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THEORY OF POSITRON CREATION IN SUPERCRITICAL QUASIMOLECULES

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ABSTRACT

Collisionsof very heavy ions at energies close to the Coulomb barrier are discussed as a unique tool to study the behaviour of the electron-positron field in the presence of strong external electromagnetic fields. Theoretical predictions for positron production rates are compared with experimental results. Emphasize is laid on effects due to nuclear time delay and on the influence of electron screening for both sub- and supercritical collision systems. Internal electron-positron pair production from electric monopole transitions is investigated for atoms with charge number up to $Z \approx 170$.

1. INTRODUCTION

The behaviour of electronic states in a strong external electrostatic Coulomb-potential $V(r) = -Z\alpha/r$ is a matter of investigation since the beginnings of relativistic quantum mechanics (for more details cf. the review articles 1 and Ref. 2). It is well-known that the spectrum of the Dirac equation with a weak external

^{*}) Talk presented by J. Reinhardt.

potential consists of a set of bound levels, within $-mc^2 < E_n < +mc^2$, and two continua with positive and negative frequencies. However, at $Z = 1/\alpha \approx 137$ the Dirac equation for point nuclei ceases to have stable s - and $p_{1/2}$ -wave solutions, in general the Sommerfeld finestructure-formula yields complex energy eigenvalues for $Z\alpha > |\kappa|$. This singular behaviour is cured by the finite extension of the nucleus, the bound states then can be traced³ beyond the value $Z\alpha = |\kappa|$. Note that, as a consequence, the wavefunctions of s - and $p_{1/2}$ -wave electrons become extremely sensitive to the nuclear charge distribution.

At $Z \approx 150$ (assuming normal nuclear density) the K-shell electrons have lost all their rest energy due to strong binding in the Coulomb potential, $E_{1s} \approx 0$. We enter the truly relativistic region where relativistic effects are no longer a small perturbation, but dominate the behaviour of the inner-shell electrons. Increasing the nuclear charge up to $Z \approx 173$ the $1s$ -state enters the negative energy continuum, $E_{1s} < -mc^2$; the binding energy has reached the threshold where spontaneous pair production becomes possible. At this point the spectrum of eigenstates of the Dirac equation is subject to a characteristic change: The $1s$ -state becomes a resonance in the lower energy continuum.

In a Gedankenexperiment it is possible to increase the charge of a bare nucleus from $Z < Z_{cr} \approx 173$ to $Z > Z_{cr}$, the unoccupied $1s$ -state will then be filled under emission of two (due to spin-degeneracy) positrons. The new stable ground state of the system consists of the nucleus plus two electrons in the K-shell; it is called the charged vacuum^{4, 5}. The experimental exploration of this new phenomenon would constitute an important test of the theory of Quantum Electrodynamics (QED) in the region of strong fields.

At present, however, the unique way of extending the periodic table of elements, where these atomic effects can be pursued, lies in heavy ion collisions. By now it is well-known and generally accepted that during slow (in terms of the K-shell electron velocity) collisions of heavy atoms a quasimolecule is formed and finally, at closest approach, a quasiautom. We have learnt that this becomes the closer to the truth the heavier the ions are. Experimentally the upper boundary is given by the heaviest available combination of projectile and target, presently U-Cm, i.e. $Z = 188$.

In such scattering experiments, however, the dynamics of the collision becomes extremely important. The time-scale must be sufficiently long to allow the electron (positron) to adjust to the variation of the combined Coulomb field of the two nuclei. Since the typical velocities required to bring the nuclei close together are about $v/c \approx .1$, the adiabatic picture is meaningful only for electrons in relativistic motion. The theoretical description of the dynamics of the electronic states is outlined in Section 2.

In this contribution we will concentrate on positron creation, while various other excitation processes in collisions of very highly charged systems will be discussed by G. Soff et al.⁶. In particular, it is our aim to give an adequate description for positron production in supercritical collision systems, where $Z_T + Z_P$ exceeds 173. The theoretical basis is laid in Section 2, where also a method, based on a projection operator technique, is introduced to treat the time dependent $1s\sigma$ -resonance. The theoretical description is extended to the case of nuclear reactions with time delay in Section 3.

Numerical results for positron production in several collision systems are discussed in Section 4 and compared with presently available experimental data. The effects of a nuclear time delay (Section 5) and the influence of electron screening (Section 6) on positron production rates are investigated separately.

Section 7 was motivated by the recently observed structure^{7,8} in the energy distribution of positrons emitted in collisions of very heavy ions like U-Th, U-U, and U-Cm. Having discussed the observations in terms of the mechanism of spontaneous positron emission, we investigate in addition if internal electron-positron pair conversion from electric monopole transitions can explain the observed spectra.

2. DYNAMICS OF ELECTRONIC STATES IN THE COMBINED FIELD OF TWO COLLIDING HEAVY IONS

To describe the production of positrons - as well as K-hole formation or δ -electron excitation⁹ - in a heavy ion collision it is essential to account for the strong Coulomb force experienced by the electron-positron field in the vicinity of the colliding nuclei. Since the adiabaticity criterion is satisfied for the motion of innershell electrons, it is most natural to adopt the quasi-molecular picture¹⁰. In the semiclassical approximation we have to solve the time-dependent two-centre Dirac equation

$$i\hbar \partial/\partial t \phi_i(\vec{R}(t)) = H_{TCD}(\vec{R}(t)) \phi_i(\vec{R}(t)), \quad (1)$$

where $H_{TCD}(\vec{R}(t))$ is the two-centre Hamiltonian depending on the time-dependent internuclear distance $\vec{R}(t)$. Eq. 1 determines the motion of a single electron (initially in state i) in the external time-varying electromagnetic field generated by the two colliding ions.

The time-dependent wavefunction $\phi_i(\vec{R}(t))$ is expanded in terms of the complete basis of stationary molecular states $\phi_j(\vec{R})$

$$\phi_i(\vec{R}(t)) = \sum_j a_{ij}(t) \phi_j(\vec{R}(t)) \exp \{-i\chi_j(t)\}, \quad (2)$$

where the states $\phi_j(\vec{R})$ are solutions of the stationary two-centre

Dirac equation calculated for fixed internuclear distances $\vec{R}$ with energy eigenvalues $E_j(R)$. The sum includes the set of bound states as well as the upper and lower continua. The phase factor $\chi_j(t)$ is conveniently chosen so as to eliminate the diagonal matrix element of the Hamiltonian, i.e.

$$\chi_j(t) = 1/\hbar \int^t dt' \langle \phi_j(t') | H(\vec{R}(t')) | \phi_j(t') \rangle . \quad (3)$$

The resulting system of coupled differential equations for the expansion amplitudes a_{ij} , equivalent to eq. 1, is

$$\begin{aligned} \dot{a}_{ij}(t) = & - \sum_{k \neq j} a_{ik}(t) \langle \phi_j | \partial/\partial t + i/\hbar H | \phi_k \rangle \cdot \\ & \cdot \exp \{-i(\chi_k - \chi_j)\} \end{aligned} \quad (4)$$

with the initial condition $a_{ij}(-\infty) = \delta_{ij}$. Solving the coupled channel eqs. 4 numerically we make the following approximations: magnetic¹¹ and retardation effects are neglected, the multipole expansion of the two-centre potential¹² is restricted to the monopole- ($l=0$) term. Therefore the time derivative operator $\partial/\partial t$ in eq. 4 can be reduced to $\vec{R}\partial/\partial R$ (radial coupling). Concerning the problem of spurious long-range couplings which can be corrected by using "translation factors", we refer to the work of Heinz et al.¹³. Details on the computation and properties of wavefunctions and matrix elements are given in Refs. 12, 14, 15.

To calculate the various possible excitations one has to solve a problem with infinitely many particles, since according to Dirac's hole picture the negative continuum is occupied with electrons. In addition various bound states may be occupied. If one neglects the electron-electron two-body-interaction it turns out, however, that it is sufficient to solve the one-electron problem and to include the effect of the Pauli-principle only afterwards^{14, 16}.

The probability for excitation of particles in a level above the Fermi surface $p > F$ or holes in a state $q < F$ can be deduced completely from the single particle transition amplitudes

$$N_p = \sum_{r < F} |a_{rp}|^2 \quad \text{for } p > F, \quad (5)$$

$$N_q = \sum_{r > F} |a_{rq}|^2 \quad \text{for } q < F, \quad (6)$$

where F denotes the Fermi level up to which the states are occupied prior to the collision.

