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Partial wave expansions of modified one-particle-exchange diagrams

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ABSTRACT

Expressions are derived for the partial wave expansion coefficients of the scattering matrix elements corresponding to one-particle-exchange Feynman diagrams which are modified by form factors or by Reggeization. A few examples are presented, including the application to nucleon-nucleon scattering. An evaluation of the radius of the cloudy bag of the nucleon is presented. A value of $R \approx 0.8$ fm seems to be consistent with the phase-shift analysis.

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I. INTRODUCTION

In practical applications of Feynman diagrams one often needs the partial wave expansions of S-matrix elements corresponding to one-particle-exchange diagrams. In the spinless case the result is rather simple and well known. If we denote by θ the c.m. scattering angle the amplitude is proportional to $1/(z_0 - \cos\theta)$, where z_0 depends on the momenta and masses of the particles involved. In this case the partial wave expansion is proportional to

$$1/(z_0 - \cos\theta) = \sum_{k=0}^{\infty} (2k+1) Q_k(z_0) P_k(\cos\theta), \quad (1.1)$$

and

$$Q_k(z_0) = \frac{1}{2} \int_{-1}^1 [P_k(\cos\theta)/(z_0 - \cos\theta)] d \cos\theta, \quad (1.2)$$

where P_k are the Legendre polynomials and Q_k are the Legendre functions of the second kind. In the case of scattering of particles with spin the amplitudes may involve spherical harmonics (associated Legendre polynomials) or Wigner $d_{\lambda\mu}^j(\theta)$ functions (in the helicity representation) and the expansion coefficients are combinations of the $Q_k(z_0)$ functions of different orders.

The subject of this paper is the partial wave expansion corresponding to one-particle-exchange diagrams modified by form factors at the vertices or by Reggeization. In general this modification will lead to amplitudes proportional to

$$f(t)/(z_0 - \cos\theta), \quad (1.3)$$

where $(-t)$ is the four-momentum transfer squared and f is a related function. In general it will be sufficient to replace the $Q_k(z_0)$ functions appearing in the expansion coefficients (even in the case of the scattering of particles having spin) by the new functions

$$A_g(z_0) = \frac{1}{2} \int_{-1}^1 [f(t) P_g(\cos\theta)/(z_0 - \cos\theta)] d \cos\theta. \quad (1.4)$$

In Sec. II we derive formulae suitable for an evaluation of these integrals and in Sec. IV a few examples are discussed. In Sec. III a simple recurrence relation for the $A_g(z_0)$ is derived for the case where the partial wave expansion of $f(t)$ is known. As an important application of our scheme we discuss in Sec. V the possibility of determining the radius R of the cloudy bag nucleon from the nucleon-nucleon phase-shift analysis. An estimate of $R \approx 0.8$ fm is obtained.

II. GENERAL FORMULAE

In this section we derive general formulae for evaluating the integrals of Eq. (1.4). Let us assume that we can expand

$$f(t) = F(\cos\theta) = \sum_{n=0}^{\infty} a_n \left(\frac{\gamma - \cos\theta}{2} \right)^n. \quad (2.1)$$

Let us denote $x = \cos\theta$ and

$$S_N(x) = \sum_{n=0}^N a_n \left(\frac{\gamma - x}{2} \right)^n. \quad (2.2)$$

In Appendix A we prove that

$$A_n(z) = \frac{1}{2} \int_{-1}^1 \frac{F(x) P_n(x) dx}{z-x} = F(z) Q_n(z) + \frac{1}{2} \sum_{m=n}^{\infty} \left\{ [F(z) - S_m(z)] / \left(\frac{\gamma - z}{2} \right)^{m+1} \right\} C_{mn}(\gamma), \quad (2.3)$$

where

$$C_{mn}(\gamma) = \frac{1}{2} \int_{-1}^1 \left(\frac{\gamma - x}{2} \right)^m P_n(x) dx. \quad (2.4)$$

The advantage of this formula appears when $F(x)$ is regular, then all singularities are retained in the Legendre function $Q_n(z)$ for which the properties are well known.

