsewhere. Our results agree with those of Refs. 10), 11). In principle we could evaluate the dispersion integrals analytically. In the unequal mass case the expressions become very lengthy. They contain many di- and tri-logarithms which have to be evaluated on a computer anyway. We have therefore chosen to perform the dispersion integrals numerically.

*) Equations (11) and (12) are only valid if $m_1 \geq m_2$.

In the small mass approximation, the formulas for the mass shift simplify enormously. For the u, d quark combination e.g., including $O(\alpha_s)$ they are given by :

$$\frac{\delta M_W^2}{M_W^2} = \frac{g^2}{48\pi^2} \left[\left(1 + \frac{\alpha_s}{\pi}\right) \left(2 \ln \frac{M_W^2}{m_u^2} + \ln \frac{M_W^2}{m_d^2}\right) - 5 + \frac{\alpha_s}{\pi} \left(12 \left\{ \frac{3}{2} \right\} - \frac{5}{2} \right) \right] \quad (13)$$

$$\begin{aligned} \frac{\delta M_Z^2}{M_Z^2} = & \frac{g^2}{64\pi^2 \cos^2 \theta_W} \left[\left(1 + \frac{\alpha_s}{\pi}\right) \left\{ \left(\frac{8}{3} - \frac{16}{3} \sin^2 \theta_W + \frac{64}{9} \sin^4 \theta_W \right) \ln \frac{M_Z^2}{m_u^2} \right. \right. \\ & + \left. \left(\frac{4}{3} - \frac{8}{3} \sin^2 \theta_W + \frac{16}{9} \sin^4 \theta_W \right) \ln \frac{M_Z^2}{m_d^2} \right\} + \left(-\frac{20}{3} + \frac{40}{3} \sin^2 \theta_W - \frac{400}{27} \sin^4 \theta_W \right) \\ & + \left. \frac{\alpha_s}{\pi} \left\{ -\frac{10}{3} + 16 \left\{ \frac{3}{2} \right\} + \left(\frac{20}{3} - 32 \left\{ \frac{3}{2} \right\} \right) \sin^2 \theta_W + \left(\frac{320}{9} \left\{ \frac{3}{2} \right\} - \frac{200}{27} \right) \sin^4 \theta_W \right\} \right] \end{aligned} \quad (14)$$

We have used these approximations to check the stability of the numerical programme that performed the dispersion integrals.

If the mass of the top quark is in the neighbourhood of $M_W - m_b$ or $M_Z/2$, respectively, we find terms like :

$$\begin{aligned} \frac{\delta M_W^2}{M_W^2} & \sim \pi^2 \ln |M_W^2 - (m_t + m_b)^2| \\ \frac{\delta M_Z^2}{M_Z^2} & \sim \pi^2 \ln |M_Z^2 - 4m_t^2| \end{aligned} \quad (15)$$

This enhancement near the threshold finds its origin in the Coulomb singularity. It can be traced back to the vertex insertion in Graph 1b. The real part of the electromagnetic form factor in the neighbourhood of the threshold behaves like ¹²⁾

$$\text{Re } F_1(s) \sim \frac{\pi}{\sqrt{|s - (m_1 + m_2)^2|}} \quad (16)$$

After insertion in the one-loop graphs (Fig. 1a), $\text{Im}\Pi_1$ acquires an extra factor $(s-(m_1+m_2)^2)^{\frac{1}{2}}$ (s wave) from the vector-vector and a factor $(s-(m_1+m_2)^2)^{\frac{3}{2}}$ (p wave) from the axial-axial combination, so that

$$\begin{aligned} \text{Im}\Pi_1^V &\sim \pi^2 \\ \text{Im}\Pi_1^A &\sim \pi^2 (s - (m_1 + m_2)^2) \end{aligned} \quad (17)$$

After insertion in the dispersion integral, we see that only the V-V combination Π_1^V obtains a logarithmic enhancement (singular) near the threshold in (17). Notice that the top quark contribution (in the neighbourhood of the threshold) leads to a negative mass-shift of the vector bosons.

We will now present numerical results for the $O(\alpha_s)$ mass corrections. For the quark masses we take 0.25 GeV and 0.25 GeV for the up and down, 0.3 GeV and 1.5 GeV for the strange and charm and 5.0 GeV and 20.0 GeV for bottom and top. For $\sin^2\theta_W$ we took 0.238¹⁾. The mass-shifts corresponding to the three families are given in the Table, both for the lowest order and for the $O(\alpha_s)$ result. As can be seen, the $O(\alpha_s)$ are generally small. For α_s we took the running coupling constant at M_W resp. M_Z for six flavours, $\Lambda_{\overline{\text{MS}}} = 200$ MeV. We also found that the results of the exact calculation are well-approximated by formulas (15) and (16) up to top quark masses of 20 GeV (the difference between exact and approximate results is here 10%).

In Fig. 2a we give the mass shift to the W boson in lowest order as a function of the mass of the top quark (only the contribution from the b, t doublet). In Fig. 2b, we plot the analogous α_s correction. In the same way, in Figs. 3a and 3b, we see the lowest order and order α_s corrections to the Z mass due to the b, t doublet. Notice, in Figs. 2b and 3b, the Coulomb peaks which we discussed above.

Summarizing, we see that the order α_s corrections to the Z and W boson masses are of the order of 0.15 GeV (all doublets). This is of the same order of magnitude as the lowest order contribution of b, t alone. If in the future the masses of Z and W can be measured with an accuracy of 10 MeV, say, the corrections outlined in this letter should be observable.

	W		Z	
	One loop	$O(\alpha_s)$	One loop	$O(\alpha_s)$
e, ν_e	0.6890		0.7872	
μ , ν_μ	0.3576		0.4130	
τ , ν_τ	0.2128		0.2420	
u, d	0.9120	0.0691	0.9158	0.0672
c, s	0.6774	0.0604	0.6790	0.0562
t, b	0.1493	0.0363	0.1670	0.0293
ΔM in GeV ; $M_W^{(0)} = 76,429$ GeV ; $M_Z^{(0)} = 87.555$ GeV				

- Table -

Mass shifts in GeV of W, Z in lowest-order and in order α_s . Also included are the zeroth order mass values for W and Z.

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FIGURE CAPTIONS

- Figure 1 a) One-fermion loop graph contributing to the mass shift.
 b) Graphs contributing to α_s .
- Figure 2 a) Mass shift of W boson due to t, b in lowest order.
 b) Mass shift of W boson due to t, b in order α_s . The value
 of α_s is the running coupling constant taken at $\Lambda = 0.1$;
 0.2 GeV.
- Figure 3 a) Mass shift of Z boson due to t, b in lowest order.
 b) Mass shift of Z boson due to t, b in order α_s . The value
 of α_s is the running coupling constant taken at $\Lambda = 0.1$;
 0.2 GeV.

Fig. 1a

Fig. 1b

Fig. 2a

Fig. 2b

Fig. 3a

Fig. 3b