The number of correlated particle-hole pairs is given by

$$N_{p,q} = N_p \cdot N_q + \left| \sum_{r < F} a_{rp}^* a_{rq} \right|^2. \quad (7)$$

The same formula holds also in the case of particle-particle or hole-hole-correlations if the sign of the second term is inverted. The first term of eq. 7 describes statistical coincidences while the sum contains coherent correlation effects. This formula should, e.g., be applied to analyze experiments like those presently performed for coincidences between positrons and δ -electrons.

We emphasize the fact that eq. 4 exhibits two kinds of coupling operators. If the set $\phi_i(\vec{R})$ consists of exact eigensolutions of the adiabatic Hamiltonian, as assumed so far, only dynamical couplings due to the time derivative operator $\partial/\partial t$, acting on the parametric time-dependence of the wave functions, are present. Otherwise, the second term will cause a potential-coupling for each non-eigenstate. This will be of importance in the following.

In supercritical collisions the resonance property of the $1s\sigma$ -state must be handled with care. In Refs. 14, 17 a formalism is developed, which avoids those difficulties and moreover has heuristic value for the interpretation of the positron creation process. The method is based on the observation that the continuum wavefunction of the supercritical system ($Z_T + Z_P \geq 173$) at resonance energy $E_p = E_{res}$ is quite similar to the discrete $1s\sigma$ -state in the subcritical case except for an oscillating tail - small in amplitude - reaching out to infinity. This structure reflects the occurrence of a tunneling process through the gap separating the particle- and antiparticle solutions of the Dirac equation (cf. the problem of Klein's paradox^{18, 19}).

Apart from the asymptotic behaviour the $1s\sigma$ -wavefunction retains much of its identity, e.g. the strongly localized charge distribution having the extension of the atomic K-shell. Many properties, e.g. the radial matrixelements for ionization, may be continued smoothly to the supercritical region just by neglecting the tail of the wavefunction.

This idea can be used to develop a general method to treat resonance scattering. In this context Wang and Shakin²⁰ introduced a projection formalism for resonances in the nuclear continuum shell model: After having defined a normalizable quasibound wavefunction ϕ_R , a new continuum $\tilde{\phi}_{Ep}$ is constructed which spans a subspace orthogonal to ϕ_R and replaces the old continuum ϕ_{Ep} . Here ϕ_{Ep} satisfies the Dirac equation

$$(H_{TCD} - E_p) |\phi_{Ep}\rangle = 0 \quad (8)$$

and contains the resonance; E_p denotes the energy of the positron. The modified continuum wavefunctions $\tilde{\varphi}_{Ep}$ are eigenstates of a projected Hamiltonian

$$(P H P - E_p) |\tilde{\varphi}_{Ep}\rangle = 0, \quad (9)$$

with

$$P = 1 - Q := \int dE_p |\tilde{\varphi}_{Ep}\rangle \langle \tilde{\varphi}_{Ep}|, \quad (10)$$

$$Q = |\phi_R\rangle \langle \phi_R| + \sum_{\alpha} |\varphi_{\alpha}\rangle \langle \varphi_{\alpha}|,$$

where Q is an operator projecting on the resonance state ϕ_R and on all higher bound and electron continuum states φ_{α} . If ϕ_R was chosen judiciously (this problem is discussed in Ref. 14) the newly defined modified continuum $\tilde{\varphi}_{Ep}$ will no longer show resonance behaviour. Using eqs. 10 and the orthogonality relations, eq. 9 may be transformed to a more explicit form

$$(H - E_p) |\tilde{\varphi}_{Ep}\rangle = \langle \phi_R | H | \tilde{\varphi}_{Ep} \rangle |\phi_R\rangle. \quad (11)$$

The modified continuum states satisfy the original Dirac equation supplemented by an inhomogeneous term containing an integral over the solution $\tilde{\varphi}_{Ep}$.

If the states ϕ_R and $\tilde{\varphi}_{Ep}$ are used as part of the basis in eq. 2 the $1s\sigma$ -state couples to the positron continuum by two separate coupling operators

$$\dot{R} \langle \tilde{\varphi}_{Ep} | \partial / \partial R | \phi_R \rangle + i/\hbar \langle \tilde{\varphi}_{Ep} | H | \phi_R \rangle. \quad (12)$$

The second matrixelement arises since ϕ_R and $\tilde{\varphi}_{Ep}$ are not exact eigenstates of the two-centre Hamiltonian H . It does not depend on the nuclear motion and leads, in the static limit $R(t) = \text{const}$ $\langle R \rangle^{21}$, to an exponential decay of a hole prepared in ϕ_R with the width

$$\Gamma = 2\pi |\langle \tilde{\varphi}_{E_{\text{res}}} | H | \phi_R \rangle|^2. \quad (13)$$

The values of Γ deduced by this method agree well with the exact widths obtained by a phase shift analysis of the old continuum φ_{Ep} .

The developed formalism thus has led to the emergence of "induced" and "spontaneous" positron coupling, the latter resulting from the presence of an unstable state ϕ_R in the expansion basis. In practice, however, this does not result in a threshold behaviour

at the border of the supercritical region. Firstly both coupling matrixelements enter via their Fourier transforms depending on the time development of the heavy ion collision. Their contributions have to be added coherently so that in a given collision there is no physical way to distinguish between them. Secondly in Coulomb collisions the rapid variation of the quasimolecular potential causes significant contributions from the dynamical coupling, whereas the period of time for which the internuclear distance $R(t)$ is less than R_{cr} is usually very short ($\sim 10^{-21}$ sec) compared to the decay time of the $1s$ -resonance ($\sim 10^{-19}$ sec).

Therefore, the predicted production rates and energy spectra of positrons continue smoothly from the subcritical to the supercritical region (cf. Sec. 4). It turns out that the emergence of a new coupling in eq. 12 is accompanied by a reduction of the radial matrixelements $\langle \tilde{\varphi}_{Ep} | \partial/\partial R | \phi_R \rangle$ and both effects nearly cancel in collisions on Coulomb trajectories. This can be understood from Fig. 1, where the strength of both couplings is shown as a function of time.

Fig. 1. The matrixelements of "radial coupling" and "spontaneous decay", $\langle \tilde{\varphi}_{Ep} | \dot{R} \partial/\partial R | \phi_R \rangle$ and $\langle \tilde{\varphi}_{Ep} | H | \phi_R \rangle$, joining the $1s$ -resonance and the positron state of energy $E = -2 mc^2$ is shown as a function of collision time t . The nuclei are assumed to stick together for a time $T = 2 \cdot 10^{-21}$ sec. To demonstrate the effect most clearly, a head-on U-Cf collision at $2a = 18$ fm has been chosen. In pure Rutherford scattering the delay time T is obsolete.

As discussed in the following sections qualitative deviations of the positron production rate in supercritical collision systems are expected only under favourable conditions, i.e. in encounters with a prolonged interaction time.

3. THE INFLUENCE OF A TIME DELAY ON EXCITATION PROBABILITIES

Since the "spontaneous" and "dynamical" couplings exhibit a different functional dependence on the nuclear motion (the "induced" coupling is proportional to the velocity $\dot{R}$ while the "spontaneous" part depends only on the distance R), an increase in collision time can be expected to provide a clear signature for supercritical collisions. Therefore Rafelski et al.²² suggested the study of positron emission in heavy ion reactions at bombarding energies above the Coulomb barrier, where the formation of a dinuclear system or of a compound nucleus would eventually lead to a time delay within the bounds of the critical distance R_{cr} . During this delay time T the spontaneous decay of the $1s\sigma$ -resonance, by filling dynamically created K-shell holes under emission of positrons, might be strongly enhanced.

In addition to the general interest this effect could also prove to be important for the study of nuclear reactions. Although several other methods²³⁻²⁷ have been proposed to investigate the reaction time in deep inelastic nuclear collisions or the life time of compound nuclei by its influence on atomic processes, positron spectroscopy may be the most sensitive method (in very heavy systems) because the delay could result in an increase of the yield at resonance by one or two orders of magnitude.

In order to obtain a rough estimate for the effect of a time delay during the collision we discuss the solution of eq. 4 in time-dependent perturbation theory (although this approximation is not sufficient to obtain quantitatively reliable results).

In a schematic model the nuclear motion starts on a Rutherford hyperbola with initial kinematic parameters $\{E_i, b_i\}$ (energy and impact parameter), stops at $t=0$ and sets in again at $t=T$, i.e. $R=R_{min}$ for $0 \leq t \leq T$. The outgoing path is assumed to be a second Coulomb trajectory with final kinematic parameters $\{E_f, b_f\}$.

