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ABSTRACT

A systematical investigation of $1s\sigma$ -ionization probabilities and δ -electron production rates in heavy ion collisions is performed. We present their charge dependence from $Z_1+Z_2=134$ up to $Z_1+Z_2=184$ and compare our results with recent experimental data. The influence of electron screening on vacancy formation and the facility to use δ -electron spectra in collisions with nuclear sticking as an 'atomic clock' are also discussed. Strong magnetic fields ($|B_{\max}| \sim 10^{16} \text{G}$) in heavy ion collisions are expected to cause spin polarizations of the $1s\sigma$ -vacancies and emitted δ -electrons in the order of 5-10%.

1. INTRODUCTION

Collisions of very heavy ions with bombarding energies close to the Coulomb barrier offer us the possibility to perform a spectroscopy of electronic states in transient superheavy systems within a charge range of $100 \leq Z \leq 190$ ($92\text{U}+98\text{Cf}$).

^{x)} Talk presented by G. Soff.

The pivotal question to answer is whether the binding energy of the strongest bound electronic state can reach or exceed twice the electron rest mass.

An important task in this connection is the investigation of electron excitation processes in superheavy systems. Two possible excitation mechanisms are visualized in Fig. 1. As an example we consider the schematic level structure for a head-on collision of Pb + Cm with a total charge $Z = 178$ at 5.9 MeV/u bombarding energy. Binding energies of the adiabatic $1s\sigma$ - and $2s\sigma$ - states¹ are displayed versus the internuclear separation R . The turning point between incoming and outgoing trajectory is given by $R_{\min} = 17$ fm.

A typical one-step excitation process is the direct ionization from the $1s\sigma$ - state to the positive continuum. In addition the multi-step processes play an important role. Actually they form the dominant part of inner shell vacancy production probabilities. An example for a two-step process is depicted in Fig. 1, where a $2s\sigma$ - electron is ionized to the continuum. The remaining hole is filled by the $1s\sigma$ - electron which leaves a vacancy in the K-shell. Since initial and final state of both processes are indistinguishable, they have to be treated coherently. This is achieved by solving coupled channel equations for the electron occupation amplitudes of the quasimolecular adiabatic states.

Fig. 1. Binding energies of the $1s\sigma$ and $2s\sigma$ state versus internuclear separation for the Pb+Cm system. One-step and two-step electron excitation processes are indicated qualitatively. On the right hand side characteristic numbers describing the collision are given.

A typical feature in collisions of very heavy ions is the shrinkage of the $1s\sigma$ - wavefunction at shortest internuclear distances. Sticking to our example Pb + Cm at $E_{lab} = 5.9$ MeV/u we find a radial expectation value for the $1s\sigma$ orbital of about 120 fm at the distance of closest approach.

Due to this extreme localization (within the Compton wavelength of an electron, $\lambda_e = 386$ fm) transfer of high momenta components to the strongly bound electrons resulting from the nuclear motion becomes possible. Therefore δ - electrons up to kinetic energies of 2 MeV can be observed experimentally², whereas the classical allowed maximum energy transfer to an electron at rest is about 10 keV.

After a short theoretical survey in Chapter 2 we present the systematics of the $1s\sigma$ - binding energies, K- vacancy formation and δ - electron production (Chapter 3) with respect to the total nuclear charge Z . In Chapter 4 the experimental vacancy production rates are compared with our coupled channel results. The influence of an initially unsharp Fermi surface and electron screening on K-hole production is discussed in Chapter 5.

We compare measured δ - electron cross sections of superheavy systems with our calculations in Chapter 6.

In collisions where the combined nuclear charge $Z_1 + Z_2$ exceeds 174 the $1s\sigma$ - state is expected to 'dive into' the negative energy continuum. The very strong time-varying Coulomb field which causes this effect amounts to about 10^{19} V/cm. Furthermore the nuclear motion is a source of an extremely strong magnetic field in the order of 10^{16} G. The consequences of this fact which causes a partial polarization of K-holes and emitted δ -electrons³ are discussed in Chapter 7.

2. THEORETICAL RUDIMENTS

In this chapter we summarize the essential equations where our calculations are based on. We emphasize that in superheavy quasi-molecules a full relativistic treatment of the electron motion is absolutely necessary since the combined charge $Z=Z_1+Z_2 \gtrsim 1/\alpha$ and the electron binding energies become comparable with the electron rest mass or even exceed it.

In a complete adiabatic picture electronic binding energies and wave functions therefore have been evaluated by the stationary solutions of the two centre Dirac equation ($\hbar=c=1$)

$$[\vec{\alpha} \cdot \vec{p} + \beta m_e + V_{TC} - E_n(R(t))] \phi_n(r, R) = 0 \quad (1)$$

with

$$V_{TC} = -Z_1 e^2 / |\vec{r} - \vec{R}/2| - Z_2 e^2 / |\vec{r} + \vec{R}/2| \quad (2)$$

After expanding the two-centre potential of eq.(2) into multipoles and restricting ourselves to the dominant monopole part V_0 we obtain two first-order coupled differential equations

$$\begin{aligned} d/dr u_1 &= -\kappa/r u_1 + (E + m - V_0) u_2, \\ d/dr u_2 &= -(E - m - V_0) u_1 + \kappa/r u_2, \end{aligned} \quad (3)$$

where $u_1 = r \cdot g(r)$ and $u_2 = r \cdot f(r)$.

The radial dependence of the upper and lower components of a $1s\sigma$ -wave function for a combined nuclear charge $Z=169$ ($Z\alpha > 1$) is shown in Fig. 2. A striking feature is the almost equal absolute amplitude of both components between 10 and 100 fm emphasizing the necessity to perform relativistic calculations.