The integrals which appear in Eq. (2.4) can be evaluated for

$\gamma = 0, \pm 1$:

$$C_{mn}(1) = (-1)^n (m!)^2 / [(m+n+1)! (m-n)!] \quad (2.5)$$

$$C_{mn}(-1) = (-1)^{m+n+1} (m!)^2 / [(m+n+1)! (m-n)!] \quad (2.6)$$

$$C_{mn}(0) = 0 \text{ if } m-n \text{ is odd,} \quad (2.7)$$

otherwise

$$C_{mn}(0) = (-1)^n m! / 4^m \left(\frac{m-n}{2} \right)! \left(\frac{3}{2} \right)^{(n+m)/2}, \quad (2.8)$$

where

$$\left(\frac{3}{2} \right)_i = \frac{3}{2} \left(\frac{3}{2} + 1 \right) \dots \left(\frac{3}{2} + i - 1 \right). \quad (2.9)$$

For $\gamma \neq 0, \pm 1$, the binomial formula can be used in order to evaluate the integral, (2.4). Thus for $\gamma = 0$ we have from Eqs. (2.7), (2.8) and (2.3):

$$A_n(z) = F(z) Q_n(z) + \frac{(-1)^n}{2} \frac{n!}{4^n 0! (3/2)_n} \left[a_{n+1} + a_{n+2} \left(\frac{-z}{2} \right) + a_{n+3} \left(\frac{-z}{2} \right)^2 + \dots \right] + \frac{(n+2)!}{4^{n+2} 1! (3/2)_{n+1}} \left[a_{n+3} + a_{n+4} \left(\frac{-z}{2} \right) + \dots \right]. \quad (2.10)$$

Similarly for $\gamma = 1$

$$A_n(z) = F(z) Q_n(z) + \frac{(-1)^n}{2} \frac{(n!)^2}{0! (2n+1)!} \left[a_{n+1} + a_{n+2} \left(\frac{1-z}{2} \right) + \dots \right] + \frac{[(n+1)!]^2}{1! (2n+2)!} \left[a_{n+2} + a_{n+3} \left(\frac{1-z}{2} \right) + \dots \right] + \dots \quad (2.11)$$

Equation (2.11) can be rearranged in the form

$$A_n(z) = F(z) Q_n(z) + \frac{(-1)^n}{2} \frac{(n!)^2}{(2n+1)!} \sum_{k=1}^{\infty} a_{n+k} B_{k,n}(z). \quad (2.12)$$

Where $B_{k,n}(z)$ are polynomials of degree $k-1$ in z , they satisfy the following recursion relations

$$B_{k+1,n}(z) = \left(\frac{1-z}{2} \right) B_{k,n}(z) + b_{k+1,n} - \delta_{k1}, \quad b_{k+1,n} = b_{k,n} \frac{(n+k)^2}{(2n+k+1)k}, \quad B_{1,n} = 1, \quad b_{1n} = 1, \quad (2.13)$$

where δ_{k1} is the Kroneker delta.

The case $\gamma = z$ is obviously simple. Substituting Eq. (2.1) in Eq. (1.4) we obtain

$$F(z)Q_n(z) + \frac{1}{4} \sum_{k=n+1}^{\infty} d_k \int_{-1}^1 \left(\frac{z-x}{2}\right)^{k-1} P_n(x) dx. \quad (2.14)$$

of Eq. (2.14) can be majorized

$$\left| \int_{-1}^1 \left(\frac{z-x}{2}\right)^m P_n(x) dx \right| < 2 \left(\frac{z+1}{2}\right)^m, \quad (2.15)$$

and the expansion (2.14) will be convergent if

$$\lim_{m \rightarrow \infty} \left| \frac{a_{m+1}}{a_m} \right| < \frac{z+1}{2}. \quad (2.16)$$

III. RECURSION RELATIONS

If the expansion of the modifying function $F(x)$ in terms of Legendre polynomials is known, one can derive recursion relations for the $A_k(z)$ coefficients in the following way. We start with the Christoffel-Darboux formula²

$$\sum_{n=0}^N (2n+1) P_n(x) P_n(z) = (n+1) [P_n(x) P_{n+1}(z) - P_{n+1}(x) P_n(z)] / (z-x) \quad (3.1)$$

and assuming that

$$F(x) = \sum_{m=0}^{\infty} (2m+1) \phi_m P_m(x) \quad (3.2)$$

is known, we multiply Eq. (3.1) by $F(x)$ and integrate both sides from -1 to 1. If we rewrite Eq. (1.4) as