Then eq. 4 is solved by a time integral between $-\infty$ and $+\infty$, which can be split in three parts: $-\infty < t < 0$, $0 \leq t \leq T$, and $T < t < +\infty$. Due to the vanishing radial velocity $\dot{R}$ in the central region, only the potential or "spontaneous" coupling between the $1s\sigma$ -state and the positron continuum is effective here. This is illustrated in Fig. 1 for the example of a time delay $T = 2 \cdot 10^{-21}$ sec.

With the abbreviation

$$M_{ij} = \langle \varphi_i | R \partial / \partial R + i/\hbar H | \varphi_j \rangle = D_{ij} + iH_{ij} \quad (14)$$

and the relation

$$M_{ij}(R(t)) = - M_{ij}^*(R(-t)) , \quad (15)$$

we obtain for the first order transition amplitude a_{ij} for $t \rightarrow \infty$

$$\begin{aligned} a_{ij}^{(1)}(\infty) &= \int_{-\infty}^{\infty} dt M_{ji}(t) \exp\{-i(\chi_i(t) - \chi_j(t))\} \\ &= a_{ij}^{(1)}(0) - e^{-i(E_i - E_j)T/\hbar} \tilde{a}_{ij}^{(1)*}(0) \\ &\quad - e^{-i(E_i - E_j)T/\hbar} \frac{-1}{E_i - E_j} H_{ji}(R_{\min}) , \end{aligned} \quad (16)$$

where the third term is present only in supercritical collisions and it only couples the $1s$ and positron continuum states. The tilde, $\tilde{a}_{ij}^{(1)}$, indicates that the amplitude has to be taken for a half-trajectory with the kinematic parameters of the outgoing branch $\{E_f, b_f\}$ (for an elastic collision we have $\tilde{a}_{ij}^{(1)}(0) = a_{ij}^{(1)}(0)$). The energies are those at the distance of nuclear delay: $E_i = E_j(R_{\min})$. Eq. 16 applies only if state i is brought empty in the collision, whereas in a realistic experiment inner shell vacancies have to be created dynamically during the collision. Therefore multistep excitation, i.e. higher - order processes, must be investigated.

Since the excitation probabilities are given by the square of the amplitudes, the time delay will lead to oscillations as a function of delay time T or of particle energy E . They are due to interference of the excitations taking place on the incoming and outgoing branch of the trajectory, with relative phases depending on T and E . Equivalent interference effects have been discussed for δ -electron spectra²⁷ and K-hole formation²⁶. In addition eq. 16 contains a term increasing monotonically with T , which is - for the case $i \equiv 1s$, $j \equiv \tilde{\varphi}_{Ep}$ - proportional to $H_{Ep,1s}(R_{\min})$. It has to be added coherently to the combination of amplitudes in eq. 16. An estimate of the influence of this term that is appropriate for values of T large compared with the Coulomb collision time, but shorter than the spontaneous decay time of the $1s\sigma$ -resonance, is obtained by simply disregarding the excitations in the region outside $0 \leq t \leq T$. The number of positrons with energy E then reads

$$\frac{dP^{e+}}{dE} = \sum_{i>F} |a_{i,E}^{(\infty)}|^2 \approx$$

$$\approx P_{1s}(0) \frac{\Gamma}{2\pi} T^2 \cdot \left(\frac{\sin(E-E_{1s})T/2\hbar}{(E-E_{1s})T/2\hbar} \right)^2 \quad (17)$$

The spectrum of positrons thus develops a line at the position of the resonance, $E_{1s\sigma}$, which becomes increasingly narrow with growing delay time T . The width of the line is $\Delta E_{1/2} \sim 5.56 \hbar/T$ (FWHM). The total intensity

$$P^{e+} = \int_{-\infty}^{-mc^2} dE (dP^{e+}/dE) \approx P_{1s}(0) \Gamma T/\hbar \quad (18)$$

grows linearly with T . We note that eqs. 17 and 18 are valid only for $T \ll \hbar$ since the filling of the K-vacancy has been neglected. In the case of very long times T the spectrum approximately takes on a Breit-Wigner shape with the natural line width Γ .

As mentioned above it is not sufficient to solve eq. 4 in time-dependent perturbation theory, because quantitatively reliable results cannot be obtained. Multi-step processes are of great importance, therefore coupled channel calculations have to be carried out. On the other hand the need for numerical calculations can be substantially reduced by use of the identity²⁸

$$a_{ij}^{(\infty)} = \sum_{kl} a_{ik}(0) A_{kl}(T) \tilde{a}_{jl}(0), \quad (19)$$

with

$$A_{kl}(T) = \delta_{kl} \exp \{-i/\hbar E_k (R_{\min}) \cdot T\}, \quad (20)$$

which connects the final transition amplitude $a_{ij}^{(+\infty)}$ with those at time $t=0$, denoted by $a_{ik}(0)$. Within the schematic model eq. 19 allows easily for variation of the free parameter T , once the set of expansion coefficients for half-trajectories with incoming and outgoing kinematics has been calculated.

As it stands, eq. 19 is valid only for subcritical collisions, where no transitions take place at $0 \leq t \leq T$. If, e.g., the $1s\sigma$ -state is supercritical during nuclear contact, the exponential factor in eq. 19 must be replaced by an off-diagonal matrix for those states that participate in the spontaneous decay²⁹. The transition amplitudes can also be integrated numerically to time $t=T/2$, the matrix A_{kl} in eq. 19 then simply becomes the unit matrix δ_{kl} . Trivially the same is true in the case of undelayed collisions with $T=0$.

Up to now the discussion was based on the very simple schematic model of the reaction described above. Most of our results were calculated within this model, where heavy-ion collisions at energy above the barrier are considered. On the other hand, at nuclear contact attractive and dissipative nuclear forces begin to act, the trajectory $R(t)$ deviates from the usual Rutherford orbit but the

radial velocity R does not necessarily vanish for $0 \leq t \leq T$. Therefore more realistic calculations have to be performed for trajectories taking into account nuclear forces. These results will be presented in Section 5.

4. POSITRON CREATION - THEORETICAL PREDICTIONS AND EXPERIMENTAL RESULTS

Since beams of very heavy ions at energies close to the Coulomb barrier or even above have become available at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, a number of experiments has been performed to study positron emission in highly charged collision systems. At the same time, experiments measuring K-hole and δ -electron production have yielded independent information on the excitation mechanism and have served to strengthen confidence in the underlying theoretical framework, see Ref. 9.

Before we will reflect on experimental results for positron creation and their comparison with theory, we first want to discuss the main theoretical predictions. The first calculations had been performed in the framework of perturbation theory^{14, 30} which describes much of the physics involved in the excitation processes. However, the growing importance of multi-step processes indicates that perturbative calculations are of limited validity. In the following, therefore, we will present the results of coupled channel calculations¹⁴.

We have solved the system of differential equations using the monopole approximation including up to 8 bound states and ~ 15 states in the upper continuum, separately for the angular momentum channels $\kappa = +1$ and $\kappa = -1$. The integration was performed with a standard Hamming predictor-corrector routine taking about 1500 steps in time. Since the probability for positron excitation is small compared to unity, it is sufficient to include only one positronic state at a time when varying E_p , i.e. the lower continuum can be coupled in perturbation theory^p.

We will present coupled-channel calculations in four different heavy ion collision systems, Pb-Pb, Pb-U, U-U, U-Cf with the total charges $Z_T + Z_P = 164, 174, 184, 190$ (Fig. 2). The impact energy per nucleon was kept constant at $E_{lab} = 5.9$ MeV/u.

Positron emission rates increase very fast with total nuclear charge, flattening somewhat for the highest Z -values. If parametrized by a power law $(Z_T + Z_P)^n$ the power takes values of 20 down to 13. Here a Fermi-level at $F = 3s\sigma, 4p_{1/2}\sigma$ is assumed, i.e. the states above the $3s\sigma$ - and $4p_{1/2}\sigma$ - levels are empty, which should give an upper bound for positron production rates. If the distance of closest approach R_{min} is kept fixed instead of the impact velocity, n becomes still larger. In collisions of bare nuclei

Fig. 2. a) Energy spectra of positrons created in 5.9 MeV/u head-on collisions of Pb-Pb, Pb-U, U-U, and U-Cf. The results are calculated in the coupled-channel approach assuming initial occupation up to the states $3s\sigma$ and $4p_{1/2}\sigma$ ($F=3$).
 b) Emission probability of positrons in 5.9 MeV/u collisions as a function of distance of closest approach $R_{\min}$ ($F=3$).

