VACUUM ALIGNMENT BY BROKEN GAUGE INTERACTIONS

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ABSTRACT

We study the problem of vacuum alignment by the extended technicolour (ETC) forces which have been invoked to give mass to ordinary fermions (quarks and leptons) in technicolour theories. We find that the ETC interactions prefer to align the vacuum in such a way as to give the maximum possible mass to the ordinary fermions. We show that the P^0 , P^3 , $P^\pm$, P_θ^0 , P_θ^3 , and $P_\theta^\pm$ pseudo-Goldstone bosons in one technifamily ETC models cannot acquire any mass from ETC interactions alone. Finally, we point out that the Pati-Salam unification which has been invoked to give mass to the P^0 and P^3 implies a remaining discrete degeneracy of the vacuum.

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1. INTRODUCTION

In principle, scalarless gauge theories have great predictive power. Indeed a theory built out of gauge fields and spin $\frac{1}{2}$ fermions has only gauge coupling constants as arbitrary parameters. Moreover, by unifying gauge groups the gauge coupling constants can be related to one another¹⁾. Finally, it is well known that scalarless gauge theories can predict masses by dimensional transmutation. One can thus hope for the existence of a scalarless gauge theory that would reproduce the successful phenomenology of the standard model but in addition predict the masses of the $W^\pm$, Z^0 , the quarks and leptons. No successful model of this kind has been constructed yet, but it certainly is an aim worth striving for.

This paper is devoted to the issue of vacuum alignment²⁾ in scalarless gauge theories³⁾. Consider an unbroken asymptotically free gauge group G^{TC} (TC stands for "technicolour", although the reader need not attach any particular meaning to the name at this point) and the technifermion representation content:

$$F_{\alpha i} \sim N(n)_L + N(n)_R \text{ of } G^{TC}, \quad (1.1)$$

where $i = 1, \dots, n$ is the technicolour index on which G^{TC} acts, and $\alpha = 1 \dots N$ is the techniflavour index. The global symmetry of the technicolour gauge interactions G^{TF} is $SU_L(N) \times SU_R(N) \times U_V(1)$ or $SU(2N)$ depending on whether the technicolour representation (n) is complex or real. Now, we assume that at the scale Λ^{TC} , where the technicolour gauge coupling constant becomes strong, the following technifermion condensates form:

$$\langle \bar{F}_{\alpha i} F_{\beta j} \rangle_0 \simeq \delta_{\alpha\beta} (\mu_{TC})^3, \quad (1.2)$$

where μ_{TC} is of order Λ^{TC} . The technifermion condensates (1.2) spontaneously break $G^{TF} \rightarrow H^{TF}$:

- i) $SU_L(N) \times SU_R(N) \times U_V(1) \rightarrow SU_V(N) \times U_V(1)$ if technicolour is complex;
- ii) $SU(2N) \rightarrow O(2N)$ if technicolour is real with a symmetric metric (hereafter called real);
- iii) $SU(2N) \rightarrow Sp(2N)$ if technicolour is real with an antisymmetric metric (hereafter called pseudoreal).

We have good reason to believe that the condensates (1.2) do occur in the theories specified by (1.1). We know it is true in the case of QCD. Moreover, Coleman and Witten⁴⁾ have shown it to be true in lowest order of the $1/n$ expansion. This means that the condensates have *exactly* the form (1.2) for finite n provided n is large enough⁵⁾.

Now let us suppose that in addition to the technicolour interactions a weak perturbation is present that explicitly breaks the global symmetry group G^{TF} of the technicolour interactions. This situation raises the issue of "vacuum alignment": how does H^{TF} orient itself with respect to the privileged directions the weak perturbation establishes in G^{TF} . Of course, the problem of vacuum alignment is very much related to the question of which Goldstone bosons accompanying the $G^{TF} \rightarrow H^{TF}$ spontaneous symmetry breakdown acquire a mass from the weak perturbation, thus becoming pseudo-Goldstone bosons.

Since we restrict ourselves to scalarless gauge theories, the weak perturbation can only be a gauge interaction. There are two possibilities: either this gauge interaction is based on a gauge group that is already broken when the technifermion condensates (1.2) form, or it is based on a gauge group that is still unbroken just before the technifermion condensates (1.2) form. The second case (vacuum alignment by unbroken gauge interactions) has already been given a quite general treatment by Weinberg³⁾, Peskin⁶⁾ and Preskill⁷⁾. It is our purpose in this paper to give the most general treatment we can of vacuum alignment by broken gauge interactions. Of course, vacuum alignment is an important issue in technicolour versions^{8,9)} of the standard model, in which $\Lambda^{TC} \sim 1$ TeV and technifermion condensates of the type (1.2) spontaneously break $SU_L(2) \times U_Y(1) \times SU^C(3) \rightarrow U^{em}(1) \times SU^C(3)$. In such theories¹⁰⁾ the vacuum (or equivalently the technifermion condensates) is aligned not only by the unbroken (at scales above Λ^{TC}) $SU_L(2) \times U_Y(1) \times SU^C(3)$ gauge interactions but also by the broken extended technicolour (ETC) interactions⁹⁾ which are introduced to give mass to the ordinary quarks and leptons.

A very simple criterion has been established to decide how the vacuum aligns under the effect of an unbroken gauge interaction. The situation is represented in Fig. 1. Suppose one has gauged a G^W subgroup of G^{TF} which spontaneously breaks down to H^{TF} because of (1.2). We must also suppose that the gauge coupling constant of G^W is small at the scale Λ^{TC} . Otherwise the roles of G^W and G^{TC} must obviously be reversed. The criterion^{3,6,7)} says that the vacuum aligns in such a way that G^W is as much as possible a subgroup of H^{TF} . More precisely, the alignment is such that the trace of the $(\text{mass})^2$ matrix of the G^W gauge fields is minimized. This criterion makes a lot of intuitive sense: the true vacuum is the one which is most attractive from the point of view of the G^W interactions; that is to say that the technifermion condensates form the lowest possible G^W representation content, i.e. they break G^W as little as possible. Peskin⁶⁾ and Preskill⁷⁾ studied the vacuum alignment and the pseudo-Goldstone boson (PGB) mass generation by the $SU_L(2) \times U_Y(1) \times SU^C(3)$ gauge interactions in the one technifamily technicolour model¹¹⁾. They found that the vacuum aligns properly only if technicolour is complex or real (the vacuum breaks $U^{em}(1)$ if technicolour is pseudoreal). Their results -- including subsequent corrections calculated by Chadha and Peskin¹²⁾ -- are shown in Table 1. Note that the PGBs P^0 and P^3 do not acquire any mass from these colour and electroweak perturbative effects, whereas the mass acquired by $P^\pm$ is between 5 and 8 GeV or 8 and 14 GeV, depending on whether technicolour is complex or real. Being the lightest PGBs, P^0 , P^3 and $P^\pm$ are of great phenomenological interest^{13,14)}. However, the study of their properties has been obscured so far by the fact that the contributions from ETC interactions to their masses have never been properly analysed. It was said that the ETC contributions to the $(\text{mass})^2$ of PGBs was typically of order $(30 \text{ GeV})^2$, but otherwise very model-dependent.

As was already announced above, it is precisely our purpose to study ETC contributions to PGB masses, ETC being a particular case of a gauge interaction broken at energy scales above Λ^{TC} . In Section 2, we study a simple ETC toy model. Our purpose there is to visualize clearly the issues involved in ETC vacuum

alignment and to develop a procedure for solving them. The ETC mechanism consists in placing "ordinary" -- i.e. technicolour singlet -- fermions and technifermions into common multiplets of a gauge group G^{ETC} which spontaneously breaks down to G^{TC} at energies $\mu_{ETC} \gg \Lambda^{TC}$. We will show that in the toy model, the technifermion condensates align in such a way that the ordinary fermions acquire the maximum possible mass from the technifermion condensates. We believe that this is a general property of vacuum alignment by ETC interactions. As a result of heavy ETC vector boson exchange the technifermion condensates align along the maximally attractive direction of ETC. This direction of condensate orientation is, in all the cases examined by us, the one in which the ordinary technisinglet fermions acquire the maximum possible mass. An appendix is devoted to an example which clarifies the way this occurs.

In Section 3 we consider those contributions to the masses of PGBs in the one technifamily models which may arise from the ETC interactions. We show that the 36 PGBs P^0 , P^3 , $P^\pm$, P_8^0 , P_8^3 and $P_8^\pm$ cannot acquire any mass from the ETC interactions alone. This result is ETC model independent. There remains, however, the possibility of higher order mass contributions to these PGBs from a combination of ETC and electroweak or strong interactions. This is perhaps uninteresting for the octet PGBs since they have already acquired leading order masses from coloured gluon exchange^{6,7,11}). But we find that it is possible for the $P^\pm$ to acquire $O(\alpha_w \alpha_{ETC})$, but not $O(\alpha_w' \alpha_{ETC})$ contributions to their squared masses*. On the other hand, P^0 and P^3 remain massless to all orders. One can give¹⁵) them a small mass ($\lesssim 2$ GeV) by unifying (techni-)quarks and (techni-)leptons into multiplets of the Pati-Salam $SU^{PS}(4)$ ¹⁶). We point out however that after Pati-Salam unification the vacuum is still discretely degenerate. At least some of the discrete vacua imply spontaneous CP violation. However, $SU^C(3)$ colour instantons may slightly lower the energy of the CP conserving vacuum. We briefly discuss the cosmological implications of this picture.

* α_w , α_w' and α_{ETC} are the $SU(2)_L$, hypercharge and ETC coupling constants respectively.

2. THE TOY MODEL

In this section we consider in detail an unrealistic toy model of the ETC interactions. Let the ETC gauge group be $SU^{ETC}(3)$ whose generators are denoted by $\frac{1}{2} \lambda_a$ ($a = 1, \dots, 8$), where λ_a are the usual Gell-Mann matrices. The corresponding gauge bosons are called E_μ^a . It is convenient to consider all fermions to be two component left-handed spinors. We assume that the fermion content of the model consists of two triplets and two antitriplets. These fermions are denoted by ψ_ℓ^r and $\psi_{\ell+2,r}$, respectively, where $\ell = 1, 2$ is a "flavour" index and r is an (anti-)triplet index of $SU^{ETC}(3)$. Occasionally the flavour indices will also be collectively denoted by $\alpha = 1, \dots, 4$.

