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STATISTICAL AND DYNAMICAL ASPECTS OF HEAVY-ION
COLLISIONS

W. NÖRENBERG

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Gesellschaft für Schwerionenforschung mbH
Planckstr. 1 · Postfach 110541 · D-6100 Darmstadt 11 · Germany

STATISTICAL AND DYNAMICAL ASPECTS OF HEAVY-ION COLLISIONS

W. Nörenberg

Gesellschaft für Schwerionenforschung (GSI)

D-6100 Darmstadt, Germany

The success and limitation of current transport theories which assume local equilibrium throughout the collision, is reviewed and new results on the equilibration of the neutron-to-proton ratio and on the shell effects in the kinetic energy loss are presented. Treating the strong coupling dynamically, a general transport theory is formulated which can account explicitly for the local equilibration process. Apart from ordinary friction a new dissipation mechanism arises which is the result of a combined effect from diabatic single-particle motion and dissipative two-body collisions. This dissipative diabatic dynamics (DDD) is discussed within simple models. An important feature is a considerable retardation of dissipation for realistic values of the mean free path of nucleons.

I. INTRODUCTION

We consider collisions between heavy nuclei with energies typically a few MeV per nucleon above the interaction barrier. The observation of dissipative phenomena in the loss of relative kinetic energy and the transfer of nucleons between the nuclei have stimulated the application and development of transport theories to heavy-ion collisions. These dissipative collisions cover the range between direct reactions and compound-nucleus formation. Whereas only a few degrees of freedom participate in direct reactions, all degrees of freedom are involved in compound-nucleus formation. Therefore, dissipative collisions carry information about the relaxation processes which lead from simple nuclear states to more complex configurations. Such phenomena have also been observed in precompound reactions and fission, but only in heavy-ion collisions a rich variety of such phenomena have been discovered. In this respect the study of dissipative collisions has opened a new field of nuclear research.

As illustrated in fig. 1, the nuclear part of a heavy-ion collision can be roughly divided into three stages.

(1) The *initial stage* is characterized by the mutual approach of the nuclei in their ground states. Because of the long mean free path the

Fig. 1: Evolution of the collision complex in a dissipative collision. For details see text.

motion of the nucleons during this first stage of the collision is expected to be governed only by their self-consistent single-particle potential which evolves slowly in time. This stage should therefore be well described by the time-dependent Hartree-Fock theory.

(2) By residual interactions the Slater determinant of time-dependent single-particle states decays to more complex configurations. This decay leads to a '*local*' statistical equilibrium. At the end of this decay the system occupies the total phase space (total configuration space) which is 'locally' available, i.e. available for fixed values of the macroscopic (collective) degrees of freedom.

(3) During the third stage the slow macroscopic (collective) variables relax towards their equilibrium distributions. The complexity of the wave function in this stage of the process suggests to take advantage of random properties of the system as implied by the local statistical equilibrium. This leads naturally to the formulation of *transport theories*.

The paper is divided into two main parts. Section 2 gives a reminder and new results of the 'old' transport theory where local equilibrium is assumed throughout the collision. The statistical treatment of strong coupling between the reaction channels is replaced in section 3 by a dynamical treatment for the single-particle motion. The new dissipation mechanism which is discovered in this formulation, is treated in a diabatic limit for the single-particle motion including two-body collisions as the major agent for the local equilibration process (*dissipative diabatic dynamics*).

2. STATISTICAL ASPECTS

Transport theories of dissipative heavy-ion collisions have been formulated by several groups [cf. 1-3]. So far all derivations start from an assumed separation of the degrees of freedom into *slow macroscopic* (collective) and *fast equilibrating* '*intrinsic*' variables. The corresponding local statistical equilibrium is either represented by a heat bath of given temperature or by random properties of the coupling matrix elements. In the following we limit our discussion to the non-perturbative quantum-statistical approach [2,4] which uses the random matrix assumption.

In § 2.1 the elements of the derivation of transport equations are discussed. In § 2.2 a short summary of the applications of Fokker-Planck equations for describing the relaxation phenomena in heavy-ion collisions is given. New results on proton and neutron diffusion and on shell-structure effects are presented in §§ 2.3 and 2.4.

2.1 Derivation of transport equations [2]

There are three essential steps in the derivation of transport equations. For simplicity we treat the relative motion classically and hence assume that the trajectory $\vec{r}(t)$ and the coupling $V(t)$ between excited states of the interacting nuclei are known. For a more general discussion see [5].