($F=0$) positron production is increased by up to two orders of magnitude. Here also the Z -dependence is extremely steep ($n \approx 29$). Mainly responsible for this effect is the contribution of the $1s$ -state which in normal collisions ($F > 0$) is suppressed due to the small K -vacancy probability. If the K -shell is empty it becomes the dominant final state for pair production. This clearly reflects the strong coupling between the $1s$ -state and the antiparticle continuum which it approaches and even enters in the supercritical region. In all cases investigated the channels $\kappa = -1$ and $\kappa = +1$ contribute about equally to the total result.

Fig. 2a shows the energy spectra of positrons dP/dE_p produced in 5.9 MeV/u head-on collisions ($F=3$). As is well known⁷, the emission of low-energy positrons is suppressed by Coulomb repulsion while at high energies the spectra fall off exponentially in analogy to the spectra of δ -electrons. The shapes do not differ qualitatively from the perturbative results. Obviously they are practically independent of the charge of the collision system.

As a further theoretical prediction Fig. 2b displays the impact parameter dependence of positron production. The abscissa is the distance of closest approach $R_{\min}$, which is related to impact parameter b by $R_{\min} = a (1 + \sqrt{1 + (b/a)^2})$. A nearly perfect exponential decrease of the curves is noted. The high value of the slope is understandable in terms of the energy transfer required to produce a pair. Therefore the impact parameter dependence is much weaker in the case $F=0$ where the gap between the lowest empty state ($1s\sigma$) and the positron continuum becomes small or vanishes.

The displayed results demonstrate remarkable features of the pair production mechanism in "slow" collisions of highly charged nuclei. In particular, the large excitation rates and their high sensitivity on total nuclear charge in the region $Z_T + Z_P \approx Z_{cr}$ are characteristic for the action of the time-dependent strong Coulomb field.

Now we turn to the discussion of experimental results and their comparison with the described predictions. But first we want to point out a major problem in analyzing the experimental data.

Already for bombarding energies well below the Coulomb barrier ($E \sim 0.8E_C$) the nuclei can be excited by Coulomb excitation (which even may cause "Coulomb-fission"^{31,32}), the emitted photons with energy above 1022 keV can undergo pair conversion.¹³ Although this process takes place long after the collision ($\sim 10^{-13}$ sec, while "atomic" positrons are emitted within $\sim 10^{-20}$ sec), it cannot be distinguished experimentally from the quasimolecular mechanism by ordinary methods (The use of the nuclear blocking effect has been suggested by Kaun and Karamyan³³), resulting in a background which is difficult to handle. For nuclei with a simple level-structure (e.g., ^{208}Pb : $2^+, 3^-$) Coulomb excitation can be calculated. The resulting pair creation is then deduced from the theoretically³⁴ known conversion coefficients³⁵. Otherwise one has to measure simultaneously the γ -spectrum and fold it with the conversion coefficients³⁶. Here one has to know - or to assume - the γ -ray multipolarity. Monopole conversion (see Section 7) cannot be handled by this method.

In collisions with lighter targets, where the combined charge of projectile and target nucleus, $Z_u = Z_P + Z_T$, is well below the critical charge so that no significant contribution of atomic positrons is expected, the γ -ray spectra have been measured. The data can be fitted nicely by pair conversion if E_γ multipolarity is assumed. Beginning in the region $Z_P + Z_T \geq 160$, however, all experiments have found an increase which could not be explained by nuclear conversion alone (Fig. 3). This was the first hint for "atomic" positron production in heavy-ion collisions and, up to now, all conclusions had to rely on the described procedure for background

Fig. 3. In the left part the ratio between measured positron production rates and the expected pair conversion rates deduced from the nuclear γ -spectra, $Z_{e^+}^{\text{exp}} [Z_{e^+}^{\text{calc}} (\gamma, E1) \cdot f]$, is displayed as a function of the united charge $Z_u = 92 + Z_T$. If E1 multipolarity is assumed, $f \approx 1$. For $Z \geq 170$ an increase in positron emission is observed, which cannot be explained by nuclear excitation processes³⁷. The right part shows theoretical predictions for the positron cross section, $\sigma_{e^+}(Z)$, in heavy ion collisions at $E = 5.9 \text{ MeV/u}$. A strong increase is expected.

subtraction.

The first generation of experiments further established the dependence of positron excitation rates on the kinematic conditions as well as on the combined charge Z_u . Fig. 4 shows results of Kozhuharov et al.³⁸ for three collision systems, Pb-Pb, U-Pb, and U-U, at 5.8 MeV/u bombarding energy, measured with an orange-type β -spectrometer. The positron production probability in an energy window $E(e^+) = 490\text{--}50 \text{ keV}$ is shown as a function of projectile cm-scattering angle θ_{cm} . The nuclear background is subtracted. Because target and projectile cannot be distinguished in the experiment, the theoretical values have been symmetrized with respect to forward and recoil scattering.

The shape of the theoretical curves is in quite good agreement with the experimental data. Also the Z -dependent increase is well described, which spans an order of a magnitude while $\Delta Z/Z$ is only 12%. On the other hand the absolute magnitude of the theoretical

Fig. 4. For 5.8 MeV/u Pb-Pb, Pb-U, and U-U collisions positron production probability in an energy window $E_e^+ = 490 \pm 50$ keV is displayed as a function of projectile scattering angle θ_{cm} . The experimental data are taken from Kozhuharov et al.³⁸. Here, the nuclear background is subtracted.

Fig. 5. Spectra of emitted positrons in 5.9 MeV/u collisions measured by Backe et al.^{41, 42} in coincidence with ions scattered in the angular window $\theta_{lab} = 45^\circ \pm 10^\circ$. The spectrum in the lightest system, U-Pd, is explained by nuclear pair conversion alone (light line). In the U-Pb and U-U systems the sum of nuclear and calculated atomic positron production rates (full lines) is displayed.

values is generally too high.

Using another type of experimental set-up, a solenoidal spectrometer, Backe et al.³⁹ obtained integrated and differential positron probabilities for various impact parameters for the heavy-ion collision systems Pb-Pb, Pb-U, U-U, and U-Cm. As in the experiment described above general agreement with theory is found. In the Pb-Pb case and, for smaller distances of closest approach, even in Pb-U and U-U collisions the data seem to agree also in absolute values,

in contrast to the experiment discussed above. But in the heaviest accessible system U-Cm ($Z=188$) and for larger distances $R_{\min}$ theory again has a tendency to overestimate the measured data. The experimental slopes are somewhat steeper than predicted, while the Z-dependence is overestimated - a trend, which seems to be continued also in new experimental data⁴⁰.

For given kinematic parameters $\{E, b\}$ the Z-dependence of the total positron probabilities is given by Z^{17} , for lighter systems as well as for heavier ones⁴¹. From these data no qualitative signature for the "diving" of the $1s\sigma$ -state in U-U, U-Cm collisions can be extracted, in agreement with theoretical predictions.

More sensitive information can be obtained by the measurement of energy spectra of positrons detected in coincidence with the scattered ions. Their knowledge is most useful if one wants to find deviations hinting to the positron creation mechanism.

Fig. 5 shows the first published positron spectra of Backe et al.^{41, 42} for three collision systems, U-Pd, U-Pb, and U-U, at 5.9 MeV/u bombarding energy; the ions are detected in an angular window $\theta_{\text{lab}} = 45^\circ \pm 10^\circ$. For U-Pd ($Z=138$) no atomic positrons are expected, the data can be fully accounted for by nuclear conversion (light curve). Extrapolating this procedure to the U-Pb system (light curve) the sum of background and calculated QED positron rates (full curve) is in excellent agreement with the observed emission rates. In the spectrum of the supercritical U-U system ($E_{1s} \approx 1200$ keV for $R_{\min} \approx 21$ fm) some deviations are seen.

Two possible sources which could cause deviations in the shape of the positron spectra from the results presented so far will be discussed in the next two sections.

5. TIME DELAY EFFECTS

To obtain theoretical predictions for positron production it is essential to include the "spontaneous" coupling for collisions where the $1s\sigma$ -state joins the lower continuum. If it is left out of the calculation the resulting positron spectra would be strongly altered: The "induced" radial coupling is changed at the same time as the spontaneous coupling becomes important. Both contributions add up coherently and cannot be observed separately¹⁴. A promising strategy to get a clear qualitative signature for the diving process will be to modify the time structure and to select heavy-ion collisions with prolonged nuclear contact time²². Such nuclear reactions are expected to occur at energies close to or above the Coulomb barrier. The nuclear delay time T should provide a handle to distinguish supercritical systems.