If we want to describe the dynamical evolution of an electron involving two colliding nuclei we thus have to solve the time-dependent two-centre Dirac equation

$$i\partial/\partial t \phi_i(\vec{R}(t)) = H_{TCD}(\vec{R}(t)) \phi_i(\vec{R}(t)) \quad (4)$$

H_{TCD} is the relativistic two-centre Hamiltonian which depends sensitively on the internuclear separation $R(t)$. For this reason it is

Fig. 2. Radial components of the $1s\sigma$ -wave function for a total nuclear charge $Z = 169$.

useful to expand the total wave function Φ_i into Born-Oppenheimer states ϕ_n which are represented by the stationary molecular states of eq. (1). The expansion reads

$$\Phi_i(t) = \sum_j a_{ij}(t) \phi_j(\vec{R}(t)) \exp(-i\chi_j(t)) , \quad (5)$$

where the sum includes an integration over continuum states with positive and negative energies. The phase factors χ_j are chosen as

$$\chi_j(t) = \int^t dt' \langle \phi_j(\vec{R}(t')) | H_{TCD}(\vec{R}(t')) | \phi_j(\vec{R}(t')) \rangle . \quad (6)$$

Inserting the expansion of eq. (5) into eq. (4) and projecting with stationary eigenfunctions of eq. (1) we obtain the following set of first-order coupled differential equations for the occupation amplitudes $a_{ij}(t)$

$$\dot{a}_{ij}(t) = - \sum_k a_{ik}(t) \langle \phi_j | \partial/\partial t | \phi_k \rangle \exp \{+i(\chi_j - \chi_k)\} . \quad (7)$$

The time derivative operator can be expressed in terms of a radial and a rotational coupling

$$\partial/\partial t \rightarrow \dot{\vec{R}} \cdot \partial/\partial \vec{R} - i\dot{\omega} \cdot \vec{J} . \quad (8)$$

Our calculations are restricted to radial couplings only.

Concerning the many electron problem we first state that the full time-dependent wave functions $\Phi_i(t)$ form a complete and orthogonal basis set at each time t . This can be concluded from the unitarity of the time-development operator

$$\langle \Phi_j(t) | \Phi_i(t) \rangle = \delta_{ij} \quad (9)$$

or equivalently

$$\sum_i a_{ki}^* a_{li} = \sum_i a_{ik}^* a_{il} = \delta_{kl} . \quad (10)$$

Therefore the set $\Phi_i(t)$, already containing the dynamical excitations, can be used as a basis for solving the many-particle problem. To do this, one can expand the total wave function in a basis of many-electron configurations which may be represented by Slater determinants of the single particle basis functions ϕ_i . The amplitude for exciting a final configuration starting from a given initial configuration turns out to be just the determinant of the corresponding single-particle amplitudes $a_{ij}(t)$.

Therefore the number of physical particles p or holes q in a particular level can be deduced completely from the single-particle amplitudes

$$N_p = \sum_{r < F} |a_{rp}|^2 \quad \text{for } p > F , \quad (11)$$

$$N_q = \sum_{r>F} |a_{rq}|^2 \quad \text{for } q < F. \quad (12)$$

F denotes the Fermi surface of occupied states. For the number of correlated particle - hole pairs $N_{p,q}$ one finds

$$N_{p,q} = N_p \cdot N_q + \left| \sum_{r<F} a_{rp}^* a_{rq} \right|^2. \quad (13)$$

This formula should be applied to analyze experiments like those presently performed for coincidences between electrons and $1s\sigma^-$ vacancy formation. Eq. (13) holds also for particle-particle or hole-hole correlations if the sign of the second term is inverted.

The differential probability for the emission of an $Es\sigma^-$ δ^- electron with energy E and definite spin projection measured in coincidence with a $1s\sigma^-$ vacancy having the same spin projection is given by

$$\begin{aligned} (dP/dE)_{\text{coinc}} = & \sum_{r<F} |a_{r,Es\sigma}|^2 \left(1 - \sum_{r'<F} |a_{r',1s\sigma}|^2 \right) \\ & + \left| \sum_{r<F} a_{r,Es\sigma}^* a_{r,1s\sigma} \right|^2. \end{aligned} \quad (14)$$

In coincidence experiments with a particle p and an additional hole q a problem arises from accidental coincidences since in measurements there is usually no distinction between a δ^- electron with spin up and a $1s\sigma^-$ vacancy with spin down.

In order to obtain the total number of particle-hole pairs we have to perform a summation before comparing with experimental data

$$N_{p,q}^{\text{tot}} = 2 (N_{p,q} + N_p \cdot N_q), \quad (15)$$

where the last term describes accidental coincidences.

3. SYSTEMATICAL INVESTIGATION OF K-HOLE FORMATION AND δ^- -ELECTRON PRODUCTION

In this chapter we present our theoretical results for the inner-shell ionizations and δ^- electron production probabilities. An essential ingredient hereby is the precise knowledge about the $1s\sigma^-$ binding energy and wave function. Fig. 3a shows the dependence of $1s\sigma^-$ total energy on the internuclear separation R of two

Fig. 3. (a) Binding energies of the $1s\sigma$ - state in dependence on the internuclear separation R for various total charges Z . - (b) Corresponding radial expectation values $\langle r_{1s\sigma} \rangle$.

colliding nuclei in a semilogarithmical representation. The different curves belong to different united charges Z_1+Z_2 denoted by $Z=134$ to $Z=184$. We observe that the binding energy increases with decreasing nuclear distance, but it increases the more the heavier the system is. For the system U+U with united charge $Z=184$ the binding energy rises by more than 1 MeV in the considered range of internuclear separation. Here we enter the truly relativistic region where relativistic effects are no longer a small perturbation, but dominate the behaviour of inner-shell electrons, cf. Fig. 2.

In Fig. 3b we have drawn the radial expectation value $\langle r_{1s\sigma} \rangle$ of the $1s\sigma$ - level in dependence on the internuclear separation R for combined charges $Z=134$ up to $Z=184$. For fixed distance R the $1s\sigma$ -wave function shrinks the heavier the systems are.

Binding energies as well as the radial expectation value $\langle r_{1s\sigma} \rangle$ play an important, but contrary role when calculating K-hole formation and δ - electron production. Whereas a strong increase in the binding energy hinders, e.g., the ionization of an electron from the K-shell - thus producing a K-hole and, eventually, a δ - electron - the strong decrease of $\langle r_{1s\sigma} \rangle$ leads to high Fourier frequencies in the pulse spectrum - thus stimulating K- shell ionization. Therefore one can predict a maximum for K-hole production rates, $P_{1s\sigma}$, around $Z \sim 170$ (Fig. 4).

Subsequently we investigate this effect more systematically for K-hole formation as well as for δ - electron production rates. For this purpose we evaluated K-vacancy production rates for various combined nuclear charges $Z=134$ up to $Z=184$. We present coupled

Fig. 4. The dependence of $P_{1s\sigma}(b=0)$ on total nuclear charge Z_1+Z_2 in superheavy systems. Collisions with a distance of closest approach $R_{\min} = 15, 20$, and 25 fm are considered.

channel calculations stressing the Z -dependence of $P_{1s\sigma}(b)$ in Fig. 5. Part a shows K-hole production rates for charges $Z \leq 164$. We observe an increasing slope with rising binding energy. Vacancy formation increases especially for small impact parameters as a consequence of the sharp localized $1s\sigma$ -state. The latter effect allows the transfer of high Fourier frequencies to the K-shell originating from the nuclear motion, thus leading to increased K-hole production.