We now assume that at some scale μ_{ETC} the ETC gauge group breaks down to the TC gauge group taken to be $SU^{TC}(2)$. For definiteness we may consider this latter group to be generated by $(\lambda_1, \lambda_2, \lambda_3)$. Under the $SU^{TC}(2)$ gauge group, each triplet and antitriplet of fermions decomposes into a doublet and a singlet. We thus have altogether four doublets of technifermions $F_\alpha^i = (\psi_\ell^i, \tau_2^{ij} \psi_{\ell+2,j})$, $i = 1, 2$, and four singlets of usual fermions $f_\alpha = (\psi_\ell^3, \psi_{\ell+2,3})$. Moreover, as a result of the gauge symmetry breakdown $SU^{ETC}(3) \rightarrow SU^{TC}(2)$, the gauge bosons $E_\mu^4, \dots, E_\mu^8$ corresponding to the broken generators become massive.

Let us review the various global flavour chiral symmetries of the model. The flavour symmetry of the technifermions is clearly $G^{TF} = SU(4)$, which are the special unitary transformations that can be performed on the four flavours of technidoublets F_α^i . At a scale $\Lambda^{TC} < \mu_{ETC}$, where the gauge coupling constant associated with the $SU^{TC}(2)$ becomes strong, the G^{TF} symmetry is spontaneously broken dynamically by the effects of the technicolour forces themselves. We assume that the fermion condensate responsible for this breaking is

$$\sum_{\alpha\beta} = \langle F_\alpha^i \sigma_2 F_\beta^j \epsilon_{ij} \rangle = - \sum_{\beta\alpha} \quad , \quad \alpha, \beta = 1, \dots, 4 \quad (2.1)$$

$$\simeq (\Lambda^{TC})^3$$

which is a technicolour gauge singlet and a 4×4 antisymmetric matrix in flavour space. The surviving unbroken flavour symmetry of the technifermions is thus $H^{TF} = Sp(4)$.

The condensate, Eq. (2.1), also breaks the flavour symmetry of the ETC sector. Since the model contains two triplets and two antitriplets of fermions under $SU^{ETC}(3)$ this flavour symmetry is $G^{ETF} = SU(2) \times SU(2) \times U(1)$. The first $SU(2)$ group acts only on the triplet fermions and the second $SU(2)$ only on the antitriplet fermions. The $U(1)$ is a phase rotation that acts oppositely on the triplet and the antitriplet. Another possible $U(1)$ that acts similarly on the triplet and the antitriplet is anomalous. To examine the effect of the condensate Σ on the G^{ETF} symmetry it is convenient to write it in the form

$$\Sigma = \left(\begin{array}{c|c} M_1 & M_3 \\ \hline -M_3^T & M_2 \end{array} \right), \quad M_1 = -M_1^T, \quad M_2 = -M_2^T, \quad (2.2)$$

where M_1 and M_2 are antisymmetric 2×2 matrices, and M_3 is assumed to be proportional to the 2×2 unit matrix 1_2

$$M_3 = \kappa 1_2 \quad (2.3)$$

Now, under the action of a G^{ETF} transformation, the condensate transforms to

$$\Sigma \rightarrow \left(\begin{array}{c|c} e^{i\phi} U M_1 U^T & U M_3 V^T \\ \hline -V M_3^T U^T & V M_2 V^T e^{-2i\phi} \end{array} \right). \quad (2.4)$$

Here U and V are special unitary matrices belonging to the two $SU(2)$ subgroups of G^{ETF} , respectively, and $e^{i\phi} \in U(1)$. We see therefore that if the condensate is such that $M_1 = 0$, $M_2 = 0$, $M_3 = \kappa 1_2$, then the $U(1)$ remains unbroken and M_3 remains invariant under the diagonal $SU(2)$ subgroup of $SU(2) \times SU(2)$ defined by the condition $V = U^*$. Thus, in this case, the unbroken ETC flavour symmetry is $H^{ETF} = SU_V(2) \times U(1)$. In the opposite case, however, when $M_1 \neq 0$, $M_2 \neq 0$, $M_3 = 0$, the $U(1)$ group is clearly broken, but M_1 and M_2 remain invariant under all the other transformations of G^{ETF} . In this case, then, $H^{ETF} = SU(2) \times SU(2)$.

The preceding discussion brings to the fore the crucial question of the "alignment of the vacuum" in this model. The point is simply this: the various

orientations of the condensate are equivalent from the point of view of the technicolour interactions, but not when the ETC interactions are taken into account. In fact, the effective Hamiltonian density $\delta\mathcal{H}_{\text{ETC}}(0)$, which describes the ETC interactions of the technifermions at low energy, breaks this degeneracy and orients the condensate Σ in a specific direction, thus defining the true vacuum. We wish to find this preferred orientation. $\delta\mathcal{H}_{\text{ETC}}(0)$ may be reliably calculated perturbatively at a suitably high energy where the ETC gauge coupling constant g is still weak. Then this $\delta\mathcal{H}_{\text{ETC}}(0)$ may be renormalized down to low energies. For the moment we neglect these renormalization corrections and consider them in detail later. At low energies, $\Lambda \lesssim \Lambda^{\text{TC}}$, the momentum dependence in the propagators of the heavy ETC gauge bosons ($E_\mu^4, \dots, E_\mu^8$) may be neglected, and their contribution to the perturbation potential represented in the form of effective four fermion interactions:

$$\delta\mathcal{H}_{\text{ETC}}(0) = \sum_{a=4}^8 \frac{1}{2m_a^2} j_\mu^a j^{\mu a}(0), \quad (2.5)$$

where m_a is the mass of the gauge boson E_μ^a , and the current j_μ^a is given by

$$j_\mu^a = g \left(\psi_\ell^* \sigma_\mu \frac{1}{2} \lambda_a \psi_\ell - \psi_{\ell+2}^* \sigma_\mu \frac{1}{2} \lambda_a^T \psi_{\ell+2} \right) \quad (2.6)$$

$$\sigma_\mu = (1, \vec{\sigma}) , \quad (2.7)$$

$\vec{\sigma}$ being the usual Pauli matrices. For the purposes of vacuum alignment, we are interested in the contribution of only those processes to the potential which involve the technifermions as external particles. In the lowest order in g such processes arise from the diagrams depicted in Fig. 2. These diagrams, involving only the gauge boson E_μ^8 , correspond to the term

$$\begin{aligned} \delta\mathcal{H}_{\text{ETC}}(0) = & \frac{g^2}{24m_8^2} \left(F_\ell^{i*} \sigma_\mu F_\ell^i - F_{\ell+2}^{i*} \sigma_\mu F_{\ell+2}^i \right) \\ & \times \left(F_m^{j*} \sigma^\mu F_m^j - F_{m+2}^{j*} \sigma^\mu F_{m+2}^j \right) + \dots \end{aligned} \quad (2.8)$$

The preferred orientation of the condensate Σ is that which minimizes $\delta\mathcal{H}_{\text{ETC}}(0)$.

It is worth emphasizing at the outset that, quite generally in any model of the sort we are considering here, the physical implications are critically dependent on the preferred orientation of the technicolour condensate. For instance, in this specific model, if the vacuum chosen is that corresponding to $M_1 = 0$, $M_2 = 0$, $M_3 \neq 0$, then the technisinglets f_α acquire masses through the ETC mechanism⁹). A diagram illustrating this in lowest order is shown in Fig. 3. The order of magnitude of the masses acquired by the fermions is given by

$$m_f \approx g^2 \frac{(\Lambda^\pi)^3}{m^2}, \quad (2.9)$$

where m generically denotes the mass of the exchanged gauge boson. By contrast, we note that if $M_1 \neq 0$, $M_2 \neq 0$, $M_3 = 0$, then a diagram like that of Fig. 3 is not possible, since the same gauge boson cannot couple at both vertices of the diagram. Thus, with this orientation of the condensate, the technisinglets f_α cannot acquire any masses in lowest order. Besides the mass matrix of the fermions, the mass matrix of the Goldstone bosons which arise as a result of the spontaneous breakdown $G^{TF} \rightarrow H^{TF}$ also depends crucially on the condensate orientation. This point will be discussed more fully later when the Goldstone boson mass matrix in this model is explicitly calculated.

Let us now return to Eq. (2.8) to determine how the condensate Σ orients itself in this illustrative example. For this purpose we transform $\delta\mathcal{H}_{ETC}(0)$ with the aid of the Fierz theorem in such a way that the transformed expression contains the products of only (pseudo-)scalar fermion bilinears. Then $\delta\mathcal{H}_{ETC}(0)$ becomes

$$\begin{aligned} \delta\mathcal{H}_{ETC}(0) = & \frac{g^2}{12m_g^2} \left[(F_e^i \sigma_2 F_m^j) (F_m^{j*} \sigma_2 F_e^{i*}) \right. \\ & + (F_{e+2}^i \sigma_2 F_{m+2}^j) (F_{m+2}^{j*} \sigma_2 F_{e+2}^{i*}) \\ & \left. - 2 (F_e^i \sigma_2 F_{m+2}^j) (F_{m+2}^{j*} \sigma_2 F_e^{i*}) \right] + \dots \quad (2.10) \end{aligned}$$

In the vacuum Eq. (2.2) this potential has a nonzero value, which may be obtained by approximating the vacuum expectation value of each product of two scalar fermion bilinears by the product of their vacuum expectation values

$$\langle \delta \mathcal{H}_{\text{ETC}}(0) \rangle_0 = \frac{g^2}{24m_g^2} \left[\text{Tr}(M_1 M_1^\dagger) + \text{Tr}(M_2 M_2^\dagger) - 2 \text{Tr}(M_3 M_3^\dagger) \right] + \dots \quad (2.11)$$

Clearly then the vacuum energy contributed by the ETC interactions is minimized provided

$$M_1 = 0, \quad M_2 = 0, \quad M_3 \neq 0. \quad (2.12)$$

This specifies the preferred direction of the condensate Σ . Significantly this coincides with the maximally attractive direction with respect to the ETC interactions viz. the condensation of a (3) and a ($\bar{3}$). From our previous analysis we now know that in this model the ETC flavour symmetry is broken down to $H^{\text{ETF}} = \text{SU}_V(2) \times \text{U}(1)$. A summary of the breakdown of the various flavour symmetries is pictorially depicted in Fig. 4. Furthermore the fermions f_α acquire masses of order Eq. (2.9) as a result of the choice Eq. (2.12). Indeed the quantity $\text{Tr}(m_f^\dagger m_f)$, where m_f is the consequent fermion mass matrix, is maximized.