Within the schematic trajectory model which was described in

Section 3 we have performed coupled channel calculations for the four heavy-ion collision systems Pb-Pb, Pb-U, U-U, and U-Cm, corresponding to $Z_{\text{united}} = 164, 174, 184, \text{ and } 188$, respectively. As in the preceding section the contributions of s- and $p_{1/2}$ -partial waves ($\kappa=\pm 1$), which are nearly equal for Coulomb-trajectories^{14, 30}, are taken into account.

Independently of assumptions on the incoming and outgoing path, of dissipation of nuclear kinetic energy or angular momentum, and of the position of the Fermi level, all positron spectra exhibit the following features:

In subcritical collision systems ($Z_T + Z_P \leq 173$) a delay time causes modulations in the positron spectrum with a width $\Delta E = 2\pi\hbar/T$. In Fig. 6a positron spectra are displayed for a Pb-Pb collision ($E_{\text{lab}} = 8.73 \text{ MeV/u}$, $b = 7.11 \text{ fm}$, $F = 3\sigma$, $4p_{1/2}\sigma$) with delay times $T = 0$ (pure Rutherford scattering), $3 \cdot 10^{-21}$, $6 \cdot 10^{-21}$, and $10 \cdot 10^{-21} \text{ sec}$. The modulations are due to interference effects in much the same way as predicted for the δ -electron spectra in deep inelastic heavy-ion collisions¹⁵. Fig. 6b shows the corresponding spectra for a Pb-U collision ($E_{\text{lab}} = 8.97 \text{ MeV/u}$, $b = 7.27 \text{ fm}$, $F = 3$), a system, which is just at the border to supercriticality. The spectra of Figs. 6a, b are qualitatively similar, although the heavier system is characterized by the presence of constructive interference.

In addition to the oscillatory interference patterns an enhancement of positron production in time-delayed supercritical collisions is observed, where the binding energy of the lowest bound states exceeds the value $2mc^2$. For long delay times a distinct peak in the positron spectrum is found at the location of the supercritical bound state resonance (binding energy minus $2mc^2$) due to the spontaneous pair-creation mechanism, as discussed in Section 3. A detailed analysis of the spectra reveals that this peak emerges gradually as $Z_u = Z_T + Z_P$ exceeds Z_{cr} .

Positron spectra for the supercritical system U-U ($E_{\text{lab}} = 7.35 \text{ MeV/u}$, $b = 3.72 \text{ fm}$, $F = 3$) are shown in Fig. 6c. With increasing delay time the position of the maximum drifts slowly from the kinematic maximum to the "resonance energy", which depends on the combined charge, the separation of the two nuclei and on the nuclear charge distribution (cf. Ref. 29).

The integrated positron emission probability P_e^+ as a function of T for these three collision systems and for U-Cm ($E_{\text{lab}} = 7.5 \text{ MeV/u}$, $b = 3.74 \text{ fm}$, $F = 3$) is displayed in Fig. 6d, where the solid lines denote s-state-, the dashed lines $p_{1/2}$ -state contributions. The absolute values, $P_e^+(T=0)$, should not be compared since the impact energies are chosen differently. However, the variation of P_e^+ with delay time T is of particular interest. In the investigated

Fig. 6. Spectra of positrons created in subcritical (parts a,b) and supercritical (part c) heavy ion collisions assuming grazing Coulomb trajectory (full lines) and nuclear reactions leading to delay times $T=3\cdot, 6\cdot,$ and $10\cdot 10^{-21}$ sec, resp., using the schematic model for the trajectory. The

kinematic parameters $\{E, b\}$ chosen are displayed within the figures, dissipation of energy or angular momentum is not taken into account. A Fermi level $F=3s\sigma$, $4p_{1/2}\sigma$ is assumed. Whereas for the lighter collision systems modulations in the positron spectra are present, a distinct peak at the "resonance" energy $E_{ls}(R_{min})$ builds up for systems with $Z > 173$. - Part d: Probability for the emission of positrons, integrated over kinetic positron energy, as a function of reaction time T . Full lines: contribution of s-partial waves, dashed lines: $p_{1/2}$ -partial waves' contribution. For the supercritical systems U-U and U-Cm $P_{e,s}^+$ increases strongly with T .

subcritical systems, $P_{e,s}^+(T)$ remains roughly unchanged or is even reduced by up to a factor two due to destructive interference between incoming and outgoing path. In the supercritical systems the probability rises approximately in linear relation with T , which reflects the increasing contribution of the spontaneous decay (cf. eq. 18). In the schematic model its slope depends on the separation of the two nuclei, R_{min} ; in general it depends also on the time-dependent charge distribution during nuclear contact. At small values of T interference effects seem to prevail even in supercritical collisions.

The results described so far were obtained within the schematic model for the nuclear motion. It facilitates a systematic study of the time delay effect and allows for an investigation of the conceptually interesting limit of large sticking times.

To analyze a given experiment, however, the employed nuclear trajectories should be consistent with the elastic and inelastic heavy-ion scattering data. Many reaction models with different degrees of refinement have been discussed in the literature. We have calculated²⁹ trajectories with the macroscopic models of Blocki et al., using the proximity potential and friction force⁴³, of Gross and Kalinowski⁴⁴ and of Schmidt et al.⁴⁵, which accounts for neck formation. The model parameters are determined in Refs. 43-45 by fitting the measured deflection function and energy loss. Fig. 7a displays the trajectories, compared with a Rutherford orbit, for a head-on U-U collision with bombarding energy $E_{lab}=7.5$ MeV/u. Strong deviations from Coulomb trajectory are found, energy loss up to ~30% (for model III) can be obtained.

We have performed coupled channel calculations with the trajectories of models I and III which are "extreme" with respect to minimal approach, time delay and energy loss. The resulting modified spectra are shown in Fig. 7b (s- plus $p_{1/2}$ -states). As expected a longer delay time ΔT causes an increase of positron production in the s-channel. On the other hand the change in kinematics causes a drift to lower kinetic energies in the $p_{1/2}$ -partial

Fig. 7. Left part: Nuclear trajectories for a head-on U-U collision at 7.5 MeV/u lab. energy assuming a pure Coulomb orbit (full line) and various models for the dissipative nucleus-nucleus interaction. Trajectory I: Proximity potential of Błocki et al.⁴³, II: friction model of Gross and Kalinowski⁴⁴, and III: model of Schmidt et al.⁴⁵ including neck formation.- Right part: Spectra of positrons (s- plus $p_{1/2}$ -partial waves) for Rutherford and nuclear trajectories depicted in part a.

wave spectra due to destructive interference. Both effects taken together lead to an enhancement of the maximum and a drift towards lower energies also in the total spectrum.

New measurements of Backe et al.⁴⁶ seem to indicate such tendencies: In U-U and U-Cm collisions at $E_{lab} = 7.5$ MeV/u, i.e. above the Coulomb barrier, positron spectra have been measured in coincidence with fission fragments in order to get a signature for close nuclear contact. A first preliminary analysis shows an enhancement of dP/dE_+ at lower kinetic energies in qualitative agreement with Fig. 7b.^e

For a quantitative comparison one has to integrate the impact parameter-dependent positron spectra over all values of b which lead to a nuclear reaction, weighted by the corresponding probability $w(b)$ to induce nuclear fission

$$\frac{dP_{e^+}}{dE_{e^+}} = \int b \, db \, \frac{dP_{e^+}}{dE_{e^+}}(b) w(b) / \int b \, db \, w(b) . \quad (21)$$

Calculating the positron spectra for different impact parameters within model III and performing the integration eq. 21 with a primitive weight factor $w(b)=1$ for $b < b_{\text{grazing}}$ and $w(b)=0$ elsewhere, there remains, however, an unexplained energy shift of ~ 50 keV in the experimental data in comparison with the theoretical curves. Furthermore, as mentioned in Section 4, the theoretical values have to be reduced by an overall factor $\sim 2/3$. For the influence of electron screening see Section 6.