In Fig. 5b combined nuclear charges beyond $Z=164$ are considered. In this region the ionization probabilities decrease due to the extremely strong binding energy which amounts nearly twice the electron rest mass, reflecting the preceding remarks concerning Fig. 4.

Another observable quantity where the influence of $1s\sigma$ -binding energies and the radial expectation value $\langle r_{1s\sigma} \rangle$ can be demonstrated is the differential emission probability of δ -electrons as a function of the kinetic electron energy. In Fig. 6 δ -electron cross sections are displayed for systems with united charge $Z=134$ up to $Z=184$. The δ -electrons have to be measured in coincidence with $1s\sigma$ -vacancies according to eq. (14). In all calculations we chose a Fermi level of $F=3s\sigma$ and a bombarding energy $E_{\text{lab}}=4.7$ MeV/u. Asymmetric systems are considered.

In the region of united charge $Z < 164$ the δ -electron cross section rises with increasing Z . The high-kinetic part with $E_e > 500$ keV grows faster than the low-kinetic one. This is due to the stronger localization of the $1s\sigma$ -orbit which allows for high momenta transfer as described above. But if the united charge exceeds $Z \sim 170$

Fig. 5. Number of created $1s\sigma$ -vacancies versus impact parameter b . The different curves belong to different total charges Z . Asymmetric systems are assumed. Part (a) for $Z \leq 164$, part (b) for $Z \geq 164$.

the influence of the rapidly increasing $1s\sigma$ -binding energy becomes dominant. Thus the emission cross section even diminishes which is shown for $Z=184$.

Therefore it would be highly desirable to measure more systematically K-hole formation and δ -electron cross sections in dependence on the united charge Z in order to verify the interplay of strong binding and sharp localization in ionization processes.

4. VACANCY FORMATION IN COMPARISON WITH EXPERIMENTAL DATA

Now we turn to the discussion of K- and L-hole production in heavy and superheavy systems. According to eq. (12) we calculated the number of created $1s\sigma$ -vacancies per collision for the systems Xe+Au at $E_{lab}=4.7$ MeV/u and Xe+Pb at different bombarding energies. In our calculations we chose $F=3$ corresponding to the last bound state occupied initially.

The results for Xe+Au are plotted in comparison with experimental data of D. Liesen et al.^{4,5} in Fig. 7a. We find fair agreement up to impact parameters $b \lesssim 90$ fm. Beyond this value the measured data are underestimated.

Fig. 6. δ - electron distributions in coincidence with $1s\sigma$ - vacancies versus the kinetic electron energy for different combined charges Z .

Similar agreement is achieved for the dependence on the bombarding energy E at fixed impact parameters b which is shown in Fig. 7b.

A system of comparable combined charge is Xe+Pb. Experimental data of Anholt et al.^{6,7} are in remarkable agreement with our coupled channel results for $P_{1s\sigma}(b)$ at $E_{lab}=4.6$ MeV/u (solid line in Fig. 8a). At $E_{lab}=7.2$ MeV/u we only find fair agreement for small impact parameters. Furthermore K-vacancy production within first order time-dependent perturbation theory is presented (dashed line). In comparison to our coupled channel results these values are off by a factor 3 - 5, stressing that it is indispensable to take into account multi-step excitations. Scaling laws which are based on simplifying approximations within the framework of first-order time-dependent perturbation theory are useful to determine the dependence of $1s\sigma$ -vacancy production on kinematical parameters $\{b, E_{ion}\}$. They fail, however, in predicting the measured data quantitatively.

A systematical investigation of vacancy formation should also comprise $2p_{1/2}\sigma$ - hole measurements which are rarely available up to now. In Fig. 8b we also display $2p_{1/2}\sigma$ - vacancy production rates for the system Xe+Pb versus the impact parameter at $E_{lab}=3.6$ MeV/u

Fig. 7. (a) Number of created $1s\sigma$ -vacancies per Xe+Au collision at 4.7 MeV/u laboratory energy as a function of impact parameter b . Also shown are experimental data of D. Liesen et al.^{4,5}. (b) Bombarding energy dependence of the number of created $1s\sigma$ -vacancies in the Xe+Au system⁵ for some specific impact parameters.

choosing $F=3p_{1/2}\sigma$. The prevailing difference from K-vacancy rates is a drastical decrease of the slopes which stems - beside other things - from the increased spatial-extended $2p_{1/2}\sigma$ -orbital. Since L-shell electrons have a smaller binding energy the concerning vacancy yields increase at fixed bombarding energy. The dashed lines indicate the effect of electron screening and are calculated by means of the model discussed in Chapter 5.

One of the heaviest systems accessible to experimental investigation concerning $1s\sigma$ -ionization is Pb+Cm with a combined nuclear charge of $Z_1+Z_2=178$. In Fig. 9a we compare our results with experimental data of D. Liesen et al.⁸. Calculations including multi-step excitation processes are indicated by the full line. $F=4s\sigma$ is assumed. In order to estimate the effect of electron screening we use reduced binding energies adjusted to the Hartree-Fock values^{9,10} known in the limits $R \rightarrow 0, \infty$ while retaining the coupling matrix elements. A considerable increase of $P(b)$ is found, shown by the dashed lines in Fig. 9. At $E_{lab}=5.9$ MeV/u the results with reduced binding energies are in fair agreement with experiment except for small impact parameters and values $b \geq 90$ fm. Calculations for Pb+Cm including matrix elements which are determined by Hartree-Fock wave functions are in progress. However, it is not expected that the new results will deviate drastically from the corrected values presented here.

Fig. 8. (a) $1s\sigma$ - vacancy production rates in a Xe+Pb collision at 4.6 MeV/u and 7.2 MeV/u versus impact parameter b (solid lines). Also indicated is a result obtained with in first-order time-dependent perturbation theory (dashed line). The experimental data are taken from Anholt et al.^{6,7}. (b) The same system investigated at $E_{lab}=3.6$ MeV/u, $2p_{1/2}\sigma$ - and $1s\sigma$ - vacancy production rates are compared. Results including electron screening are indicated by dashed lines.