Armed with our experience of the toy model we now wish to discuss the question of the orientation of the vacuum condensate by the ETC interactions more generally. To the extent that $\alpha_{\text{ETC}} = g_{\text{ETC}}^2(\mu_{\text{ETC}})/4\pi \ll 1$ the effects of the ETC interactions are reliably calculable in perturbation theory (μ_{ETC} is the scale at which $G^{\text{ETC}} \rightarrow G^{\text{TC}}$). Clearly the technifermion condensates will align themselves so as to minimize the effective energy density $\delta \mathcal{H}_{\text{ETC}}(0)$. Our first task, therefore, is to develop a general expression for this quantity.

Towards this end, we consider technifermion condensates of the type

$$\langle F^\dagger \sigma_2 F' \rangle \simeq (\Lambda^{\text{TC}})^3, \quad (2.13)$$

where F and F' belong to the ETC multiplets $\psi = \{F, f\}$ and $\psi' = \{F', f'\}$ which transform as the representations (n) and (n') of G^{ETC} , and f and f' are the accompanying ordinary technisinglet fermions. The condensate (2.13) is a Lorentz scalar techni-

singlet which may originate from one or several ETC multiplets (n_K). It is also convenient to divide the gauge fields of G^{ETC} into four categories as follows:

- G : the technigluons of G^{TC}
- E : the heavy vector bosons that couple the technifermions to the ordinary fermions
- E' : the heavy vector bosons that only couple the ordinary fermions to the ordinary fermions
- E'' : the heavy vector bosons that couple both technifermions to technifermions and fermions to fermions.

For example, in the case of the toy model, we have

$$\begin{aligned} SU^{\text{ETC}}(3) &\longrightarrow SU^{\text{TC}}(2) \\ (8) &= (3)_G + (2)_E + (2)_{E'} + (1)_{E''} \end{aligned} \quad (2.14)$$

Then, in the lowest order (single E'' exchange), the energy density $\delta\mathcal{H}_{\text{ETC}}(0)$ is given by

$$\begin{aligned} \langle \delta\mathcal{H}_{\text{ETC}}(0) \rangle_0 &= \frac{1}{2} g^2 \sum_{a \in \{E''\}} \frac{2^{1-\delta_{FF'}}}{m_a^2} \langle (F_i^\dagger \sigma_\mu \Lambda_i^a \delta F_j) (F_R'^\dagger \sigma^\mu \Lambda_R'^a \delta F_l') \rangle_0 \\ &= g^2 \sum_{a \in \{E''\}} \frac{2^{1-\delta_{FF'}}}{m_a^2} \langle F_i^\dagger \sigma_2 F_R' \rangle_0^* \langle F_j^\dagger \sigma_2 F_l' \rangle_0 \Lambda_i^a \delta \Lambda_R'^a, \end{aligned} \quad (2.15)$$

where we have used our previously stated assumption that the v.e.v. of a 4 fermion operator can be evaluated by Fierz transforming that operator into a product of (pseudo-)scalar $\times$ (pseudo-)scalar bilinears and inserting the vacuum. In general, the E'' vector bosons fall into a small number of G^{TC} multiplets. Let us assume that all these multiplets have the same mass m . The orientation of the vacuum should not depend on this simplifying assumption. Thus:

$$\langle \delta\mathcal{H}_{\text{ETC}}(0) \rangle_0 \simeq \frac{g^2}{m^2} 2^{1-\delta_{FF'}} \sum_{a \in \{E''\}} \langle F_i^\dagger \sigma_2 F_R' \rangle_0^* \langle F_j^\dagger \sigma_2 F_l' \rangle_0 \Lambda_i^a \delta \Lambda_R'^a. \quad (2.16)$$

In (2.16) we can let the sum run also over the set of technigluons G since we know that massless or massive technigluon exchange does not lift the degeneracy of the

vacuum. Finally, since single E or E' exchange produces 4 fermion operators of zero v.e.v., we can also include the corresponding terms in (2.16). Thus:

$$\langle \delta \mathcal{H}_{\text{ETC}}(0) \rangle \simeq \frac{g^2}{m^2} 2^{1-\delta_{FF'}} \sum_{\text{all } a} \langle \Psi_i^T \sigma_2 \Psi_R' \rangle_0^* \langle \Psi_j^T \sigma_2 \Psi_L' \rangle_0 \Lambda_i^{a,j} \Lambda_R^{a,p} \quad (2.17)$$

Now let $\Gamma_{ik}^{K,r}$ be the Clebsch-Gordan coefficients associated with the Kronecker series:

$$(n) \times (n') = (n_1) + \dots + (n_p) \quad (2.18)$$

Then we have:

$$\langle \Psi_i^T \sigma_2 \Psi_R' \rangle_0 = \sum_{K=1}^P \sum_{r=1}^{n_K} \Gamma_{iR}^{K,r} \Phi_r^{(K)} \quad (2.19)$$

$$\begin{aligned} \Phi_r^{(K)} &= \sum_{i=1}^n \sum_{R=1}^{n'} \Gamma_{iR}^{K,r*} \langle \Psi_i^T \sigma_2 \Psi_R' \rangle_0 \\ &= \sum_{i,R=1}^n \Gamma_{iR}^{K,r} \langle F_i^T \sigma_2 F_R' \rangle_0, \end{aligned} \quad (2.20)$$

where n is the number of technicolours. The RHS of Eq. (2.17) can be put into the form:

$$\langle \delta \mathcal{H}_{\text{ETC}}(0) \rangle \simeq -\frac{g^2}{2m^2} 2^{1-\delta_{FF'}} \sum_{K=1}^P (C + C' - C_K) \sum_{r=1}^{n_K} \Phi_r^{(K)*} \Phi_r^{(K)}, \quad (2.21)$$

where C, C' and C_K are the Casimir operators of representations (n), (n') and (n_K) respectively.

At this stage we make use of the fact that the condensates are technisinglets, which permits us to write

$$\langle F_i^T \sigma_2 F_R' \rangle_0 = \frac{1}{n} g_{iR} \langle F^T \sigma_2 F' \rangle_0, \quad (2.22)$$

where $\langle F^T \sigma_2 F' \rangle_0$ is a number of order $(\Lambda^{\text{TC}})^3$ and g_{ik} refers to the technicolour metric, which is either symmetric or antisymmetric. Now Eq. (2.20) reads

$$\Phi_r^{(K)} = \frac{1}{n} \sum_{i,R=1}^n \Gamma_{iR}^{K,r*} g_{iR} \langle F^T \sigma_2 F' \rangle_0 \equiv a_r^{(K)} \langle F^T \sigma_2 F' \rangle_0, \quad (2.23)$$

whence

$$\langle \delta \mathcal{H}_{ETC}(0) \rangle_0 = - \frac{g^2}{m^2} 2^{-\delta_{FF'}} \left[\sum_{k=1}^P (c+c'-c_k) \left(\sum_{r=1}^{n_k} |a_r^{(k)}|^2 \right) \right] |\langle F_1^T \sigma_2 F_1' \rangle_0|^2. \quad (2.24)$$

This expression now furnishes us with a quantitative criterion for vacuum alignment in the general case:

Those technifermions in the ETC representations (n)

and (n') condense which minimize the quantity

$$A(n, n') = - \sum_{k=1}^P (c+c'-c_k) \left(\sum_{r=1}^{n_k} |a_r^{(k)}|^2 \right). \quad (2.25)$$

Note that $A(n, n')$ depends solely on (n) and (n') and involves a summation over all ETC representations contained in the product (n) $\times$ (n').

To clarify the formula (2.25), we come back to the toy model. There we have to choose between two channels:

$$(3) \times (\bar{3}) = (1) + (8) \quad \text{and} \quad (3) \times (3) = (\bar{3}) + (6).$$

In the first case,

$$\begin{aligned} \langle \delta \mathcal{H}_{ETC}(0) \rangle_0 &= - \frac{g^2}{m^2} \left[(c_3 + c_{\bar{3}} - c_1) |a_1^{(1)}|^2 + (c_3 + c_{\bar{3}} - c_8) \sum_{r=1}^8 |a_r^{(8)}|^2 \right] |\langle F_{1R}^T \sigma_2 F_{2R}^R \rangle_0|^2 \\ &= - \frac{g^2}{m^2} \left[\left(\frac{4}{3} + \frac{4}{3} - 0 \right) \frac{1}{3} + \left(\frac{4}{3} + \frac{4}{3} - 3 \right) \frac{1}{6} \right] |\langle F_{1R}^T \sigma_2 F_{2R}^R \rangle_0|^2 \\ &= - \frac{g^2}{m^2} \frac{5}{6} |\langle F_{1R}^T \sigma_2 F_{2R}^R \rangle_0|^2, \end{aligned} \quad (2.26a)$$

while in the second one,

$$\begin{aligned} \langle \delta \mathcal{H}_{ETC}(0) \rangle_0 &= - \frac{g^2}{m^2} \left[(c_3 + c_{\bar{3}} - c_{\bar{3}}) \sum_{r=1}^3 |a_r^{(\bar{3})}|^2 + (c_3 + c_3 - c_6) \sum_{r=1}^6 |a_r^{(6)}|^2 \right] |\langle \epsilon^{ij} F_{1i}^T \sigma_2 F_{2j}^R \rangle_0|^2 \\ &= - \frac{g^2}{m^2} \left[\left(\frac{4}{3} + \frac{4}{3} - \frac{4}{3} \right) \frac{1}{2} + \left(\frac{4}{3} + \frac{4}{3} - \frac{10}{3} \right) 0 \right] |\langle \epsilon^{ij} F_{1i}^T \sigma_2 F_{2j}^R \rangle_0|^2 \\ &= - \frac{g^2}{m^2} \frac{2}{3} |\langle \epsilon^{ij} F_{1i}^T \sigma_2 F_{2j}^R \rangle_0|^2. \end{aligned} \quad (2.26b)$$

It is therefore clear that the $(3) \times (\bar{3}) = (1) + (8)$ condensation is preferred since it minimizes the energy.