$$A_k(z) = \frac{1}{2} \int_{-1}^1 dx F(x) P_k(x) / (z-x), \quad (3.3)$$

we obtain:

$$\sum_{m=0}^N (2m+1) \phi_m P_m(z) = (n+1) [A_n(z) P_{n+1}(z) - A_{n+1}(z) P_n(z)] \quad (3.4)$$

or

$$A_{n+1}(z) = \frac{1}{P_n(z)} \left\{ P_{n+1}(z) A_n(z) - \frac{1}{n+1} \sum_{m=0}^n (2m+1) \phi_m P_m(z) \right\}, \quad (3.5)$$

and if

$$A_{n+1}(z) = Q_{n+1}(z) + R_{n+1}(z), \quad (3.6)$$

then

$$R_{n+1}(z) = \frac{1}{P_n(z)} \left\{ P_{n+1}(z) P_n(z) + \frac{1}{n+1} - \frac{1}{n+1} \sum_{n=0}^n (2m+1) \phi_m P_m(z) \right\}. \quad (3.7)$$

We can evaluate the first term $A_0(z)$ using the expansion

$$1/(z-x) = \sum_{m=0}^{\infty} (2m+1) Q_m(z) P_m(x). \quad (3.8)$$

Using Eqs. (3.8) and (3.2) we obtain

$$A_0(z) = \frac{1}{2} \int_{-1}^1 dx F(x) / (z-x) = \sum_{m=0}^{\infty} (2m+1) \phi_m Q_m(z). \quad (3.9)$$

Equation (3.9) can be converted to a form similar to Eq. (2.2) by noting that

$$Q_n(z) = P_n(z) Q_0(z) + W_{n-1}(z), \quad (3.10)$$

where $W_{n-1}(z)$ are well known polynomials² of degree $n-1$.

Substituting Eqs. (3.10) and (3.2) in (3.9) we obtain

$$A_0(z) = F(z) Q_0(z) + \sum_{m=0}^{\infty} (2m+1) \phi_m W_{m-1}(z). \quad (3.11)$$

IV. EXAMPLES

A. Example 1

Let us assume

$$F(x) = (y-z)/(y-x), \quad (4.1)$$

then

$$F(x)/(z-x) = 1/(z-x) - 1/(y-x) \quad (4.2)$$

and

$$\frac{1}{2} \int_{-1}^1 dx P_n(x) F(x)/(z-x) = Q_n(z) - Q_n(y). \quad (4.3)$$

Therefore this case will lead us to an expansion for $Q_n(y)$. For this purpose we will use Eq. (2.3) with $y=1$. Expanding Eq. (4.1) we have

$$F(x) = \sum_{n=0}^{\infty} a_n \left(\frac{1-x}{2}\right)^n; \quad a_n = \frac{y-z}{y-1} \left(\frac{-2}{y-1}\right)^n, \quad F(z) = 1, \quad (4.4)$$

and

$$S_n(z) = \sum_{m=0}^n a_m \left(\frac{1-z}{2}\right)^m = 1 - \left(\frac{z-1}{y-1}\right)^{n+1} \quad (4.5)$$

$$[F(z) - S_n(z)] / \left(\frac{1-z}{2}\right)^{n+1} = \left(\frac{-2}{y-1}\right)^{n+1}. \quad (4.6)$$

Substituting Eq. (4.4) in Eq. (2.3) we obtain from Eqs. (4.3) and (2.11)

$$Q_n(y) = - \frac{(-1)^n}{2} \sum_{m=n}^{\infty} \frac{(m!)^2}{(m+n+1)! (m-n)!} \left(\frac{-2}{y-1} \right)^{m+1} \quad (4.7)$$

which is a well known result. The convergence in this case is limited to $y > 3$. This has to be linked with the singularity of $F(x)$ at $x=y$ in Eq. (4.1). One can notice that the expansion (4.4) is uniformly convergent for $-1 \leq x \leq 1$ and for $y > 3$. In this case one can conjecture that the convergence of the expansion (2.4) is related to the uniform convergence of the expansion (2.1) in $-1 \leq \cos\theta \leq 1$ and for $\gamma=1$.