Fig. 9b shows $P_{1s\sigma}(b)$ for the smaller bombarding energy $E_{lab}=3.6$ MeV/u. Beside a consistent underestimation of the ionization probability for small impact parameters and values of $b \geq 90$ fm (compare Fig. 7a with Fig. 9a) we now find a clear discrepancy between calculated and measured data. In order to clarify this deviation it would be desirable to obtain also δ - electron spectra in coincidence with characteristic Cm - K - X- rays in dependence on b . Although screening effects cannot account for this discrepancies it is necessary to incorporate them into future calculations. For a further discussion of screening effects we refer to Chapter 5.

Another essential point to be considered is the influence of two-centre effects like translation factors and level crossings. If we want to incorporate them it is indispensable to evaluate the exact two-centre continuum states.

Fig. 9. (a) The system Pb+Cm is investigated concerning K-holes at 5.9 MeV/u lab energy. The dashed-dotted line represents results where the binding energies have been modified to include electron screening corrections. (b) The same as in part a for 3.6 MeV/u. Experimental data are taken from Ref. 8.

5. THE EFFECTS OF AN INITIALLY STATISTICAL OCCUPATION AND OF ELECTRON SCREENING ON VACANCY PRODUCTION

Comparing the measured K-hole production rates of the system Pb+Cm at 5.9 MeV/u with our theoretical predictions, represented by the solid line in Fig. 10a, one observes an overall slight underestimation of the experimental data. At lower bombarding energies (e.g., $E_{lab}=4.7$ MeV/u) the discrepancies between experiment and theory increase especially for small impact parameters.

We investigated the question, how a change in the initial electron distribution might modify the shape of the K-shell vacancy production.

For this purpose we assume the initial state $|F_n\rangle$ to be a mixture of many-body states which are occupied¹¹ with a possibility w_n instead of being clearly distinguishable states filled up to a certain Fermi level. We express the number of particles in a level

p by means of the creation and annihilation operators for particles

$$N_p = \sum_n w_n \langle F_n | \hat{b}_p \hat{b}_p^\dagger | F_n \rangle = \sum_n w_n \sum_{r \in F} |a_{rp}|^2 \quad (16)$$

where a_{rp} are the expansion coefficients used in eq. (7). The occupation probabilities w_n can be taken from statistical mechanics¹² if we demand our system to be in a thermal equilibrium at $T = \text{const}$

$$w_i = \frac{\exp \{-\beta (E_i - \mu N_i)\}}{\sum_n \exp \{-\beta (E_n - \mu N_n)\}} = \rho_i / Z \quad (17)$$

Here $\beta = 1/kT$ is the inverse temperature, μ the chemical potential and N_i the number of particles in the many-body level i . The grand distribution function ρ_i is expressed by

$$\rho_i = \langle F_i | \hat{\rho} | F_i \rangle, \quad \hat{\rho} = \exp \{-\beta (\hat{H} - \mu \hat{N})\} \quad (18)$$

Thus

$$w_i = \langle F_i | \hat{\rho} | F_i \rangle / (\text{tr} \hat{\rho}) \quad (19)$$

since $Z = \text{tr} \hat{\rho}$.

Requiring a system of noninteracting fermions, i.e. $\{\hat{b}_k, \hat{b}_j\} = \delta_{kj}$, for which we denote the grand distribution function as ρ_i^0 , one obtains

$$\rho_i^0 = \langle F_i | \exp \{-\beta \sum_k (\epsilon_k - \mu) n_k^i\} | F_i \rangle \quad (20)$$

n_k^i has the meaning of an occupation factor for the k^{th} level in the many-body state i (0 or 1) and μ now indicates the Fermi energy. Using eqs. (19) and (17) we arrive at

$$w_i = \frac{\prod_k \exp \{-\beta (\epsilon_k - \mu) n_k^i\}}{\prod_k (1 + \exp \{-\beta (\epsilon_k - \mu)\})} \quad (21)$$

For the number of particles and holes in the levels p and q one obtains¹¹

$$N_p = \sum_r |a_{rp}|^2 (1 + \exp \{\beta (\epsilon_r - \mu)\})^{-1} \quad (22)$$

$$N_q = \sum_r |a_{rq}|^2 (1 + \exp \{-\beta (\epsilon_r - \mu)\})^{-1} \quad (23)$$

Eqs. (22) and (23) reproduce the familiar results¹³ for vanishing temperature $T = 1/k\beta \rightarrow 0$. These expressions are restricted to noninteracting fermions but remain valid in cases where the interaction can be described by a mean potential.

Using eq. (23) we investigated the behaviour of K-vacancy production rates for the system Pb+Cm at bombarding energies of $E_{lab} = 5.9$ MeV/u and 4.7 MeV/u at different temperatures T .

The full line in Fig. 10a indicates calculations of Refs.13,14; although including multi-step excitation processes the experimental data are still underestimated. 'Switching on' temperature a general increase of the K-vacancy production can be observed. In Fig. 10a the curve for $kT=12$ keV fits quite well the experimental data of Ref. 8 but the discrepancies at small impact parameters b still remain. They aggravate with decreasing bombarding energy as displayed in Fig. 10b. In both pictures we observe a flattening of the curves with rising temperature at high impact parameters causing an over-estimation of the experimental data. In our model this is caused by those K-holes which are already present before the collision of the two heavy ions takes place, since they are generated by the Fermi distribution of eq. (23). These curves at very high temperatures are

Fig. 10. (a) The increase of $1s\sigma$ -vacancy production with rising temperature T for the system Pb+Cm at $E_{lab}=5.9$ MeV/u is indicated by the dashed lines. Numbers of K-holes initially present due to eq. (23) are listed in the third column. Also shown are experimental data of Ref. 8. The full line is obtained without temperature modification. (b) The same as before but at smaller bombarding energy $E_{lab}=4.7$ MeV/u.

of course not realistic because they result from K-vacancy probabilities up to $2.7 \cdot 10^{-2}$ initially present. But even in such cases the strong increase at small impact parameters cannot be reproduced.

Therefore this simple ansatz of an initially statistical Fermi surface due to a temperature-dependent occupation distribution surely is inapt to account for the deviations between theoretical predictions and experimental data concerning K-vacancy production rates at small impact parameters.

However we can see that the slopes of $P_{1so}(b)$ in Fig. 10 are reproduced quite well for distant collisions ($b > 40$ fm) assuming a temperature of $kT \sim 12$ keV. Probably this is an indication for K-holes initially present due to thermal excitation.

Referring to the preceding remarks it appears desirable to find out the influence of electron screening effects on vacancy production rates. From common sense an increase of K-hole formation is expected.