It is quite interesting to compare this criterion with the MAC criterion developed for tumbling gauge theories¹⁷⁾. In the case of tumbling, we simply look for representations (n), (n') and (n_k) that minimize the quantity

$$= (C + C' - C_K). \quad (2.27)$$

In the toy model, the maximally attractive channel is $(3) \times (\bar{3}) \rightarrow (1)$. However, in the general case, the condensate need not be a gauge singlet. In our ETC alignment problem, the condensate is constructed from the technifermions alone and is chosen to be a technisinglet. Thus it is necessary for us to sum over all ETC channels containing the same technicolour condensate in the manner of Eq. (2.25). For example, in (2.26a), according to the MAC criterion of tumbling gauge theories, channel $(3) \times (\bar{3}) \rightarrow (1)$ is attractive whereas channel $(3) \times (\bar{3}) \rightarrow (8)$ is repulsive; but we still have to add their contributions to compute the value of $\delta\mathcal{H}_{\text{ETC}}$ corresponding to the condensate $(3) \times (\bar{3})$.

Now we analyzed several different ETC models in accordance with the criterion of minimizing $A(n, n')$ [Eq. (2.25)]. In all cases the representations (n) and (n') were those which contained the maximally attractive channel (MAC)¹⁷⁾ with respect to ETC. Moreover the vacuum orientation thus determined gave the technifermions the most mass in the sense of maximizing $\text{Tr}(m_f^+ m_f)$ where m_f is the mass matrix of the left-handed ordinary fermion fields f :*

$$\mathcal{L}_{\text{mass}} = - \bar{f}^T \sigma_2 m_f \bar{f} + \text{h.c.} \quad (2.28)$$

We should note that in the limit $\mu_{\text{ETC}} \rightarrow 0$, the single E , E' , and E'' vector boson and single G technigluon exchange diagrams become the usual one gluon exchange diagrams that yield the MAC criterion in a "tumbling" theory. The masses that the ordinary fermions f acquire from the technifermion condensates become the dynamical masses they acquire from participating in the tumbling condensates. Therefore the maximization of $\text{Tr}(m_f^+ m_f)$ would follow if MAC in tumbling gauge theories is also the channel of maximum possible dynamical masses for the fermions. But this is in general true.

Next we would like to briefly discuss the higher order corrections to $\delta\mathcal{H}_{\text{ETC}}^{(0)}$. Let us first consider the box diagrams of Fig. 5, mediated by the E vector bosons that couple ordinary fermions to technifermions. These box diagrams are very much

* Details illustrating this mechanism in a general example are given in the Appendix.

associated with the ETC mechanism since they are essentially an iteration of the diagram (cf. Fig. 3) through which the ordinary fermions acquire mass. As a matter of fact, one can in general determine the $(\text{mass})^2$ matrix of the E vector bosons from the mass matrix of the ordinary fermions. Finally, although these box diagrams are of second order, it is possible that if m_E is lighter than $m_{E''}$ and $g^2(\mu_{\text{ETC}})/4\pi^2$ not so much smaller than one, their contribution to $\delta\mathcal{H}_{\text{ETC}}(0)$ may upset the alignment obtained from the lowest order effects discussed above.

It turns out that in our simple toy model, the contribution from Fig. 5a is zero, whereas from Fig. 5b:

$$\langle \delta\mathcal{H}_{\text{ETC}}(0) \rangle = \frac{-g^4}{128\pi^2 m^2} \left[\text{Tr}(M_1 M_1^\dagger) + \text{Tr}(M_2 M_2^\dagger) + 16 \text{Tr}(M_3 M_3^\dagger) \right], \quad (2.29)$$

where m is the mass of the E ($a = 4, 5, 6, 7$) vector bosons. The direction of alignment implied by Eq. (2.29) is clearly identical with the one implied by the lowest order diagrams, Eq. (2.11). Again, the vacuum aligns in the direction that gives the most mass to the ordinary fermions. Once more, we believe this to be a very general property of the box diagrams of Fig. 5, as the following simple model shows.

Consider two real scalar fields with an $O(2)$ symmetric potential:

$$V(\phi_1^2 + \phi_2^2) = -\frac{\mu^2}{2}(\phi_1^2 + \phi_2^2) + \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2. \quad (2.30)$$

The v.e.v. of the scalar fields lies anywhere on a circle of radius $\mu/\sqrt{\lambda}$ centred about $\phi_1 = \phi_2 = 0$. There is a Goldstone boson due to the spontaneous breaking of the $O(2)$ symmetry. We now introduce a Yukawa coupling of ϕ_1 to a fermion field ψ :

$$\mathcal{L}_Y = K \bar{\psi} \psi \phi_1 \quad (2.31)$$

and ask the question: how does this orient the vacuum? We must evaluate the loop corrections to the Higgs potential which are given by the diagram of Fig. 6 in lowest order. Since this diagram is divergent we explicitly introduce a cut-off Λ .

In ETC models however this cut-off is automatically present and is typically of order m_E . We obtain:

$$\delta V = - \frac{\kappa^2}{16\pi^2} \Lambda^2 \phi_1^2. \quad (2.32)$$

Therefore, the minimum lies in the ϕ_1 direction.

The vacuum aligns itself in such a way as to give the most mass to the fermion ψ .

Finally, we consider the higher order corrections to $\delta\mathcal{H}_{\text{ETC}}(0)$ due to the technigluon renormalization (Fig. 7) of the lowest order diagrams (Fig. 2). In the toy model, these corrections induce new operators of the type:

$$\begin{aligned} {}^5O_1 &= (F_{\ell i}^\dagger \sigma_\mu F_{\ell i} + F_{\ell+2}^{i\dagger} \sigma_\mu F_{\ell+2}^i) (F_{m i}^\dagger \sigma^\mu F_{m i} + F_{m+2}^{i\dagger} \sigma^\mu F_{m+2}^i) \\ {}^5O_2 &= (F_{\ell i}^\dagger \sigma_\mu F_{\ell j} + F_{\ell+2}^{j\dagger} \sigma_\mu F_{\ell+2}^i) (F_{m j}^\dagger \sigma^\mu F_{m i} + F_{m+2}^{i\dagger} \sigma^\mu F_{m+2}^j). \end{aligned} \quad (2.33)$$

The way to handle this renormalization is standard (see for example Ref. 19): compute the anomalous dimension matrix, diagonalize it and solve the renormalization group equations in the basis thus obtained. It turns out that, as one would expect, the v.e.v. of $\delta\mathcal{H}_{\text{ETC}}(0)$ gets multiplicatively renormalized:

$$\langle \delta\mathcal{H}_{\text{ETC}}(0) \rangle_0 = \frac{g^2(m_g)}{24m_g^2} \left(\frac{g^2(\mu)}{g^2(m_g)} \right)^{g/2b} \left[\text{Tr}(\Sigma^\dagger \Sigma) - 4 \text{Tr}(M_3^\dagger M_3) \right], \quad (2.34)$$

where b is defined by:

$$\begin{aligned} \beta(g) &= \mu \frac{dg}{d\mu} = -b \frac{g^3}{16\pi^2} \\ b &= \frac{22}{3} - \frac{2}{3} \cdot 2 = 6. \end{aligned} \quad (2.35)$$

Therefore, in our toy model, the effect of technigluon renormalization is to stabilize the vacuum orientation and to increase the PGB masses.

It only remains to discuss the various (pseudo)Goldstone bosons in this model. Accompanying the spontaneous breakdown of the TC flavour group $SU(4)$ to $Sp(4)$ are $15 - 10 = 5$ Goldstone bosons corresponding to the five broken generators. A suitable explicit set of 15 generators $G_a = \{T_p \ (p = 1, \dots, 10), X_x \ (x = 1, \dots, 5)\}$ of $SU(4)$ which is well adapted to this breakdown are as follows:

$$\begin{aligned}
 T_p = & \frac{1}{2} \left(\begin{array}{c|c} \tau_a & 0 \\ \hline 0 & -\tau_a^T \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 1_2 & 0 \\ \hline 0 & -1_2 \end{array} \right), \\
 & \frac{1}{2} \left(\begin{array}{c|c} 0 & 1_2 \\ \hline 1_2 & 0 \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 0 & i1_2 \\ \hline -i1_2 & 0 \end{array} \right), \\
 & \frac{1}{2} \left(\begin{array}{c|c} 0 & \tau_1 \\ \hline \tau_1 & 0 \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 0 & i\tau_1 \\ \hline -i\tau_1 & 0 \end{array} \right), \\
 & \frac{1}{2} \left(\begin{array}{c|c} 0 & \tau_3 \\ \hline \tau_3 & 0 \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 0 & i\tau_3 \\ \hline -i\tau_3 & 0 \end{array} \right),
 \end{aligned}
 \tag{2.36}$$

$$X_x = \frac{1}{2} \left(\begin{array}{c|c} \tau_a & 0 \\ \hline 0 & \tau_a^T \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 0 & \tau_2 \\ \hline \tau_2 & 0 \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 0 & i\tau_2 \\ \hline -i\tau_2 & 0 \end{array} \right).$$

The T_p are the 10 unbroken generators which constitute a basis for the symplectic group $Sp(4)$. The X_x are the five broken generators. All the generators are normalized such that $\text{Tr}(G_a G_b) = \delta_{ab}$. The five Goldstone bosons corresponding to the axial generators X_x transform as an irreducible five dimensional representation of $Sp(4)$; this fact implies the equality of their decay constants. Furthermore, the set of generators

$$\frac{1}{2} \left(\begin{array}{c|c} \tau_a & 0 \\ \hline 0 & \tau_a^T \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} \tau_a & 0 \\ \hline 0 & -\tau_a^T \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 1_2 & 0 \\ \hline 0 & -1_2 \end{array} \right) \tag{2.37}$$

may be identified as a basis of the $SU(2) \times SU(2) \times U(1)$ flavour symmetry group of ETC. The condensate Σ , Eq. (2.12), remains invariant under the last four of these; these therefore comprise the generators of the $H^{ETF} = SU_V(2) \times U(1)$ group.