As another interesting theoretical problem one might speculate about the presence of nuclear collisions with very long reaction times. How would positron spectra look like if, at a given scattering angle, a superposition of Rutherford scattering and long-lasting nuclear reactions is assumed? Obviously, the positron creation probability $P_{e^+}(\theta)$ is defined as the ratio

$$P_{e^+}(\theta) = \left(\frac{d\sigma^{e^+}}{d\theta}(\theta) + \frac{d\sigma^{e^+}}{d\theta}(\pi-\theta) \right) / \left(\frac{d\sigma}{d\theta}(\theta) + \frac{d\sigma}{d\theta}(\pi-\theta) \right), \quad (22)$$

where the second terms are always present when discrimination between projectile and target nuclei is not experimentally feasible. If, for the sake of simplicity, $d\sigma^{e^+}/d\theta$ and $d\sigma/d\theta$ are assumed to be a superposition of just two contributions, one for Rutherford and one for nuclear scattering with definite delay time T , the positron probability assigned to the "line" then reads

$$\begin{aligned} P_{e^+}(\theta) &= \frac{\frac{d\sigma^N}{d\theta}(\theta) P_{e^+}^+(T) + \frac{d\sigma^N}{d\theta}(\pi-\theta) P_{e^+}^+(T)}{\frac{d\sigma^R}{d\theta}(\theta) + \frac{d\sigma^N}{d\theta}(\theta) + \frac{d\sigma^R}{d\theta}(\pi-\theta) + \frac{d\sigma^N}{d\theta}(\pi-\theta)} \\ &\approx \frac{\frac{d\sigma^N}{d\theta}(\theta) + \frac{d\sigma^N}{d\theta}(\pi-\theta)}{\frac{d\sigma^R}{d\theta}(\theta) + \frac{d\sigma^R}{d\theta}(\pi-\theta)} P_{e^+}^+(T) \equiv q \cdot P_{e^+}^+(T) . \end{aligned} \quad (23)$$

Thus the value of q can be used as a measure of the relative cross section of reactions leading to long contact times.

Motivated by recent experimental results^{7, 8}, which will be discussed below, we performed calculations for the heavy-ion collision

system U-U at 5.79 MeV/u bombarding energy. Assuming an isotropic break-up of the long-living compound system ($T=4 \cdot 10^{-20}$ sec), a ratio $q \sim 2.4 \cdot 10^{-3}$, and a shrinking of the internuclear distance $R(t)$ to values smaller $2a$ (corresponding to a "resonance" energy at 370 keV), one gets Fig. 8. Due to the isotropic angular distribution $d\sigma^N/d\Theta$ of the nuclear reaction component, a line is observable in the positron spectrum at all ion scattering angles. However, it is most pronounced under $\Theta_{cm} = 90^\circ$, being suppressed at other angles relative to the elastic scattering cross section. If the long-living compound system fragmentates into a rather narrow angular window, the peak structure would disappear at forward (backward) scattering angles.

Fig. 8. Spectra of positrons emitted in 5.79 MeV/u U-U collisions in coincidence with a scattered nucleus for three selected lab-ion angles. The fully drawn curves are calculated assuming Rutherford scattering only. The dashed lines show the effect of an additional nuclear reaction with a lifetime $T=4 \cdot 10^{-20}$ sec. A relative fraction of $q=2.4 \cdot 10^{-3}$ reactions per elastically scattered ion (at 45°) has been assumed. The curves at smaller angles have been drawn under the assumption of isotropic distribution of the reaction fragments in the reaction plane.

As mentioned above two experimental groups^{7, 8} recently have performed experiments with U-Th, U-U, and U-Cm at energies close to the Coulomb barrier. Contrary to the results of Backe et al.^{4, 6} their positron spectra seem to show remarkable structures.

C. Kozhuharov, P. Kienle et al.⁷ measured at 5.8 MeV/u beam-energy positron spectra in coincidence with ions scattered into various narrow angular windows, using an orange-type β -spectrometer. A preliminary analysis shows sharp maxima in the spectra that are most pronounced under laboratory scattering angles around $\Theta_{lab} \sim 45^\circ$. In the U-U measurement the position of the peak was found to be located at ~ 370 keV with a width of about ~ 90 keV (FWHM), for U-Th the effect is less well established. After subtraction of a smooth background the number of positrons per detected ion emitted in the peak is roughly 10^{-5} at $\Theta \sim 45^\circ$.

If the observed structure is of quasimolecular origin, it must be produced in very long-lasting nuclear reactions. If one compares the experimental width with spectra from coupled channel calculations based on the schematic sticking model a minimum value $T \sim 4 \cdot 10^{-20}$ sec is required. Should the observed line width have instrumental origins or be due to additional broadening effects the reaction time T would have to be even longer. Taking the coupled channel results the probability for positron production in a delayed ($T = 4 \cdot 10^{-20}$ sec) collision should be $\sim 4.7 \cdot 10^{-3}$, which have to be compared with the observed probability $\sim 10^{-5}$. Thus a fraction of $q = P_e^{(exp)} / P_e^{(theor)} \approx 2 \cdot 10^{-3}$ delayed collisions per elastically scattered ion is sufficient to produce the observed effect (at 45°). This number should serve only for a general orientation, since it depends on the details of the model assumed.

Similar features were detected in the experiment of H. Bokemeyer, J.S. Greenberg et al.⁸ in U-U and U-Cm collisions. When displaying their data for U-U for different narrow scattering angle windows Θ_{lab} the differential positron spectra exhibit prominent peak structures at different kinetic energies E_e .

Thus the three experimental groups performing positron-spectroscopy at GSI, Darmstadt, so far have obtained different spectra. Backe's data^{4, 6} show no significant structures, while Berdermann et al.⁷ and Bokemeyer et al.⁸ reported similar, but not identical observations on peak structure. One might suppose that those peaks are caused by nuclear background processes. However, nuclear transitions of, e.g., multipolarity E1 or E2 should also be observable in the emitted photon spectra, provided that proper Doppler shift corrections are performed^{4, 7}. Whether the peaks can be caused by EO-processes will be discussed in Section 7.

On the other hand, if a sharply focussed nuclear reaction takes

place it is not surprising that an experiment not triggering for the optimal kinematic conditions might smear out any evidence for structure. Further investigations are needed to settle this question. Should the observed phenomena indeed be caused by reactions with a very long time scale, this would have far reaching consequences for the physics of nuclear systems in the superheavy region.

6. THE EFFECTS OF ELECTRON SCREENING ON POSITRON PRODUCTION RATES

In our discussion electron screening has been neglected so far. In order to clarify its gross features we choose the simple non-relativistic Thomas-Fermi model⁴⁸ and apply it to the calculations of basis wavefunctions in the framework of the monopole approximation. According to this model the semiclassical electron density reads

$$\rho(r) = 1/(3\pi^2\hbar^3) \cdot (2m)^{3/2} (E_F - V_{\text{eff}}(r))^{3/2}. \quad (24)$$

The potential is expressed by

$$V_{\text{eff}}(r) = V_{\text{nuc1}} + V_{\text{ee}} = -Ze^2/r \cdot \phi(r), \quad (25)$$

where $\phi(r)$ is a screening function. Using Poisson's equation we arrive at the nonrelativistic Thomas-Fermi equation

$$\frac{d^2\phi(x)}{dx^2} = \frac{\phi^{3/2}(x)}{x^{1/2}}. \quad (26)$$

For practical calculations we used an approximation to the solution of eq. 26 proposed by Latter⁴⁹

$$\phi(x) = \left(\sum_{j=0}^6 a_j x^{j/2} \right)^{-1}, \quad (27)$$

where the a_j are given constants⁵⁰. Strictly speaking this approximation is valid only in the nonrelativistic case for pointlike nuclei. But it was shown⁵¹ that relativistic corrections are partially cancelled by the electron exchange terms for large nuclear charge Z . Eq. 26 yields vanishing electron screening at the origin, which is wrong for extended nuclei. We circumvented this problem by choosing the following local modified nonrelativistic Thomas-Fermi potential

$$U(r,R) = \phi(x) \left[1 - \frac{a(R)R_+}{(r+R_+)} \right] V_0(r,R), \quad (28)$$

where $V_0(r,R)$ is the monopole part of the two-centre potential, R_+ denotes $R/2 + r_+$ with R the two-centre distance and r_+ the main nuclear radius. In eq. 26 $x=r/b$ is a dimensionless quantity, with

$$b = 1/4 \cdot (9\pi^2/2Z)^{1/3} a_0, \quad (29)$$

a₀ the Bohr radius. $a(R)$ is a fit parameter which matches the $1s\sigma$ -binding energy of the heavy-ion system to Hartree-Fock-Slater values. It turns out that then even the binding energies of the weaker bound states, e.g. up to $5s\sigma$, correspond within 7% in the TF and HFS models.