The most simple ansatz for describing screening is the non-relativistic Thomas-Fermi model which can serve to discuss the main trend of the electron-electron interaction. According to Ref. 15 the semiclassical electron density becomes

$$\rho(r) = 1/3\pi^2 \hbar^3 (2m)^{3/2} (E_F - V_{\text{eff}}(r))^{3/2}. \quad (24)$$

V_{eff} contains the Coulomb potential and the electron-electron interaction part. From a variational principle one minimizes the total energy and finds $E_F = -\lambda$, where λ is a Lagrange multiplier taking into account the conservation of the total particle number. E_F can be recognized as the chemical potential at vanishing temperature $T=0$ K. Thomas and Fermi chose the ansatz

$$V_{\text{eff}}(r) = V_n + V_{ee} = -Ze^2/r \Phi(r), \quad (25)$$

which is the limit for extended nuclei if $r > R_+$, cf. eq. (29). $\Phi(r)$ denotes the screening function. Inserting this into the Poisson equation one is led to the non-relativistic Thomas-Fermi equation

$$d^2/dx^2 \Phi(x) = \Phi(x)^{3/2}/\sqrt{x}. \quad (26)$$

The dimensionless variable x is defined by eq. (30). In our calculations we used an approximation proposed by Latter¹⁶

$$\Phi(x) = \left(\sum_{j=0}^6 a_j x^{j/2} \right)^{-1}, \quad (27)$$

where a_j are familiar constants¹⁷.

The asymptotic behaviour of $\phi(x)$ reads

$$\phi(0) = 1, \quad \phi(x \gg 1) \sim x^{-3}, \quad \phi''(x \ll 1) \sim x^{-1/2}.$$

Thus

$$\rho(x \ll 1) \sim x^{-3/2}, \quad \rho(x \gg 1) \sim x^{-6}. \quad (28)$$

$\phi(x)$ is valid for pointlike nuclei and yields vanishing screening for $x \rightarrow 0$ which is wrong for extended nuclei. For supercritical systems we achieve finite screening at the origin with eq. (29).

Another problem arises from the non-relativistic treatment, but fortunately it was shown¹⁸ that the electron exchange terms nearly cancel the relativistic corrections for large Z . Therefore in the case of large nuclear charges the non-relativistic TF-model should be a reasonable approximation with the exception of small inter-nuclear distances.

We investigated the systems Xe+Pb and Pb+U using eq. (27). Taking into account electron-electron interaction it was necessary to evaluate modified wave functions using eq. (1) with the potential for extended nuclei multiplied by the screening function of eq. (25). The dominant changes in binding energies and matrix elements $\langle \phi_i | \partial/\partial R | \phi_k \rangle$ can be briefly summarized as follows:

a) For bound states we find a strong decrease of the binding energies with increasing main quantum numbers (Fig. 11a) due to the reduced Coulomb attraction. This causes a reduction of the transition probabilities, e.g., between the $1s\sigma^-$ state and weaker bound levels as shown in Fig. 11b.

b) The probability-density for positron wave functions of low kinetic energy increases drastically in the vicinity of the centre of charge origin. Transitions involving such states therefore yield larger matrix elements.

c) Since screening causes smaller binding energies the $1s\sigma^-$ level does not 'dive' as deep into the negative energy continuum as it would do in the case of a pure Coulomb potential.

Inserting these binding energies and matrix elements into the coupled channel equations we evaluated K- and L- vacancy production rates for the systems Xe+Pb and Pb+U at different bombarding energies choosing a Fermi level $F=3$. Referring to an impact parameter range $0 \leq b \leq 20$ fm we find an increase of about 13% for K- and L- hole production at $E_{lab}=5.9$ MeV/u considering the system Pb+U (Fig. 12a).

The same system was also investigated at $E_{lab}=3.6$ MeV/u and

Fig. 11. (a) Binding energies for bound states versus internuclear separation R for total nuclear charge $Z=174$. The dashed lines indicate results including electron screening. (b) Radial coupling matrix elements between bound states for the same system. Now the full lines indicate calculations obtained by the potential of eq. (25).

$E_{lab} = 8.9$ MeV/u. We find that the higher the lab energy is chosen the weaker the screening effects turn out. Since at high bombarding energies the inner-shells play a dominant role this is plausible, because the TF-potential hardly changes the binding energies of, e.g., $1s\sigma$ - and $2s\sigma$ - states.

We also observe a decreasing slope with increasing impact parameter as a result of the reduced binding energies for weaker bound states.

Another important feature of screening is its different influence on s- and p- states which emerges especially at lower combined charge Z . Calculations for the system Xe+Pb which were performed at $E_{lab} = 3.6$ MeV/u, 4.7 MeV/u, and 5.9 MeV/u will elucidate this point. Plotting the data for $E_{lab} = 4.7$ MeV/u (Fig. 12b) we find a tremendous increase of the $2p_{1/2}\sigma$ - vacancy production rates which exceed calculations using a pure Coulomb potential by about 40%. In contrast to this the $1s\sigma$ - vacancy formation only rises by approx-

Fig. 12. (a) Comparison between $1s\sigma$ - and $2p_{1/2}\sigma$ - vacancy production rates for the system Pb+U at $E_{lab}=5.9$ MeV/u. The results including electron screening are indicated by dashed lines. (b) The same as in part a, but for the lighter system Xe+Pb for $E_{lab}=4.7$ MeV/u.

imately 10%. Thus it would be of great interest to have the facility of a comparison with experimental data, as one has for K- holes, in order to affirm or to reject this prediction.

It was already mentioned above that the simple non-relativistic Thomas-Fermi model for pointlike nuclei yields a vanishing screening function $\Phi(x)$ at the origin. This can be circumvented by using the following modified potential

$$U(r,R) = \Phi(x) [1 - a(R)R_+/ (r+R_+)] V_0(r,R) , \quad (29)$$

where $V_0(r,R)$ is the monopole part of the two-centre potential and $R_+ = R/2 + R_n$. R denotes the two-centre separation and R_n the nuclear radius. The spatial distance is dimensionless according to

$$r = bx, \quad (30)$$

where

$$b = 1/4 (9\pi^2/2Z)^{1/3} a_0$$

and a_0 denotes the Bohr radius.

For two-centre distances smaller than a critical value $9.10 R_{cr}$ the $1s\sigma$ - state 'dives' into the negative continuum ($E < -m_e c^2$). This occurs for the systems U+U and U+Cm. Therefore we have to apply the projection formalism developed in Ref. 11 in order to take into account also spontaneous couplings $\langle \phi_{Ep} | H | \phi_R \rangle$. We calculated K- hole production rates for the systems U+U at $E_{lab}=5.9$ MeV/u and U+Cm at $E_{lab}=5.8$ MeV/u using the potential of eq. (29), cf. Fig. 13. We again find an increase in K- hole formation for different Fermi surfaces $F=2$ till $F=4$, respectively, by about 15-20%.