In the absence of the ETC interactions all the Goldstone bosons are massless. However, by virtue of the effective four fermion interactions contained in $\delta\mathcal{H}_{ETC}(0)$, Eq. (2.10), some of them become massive. Since $\delta\mathcal{H}_{ETC}(0)$ is invariant under the $G^{ETF} = SU(2) \times SU(2) \times U(1)$ group, it is obvious that the three Goldstone bosons corresponding to the broken generators

$$\frac{1}{2} \left(\begin{array}{c|c} \tau_a & 0 \\ \hline 0 & \tau_a^T \end{array} \right) \quad (2.38)$$

cannot acquire any mass. In the terminology of Weinberg, these are the true Goldstone bosons of the model. On the other hand, the Goldstone particle-antiparticle pair corresponding to the generators

$$\frac{1}{2} \left(\begin{array}{c|c} 0 & \tau_2 \\ \hline \tau_2 & 0 \end{array} \right), \quad \frac{1}{2} \left(\begin{array}{c|c} 0 & i\tau_2 \\ \hline -i\tau_2 & 0 \end{array} \right) \quad (2.39)$$

do obtain a common mass, and are termed the pseudo-Goldstone bosons. Now a chiral rotation on the fermion fields with parameters α_x is specified by

$$F_\alpha^i \longrightarrow \left(e^{i\alpha_x X_x} \right)_{\alpha\beta} F_\beta^i. \quad (2.40)$$

In the standard fashion²⁾, the mass matrix of the Goldstone bosons is determined by performing such a chiral rotation on the perturbation energy density $\delta\mathcal{H}_{ETC}(0)$ Eq. (2.10), and differentiating twice with respect to the parameters α_x :

$$M_{xy}^2 = \frac{1}{F_\pi^2} \frac{\partial^2}{\partial\alpha_x \partial\alpha_y} \langle \delta\mathcal{H}_{ETC}(0) \rangle_0. \quad (2.41)$$

Here F_π denotes the decay constant of the Goldstone bosons. Equation (2.41) may also be alternatively presented as

$$M_{xy}^2 = - \frac{1}{F_\pi^2} \langle [X_x, [X_y, \delta \mathcal{H}_{\text{ETC}}(0)]] \rangle. \quad (2.42)$$

It is now easily verified explicitly that, as anticipated above, this model contains three true Goldstone bosons and two pseudo-Goldstone bosons, the masses of the latter being given by

$$M^2 = \frac{4}{3F_\pi^2} \frac{g^2}{m_g^2} |K|^2. \quad (2.43)$$

This completes our discussion of the toy model. It hardly needs to be emphasized that this model is quite unrealistic, and was considered here only to focus attention on some of the relevant issues involved. These same issues will recur in the somewhat more realistic models of the next section.

3. PSEUDO-GOLDSTONE BOSON MASSES IN TECHNICOLOUR MODELS

One technifamily ETC models have been proposed as candidates for a dynamical theory of the $W^\pm$, Z^0 , quark, and lepton masses. No completely successful model of this type has been constructed yet. Indeed the models constructed so far all predict excessively large rates for flavour changing neutral current (FCNC) processes such as $K^0-\bar{K}^0$ mixing, $D^0-\bar{D}^0$ mixing, $K_L \rightarrow \mu e$, etc. ... ^{20,21}). On the other hand, it is not excluded that a successful one technifamily ETC model exists; we might just not have found it yet.

At any rate, the existence of such a model has rather well-defined phenomenological consequences which can be derived without knowledge of the detailed structure of the model¹⁴). One important aspect of this phenomenology which has not been discussed in detail so far is the contribution from ETC interactions to PGB masses. It has only been said that these contributions are important $O[(30 \text{ GeV})^2]$, but otherwise very ETC model dependent. However we find that things can be said which are model independent: in particular, that the P^0 , P^3 , $P^\pm$, P_8^0 , P_8^3 , and $P_8^\pm$ PGBs cannot acquire any mass from ETC interactions alone.

A one technifamily ETC model has eight four-component technifermions:

$$\{F_{\alpha i}\} = \begin{pmatrix} U_R & U_W & U_B & N \\ D_R & D_W & D_B & E \end{pmatrix} \quad (3.1)$$

($a = 1, 2, 3, 4$ and $i = 1, 2$) which have the same $SU_L(2) \times U_Y(1) \times SU^C(3)$ quantum numbers as an ordinary family u, d, e and ν , but which in addition carry technicolour. The TC gauge coupling constant becomes large at a scale Λ^{TC} of order 1 TeV. As was already mentioned in the introduction, one expects technifermion condensates to form at the scale Λ^{TC} , thereby spontaneously breaking the global flavour symmetry of the technicolour interactions¹¹⁾: $SU_L(8) \times SU_R(8) \times U_V(1) \rightarrow SU_V(8) \times U_V(1)$ if technicolour is complex; $SU(16) \rightarrow O(16)$ if technicolour is real; $SU(16) \rightarrow Sp(16)$ if technicolour is pseudoreal. There are, respectively, 63, 135, or 119 (pseudo-)Goldstone bosons produced by the spontaneous breakdown. In each case, three Goldstone bosons ($\pi^0, \pi^\pm$) will be "eaten up" by the $W^\pm$ and Z^0 vector bosons. It is the properties (masses, couplings, decay and production rates) of the remaining PGBs that constitute the phenomenology¹²⁻¹⁴⁾ to which we alluded earlier.

The PGBs are all exactly massless in the absence of any interactions other than technicolour. They will acquire mass however from the $SU_L(2) \times U_Y(1) \times SU^C(3)$ gauge interactions, the ETC interactions, or any other interactions that explicitly break some of the global symmetries of the technicolour interactions. The vacuum alignment and the PGB masses resulting from the $SU_L(2) \times U_Y(1) \times SU^C(3)$ gauge interactions have been determined^{6,7,12)}. If technicolour is complex or real, the vacuum aligns correctly, i.e.

$$\begin{aligned} \langle \bar{U}_R U_R \rangle_0 &= \langle \bar{U}_W U_W \rangle_0 = \langle \bar{U}_B U_B \rangle_0 = \langle \bar{N} N \rangle_0 \\ &= \langle \bar{D}_R D_R \rangle_0 = \langle \bar{D}_W D_W \rangle_0 = \langle \bar{D}_B D_B \rangle_0 = \langle \bar{E} E \rangle_0 \end{aligned} \quad (3.2)$$

The contributions to the PGB masses are shown in the fourth and fifth columns of Table 1 for these two cases, respectively. If technicolour is pseudoreal, the vacuum aligns in such a way as to break electric charge. Pseudoreal technicolour is thus ruled out.

Note that two physical PGBs, P^0 and P^3 , do not acquire a mass from $SU_L(2) \times U_Y(1) \times SU^C(3)$ gauge interactions. Actually, this is not quite true since colour instantons explicitly break the symmetry generated by:

$$\begin{aligned} Q_A^{15,0} &= \frac{1}{2} \int d^3x F_{ai}^\dagger (\Lambda^{15})_a^b \gamma_5 F_{bi} \\ &= \frac{1}{2\sqrt{6}} \int d^3x \left[V^\dagger \gamma_5 V + D^\dagger \gamma_5 D - 3(N^\dagger \gamma_5 N + E^\dagger \gamma_5 E) \right], \end{aligned} \quad (3.3)$$

of which P^0 is the Goldstone boson. P^0 thus acquires a mass of order:

$$(m_{P^0})^2 \sim \frac{1}{\ell^2} n(\ell) \quad (3.4)$$

from the colour gluon anomaly. ℓ is the characteristic size of the colour instantons that make the dominant contribution to this mass and $n(\ell)$ is the distribution function of instanton sizes²²):

$$n(\ell) \simeq 0.015 \left(\frac{8\pi^2}{g_c^2(\ell)} \right)^6 \exp \left(- \frac{8\pi^2}{g_c^2(\ell)} \right), \quad (3.5)$$

where $g_c(\ell)$ is the QCD coupling constant at momentum scale $1/\ell$. Presumably $1/\ell \sim \Lambda^{TC}$, and P^0 only acquires a tiny mass (~ 500 eV) from colour instantons.

PGBs can also receive masses from gauge interactions mediated by vector bosons with masses which are larger than the scale of technicolour. These interactions correspond to gauge groups which are spontaneously broken before the technifermion condensates form. The ETC interactions, which are necessary to give mass to the ordinary quarks and leptons, are of this type. The essence of the ETC mechanism was explained in Section 2. In technicolour models, it can be implemented in such

a way that several families of quarks and leptons acquire mass from one family of technifermions. For example, if G^{ETC} commutes with $SU_L(2) \times U_Y(1) \times SU^C(3)$, the multiplet structure is as follows:

$$\begin{array}{ccccccc}
 u & c & t & U_1 & \dots & U_n & \\
 d & s & b & D_1 & \dots & D_n & \\
 \nu_e & \nu_\mu & \nu_\tau & N_1 & \dots & N_n & \\
 e & \mu & \tau & E_1 & \dots & E_n &
 \end{array} \tag{3.6}$$

$\longleftrightarrow$ generation $\longleftrightarrow$ technicolour $\longleftrightarrow$
 $\longleftrightarrow$ extended technicolour $\longleftrightarrow$

where n is the number of technicolours. We should emphasize however that ETC does not necessarily commute with the colour and electroweak quantum numbers. For example, the ETC vector bosons that couple technifermions to ordinary fermions could be colour triplets if G^{ETC} unifies colour with technicolour²³):

$$G^{\text{ETC}} \xrightarrow{\mu} SU^C(3) \times G^{\text{TC}}. \tag{3.7}$$

They then couple technileptons to quarks and techniquarks to leptons and/or anti-quarks. To produce quark and lepton masses ranging from a few MeV to a few GeV, the vacuum expectation values μ_{ETC} that break G^{ETC} down to G^{TC} must range from a few TeV to about 100 TeV.