B. Example 2

Reggeization leads to functions $f(t)$ of the form

$$f(t) \simeq e^{\alpha t + \beta} \quad (4.8)$$

where α and β may be complex numbers depending on energies and $-t$ is the four-momentum transfer squared. Equation (4.7) can be rewritten as

$$F(\cos\theta) = e^{A-B\cos\theta} = e^A \sum_{n=0}^{\infty} \frac{(-B)^n}{n!} (\cos\theta)^n, \quad (4.9)$$

where A and B depend on the energies of the particles involved.

We can transform Eq. (4.9) to the form

$$\begin{aligned} F(\cos\theta) &= e^{A-Bz} e^{2B(z-\cos\theta/2)} \\ &= e^{A-Bz} \sum_{n=0}^{\infty} \frac{(2B)^n}{n!} \left(\frac{z-\cos\theta}{2} \right)^n \end{aligned} \quad (4.10)$$

and use Eq. (2.14). One can majorize the integrals of Eq. (2.14) as in Eqs. (2.15) and (2.16), and hence one can see that for Eq. (4.10) the expansion (2.14) will always be convergent. In practice the use of Eq. (2.10) or Eq. (2.11) would be more convenient. Example 2 can be treated efficiently by using the techniques of Sec. III. We can use the expansion

$$e^{-B\cos\theta} = \sum_{n=0}^{\infty} (-1)^n (2n+1) f_n(B) P_n(\cos\theta), \quad (4.11)$$

where

$$\begin{aligned} f_n(B) &= \sqrt{\pi/2z} I_{n+1/2}(B) \\ &= (i)^n j_n(iB) \text{ for } -\pi \leq \arg B \leq \frac{\pi}{2} \end{aligned} \quad (4.12)$$

and I_m and j_n are the Bessel function of the second kind and the spherical Bessel function, respectively. The functions $f_n(B)$ can be easily calculated from the recursion relations

$$f_{n-1}(B) - f_{n+1}(B) = (2n+1) f_n(B)/B \quad (4.13)$$

with

$$f_0(B) = \sinh B/B \quad (4.14)$$

and

$$f_1(B) = \cosh B/B - \sinh B/B^2 \quad (4.15)$$

C. Example 3

In the cloudy bag model of the nucleon the following form factor emerges for the $NN\pi$ vertex¹

$$F(qR) = 3j_1(qR)/(qR) = 3[\sin qR/(qR) - \cos qR]/(qR)^2, \quad (4.16)$$

where R is the bag radius and q the momentum transfer. For the one-pion-exchange diagram this form factor will appear in two vertices. Therefore, the pion propagator will be modified by

$$\begin{aligned} [F(qR)]^2 &= 9[(1-\cos 2qR)/2q^2 R^2] - \sin 2qR/(qR) \\ &\quad + (1+\cos 2qR)/2/(qR)^4 \\ &= 72 \sum_{n=0}^{\infty} \frac{(2n+2)(2n+5)(-4)^n}{(2n+6)!} (qR)^{2n} \\ &= 72 \sum_{n=0}^{\infty} \frac{(2n+2)(2n+5)(-8p^2 R^2)^n}{(2n+6)!} (1-\cos\theta)^n. \end{aligned} \quad (4.17)$$

In the last stage we have used the substitution

$$q^2 = 2p^2(1-\cos\theta), \quad (4.18)$$

where p is the c.m. momentum and θ the c.m. scattering angle. Now

Eq. (4.17) can be used in the framework of Eq. (2.11) to calculate the partial wave expansions of the one-pion-exchange diagram modified by the form factor squared. These partial wave expansions are presented in Sec. V.

V. THE MODIFIED ONE-PION-EXCHANGE DIAGRAM

In this example we present the partial waves of the modified nucleon-nucleon one-pion-exchange diagram.³ We do it in order to demonstrate a case of scattering of particles with spin where the coefficients of the partial waves are given by combinations of integrals as in Eq. (1.4). Let us introduce the notations

$$\gamma_J(z_0) = A_J(z_0) - \frac{J+1}{2J+1} A_{J+1}(z_0) - \frac{J}{2J+1} A_{J-1}(z_0) \quad (5.1)$$

$$\eta_J(z_0) = -A_J(z_0) + \frac{J}{2J+1} A_{J+1}(z_0) + \frac{J+1}{2J+1} A_{J-1}(z_0), \quad (5.2)$$