The increase of L- hole production due to screening is comparable to that of K-holes in contrast to the system Xe-Pb. By means of this feature one should be able to decide whether screening effects show up as have been discussed here. According to our results we expect notable deviations in $2p_{1/2}\sigma$ - vacancy production rates for lighter systems like Xe-Pb.

Fig. 13. (a) K- hole production in a U+U collision at $E_{lab}=5.9$ MeV/u is investigated choosing different Fermi levels $F=2s\sigma$, $3s\sigma$, and $4s\sigma$, respectively, and is compared with results obtained with the potential of eq. (29), denoted by full lines. (b) As in part a, but for U+Cm at $E_{lab}=5.8$ MeV/u.

Fig. 14. Production probabilities for δ^- electrons in coincidence with $1s\sigma$ -vacancies versus impact parameter b , considering a Pb+Pb collision at $E_{\text{lab}}=4.7$ MeV/u. The different curves belong to different kinetic electron energies.

Fig. 15. As in Fig. 14 but for the lighter system J+Pb at $E_{\text{lab}}=3.94$ MeV/u.

6. δ^- ELECTRONS - COMPARISON OF THEORETICAL PREDICTIONS WITH RECENT EXPERIMENTAL DATA

In this chapter we turn to the discussion of δ^- electron emission in superheavy collision systems. The differential production probability of δ^- electrons measured in coincidence with a $1s\sigma$ -vacancy is displayed in Fig. 14 as a function of the classical impact parameter b . The superheavy system Pb+Pb at $E_{\text{lab}}=4.7$ MeV/u is under investigation. The various curves correspond to different electron energies $E_e=300, \dots, 900$ keV. For $b \gtrsim 15$ fm the curves show an exponential fall-off, which is steepest for the highest energy E . At smaller impact parameters the curves pitch down. As initial bound states we take into account the $1s\sigma$ - up to the $4s\sigma$ - state and the $2p_{1/2}\sigma$ - up to the $4p_{1/2}\sigma$ - level, which is denoted by the Fermi level F . However, one should state that also higher bound states may lead to a considerable contribution in the δ^- ray spectrum.

Fig. 16. (a) Triple coincidence measurements¹⁹ (scattering angle, δ - electron energy, and K- hole formation) of δ - electrons versus $R_0=R_{\min}$ in the system J+Pb at $E_{\text{lab}}=500$ MeV. Different curves belong to different kinetic electron energies. (b) Total δ - electron distribution (triangles) and δ - electrons in coincidence with K- vacancies for the same system versus kinetic electron energies are results of Ref. 19. The latter data are compared with absolute values of coupled channel calculations.

For the J+Pb system with $Z_1+Z_2=135$ the δ - electron spectra are shown as a function of the kinetic electron energy at various impact parameters $b=7.5$ fm up to $b=75$ fm (Fig. 15). The dependence is again of exponential form and less steep for more central collisions.

A comparison with experimental data is presented in Fig. 16 for the system J+Pb, $E_{\text{lab}}=500$ MeV. Part a shows a triple coincidence measurement (scattering angle, δ - electron energy, and K- hole formation) by W. König, C. Kozhuharov et al.¹⁹. Good agreement is achieved for all cm- kinetic electron energies and for all impact parameters. In part b the double differential electron cross section $d^2\sigma/d\Omega dE$ is displayed¹⁹. The triangles denote the total spectrum of emitted δ - electrons. The experimental data representing coincidence measurements with K- vacancies are in overall agreement with our theoretical calculations. Also for the lower Z- system Br+Pb fair agreement with the measured data of Ref. 19 is achieved (Fig. 17).

Fig. 17. The system Br+Pb at $E_{lab}=264$ MeV is under investigation with respect to the total δ -electron distribution (triangles) and δ -ray emission in coincidence with 1σ -vacancies. The double differential cross section is displayed versus the kinetic electron energy and compared with coupled channel results for the coincidence measurement.

The δ -electron distribution of the system Pb+Sn in coincidence with 1σ -vacancies is displayed in Fig. 18. For comparison with experimental data of Kozhuharov et al.²⁰ we divided our results by 4π assuming a spherical symmetric angular distribution of the δ -ray emission. The latter assumption was verified experimentally²⁰. We obtain a theoretical cross section which slightly underestimates the measured data.

In a last comparison with experimental results we show total δ -electron production rates for a Pb+Pb collision^{14,21,22} (Fig. 19). Again the experiment exceeds the theoretical values by about a factor of three. It could be that higher bound states yield considerable contributions to the electron spectrum. Also the modification of the binding energies due to electron screening is expected to shift the theoretical predictions towards higher kinetic energy.

On the other hand it should be noted that all calculations agree with the exponential slope of the energy distribution of emitted electrons found experimentally, cf. Figs. 16-19. In general better agreement is achieved if the δ -electrons are measured in coincidence with K-vacancies of the heavier collision partner in asymmetric

Fig. 18. δ - ray spectrum for the Pb+Sn system which should be measured in coincidence with $1s\sigma$ - vacancy formation. The data are divided by 4π and compared with measurements of Kozhuharov et al.²⁰. The experimental error bars are not drawn.

Fig. 19. Total δ - electron distribution versus kinetic electron energy for the Pb+Pb quasimolecule as well as the various portions of the $1s\sigma$ - and $2p_{1/2}\sigma$ - initial bound states. Comparison with experimental data of C. Kozhuharov et al.²⁰.

systems.

Finally we discuss the influence of a nuclear time delay in deep - inelastic heavy - ion reactions on δ - electron spectra^{23,24}. To determine the excitation amplitudes in delayed collisions, we make the simplest possible ansatz for the nuclear motion. The branches of two Rutherford hyperbolas having the same distance of closest approach $R_{\min}$ are joined by a section without any radial motion. The delay time T is taken as a free parameter. The effect of the reaction then enters via an additional phase in the set of coupled differential equations (7) for the occupation amplitudes $a_{ij}(t)$.

Fig. 20. Differential emission probability of δ - electrons in dependence on the kinetic electron energy for the system Pb+Pb at $E_{\text{lab}} = 7.5 \text{ MeV/u}$ and $b = 4.7 \text{ fm}$. A nuclear sticking time of $T = 10^{-20} \text{ sec}$ is assumed. In part a the dashed-dotted line denotes the total δ - electron spectrum, whereas the dashed line indicates δ - ray emission in coincidence with $1s$ - vacancies choosing $F = 3s$. Part b shows the corresponding results for $p_{1/2}$ -partial waves.