3.1 Contributions from horizontal broken gauge interactions

We want to study in the most general way the contributions of the $G^{\text{ETC}}/G^{\text{TC}}$ gauge interactions to PGB masses. Now, there are six $SU_L(2) \times U_Y(1) \times SU^C(3)$ multiplets of technifermions: (U_L, D_L) , U_R , D_R , (N_L, E_L) , N_R , and E_R . By definition, the ETC gauge group unifies each one of these multiplets with a certain unspecified set of quarks and leptons. In other words, ETC acts horizontally in the following picture:

$$\begin{array}{llll}
 (U_L, D_L) & \longleftrightarrow & \text{some quarks and leptons} & \\
 U_R & \longleftrightarrow & " & " & " & " \\
 D_R & \longleftrightarrow & " & " & " & " \\
 (N_L, E_L) & \longleftrightarrow & " & " & " & " \\
 N_R & \longleftrightarrow & " & " & " & " \\
 E_R & \longleftrightarrow & " & " & " & "
 \end{array} \quad (3.8)$$

but not vertically. That is to say, ETC does not unify different $SU_L(2) \times U_Y(1) \times SU^c(3)$ multiplets of technifermions. This definition of ETC is very general. It does not even require "monophagy"²⁴⁾. All one technifamily ETC models that have been considered in the literature -- including the Farhi-Susskind model²³⁾ -- fall within the bounds of this definition.

A priori the six multiplets to which (U_L, D_L) , U_R^c , D_R^c , (N_L, E_L) , N_R^c and E_R^c belong have different ETC transformation properties. The *minimal* global symmetry group of the ETC interactions is thus the

$$\begin{aligned}
 & SU(6) \times SU^{\bar{U}}(3) \times SU^{\bar{D}}(3) \times SU^L(2) \times U_V^{0,0}(1) \\
 & \times U_L^{15,0}(1) \times U_R^{0,3}(1) \times U_R^{15,0}(1) \times U_R^{15,3}(1)
 \end{aligned} \quad (3.9)$$

subgroup of the $SU_L(8) \times SU_R(8) \times U_V(1)$ global symmetry group of the technicolour interactions. In our notation, $U_{R(L)}^{\alpha,r}(1)$ is generated by:

$$Q_{R(L)}^{\alpha,r} = \frac{1}{2\sqrt{2}} \int d^3x \ F_{ai}^\dagger (\Lambda^\alpha)_a^b (\tau^r)_i^j \frac{1 \pm \gamma_5}{2} F_{bj} \ , \quad (3.10)$$

where the Λ^α ($\alpha = 0, 1, \dots, 15$) are the 4×4 Hermitian matrices which generate $SU(4)$. Also Q signifies techniquark and L technilepton. Note that the generator of a global symmetry of the ETC interactions is always the sum of a generator acting on technifermions and a generator acting on ordinary fermions:

$$Q^{ETC} = Q^{ETC}(F) + Q^{ETC}(f), \quad (3.11)$$

where $Q^{ETC}(F)$ is one of the generators (3.10). But the $Q^{ETC}(f)$ piece can be forgotten about when calculating PGB masses. This should be clear from Dashen's²⁾ formula (2.42) and the fact that $Q^{ETC}(f) |0\rangle = 0$. The minimal global symmetry group of the ETC interactions spontaneously breaks down to $SU^U(3) \times SU^D(3) \times U_V^{0,0}(1) \times U_V^{0,3}(1) \times U_V^{15,0}(1) \times U_V^{15,3}(1)$ by the technifermion condensates (3.2). This means that 39 PGBs (including the three Goldstone bosons "eaten up" by the $W^\pm$ and Z^0) can *never* get a mass from the ETC interactions alone. In the case of complex technicolour, at most 24 of the 63 PGBs can get a mass from the ETC interactions. Because the minimal global symmetry group (3.9) of the ETC interactions is a subgroup of $SU_L(8) \times SU_R(8) \times U_V(1)$, the 72 additional PGBs which are present in the case of real technicolour can as a rule get a mass from the ETC interactions. It should be emphasized however that in general some of the six multiplets to which $(U_L D_L)$, U_R^C , D_R^C , $(N_L E_L)$, N_R^C , and E_R^C belong can have the same ETC transformation properties. The global symmetry group of ETC will then be larger than the minimal one given in (3.9), and even fewer PGBs will get mass contributions from the ETC interactions. We will give an example below in which this is the case.

We will now show explicitly that P^0 , P^3 , $P^\pm$, P_8^0 , P_8^3 , $P_8^\pm$ cannot get a mass from the ETC interactions alone. Thus, among the 63 PGBs which are present when technicolour is complex, only the 24 lepto-quark PGBs ($P_{\bar{Q}L}$ and $P_{\bar{L}Q}$ in Table 1) can get a mass from the ETC interactions. The $(\text{mass})^2$ submatrix of P^0 , P^3 , $P^\pm$ is:

$$(M^2)^{rs} = - \frac{1}{F_\pi^2} \langle 0 | [Q_A^{15,r}, [Q_A^{15,s}, \delta \mathcal{H}_{ETC}(0)]] | 0 \rangle, \quad (3.12)$$

where $r, s = 0, 1, 2, 3$ and $Q_A^{\alpha, r}$ is the axial (R-L) linear combination of the generators defined in (3.10). However, since $Q_L^{15, s}$ generates a symmetry of $\delta\mathcal{H}_{\text{ETC}}(0)$, we have:

$$\left[Q_A^{15, s}, \delta\mathcal{H}_{\text{ETC}}(0) \right] = \left[Q_V^{15, s}, \delta\mathcal{H}_{\text{ETC}}(0) \right]. \quad (3.13)$$

Using Jacobi's identity, (3.12) becomes:

$$\begin{aligned} (M^2)^{rs} &= \frac{1}{F_\pi^2} \langle 0 | \left[Q_V^{15, s}, \left[\delta\mathcal{H}_{\text{ETC}}(0), Q_A^{15, r} \right] \right] | 0 \rangle \\ &+ \frac{1}{F_\pi^2} \langle 0 | \left[\delta\mathcal{H}_{\text{ETC}}(0), \left[Q_A^{15, r}, Q_V^{15, s} \right] \right] | 0 \rangle. \end{aligned} \quad (3.14)$$

The first term vanishes since $Q_V^{15, r} | 0 \rangle = 0$. The second term vanishes as well since it is proportional to

$$\begin{aligned} &\langle 0 | \left[\delta\mathcal{H}_{\text{ETC}}(0), Q_A^{15, t} \right] | 0 \rangle \\ &= \langle 0 | \left[\delta\mathcal{H}_{\text{ETC}}(0), Q_V^{15, t} \right] | 0 \rangle = 0. \end{aligned} \quad (3.15)$$

Similar arguments apply to the octet PGBs P_8^0 , P_8^3 and $P_8^\pm$.

We must still consider the important question of whether some of the 36 PGBs P^0 , P^3 , $P^\pm$, P_8^0 , P_8^3 and $P_8^\pm$ can acquire higher order mass contributions from a combination of the ETC and electroweak or strong interactions. This question is perhaps not very interesting for the case of the octet PGBs since they have already acquired large leading order masses from coloured gluon exchange. We therefore proceed to examine only the P^0 , P^3 and $P^\pm$ bosons in detail.

The neutrals P^0 and P^3 , which are known not to acquire any masses from the colour and electroweak interactions (see Table 1), cannot acquire any masses even

when these interactions are combined with ETC. Thus P^0 and P^3 remain completely massless to all orders (see however Section 3.2). This is simply because the generators $U_A^{15,0}(1)$ and $U_R^{15,3}(1)$ are symmetries not only of the colour and electroweak interactions but also belong to the minimal flavour group of the ETC interactions. In the now familiar manner these invariances prevent the P^0 and P^3 from acquiring any mass at all.

The situation is different for the charged PGBs $P^\pm$ however. Here the $P^\pm$ cannot acquire any contributions to their squared masses which are proportional to $\alpha_s \alpha_{\text{ETC}}$ or $\alpha'_w \alpha_{\text{ETC}}$ (α_s is the colour coupling constant) but they can acquire contributions proportional to $\alpha_w \alpha_{\text{ETC}}$. This is easily seen as follows. The first two contributions are unaltered if we let $\alpha_w \rightarrow 0$. But in this limit the generators $Q_L^{15,1}$ and $Q_L^{15,2}$ are invariances of the Hamiltonian, and these symmetries prevent the $P^\pm$ from acquiring any mass squared contributions proportional to $\alpha_s \alpha_{\text{ETC}}$ and $\alpha'_w \alpha_{\text{ETC}}$.

Let us finally consider the contributions proportional to $\alpha_w \alpha_{\text{ETC}}$. Now the symmetries which prevent the $P^\pm$ from acquiring masses from the $SU_L(2)$ gauge bosons alone are the right-handed $Q_R^{15,1}$ and $Q_R^{15,2}$. But these right-handed generators are no longer symmetries of the ETC interactions in general. On the other hand, the symmetries which forbid the ETC interactions alone from contributing to the masses of the $P^\pm$ are left-handed: $Q_L^{15,1}$ and $Q_L^{15,2}$. It thus follows that when we consider the weak $SU_L(2)$ and ETC interactions together we do not have any symmetries which will keep the $P^\pm$ massless. Thus, *the $P^\pm$ are expected to acquire higher order mass squared contributions of order $\alpha_w \alpha_{\text{ETC}}$* . We estimate these to be $\lesssim 5$ GeV in order of magnitude, to be compared with the 5-14 GeV masses that the $P^\pm$ may obtain from the electroweak interactions*.