(1) By dividing the total channel space into subsets H_μ we define the macroscopic occupation probabilities

$$f_\mu(t) \equiv \sum_{m \in H_\mu} \rho_{mm}(t) \quad (2.1)$$

which denote the total occupation probabilities of the subsets H_μ . Rewriting the Liouville equation for the density matrix $\rho(t)$, yields the generalized master equation

$$\frac{d}{dt} f_\mu(t) = I_\mu(t, t_0) + \sum_{\nu} \int_{t_0}^{t-t_0} d\tau K_{\mu\nu}(t, \tau) d_\mu f_\nu(t-\tau) \quad (2.2)$$

where d_μ denotes the number of states in the subset μ .

The first term on the r.h.s. represents the contribution from those parts of the initial density matrix $\rho(t_0)$ which are not contained in $f_\mu(t)$. The second term on the r.h.s. describes the coupling between different subsets. In analogy to the Boltzmann equation it is referred to as the collision term. The collision term consists of gain terms ($\nu \neq \mu$) which correspond to transitions from all subsets $\nu \neq \mu$ to the subset μ , and another term $\nu = \mu$ which describes the loss from the subset μ . The kernel $K_{\mu\nu}(t, \tau)$ is non-local in time. The change of the macroscopic occupation probabilities at time t depends on all earlier times $t-\tau > t_0$. Because of this property, the kernel and its characteristic decay time are refer-

red to as the *memory kernel* and the *memory time* $\tau_{\text{mem}}^{\nu\mu}$, respectively.

(2) The second step in the derivation of a transport equation is the introduction of random properties of the coupling matrix $V(t)$. This leads to the following structure of the memory kernel,

$$K_{\mu\nu}(t, \tau) = \overline{V_{mn}^{-2}(t) V_{nm}(t-\tau) G_{\nu\mu}(t, t-\tau) + \text{c.c.}} \quad (2.3)$$

The mean correlation function $\langle V_{mn}(t) V_{nm}(t-\tau) \rangle_{\nu\mu}$ decays as function of τ within a characteristic time $\tau_{\text{cor}}^{\nu\mu}$ (correlation time). This correlation time is related to a correlation length σ by $\tau_{\text{cor}} = \sigma/|\vec{v}|$. The correlation length σ is typically given by the mean distance between the nodes of the single-particle wave functions. For strong overlap we have $\sigma \approx 1$ fm, while in the nuclear surface $\sigma \approx (3...4)$ fm [6]. Therefore, $\tau_{\text{cor}} \approx 10^{-22}$ s. The mean propagator $G_{\nu\mu}(t, t-\tau)$ for a state $n \in H_\nu$ and a state $m \in H_\mu$ has a lifetime denoted by $\tau_{\text{dec}}^{\nu\mu}$. The relative values of $\tau_{\text{cor}}^{\nu\mu}$ and $\tau_{\text{dec}}^{\nu\mu}$ determine the two limits of weak and strong coupling. The *weak-coupling* limit is defined by

$$\tau_{\text{mem}}^{\nu\mu} \equiv \tau_{\text{cor}}^{\nu\mu} \ll \tau_{\text{dec}}^{\nu\mu} \quad (2.4)$$

and the *strong-coupling* limit by

$$\tau_{\text{mem}}^{\nu\mu} \equiv \tau_{\text{dec}}^{\nu\mu} \ll \tau_{\text{cor}}^{\nu\mu} \quad (2.5)$$

Calculations [7] of the decay time for small overlap give values $\tau_{\text{dec}} \lesssim 5 \cdot 10^{-23}$ s. Thus, in the basis of asymptotic eigenstates we find $\tau_{\text{dec}} \ll \tau_{\text{cor}}$, such that the strong-coupling limit is applicable for this formulation. The initial correlations $I_{\mu\nu}(t, t_0)$ have the same structure as the memory kernel (2.3). It is usually assumed that the resulting transport equation is applied for $t-t_0 \gg \tau_{\text{mem}}^{\nu\mu}$ where $I_{\mu\nu}(t, t_0)$ can be neglected. In the strong-coupling limit large off-shell contributions are allowed in the collision kernel.