The resulting production rates for a head-on U-U collision at 5.9 MeV/u bombarding energy are displayed in Fig. 9a. A shift of the maximum towards lower positron energies by an amount of about 50 keV can be deduced. Recent, not completely analyzed measurements of Backe et al.⁴⁶ tend to confirm this shift. Fig. 9b shows the positron spectrum resulting from a U-Cm collision at 5.8 MeV/u. This system with higher combined charge shows a smaller shift of the maximum by about 30 keV.

Let us investigate now how screening affects the spectra in heavy-ion collisions where nuclear contact and time delay occurs. Fig. 9c shows positron production rates for a U-U collision, $E_{lab} = 8.5$ MeV/u, $b = 6.5$ fm (corresponding to nuclear contact for spherical nuclei), with time delay $T = 10^{-20}$ sec within the schematic model. We have to distinguish between the positron line emerging at ~ 400 keV (unscreened case) and 300 keV (screened case), respectively, and the oscillations at higher kinetic energies which are inversely proportional to the sticking time. The peak shift is essentially due to the reduction of binding energy of the supercritical $1s\sigma$ -level, which amounts to ~ 100 keV. The latter structures are also shifted by ~ 100 keV towards lower energies.

Caused by the reduction of the decay width due to screening we observe a decrease in intensity of the line by about 30% relative to previous results neglecting the screening effect. We also evaluated positron spectra of time delayed collisions for the system U-Cm at $E_{lab} = 7.5$ MeV/u and found similar results (Fig. 9d).

In Section 5 we have discussed positron creation in U-U collisions at 7.5 MeV/u considering nuclear trajectories originating from classical friction models. Fig. 10 shows the change of the positron spectrum for collisions with varying degree of nuclear contact. Part a displays the trajectories $R(t)$ calculated in the model of Schmidt et al.⁴⁵ for several orbital angular momenta from $l=0$ head-on (same as in Fig. 7a) to $l=400 \hbar$ near grazing collisions. In Fig. 10b we present a reevaluation of the positron spectrum including the influence of electron screening. As expected, an energy-shift (of ~ 55 keV) towards lower kinetic energies is observed. However, the cross section (cf. eq. 21) is reduced only by about 10%, which is not quite enough to obtain quantitative agreement with the experimental data of Ref. 46.

Fig. 9. Spectra of emitted positrons in heavy ion collisions. Coulomb potential (dashed lines) and a Thomas-Fermi screened potential (full lines) are investigated. Part a shows a head-on U-U collision at 5.9 MeV/u, part b displays a U-Cm collision, $E_{\text{Lab}} = 5.8 \text{ MeV/u}$, $b = 0$. Rutherford trajectories and $F = 3s\sigma$, $4p_{1/2}\sigma$ are assumed. - Parts c,d: In the case of time delayed supercritical collisions the positron spectra for screened (full lines) and unscreened (dashed ones) potential differ not only in their magnitude but also in the position of the "spontaneous" line. Spectra of a U-U collision, $E_{\text{Lab}} = 8.5 \text{ MeV/u}$, $b = 6.5 \text{ fm}$ (part c) and a U-Cm collision, $E_{\text{Lab}} = 7.5 \text{ MeV/u}$, $b = 3.7 \text{ fm}$, are shown assuming a delay time $T = 10^{-20} \text{ sec}$.

Fig. 10. Part a: Nuclear trajectories calculated in the friction model of Ref. 45 for 7.5 MeV/u U-U collisions at various values of the orbital angular momentum l between 0 and 400 $\hbar$. - Part b: Energy spectra of positrons emitted in U-U 7.5 MeV/u collisions assuming nuclear trajectories depicted in part a) for $l=0, 200$, and 400 $\hbar$. Screened Thomas-Fermi and unscreened Coulomb-potential are investigated. The results for s - and $p_{1/2}$ - partial waves have been added.

7. INTERNAL ELECTRON-POSITRON PAIR PRODUCTION FROM ELECTRIC MONO-POLE TRANSITIONS

Our theoretical investigations are partly motivated by the recently observed structures^{7, 8} in the energy distribution of emitted positrons which have been produced in collisions of very heavy ions like U-Th, U-U, and U-Cm. These systems are supercritical and in principle may lead to spontaneous positron production. In Section 5 we have discussed this effect in detail.

A second, more conventional explanation for this peculiar structure was proposed: Intrinsic nuclear excitations may take place either by Coulomb excitation or by transfer reactions. The nuclear transition can be accompanied by internal electron-positron pair creation^{34, 52, 53}. Contrary to EO-processes nuclear transitions of multipolarity $E1$ or $E2$ should also be observable in the emitted photon spectra provided that proper Doppler shift corrections are performed⁴⁷. Here we will prove the following assertion: If the observed structure in the positron distribution originates from EO-pair production one should definitely see also a peak in the high energy part ($E_{e^-} > 1\text{MeV}$) of the δ -electron distribution originating from K-shell conversion.

Nuclear EO-transitions are characterized by the absence of single photon emissions, because a photon must carry at least one unit of angular momentum. Therefore EO-transitions predominantly occur by bound state electron conversion or by internal electron-positron pair creation. The latter process requires nuclear transition energies $\hbar\omega$ larger than twice the electron rest mass mc^2 . The basic processes under investigation are depicted schematically in Fig. 11. The nucleus which undergoes EO-transition is labelled by its initial and final state angular momenta $J_i, J_f=J_i$ and eigen-energies $E_i, E_f=E_i-\hbar\omega$. Process a) describes the electron-positron pair creation. An electron of the negative energy continuum ($\epsilon=-E < mc^2$) with Dirac quantum number κ is lifted to the positive energy continuum. The final state energy obviously amounts to $E' = \epsilon + \hbar\omega$ whereas the angular momentum quantum number remains unchanged. Since neither the initial electron state energy nor the final state energy is fixed one expects a continuous energy distribution for the emitted positrons.

Process b) indicates the conversion of a K-shell electron ($n=1, l=0, j=\frac{1}{2}, \kappa=-1$) with energy eigenvalue $E_{1s_{1/2}}$. Thus bound states with definite energies are involved. Energy conservation then simply causes monoenergetic lepton emission for a fixed nuclear transition energy $\hbar\omega$.

Process c) symbolizes monoenergetic positron production. Here an electron of the negative energy continuum is excited to a bound state, e.g., to the $1s_{1/2}$ -state. This represents a rather rare process since it requires a vacant bound state with a large overlap with the nuclear interior. If we investigate a realistic heavy-ion collision of ^{92}U on ^{92}U with a bombarding energy close to the Coulomb barrier, the number of created K-vacancies per collision with impact parameter $b \lesssim 20$ fm is typically of the order of $10^{12, 15, 54}$. The transition time (X-ray decay) of an electron from a higher shell to the K-shell is⁵⁵ about 10^{-17} sec. This has to be compared with nuclear transition times of about 10^{-13} sec, after which conversion takes place. Hence the K-vacancy will be decayed before it could eventually be filled again due to pair conversion of a nuclear state.

Therefore the monoenergetic positron formation is suppressed in contrast to ordinary K-shell conversion by at least 7 orders of magnitude, because one can conclude⁵⁶ that the EO-conversion of a K-shell electron has a probability larger by about 3 orders of magnitude than the monoenergetic e^+ -production where the electron occupies the vacant K-shell. This ratio even increases by one order of magnitude if one considers higher bound states in relation to the positron production process. Thus process c) can be neglected completely.

Therefore, the ratio η of the two conversion probabilities P_{e^+,e^-} for electron-positron pair creation and P_{e^-} for the ionization of bound state electrons

$$\eta = \frac{P_{e^+,e^-}}{P_{e^-}} \quad (30)$$

must be investigated more closely. The differential ratio with respect to the positron energy E obviously is

$$d\eta/dE = \frac{dP_{e^+,e^-}}{dE} / P_{e^-} \quad (31)$$

and consequently

$$\eta = \frac{\hbar\omega - mc^2}{mc^2} \int \frac{d\eta}{dE} dE. \quad (32)$$

The investigation of such a ratio is motivated by the following considerations: The calculation of the nuclear EO-transition probability implies the precise knowledge of the initial and final state nuclear wavefunction, which clearly is model-dependent. Therefore it seems to be more advantageous to perform model-independent evaluations by considering the ratio of two different conversion probabilities. In this ratio the nuclear transition matrix element drops out, thereby avoiding difficulties with the specific nuclear structure.