Next, as a particular example of PGB mass generation by ETC interactions, we analyse a model²¹⁾ in the literature. In this model the ETC gauge group $SU^{\text{ETC}}(8)$ commutes with $SU_L(2) \times U_Y(1) \times SU^C(3)$ and breaks down to $O^{\text{TC}}(5)$ at energy scales ranging from a few TeV to about 100 TeV. The multiplet structure, completed by the introduction of leptons and technileptons, is the following:

* These two contributions may not correspond to the same vacuum orientation.

$$\begin{aligned}
 \begin{pmatrix} U_{L1} \dots U_{L5} & t_L & c_L & u_L \\ D_{L1} \dots D_{L5} & b_L & s_L & d_L \end{pmatrix} &\sim (8)_{\text{ETC}} \\
 \begin{pmatrix} U_{R1}^c \dots U_{R5}^c & t_R^c & c_R^c & u_R^c \\ D_{R1}^c \dots D_{R5}^c & b_R^c & s_R^c & d_R^c \end{pmatrix} &\sim (\bar{8})_{\text{ETC}} \\
 \begin{pmatrix} N_{L1} \dots N_{L5} & \nu_L'' & \nu_L' & \nu_L \\ E_{L1} \dots E_{L5} & \tau_L & \mu_L & e_L \end{pmatrix} &\sim (8)_{\text{ETC}} \\
 \begin{pmatrix} N_{R1}^c \dots N_{R5}^c & \nu_R''^c & \nu_R'^c & \nu_R^c \\ E_{R1}^c \dots E_{R5}^c & \tau_R^c & \mu_R^c & e_R^c \end{pmatrix} &\sim (\bar{8})_{\text{ETC}}
 \end{aligned} \tag{3.16}$$

The right-handed neutrinos acquire large Majorana masses (~ 100 TeV). The global symmetry group of the ETC interactions in this model is

$$SU(12) \times SU(4) \times U(1) , \tag{3.17}$$

which is spontaneously broken down to

$$O(8) \times SU(4) \times U'(1) \tag{3.18}$$

by the technifermion condensates (3.2). Comparing (3.17) with (3.18), we find that 115 of the $(63 + 72)$ PGBs present in this model do not acquire a mass from the ETC interactions alone. Arguments similar to those that accompany Eqs. (3.12) to (3.15) establish that the 20 PGBs that do acquire a mass from the ETC interactions in this model are P_{UU} , P_{UN} , P_{NN} (which are, respectively, a colour sextet, triplet, and singlet; see Table 1) and their antiparticles. The main lowest order contribution comes from the four technifermion interactions mediated by the 14 massive $SU(5)/O^{\text{TC}}(5)$ vector bosons, where $SU(5)$ is the subgroup of $SU^{\text{ETC}}(8)$ that acts

unitarily on the five technicolours. Using the methods described in detail in Section 2, we found

$$M_{\text{ETC}}^2 \simeq 5 \frac{(\langle \bar{N} N \rangle_0)^2}{F_\pi^2 v^2} \simeq (42 \text{ GeV})^2 \quad (3.19)$$

for P_{UU} , P_{UN} , and P_{NN} . The parameter v was defined in Ref. 21. This (mass)² contribution is positive because the vacuum (3.2) is from the point of view of the ETC interactions more attractive than the vacuum (3.2) rotated by those spontaneously broken symmetries of which P_{UU} , P_{UN} , and P_{NN} are the Goldstone bosons. The vacuum (3.2) corresponds to $8 \times \bar{8} \supset 1$ condensation whereas the rotated vacuum corresponds to $8 \times 8 \supset 36$ condensation.

3.2 Contributions from vertical broken gauge interactions

We have seen that even the inclusion of the ETC interactions, which are horizontal gauge interactions among the fermions (3.8), do not contribute to the masses of the P^0 and P^3 . This leaves us with massless P^0 and P^3 . (We neglect for the moment the small mass P^0 acquires from colour instantons, but will come back to it later). Massless P^0 and P^3 are phenomenologically unacceptable, because P^0 and P^3 are known²⁵⁾ to have small $O(m^2/F_\pi^2)$ induced flavour nondiagonal couplings (m is a quark mass). Unless P^0 and P^3 have masses $\geq \frac{1}{2}$ GeV, they would have been in $K \rightarrow p\pi$ decays and they would mediate too large $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ mixings.

The masslessness of P^0 and P^3 means that the vacuum is two-fold degenerate. The degenerate vacua can be labelled $|\theta^0, \theta^3\rangle$ where θ^0 and θ^3 are the angles by which one must rotate the vacuum $|0,0\rangle$ defined by Eq. (3.2) through the generators $Q_A^{15,0}$ and $Q_A^{15,3}$ defined by Eq. (3.10) to obtain the vacuum $|\theta^0, \theta^3\rangle$. Since

$$Q_A^{15,r} = \frac{1}{4\sqrt{3}} \int d^3x (Q^\dagger \tau^r \gamma_5 Q - 3 L^\dagger \tau^r \gamma_5 L), \quad (3.20)$$

the technifermion condensates in the $|\theta^0, \theta^3\rangle$ vacua are given by

$$\begin{aligned} \langle \bar{U}_c U_c + i \bar{U}_c \gamma_5 U_c \rangle_\theta &= e^{i(\theta_0 + \theta_3)} \langle \bar{F} F \rangle_0 \\ \langle \bar{D}_c D_c + i \bar{D}_c \gamma_5 D_c \rangle_\theta &= e^{i(\theta_0 - \theta_3)} \langle \bar{F} F \rangle_0 \end{aligned} \quad (3.21a)$$

for each colour $c = R, W, B$, and

$$\begin{aligned} \langle \bar{N} N + i \bar{N} \gamma_5 N \rangle_\theta &= e^{-3i(\theta_0 + \theta_3)} \langle \bar{F} F \rangle_0 \\ \langle \bar{E} E + i \bar{E} \gamma_5 E \rangle_\theta &= e^{-3i(\theta_0 - \theta_3)} \langle \bar{F} F \rangle_0, \end{aligned} \quad (3.21b)$$

where $\langle \bar{F} F \rangle_0$ is the strength of any one of the condensates given in Eq. (3.2), e.g. $\langle \bar{F} F \rangle_0 = \langle \bar{N} N \rangle_0$.

To lift this vacuum degeneracy and give mass to P^0 and P^3 we need to gauge vertically among the fermions (3.8). Indeed this will in general explicitly break the chiral symmetries of which P^0 and P^3 are the Goldstone bosons. It is obvious however that such gauging is very restricted on phenomenological grounds, e.g. by proton stability. One possibility however is Pati-Salam unification^{16,15)}

$$SU^{PS}(4) \xrightarrow{\mu_{PS}} SU^c(3) \quad (3.22)$$

with $(q_R q_W q_B \ell)$ and $(Q_R Q_W Q_B L)$ transforming as $4_{PS} = 3_c + 1_c$. Pati-Salam unification can, for example, be applied to the ETC model of Eqs. (3.16). Moreover, it can generally be implemented without violating baryon number. The absence of $K_L \rightarrow \mu e$ does, however, place a lower limit on μ_{PS} , the scale at which $SU^{PS}(4)$ spontaneously breaks to $SU^c(3)$.

To lowest order, the effective four technifermion interaction resulting from Pati-Salam unification is:

$$\begin{aligned} \delta \mathcal{H}_{PS} &= \frac{g_{PS}^2}{2\mu_3^2} (\bar{Q}_a^i \gamma_\alpha L^i) (\bar{L}^j \gamma^\alpha Q_a^j) \\ &+ \frac{g_{PS}^2}{48\mu_1^2} (\bar{Q}_a^i \gamma_\alpha Q_a^i - 3\bar{L}^i \gamma_\alpha L^i) (\bar{Q}_b^j \gamma^\alpha Q_b^j - 3\bar{L}^j \gamma^\alpha L^j), \end{aligned} \quad (3.23)$$

where μ_1 and μ_3 are the masses of the colour singlet and triplet vector bosons in $SU^{PS}(4)/SU^C(3)$. We estimate $\langle \theta | \delta \mathcal{H}_{PS} | \theta \rangle$ by Fierz transforming (3.23) in such a way that all terms become products of two (pseudo-)scalar technifermion bilinears, and by inserting the vacuum between these. We find:

$$\langle \delta \mathcal{H}_{PS} \rangle_\theta = - \frac{3g_{PS}^2}{2\mu_3^2} (\langle \bar{F}F \rangle_0)^2 [\cos 4(\theta_0 + \theta_3) + \cos 4(\theta_0 - \theta_3)]. \quad (3.24)$$

This effective interaction is minimized when

$$\theta_0 = \frac{\pi}{4} (k+k'), \quad \theta_3 = \frac{\pi}{4} (k-k'), \quad (3.25)$$

where k and k' are integers. The vacuum still has a discrete degeneracy, which will be discussed below. For the P^0 and P^3 masses we find:

$$(M^2)_{P^0} = (M^2)_{P^3} = 16 g_{PS}^2 \frac{(\langle \bar{F}F \rangle_0)^2}{\mu_3^2 F_\pi^2}. \quad (3.26)$$

For the $K_L \rightarrow \mu e$ rate we find:

$$\Gamma(K_L \rightarrow \mu e) \simeq \left(\frac{g_{PS}^2}{\mu_3^2} \right)^2 \frac{m_K^5 f_K^2}{16\pi m_s^2}. \quad (3.27)$$

The experimental upper limit B.R. $(K_L \rightarrow \mu e) < 2 \times 10^{-9}$ thus implies

$$\frac{g_{PS}^2}{\mu_3^2} < \left(\frac{1}{3.15 \text{ TeV}} \right)^2$$

for $m_s = 150 \text{ MeV}$. This implies

$$(M^2)_{P^0} = (M^2)_{P^3} \lesssim (1.4 \text{ GeV})^2, \quad (3.28)$$

where we have used

$$\begin{aligned} \langle \bar{F}F \rangle_0 &\simeq \frac{\langle \bar{q}q \rangle_0}{f_\pi^3} F_\pi^3 \frac{1}{8} \left(\frac{3}{n_\pi} \right)^{1/2} = \frac{m_\pi^2 F_\pi^3}{(m_u + m_d) f_\pi} \frac{1}{8} \left(\frac{3}{n_\pi} \right)^{1/2} \\ &\simeq (300 \text{ GeV})^3 \end{aligned} \quad (3.29)$$