We calculate δ - electron spectra in the heavy-ion collisions Pb+Pb, Pb+U, U+U, and U+Cm, where the nuclei stick together for about 10^{-21} to 10^{-20} sec . In a perturbation theoretical treatment the spectra will show interference patterns^{23,24} with an energy width $\Delta E = \hbar/T$, which was confirmed by coupled channel calculations for the collision system Xe+Pb. Investigating heavier collision systems the predicted structure is increasingly suppressed. Fig. 20 shows the spectra of emitted δ - electrons (left part s - contribution only, right part $p_{1/2}$ -partial waves only) in a Pb+Pb collision at $E_{\text{lab}} = 7.5 \text{ MeV/u}$, $b = 4.7 \text{ fm}$, choosing a delay time $T = 10^{-20} \text{ sec}$. The bound states are assumed to be initially occupied up to $3s$, $4p_{1/2}$, respectively. The dashed-dotted lines denote the total spectra, the dashed ones display δ - electron distributions in coincidence with K- vacancy formation according to eq. (14).

Although the individual emission spectra remain oscillatory patterns, the superposition of the contributions from several electron shells destroy much of the structure contrary to the case of Xe+Pb. Therefore it is advantageous to measure δ - electrons in coincidence with K- shell ionization. Under this condition the interference patterns should retain an amplitude sufficient to be observable even in very heavy collision systems if the reaction time is defined sharply enough and the severe nuclear background can be separated²⁵.

7. SPIN POLARIZATION OF ELECTRONS BY STRONG COLLISIONAL MAGNETIC FIELDS

The strongest magnetic fields on a microscopic scale accessible to experimental observation are created²⁶ in heavy ion collisions with $(Z_1+Z_2)\alpha > 1$. The maximum magnetic field strength may reach values in the order of $|B_{\max}| = 10^{16} \text{G}$.

In superheavy quasimolecules the $1s\sigma$ - electron moves almost adiabatically close to the nuclei and is highly localized. Hence it may serve as a test particle for the high- $\vec{B}$ limit of Quantumelectrodynamics. The strong collisional magnetic fields cause a spin polarization of electrons which will be discussed in this chapter.

In the following discussion we neglect electron screening and retardation effects which should have only small influence on polarization²⁷. We now have to solve the coupled channel equations (7) taking into account the scalar and vector potentials in the transverse Coulomb gauge. For this purpose we split the relativistic Hamiltonian into two parts,

$$H = H_0 + H' \quad (31)$$

where H_0 is given in eq. (1). H' contains any interaction responsible for electron excitations which is not included in H_0 . In the case of magnetic interactions it is given by

$$H' = -\vec{\alpha} \cdot \vec{A} \quad (32)$$

with the vector potential $\vec{A}$ created by the current of both colliding ions. Inserting the ansatz (5) into eq. (4) followed by projection leads to a set of first-order coupled differential equations for the occupation amplitudes $a_{ij}(t)$.

$$\dot{a}_{ij}(t) = -\sum_k a_{ik}(t) \{ \langle \phi_j | \partial/\partial t | \phi_k \rangle + i \langle \phi_j | H'(t) | \phi_k \rangle \} \exp(i\chi_{jk}) \quad (33)$$

with the phases

$$\chi_{jk} = \chi_j - \chi_k \quad (34)$$

In contrast to eq. (7) there occurs an additional magnetic interaction term resulting from the vector potential $\vec{A}$ which reads in the Coulomb gauge

$$\begin{aligned}\vec{A}_C(\vec{r}, t) &= - \sum_{i=1,2} \frac{Z_i e^{2\vec{v}_i(t)}}{2|\vec{r}-\vec{R}_i(t)|} - \\ &\quad - \sum_{i=1,2} \frac{Z_i e^{2\vec{v}_i(t)} \cdot [\vec{r}-\vec{R}_i(t)] [\vec{r}-\vec{R}_i(t)]}{2|\vec{r}-\vec{R}_i(t)|^3} \\ &= \vec{A}_C^{(1)} + \vec{A}_C^{(2)}.\end{aligned}\quad (35)$$

The corresponding result in the Lorentz-gauge $\vec{A}_L(\vec{r}, t)$ becomes

$$\vec{A}_L(\vec{r}, t) = 2\vec{A}_C^{(1)}(\vec{r}, t). \quad (36)$$

For simplicity we restrict our derivations to symmetric systems with

$$Z_1 = Z_2 = Z \quad (37)$$

and use nuclear trajectories prescribed as Rutherford hyperbolas in the x-z plane, where the z-axis connects both nuclei. The matrixelements resulting from the interaction term in eq. (32) are evaluated in the monopole approximation since exact continuum solutions of the two-centre Dirac equation are unknown up to now. Due to selection rules or triangle rules we find that all contributions to couplings between s-states with the same magnetic quantum number just cancel in the Coulomb gauge

$$\langle s' \pm \frac{1}{2} | H'_{\text{Coulomb}} | s \pm \frac{1}{2} \rangle = 0. \quad (38)$$

This is different from the Lorentz gauge, where we obtain

$$\langle s' \pm \frac{1}{2} | H'_{\text{Lorentz}} | s \pm \frac{1}{2} \rangle = - \frac{Ze^2}{3} i v_{RO} \int_0^\infty r^2 dr \frac{r}{2} (fg' - f'g). \quad (39)$$

However, also in this case there is no contribution to the Zeeman splitting: Due to the minus sign in the integrand of eq. (39) the diagonal matrix elements ($s'=s$) vanish. For the spin flip transitions we find in summary the simple result

$$\langle s' + \frac{1}{2} | H' | s - \frac{1}{2} \rangle = + \frac{Ze^2}{3} i v_\phi \int_0^\infty r^2 dr \frac{r}{2} (fg' + f'g) = \quad (40)$$

$$= +iA_{s'+, s-}, \quad (41)$$

which is the same in the Coulomb as well as in the Lorentz gauge. The only contributing rotational matrixelement in the monopole approximation leads to

$$\langle s+1/2 | -i\vec{\omega} \cdot \vec{j} | s+1/2 \rangle = + b v_{\infty} / 2R^2 . \quad (42)$$