(3) As discussed at the beginning of this section, the macroscopic variables have to be chosen as the slow modes of the system. This implies that the relaxation times for the macroscopic variables satisfy $\tau_{\text{relax}} \gg \tau_{\text{mem}}^{\nu\mu}$ and hence, justify the *Markov approximation*. It consists in replacing $f_\nu(t-\tau)$ by $f_\nu(t)$ in eq. (2.2) and yields the

the master equation for $t-t_0 \gg \tau_{\text{mem}}^{\nu\mu}$.

$$\frac{d}{dt} f_\mu(t) = \sum_{\nu \neq \mu} w_{\mu\nu}(t) [d_\mu f_\nu(t) - d_\nu f_\mu(t)] \quad (2.6)$$

with the transition probability $w_{\mu\nu}(t) \equiv \int_{-\infty}^{\infty} d\tau K_{\mu\nu}(t, \tau)$. Such a master equation was originally introduced by Pauli [8] in order to justify the *H*-theorem from the quantum-mechanical point of view. In the strong-coupling limit, however, the Markov approximation is in general not applicable without treating the energy conservation explicitly. In most applications this is taken into account by restricting all transitions to the energy shell and hence, employ an effective weak-coupling limit.

2.2 Fokker-Planck equation, transport coefficients and relaxation phenomena

A particularly convenient method of describing the solutions of the master equation (2.6) has been introduced by transforming the integral equation into a second-order differential equation, the *Fokker-Planck equation*

$$\frac{\partial f(\vec{y}, t)}{\partial t} = - \sum_{i=1}^g \frac{\partial}{\partial y_i} \{v_i(\vec{y}, t) f(\vec{y}, t)\} + \sum_{i,j=1}^g \frac{\partial^2}{\partial y_i \partial y_j} \{D_{ij}(\vec{y}, t) f(\vec{y}, t)\} \quad (2.7)$$

Here we have replaced the discrete variables μ by the continuous variable $\vec{y}$ with g components. The transport coefficients v_i (drift coefficients) and D_{ij} (diffusion coefficients) are completely determined by the second moments of the transition probabilities and the density of states [9]. Drift and diffusion coefficients are related by (generalized) Einstein relations [2,3,10].

In the further evaluation [7,10] of the microscopic expressions for the transport coefficients the method of spectral distributions is used which has been successfully applied to the calculation of mean spectroscopic nuclear quantities. This method employs the statistical properties of shell-model calculations in a large single-particle basis. It gives an accurate prescription for the evaluation of the averages $\langle |V_{nm}|^2 \rangle_{\nu\mu}$ in such a shell-model space. The evaluation

of the level densities $d(\vec{y})$, the memory or decay time $\tau_{\text{dec}}(\vec{y}, \vec{y}')$ and the transition probabilities $w(\vec{y}, \vec{y}')$ are reduced to the calculation of such averages.

The transport coefficients have been calculated for three macroscopic variables [10]: the mass A_1 of one fragment (mass-asymmetry variable), the z-component M of the total intrinsic angular momentum and the total excitation energy E^* . Drift and diffusion coefficients are simple analytical expressions depending on the macroscopic variables, on the total mass, on the relative angular momentum ℓ and on three characteristic parameters Δj , Δ and γ of the interaction matrix. The allowed change Δj of angular momentum in the transfer or excitation of a single nucleon is determined by the size of the overlap region. For touching nuclei we estimate [2] $\Delta j = 1.5 (A_1 + A_2)^{1/6}$. The corresponding energy change is determined by $\Delta = (\hbar^2 / I_{\text{rel}}) (\ell - M) \Delta j$ where I_{rel} is the moment of inertia for the relative motion. This relation is obtained from the conservation of total angular momentum and energy. The third parameter γ determines the strength of the coupling matrix elements and is expected to be strongly dependent on the relative distance between the fragments.

A considerable number of *mass* distributions measured in dissipative collisions have been analyzed [cf. ref. 2] using a Fokker-Planck equation (2.7) for the single variable A_1 . Figure 2 shows as an example the comparison of the data for ^{86}Kr (5.99 MeV/u) + ^{166}Er with the calculated distribution (dashed line) obtained by using the theoretical values for the transport coefficients. The solid line is obtained by reducing the theoretical drift coefficient to half its value.

Figure 3 compares experimental and theoretical values for the diffusion constant for various reactions [2]. In calculating the theoretical values we use $\gamma = 2.7$ MeV and the dynamical values Δj and Δ as given above with $\ell = \bar{\ell} = (2/3)\ell_{\text{gr}}$ and $M = M_{\text{st}}$ (sticking). Note that the fitted value of the strength parameter γ is in agreement with estimates for the touching configuration. Com-

paring experimental and theoretical values we realize that the dependence of the mean diffusion

Fig. 2: Calculated element distributions for the reaction $^{86}\text{Kr}(5.99 \text{ MeV/u}) + ^{166}\text{Er}$ compared with experimental data. The dashed curve corresponds to theoretical values of drift and diffusion coefficients, the solid curve to a fit by adjusting only the drift coefficient. From ref. [11].