The discrete degeneracy Eq. (3.25) of the vacuum implies that when the universe has cooled to about 1 GeV temperature, various domains form with differing values of $\theta_0 = \pi(k+k')/4$ and $\theta_3 = \pi(k-k')/4$. These domains are separated by walls of definite thickness $O(\mu_3 F_\pi / g_{PS} \langle \bar{F}F \rangle_0)$ and mass density per unit surface $O(g_{PS} F_\pi \langle \bar{F}F \rangle_0 / \mu_3)$. It must be emphasized that unless two neighbouring vacua are exactly degenerate, the wall between them will be accelerated towards the deeper vacuum. The pressure that moves the wall is equal to the difference in energy densities. Now, it is easy to show that the discrete symmetries that relate the various vacua Eq. (3.25) hold to all orders of perturbation theory in the Lagrangian which includes the standard interactions, technicolour, ETC and Pati-Salam unification. Indeed, Pati-Salam interactions conserve certain Z_4 symmetries which are precisely the ones that relate the discrete vacua. On the other hand, we have seen that the non-perturbative effects associated with $SU^C(3)$ instantons break the θ_0 (but not the θ_3) rotations. However instantons also conserve certain discrete symmetries due to the fact that θ_{QCD} is cyclic with period 2π . From this one easily concludes that instantons conserve the discrete symmetry:

$$\theta_0 \rightarrow \theta_0 + \frac{\pi}{n} \quad , \quad (3.30)$$

where n is the number of technicolours [cf. Eq. (3.6)]. If n is odd, colour instantons will lower the energy density of the $\theta_0 = 0$ vacuum relative to the $\theta_0 = \pi/4$, $\pi/2$ and $3\pi/4$ vacua by:

$$\Delta \mathcal{H} \simeq F_\pi^2 M_{P^0}^2 \Big|_{\text{instantons}} \quad (3.31)$$

The contribution to the $(\text{mass})^2$ of P^0 due to colour instantons was estimated to be of order $(500 \text{ eV})^2$ [cf. Eqs. (3.4-3.5)]. The resulting outward pressure on the walls surrounding the $\theta_0 = 0$ domains is such that they expand with the speed of light almost immediately after their formation.

4. CONCLUSIONS

To focus more easily on the issues involved in vacuum alignment by the ETC interactions, we have studied a simple ETC toy model. We found that in this model the ETC interactions orient the technicolour condensates in such a way as to give the ordinary fermions -- i.e. the technisinglet fermions which are in common ETC multiplets with the technifermions -- the maximum possible mass. We have argued that this is in fact a general property of vacuum alignment by ETC interactions.

We have also analysed the ETC contributions to PGB masses in one technifamily ETC theories. The lack of any previous analysis of this kind had left considerable uncertainties in the PGB mass spectrum, especially as far as the lightest PGBs are concerned. We have shown that the P^0 , P^3 , $P^\pm$, P_8^0 , P_8^3 , and $P_8^\pm$ PGBs can never acquire any mass from the ETC interactions alone. ETC can contribute in conjunction with the electroweak interactions in order $\alpha_{\text{ETC}}/\sin^2\theta_w \mu_{\text{ETC}}^2$. The resulting contribution to the $(\text{mass})^2$ of the phenomenologically interesting $P^\pm$ is $\lesssim 0(5 \text{ GeV})^2$.

ETC interactions correspond to horizontal gauging among the fermion multiplets (3.8). Vertical gauging is very much constrained by experimental upper limits on rare processes. We have reanalyzed the possibility of Pati-Salam unification. Our numerology produces an upper limit on the masses of P^0 and P^3 of order 1.5 GeV. We point out that after Pati-Salam unification there is still a discrete degeneracy of the vacuum. This discrete symmetry can be partially lifted by colour instantons.

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Table 1

PGB quantum numbers and the masses they acquire from the colour and electroweak interactions in perturbation theory.
Colour instantons give a small $O(\text{keV})$ mass to P^0 .

PGB	Colour	Electric charge	Mass (GeV)	
			TC complex	TC real
$P^0_8, P^3_8, P^\pm_8$	8_C	$0, 0, \pm 1$	245	245
$P^-_{EU}, P^-_{ED}, P^-_{NU}, P^-_{ND}$	3_C	$\frac{5}{3}, \frac{2}{3}, \frac{2}{3}, -\frac{1}{3}$	160	160
$P^\pm$	1_C	± 1	5 to 8	8 to 14
P^0, P^3	1_C	0	0	0
$\pi^{0,\pm}$ (eaten up by $Z^0, W^\pm$)	1_C	$0, \pm 1$	0	0
P_{UU}, P_{UD}, P_{DD}	6_C	$\frac{4}{3}, \frac{1}{3}, -\frac{2}{3}$	-	260
P_{UD}	$\bar{3}_C$	$\frac{1}{3}$	-	140
$P_{NU}, P_{ND}, P_{EU}, P_{ED}$	3_C	$\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{4}{3}$	-	140 to 170
P_{EE}	1_C	-2	-	80
P_{EN}	1_C	-1	-	60
P_{NN}	1_C	0	-	50

APPENDIX

We develop here a model where the correspondence between the direction of vacuum alignment and the direction maximizing the fermion mass matrix is realized in a less trivial way than for the toy model discussed above. We choose the ETC group to be $SU(N)$, broken to the TC group $O(n)$ ($N = n + p$). The left-handed fermion content consists of the fundamental representation (N) (ψ_i , $i = 1, \dots, N$) and its conjugate ($\bar{N}$) (ψ^i , $i = 1, \dots, N$). After breaking, we thus remain with two fundamental representations of $O(n)$ (F_i, F'_i , $i = 1, \dots, n$) and $2p$ technicolour singlets (f_α, f'_α , $\alpha = 1, \dots, p$). The technicolour flavour symmetry is thus broken from $SU(2)$ down to $O(2)$.

We write the condensate in the following matrix form:

$$\begin{pmatrix} \langle F_i \sigma_2 F_j \rangle & \langle F_i \sigma_2 F'_j \rangle \\ \langle F'_i \sigma_2 F_j \rangle & \langle F'_i \sigma_2 F'_j \rangle \end{pmatrix} = M \delta_{ij} . \quad (A.1)$$

M is symmetric and unitary and can be parametrized as

$$M = e^{i\beta} \begin{pmatrix} \cos \theta e^{i\alpha} & \sin \theta \\ \sin \theta & -\cos \theta e^{-i\alpha} \end{pmatrix} . \quad (A.2)$$

Following our criterion, the value chosen by ETC vacuum orientation is given by the $N \times \bar{N}$ channel

$$M_{\text{pref.}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \quad (A.3)$$

We intend to show that this is precisely the direction where the fermion matrix norm is maximal. We first introduce the gauge currents connecting technifermions with fermions

$$J_\mu^{(\alpha i)} = f_\alpha^\dagger \sigma_\mu F_i + F_i'^\dagger \sigma_\mu f'_\alpha \quad (A.4)$$

and their real and imaginary parts

$$J_{\mu}^{(\alpha i)} = J_{1\mu}^{(\alpha i)} + i J_{2\mu}^{(\alpha i)} . \quad (A.5)$$

This allows us to write the 4-fermion interaction giving rise to fermion masses as

$$J_{a\mu}^{(\alpha i)} \lambda_{ab}^{\alpha\beta} J_b^{(\beta i)} , \quad a, b = 1, 2, \quad \alpha, \beta = 1 \dots p, \quad (A.6)$$

where the $2p \times 2p$ matrix

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} \quad (A.7)$$

is real [the currents $J_{a\mu}^{(\alpha i)}$ are real], symmetric ($\lambda_{ab}^{\alpha\beta} = \lambda_{ba}^{\beta\alpha}$) and positive definite since

$$\Lambda = (m^2)^{-1} \quad (A.8)$$

where m^2 is the gauge boson mass matrix.

Developing (A.6) and inserting the parametrization (A.2) for the condensates, we can easily calculate the norm of the fermion mass matrix m_f :

$$\begin{aligned} \text{Tr} (m_f^\dagger m_f) = & 2 \left[\text{Tr} (\lambda_{11} - \lambda_{22})^2 + 4 \text{Tr} (\lambda_{11} + \lambda_{22})^2 \right] \\ & + 8 \sin^2 \theta \text{Tr} (\lambda_{11} \lambda_{22} - \lambda_{12}^2) . \end{aligned} \quad (A.9)$$

We are interested in the sign of the coefficient of $\sin^2 \theta$ which reads

$$\text{Tr} (\lambda_{11} \lambda_{22} - \lambda_{12}^2) = - \text{Tr} (\Lambda \varepsilon)^2 \quad (A.10)$$

where

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} . \quad (A.11)$$

Now, since Λ is a real symmetric positive definite matrix, it can be written as

$$\Lambda = P P^T \quad (\text{A.12})$$

where P is a non singular real matrix. Thus

$$\text{Tr} (\lambda_{11} \lambda_{22} - \lambda_{12}^2) = \text{Tr} (P P^T \epsilon P P^T \epsilon^T) = \text{Tr} (P^T \epsilon P) (P^T \epsilon P)^T \quad (\text{A.13})$$

$$> 0$$

The fermion mass matrix norm is therefore maximal for $\theta = \pi/2$, which corresponds to the value chosen by ETC vacuum orientation.

Figure captions

- Fig. 1 : If one gauges a subgroup G^W of G^{TF} , the G^W gauge interactions will orient the technifermion condensates in such a way as to maximize the overlap (shaded area) of H^{TF} with G^W .
- Fig. 2 : Diagrams, involving external technifermions only, which contribute to $\delta\mathcal{K}_{ETC}$ in lowest order. The thick lines are the technifermions and the wavy lines are the gauge bosons.
- Fig. 3 : A diagram illustrating the mass generating mechanism for the technisinglets (denoted by thin solid lines). The direction of the arrow represents the helicity of the particle. The cross represents the action of the condensate.
- Fig. 4 : The various flavour symmetries of the toy model of Section 2. G^{TF} is the flavour symmetry group of the technifermions which is broken down to H^{TF} by the effects of the condensate Σ . G^{ETF} is the flavour symmetry group of the ETC interactions which is broken to the subgroup H^{ETF} .
- Fig. 5 : Box diagrams giving rise to a four technifermion interaction through the exchange of ordinary fermions.
- Fig. 6 : Fermion loop correction to the Higgs potential.
- Fig. 7 : Technicolour renormalization of the effective four technifermion interaction.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7