where $A_J(z_0)$ are integrals as given by Eq. (1.4),

$$z_0 = 1 + \mu^2/(2p^2), \quad (5.3)$$

where μ is the pion mass and p the c.m. momentum of the nucleons. Let us introduce the abbreviated notation

$$\alpha = \frac{g^2}{4\pi} \langle \vec{\tau}_1 \cdot \vec{\tau}_2 \rangle / (4E_p), \quad (5.4)$$

where g is the pion nucleon pseudoscalar coupling constant ($g^2/4\pi \simeq 14.5$), $\langle \vec{\tau}_1 \cdot \vec{\tau}_2 \rangle = -3$ or 1 for states with isotopic spin 0 or 1 , respectively, and E_p is the c.m. total energy of one nucleon.

Using the bar phase-shift notation $\bar{\delta}_{LJ}$, the T -matrix elements of the modified one-pion-exchange diagram become

$$T_J = \frac{1}{2i}(e^{2i\bar{\delta}_{J-1}} - 1) = -\alpha \gamma_J(z_0)$$

$$T_{J,J} = \frac{1}{2i}(e^{2i\bar{\delta}_{JJ-1}} - 1) = -\alpha \eta_J(z)$$

$$T_{J-1,J} = \frac{1}{2i}(\cos 2\bar{\epsilon}_J e^{2i\bar{\delta}_{J-1,J-1}} - 1) = \alpha \left[\frac{J}{2J+1} \gamma_J(z_0) + \frac{J+1}{2J+1} \eta_J(z_0) \right]$$

$$T_{J+1,J} = \frac{1}{2i}(\cos 2\bar{\epsilon}_J e^{2i\bar{\delta}_{J+1,J-1}} - 1) = \alpha \left[\frac{J+1}{2J+1} \gamma_J(z_0) + \frac{J}{2J+1} \eta_J(z_0) \right]$$

$$T^J = \frac{1}{2} \sin 2\bar{\epsilon}_J \exp \left[i(\bar{\delta}_{J-1,J} + \bar{\delta}_{J+1,J}) \right] = \alpha \sqrt{J(J+1)} \times [\gamma_J(z_0) - \eta_J(z_0)] / (2J+1). \quad (5.5)$$

To the lowest order in the coupling constant

$$\bar{\delta}_J \simeq T_J, \quad \bar{\delta}_{LJ} \simeq T_{LJ} \text{ and } \bar{\epsilon}_J \simeq T^J. \quad (5.6)$$

The approximation (5.6) (which satisfies the unitarity condition) can be applied only for sufficiently small phase shifts when the result is more or less independent of the unitarization scheme.

In order to get an insight into the form factor of the πNN vertex one should consider only partial waves for which the two and more pion exchanges give very small contributions to the phase shifts. One can not go to values of L which are too high, because the effect of the form factor becomes very small, and also the experimental determination of the phase shifts is inaccurate.⁴ The two-pion-exchange contributions for the peripheral partial waves were estimated by several groups.⁵ On the basis of their results it was impossible to select partial waves satisfying the criteria mentioned above. This is understandable as we expect that the effect of the two-pion exchanges should not be smaller than the effect of the form factor. Therefore, we had to find a more refined method.

We have looked for partial waves for which the two-pion-exchange contributions are well approximated by the box diagram. For these partial waves we have used the quasipotential equation approach³ by which the ladder diagrams are generated in an approximate way. We have

selected only the partial waves for which the first iterated Born term was a good approximation to the box diagram. This led us to consider the 3D_2 , ϵ_3 , 3G_3 , 3G_4 partial waves as the best candidates. Next we used the one-pion-exchange potential modified by the form factor of Eq. (4.16). The resulting 3D_2 phase shift is displayed in Fig. 1 showing preference for a bag radius of $R \approx 0.8$ fm. The determination of the phases 3G_3 and 3G_4 in the phase-shift analysis is still not accurate. In Fig. 2 we display the ϵ_3 phase shift. The result should be interpreted with caution as the determination of the ϵ_3 phase shift is quite unstable in spite of the relatively small statistical error-bars. Nevertheless, the result $R \approx 0.8$ fm seems to be adequate for both 3D_2 and ϵ_3 phases.