The final expressions for the conversion probabilities entering eq. 31 are to lowest order in r

$$\frac{dP_{e^+,e^-}}{dE} \propto |M|^2 \sum_{K=-1}^{+1} |C_{e^+,e^-}|^2, \quad (33)$$

$$P_{e^-} \propto |M|^2 \sum_m |C_{e^-}|^2, \quad (34)$$

where M denotes the nuclear EO matrix element

$$M = \int_0^\infty dV_n \rho_n(\vec{r}_n) r_n^2, \quad (35)$$

and C is a normalization factor

$$C = \lim_{r \rightarrow 0} \frac{ff' + gg'}{r^{2j-1}}. \quad (36)$$

Here f, f' and g, g' are the radial Dirac wavefunctions belonging to the initial and final state of the electron, respectively. Σ' in eq. 34 denotes the sum over all occupied states (E_m, j_m) excluding the summation over magnetic substates; in eq. 33 only $j=1/2$ ($\kappa=\pm 1$) - states are taken into account.

Thus the ratio eq. 31 now reads

$$\frac{d\eta}{dE} = \frac{\sum_{\kappa=+1} |C_{e^+, e^-}|^2}{\sum_m |C_{e^-}|^2}, \quad (37)$$

Fig. 11. Schematic representation of electron conversion processes accompanying nuclear EO-transition from a state $\{E_i, J_i\}$ to a state $\{E_f = E_i - \hbar\omega, J_f = J_i\}$. a) Electron-positron pair production leading to a continuous energy distribution of positrons and electrons. b) Conversion of K-shell electrons - a monoenergetic electron-production mechanism. c) Monoenergetic positron-production - a negligible process.

Fig. 12. a) Differential conversion probability ratio $d\eta/dE$ with respect to the kinetic positron energy E for nuclear E0-transitions in U. Nuclear transition energies $\hbar\omega = 1323$ keV, 1423 keV, 1523 keV, and 1623 keV are considered, corresponding to maximum kinetic positron energies of $E_{\text{max}} = 300$ keV, 400 keV, 500 keV, and 600 keV. - b) Z-dependence of η for nuclear E0-transition energies $\hbar\omega = 1423$ keV, 2000 keV, and 3000 keV, resp..

where the nuclear matrixelement drops out. Due to eq. 36 the ratio η of pair conversion to bound state electron conversion is completely determined by the density of the electron wavefunctions at the nuclear origin.

According to eqs. 37 and 36 we computed the differential conversion ratio $d\eta/dE$ with respect to the positron energy E for E0-transitions. As bound state only the atomic K-shell has been taken into account. The conversion probability of higher bound states is at least one order of magnitude smaller. For the nucleus ^{92}U the energy distribution of emitted positrons is depicted in Fig. 12a. Nuclear transition energies of 1323 keV, 1423 keV, 1523 keV, and 1623 keV are considered, which correspond to maximum positron energies of $E_{\text{max}}^{\text{kin}} = 300$ keV, 400 keV, 500 keV, and 600 keV, respectively.

Fig. 13. Z-dependence of the differential conversion probability ratio $d\eta/dE$ versus the kinetic positron energy E for $E0$ -transitions with $\hbar\omega=1423$ keV (left part) and $\hbar\omega=2023$ keV (right part). The lines have been multiplied by different factors for each charge Z .

The spectra are centred around $E_{\max}^{\text{kin}}$, reflecting the strong Coulomb repulsion of low energetic positrons.

Figs. 13 display the change of $d\eta/dE$ with the nuclear charge Z for the transition energies $\hbar\omega=1423$ keV and $\hbar\omega=2023$ keV, respectively. With decreasing Z and increasing $\hbar\omega$ the energy distribution of positrons becomes more symmetric and therefore resembles the electron spectrum. The numerical integration of eq. 37 yields the dimensionless ratio η . It is presented in Fig. 12b versus the nuclear charge Z for transition energies $\hbar\omega=1423$ keV, 2000 keV, and 3000 keV. These calculations are also extended to the region of superheavy nuclei ($Z>100$). η decreases monotonically until almost critical systems ($Z\sim 170$) are reached, where the binding energy of the $1s_{1/2}$ -state exceeds twice the electron rest mass and the bound state gets imbedded as a resonance into the negative energy continuum. The onset of this behaviour is reflected in a slight increase of η around $Z\sim 160$. However, the numerator of eq. 37 and therefore the pair production probability increases by almost 4 orders of magnitude if the nuclear charge is extended from $Z=92$ to $Z=169$. In consequence the nuclear transition time due to internal

pair creation decreases by 4 orders of magnitude in the considered Z-range.

Comparing η in the region from oxygen (Z=8) to Polonium (Z=84) with experimental and other theoretical data⁵⁷ fair agreement is achieved.

Thus we can discuss the probability that the recently observed structures in positron spectra originates from nuclear EO-transitions. One convincing argument against this interpretation is related to the shape of the e^+ -energy distribution. According to Fig. 12a the halfwidth of the spectra should be at least $\Delta E \gtrsim 150$ keV. However, the observed structure is much narrower. The second argument is connected with the energy distribution of the emitted δ -electrons. The measured differential positron production probability dP_+/dE with respect to the kinetic positron energy at $E \sim 400$ keV for U- δ collisions with a bombarding energy of $E_{lab} = 5.9$ MeV/u and ion scattering angles of $\theta_{ion}^{lab} = 45^\circ \pm 10^\circ$ amounts to $dP_{e^+}/dE \approx 10^{-7}/\text{keV}$ ^{7,29}.

Assuming that these positrons are of pure nuclear origin one can easily deduce the probability P_N for nuclear excitation

$$P_N \approx \frac{dP_{e^+}}{dE} / \frac{d\eta}{dE} \approx 10^{-2}. \quad (38)$$

In principle the competing K-shell conversion should lead to a sharp line in the electron spectra at $E_{e^-} \approx 1300$ keV.

However, the experimental energy resolution for these high energy electrons is presently not better than $\Delta E_{e^-} \approx 25$ keV, which therefore yields a differential e^- -production probability of

$$dP_{e^-}/dE_{e^-} \approx P_N/\Delta E_{e^-} = 4 \cdot 10^{-4}/\text{keV}. \quad (39)$$

This is still by a factor $F=40$ larger than the expected δ -electron production rate^{15, 54} of $dP_{e^-}/dE_{e^-} \approx 10^{-5}/\text{keV}$ at a kinetic electron energy of $E_{e^-} \approx 1300$ keV, which results from dynamical electron excitations during the collision. The factor F can be increased by an order of magnitude if one measures the emitted electrons in coincidence with the decaying K-vacancy (e.g., K_α -radiation). Hence we can conclude: If the observed structure in the positron spectra is caused by nuclear EO-transitions one should also observe a distinct peak in the δ -electron distribution at $E_{e^-} \approx 1300$ keV. Furthermore we emphasize that sharp peaks in the e^- energy distribution of emitted positrons with a halfwidth of less than 100 keV may not originate from nuclear EO-conversion processes.

8. CONCLUSIONS

We have studied the mechanisms of ionization and pair production in collisions of very heavy ions within the framework of a dynamical theory of excitation based on the quasimolecular picture. Massive relativistic effects in the wavefunctions of electrons and positrons in $s_{1/2}$ and $p_{1/2}$ -states, caused by the coherent action of the Coulomb field generated by the two nuclei, are reflected in the excitation rates.

We have developed a theory which properly takes into account the resonance character of the "dived" $1s$ -state. The results of our coupled channel calculations indicate that no sharp threshold effects are to be expected at the border of the supercritical region, in accordance with the notion of dynamical collision broadening. Collisions above the Coulomb barrier characterized by a sufficiently long reaction time may help to identify the process of spontaneous positron production.

The experiments performed so far have convincingly established the predicted strong increase of positron production in collisions with very high total nuclear charge and its concentration on close collisions. There remain some discrepancies with theory in absolute magnitude, in the slope of $Pe^+(b)$, and, possibly, in the shape of the positron spectra. On the other hand, if the experimental data of Refs. 7,8 are confirmed and if no other explanation of the positron line in the region 300-400 keV can be found, then one is led to the conclusion that this already constitutes the quantitative proof for the existence of a spontaneous change of the ground state (the "vacuum decay"). The possibility that the structures in the positron spectra might be caused by $E0$ -conversion due to nuclear deexcitation has been discussed. The results of Section 7 offer a way to decide whether the observed structures can be explained by background.

The experimental findings open up a multitude of interesting questions and speculations as to the nuclear physics of the unexpected reaction mechanism, the central question being: are there meta-stable superheavy nuclei in the range $Z \sim 160-190$? More experimental and theoretical information and investigation is required before definite conclusions can be drawn.

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