Up to now we only discussed the fate of a single electron influenced by the collision dynamics. In order to describe the many-electron problem the field operator $\hat{\psi}$ is expanded²⁸ in terms of the complete adiabatic basis states ϕ_q . This leads us to expressions for the number operators of particles N_{p+}^Y and holes N_{q+}^Y with a given spin projection along a marked axis $\vec{y}$. The spin polarization of holes q along the y axis is defined by

$$\eta_q = (N_{q+}^Y - N_{q-}^Y) / (N_{q+}^Y + N_{q-}^Y) \quad (43)$$

and correspondingly for particles

$$\eta_p = (N_{p+}^Y - N_{p-}^Y) / (N_{p+}^Y + N_{p-}^Y) . \quad (44)$$

Using symmetry, time-reversal-symmetry and completeness relations of the expansion coefficients a_{i+j+} to a given spin projection we can rewrite the two preceding equations into

$$\eta_q = \frac{2\text{Im} \left(\sum_{r<F} a_{q-,r+}^* a_{q-,r-} \right)}{1 - \sum_{r<F} |a_{q-,r-}|^2 - \sum_{r>F} |a_{q-,r+}|^2} , \quad (45)$$

$$\eta_p = \frac{2\text{Im} \left(\sum_{r<F} a_{r-,p+}^* a_{r-,p-} \right)}{\sum_{r<F} |a_{r-,p-}|^2 + \sum_{r>F} |a_{r-,p+}|^2} . \quad (46)$$

The solutions for a modified set of coupled channel equations³ containing expressions for $\hat{a}_{i+j+}, \hat{a}_{i+j-}, \hat{a}_{i-j-}, \hat{a}_{i-j+}$ also take into account the rotational coupling eq. (42), the diagonal and off-diagonal magnetic interaction and as usual the radial coupling.

We only consider results for the collision Pb+Cm ($Z_1+Z_2=178$) hence being the heaviest system investigated with respect to $1s\sigma$ -ionization probabilities.

As a main result we found that the additional magnetic interaction does not change the total ionization probabilities but leads to spin polarization of electron states. The created polarization is preserved during the collision despite the importance of multi-step excitation processes. As dominant effect the Zeeman splitting of the $1s\sigma$ -state gives rise to a stronger ionization of one of the usually degenerated spin states whereas the other becomes less

Fig. 21. Zeeman splitting of the 1σ -state induced by the strong magnetic field in the collision system $Z_1+Z_2=178$.

Fig. 22. Spin polarizations of 1σ -vacancies versus impact parameter b . $E_{lab}=3.6$ MeV/u and 5.9 MeV/u, respectively.

ionized. However, the total sum remains almost exactly the same as obtained with the radial coupling only.

The Zeeman splitting of the 1σ - level as function of inter-nuclear separation R is given in Fig. 21 for a collision with $E_{lab}=5.9$ MeV/u and $b=20$ fm. Using eq. (41) it has been evaluated according to

$$\Delta E_{1\sigma} = 2 |A_{1s+\sigma, 1s-\sigma}|. \quad (47)$$

At the distance of closest approach it reaches a maximum value of $\Delta E(R=30 \text{ fm})=32$ keV.

The spin polarization η_q of created 1σ -vacancies has been calculated according to eq. (45). Its dependence on the impact parameter b is presented in Fig. 22. As can be seen, $\eta_q=0$ for head-on collisions since the rotational velocity v_ϕ is zero in this case. Its maximum is obtained for medium impact parameters between $b=30$ and 80 fm. For larger impact parameters (distances) the magnetic field is too small in order to influence ionization processes. This theoretical prediction still needs experimental verification. The

Fig. 23. The spin polarizations of emitted δ - electrons stemming directly from the $1s\sigma$ - state is displayed versus the kinetic electron energy, using eq. (46).

spin polarization η_p of δ - electrons versus kinetic electron energy E stemming directly from ionization of the $1s\sigma$ - state, cf. eq. (46), is presented in Fig. 23 for $b=20$ fm. Correlation effects are not considered in this case. Larger polarizations are found for increasing δ - electron energy.

Ionization measurements of $1s\sigma$ - electrons and δ - electron spectroscopy therefore may yield direct information about the behaviour of electrons in strong magnetic fields.

8. CONCLUSIONS

We have discussed systematically the charge dependence of vacancy formation and δ - electron production rates open to experimental verification. The steep increase at small impact parameters was explained by the strong localization of the $1s\sigma$ - state caused by relativistic effects. Furthermore we predict a maximum for vacancy and δ - electron production rates around $Z_1+Z_2=170$ due to the interplay of the extremely strong $1s\sigma$ - binding energy and the radial

expectation value $\langle r_{1s\sigma} \rangle$.

Comparing our coupled channel results with experimental data we pointed out that the inclusion of multi - step processes is indispensable to obtain reasonable absolute values. This is emphasized by results obtained within first order time-dependent perturbation theory which underestimate the measured data by factors 3-5.

Beside the expansion of $\phi(R(t))$ into stationary eigenstates a variational principle for the calculation of K- hole production rates in lighter systems is proposed in Ref. 29 which is also applicable to heavy collision systems.

Considering vacancy formation we find fair agreement with experimental data except for small impact parameters and distant collisions, where we slightly deviate from the experimental results. However, the present discrepancies in collisions far below the Coulomb barrier (e.g., Pb+Cm $E_{lab}=3.6$ MeV/u) still need clarification.

The δ - electron spectra are partially in excellent agreement with measured values of Kozhuharov and König et al.¹⁹. We find that the slope is reproduced very well for all δ - electron energies. Considering δ - electron production rates in a Pb+Pb collision in coincidence with final K- vacancies of the heavier collision partner, our calculations slightly underestimate the experimental results. We expect that electron screening will shift the present data towards higher energies.

The influence of electron screening was investigated with respect to vacancy formation. We found an increase of vacancy production up to 40% for $2p_{1/2\sigma}$ - holes considering Xe+Pb.

In nuclear reactions with a time delay T we predict interference patterns in δ -electron spectra having an energy width of $\Delta E = \hbar/T$ thus offering the opportunity to measure nuclear reaction times.

As a measurable quantity for the strong magnetic fields in heavy ion collisions we predict a Zeeman splitting of the $1s\sigma$ - state.

In future calculations we have to incorporate electron screening corrections and truly two-centre effects like translation factors and level crossings. Furthermore the influence of strong magnetic fields ($|B| \sim 10^{16}$ G) should be considered.

For more detailed comparison with measured data it would be desirable to extend the experimental investigations to a wider Z - range comprising $1s\sigma$ - and $2p_{1/2\sigma}$ -ionization probabilities.

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