In choosing the 3D_2 , ϵ_3 , 3G_3 and 3G_4 partial waves we considered only uncorrelated and correlated two-pion exchanges. At the present stage of the art it is impossible to calculate the contributions of exchanges of a higher number of mesons. Therefore, we cannot estimate how sensitive the result $R \approx 0.8$ fm is to the inclusion of 3 or more uncorrelated (irreducible) and correlated pion exchanges.

VI. SUMMARY AND CONCLUSIONS

General formulae have been derived in Sec. II [Eqs. (2.2), (2.10) and (2.11)] which allow an efficient expansion of modified one-particle-exchange diagrams. The coefficients of the partial wave expansions consist of two parts. One part is proportional to the Q_L function which would have been obtained from the one-particle-exchange diagram without modifications. The second part includes an expansion depending only on the modifications. This expansion usually converges fast if the modification is done through analytic functions. In Sec. III an

efficient scheme is developed for the case when the partial wave expansion of the modification is known. For this case we derive a simple recurrence relation, Eq. (3.7), for the partial wave coefficients of the modified diagrams. The Reggeized amplitude belongs to that category as is demonstrated in the example 2 of Sec. IV. As a further application of our scheme (perhaps the most important in our paper) we give an estimate of the radius of the cloudy bag nucleon. This particular case is considered first in example 3 of Sec. IV, and further in Sec. V by getting results via a pseudopotential equation. Our results are consistent with a bag radius of about 0.8 fm.

APPENDIX A

Here we prove Eq. (2.2). For this purpose we rewrite $A_n(z)$ as

$$A_n(z) = \frac{1}{2} \int_{-1}^1 \frac{[F(z) + [F(x) - F(z)]]}{z-x} dx. \quad (A.1)$$

We use Eq. (1.2) and substitute Eq. (2.1) in Eq. (A.1) to obtain

$$\begin{aligned} A_n(z) &= F(z)Q_n(z) + \frac{1}{2} \int_{-1}^1 \sum_{m=0}^{\infty} a_m \left[\left(\frac{y-x}{2} \right)^m - \left(\frac{y-z}{2} \right)^m \right] \frac{P_n(x)dx}{z-x} \\ &= F(z)Q_n(z) + \frac{1}{4} \int_{-1}^1 \sum_{m=1}^{\infty} a_m \left[\left(\frac{y-x}{2} \right)^{m-1} + \left(\frac{y-x}{2} \right)^{m-2} \times \right. \\ &\quad \left. \times \left(\frac{y-z}{2} \right) + \dots + \left(\frac{y-z}{2} \right)^{m-1} \right] P_n(x)dx. \end{aligned} \quad (A.2)$$

After some rearrangements of the sums in the integrand, we obtain,

$$\begin{aligned} A_n(z) &= F(z)Q_n(z) + \frac{1}{4} \int_{-1}^1 \sum_{m=1}^{\infty} \left\{ [F(z) - S_m(z)] \left(\frac{y-z}{2} \right)^{m+1} \right\} \times \\ &\quad \times \left(\frac{y-x}{2} \right)^m P_n(x)dx. \end{aligned} \quad (A.3)$$

We should note² that if $R_m(x)$ is a polynomial of degree m , then

$$\int_{-1}^1 R_m(x)P_n(x)dx = 0 \text{ if } m < n; \quad (A.4)$$

therefore, the summation in (A.3) should start with $m = n$.

REFERENCES

- 1S. Thberge, A.W. Thomas and G.A. Miller, Phys. Rev. D 22, 2838 (1980).
- 2E.D. Rainville, *Special Functions* (McMillan, New York, 1960).
- 3R.H. Thompson, A. Gersten and A.E.S. Green, Phys. Rev. D 3, 2069 (1971).
- 4R.A. Arndt and B.J. VerWest, in *Proceedings of the Ninth International Conference on the Few-Body Problem*, Eugene, Oregon, August 1980.
- 5R. Vinh Mau *et al.*, Phys. Letters 44B, 1 (1973); A.D. Jackson, D.O. Riska and B. VerWest, Nucl. Phys. A 249, 397 (1975).

FIGURE CAPTIONS

1. The $^3\text{D}_2$ bar phase shift evaluated for different bag radii R . The experimental phase shifts are from Ref. 4.
2. The ϵ_3 bar phase shift evaluated for different bag radii R . The experimental phase shifts are from Ref. 4.

Fig. 1

Fig. 2