Fig. 3: Comparison of experimental and theoretical values for the mass diffusion coefficient. The reactions are: (●) $^{86}\text{Kr}(515, 619, 703 \text{ MeV}) + ^{166}\text{Er}$, (○) $^{136}\text{Xe}(900, 1130 \text{ MeV}) + ^{209}\text{Bi}$, (■) $^{84}\text{Kr}(712 \text{ MeV}) + ^{209}\text{Bi}$, (□) $^{208}\text{Pb}(1560 \text{ MeV}) + ^{238}\text{U}$, (▲) $^{132}\text{Xe}(779 \text{ MeV}) + ^{120}\text{Sn}$, (△) $^{208}\text{Pb}(1456, 1560 \text{ MeV}) + ^{208}\text{Pb}$, (x) $^{20}\text{Ne}(168 \text{ MeV}) + ^{63}\text{Cu}$. Values from [2]

coefficient on bombarding energy, total mass and mass asymmetry is well reproduced. Large effects from shell structure in the mass diffusion coefficient are not apparent from the experimental values, probably because these values correspond to mean diffusion coefficients at rather high excitation energies.

Fig. 4: The total angular momentum $I_{\text{tot}}(Z_1)$ of the fragments as function of the fragment charge number and corresponding γ -multiplicity data for the reaction $^{86}\text{Kr}(5.99 \text{ MeV/u}) + ^{166}\text{Er}$. The dashed curve is obtained by neglecting fluctuations. From ref. [11].

Figure 4 compares the calculated *angular momentum* of the fragments with results from γ -multiplicity measurements. The dip around the projectile charge is understood as follows: The fragments close to the projectile are predominantly produced in collisions with large ℓ -values where the interaction time and hence, the dissipated angular momentum is small. Sufficiently far away from the projectile-charge number, the dissipated angular momentum saturates. These fragments are mainly populated in collisions with ℓ -values where the interaction time is large enough to reach sticking. An important feature of angular-momentum dissipation is the occurrence of large fluctuations which affect even the mean values (cf. fig. 4). These fluctuations in angular momentum together with fluctuations in the total kinetic energy loss or Q-value imply that there is no sharp correlation between impact angular momentum ℓ and the Q-value. This becomes important

in particular for small ℓ -values. Corresponding effects have been clearly demonstrated in sequential-fission measurements in bombardments of U and Pb on various targets by von Harrach et al. [12].

Returning to the time evolution of the collision complex let us discuss the *characteristic times* involved in the process (cf. fig. 1). From the analysis of angular distributions it is possible to deduce nuclear interaction times [9,13,14]. These interaction times range from about 10^{-22} s for grazing collisions up to several 10^{-21} s for close collisions (with small impact parameters). They have been used in the analyses of mass distributions and angular momentum dissipation. From a fit to experimental energy spectra the relaxation times for the loss of radial kinetic energy (τ_{rad}) and for the evolution of fragment deformations (τ_{def}) have been determined [14]. In this analysis the theoretical relaxation time τ_{ang} for the dissipation of relative angular momenta has been used (cf. fig. 4). The values for the relaxation times ($\tau_{\text{rad}} \approx 0.3 \cdot 10^{-21}$ s, $\tau_{\text{ang}} \approx 1.0 \cdot 10^{-21}$ s, $\tau_{\text{def}} \approx 4 \cdot 10^{-21}$ s) imply that the fast loss of radial kinetic energy is followed by the dissipation of relative angular momentum and finally by the evolution of fragment deformations. The analysis of mass distributions (cf. fig. 2) show that no equilibrium is reached in the mass-asymmetry coordinate for typical interaction times. The corresponding equilibration time ($\tau_{\text{mass}} \approx 2 \cdot 10^{-20}$ s) is larger by an order of magnitude and typically of the same order of magnitude as the mean time τ_{rot} necessary for a complete rotation of the collision complex.

2.3 Neutron and proton diffusion

Recently [15] we have generalized the formulation [10] for the mass transfer to treat the case where a distinction is drawn between neutrons and protons. The neutron diffusion coefficient D_{NN} is typically 3 to 5 times larger than the proton diffusion coefficient D_{ZZ} for two major reasons. Firstly because there are more neutrons

than protons taking part in most heavy-ion collisions and secondly because the protons are strongly hindered by Coulomb repulsion. On recombining D_{NN} and D_{ZZ} to obtain an effective mass diffusion coefficient we find no systematic differences from the values of D_{AA} given in [10] which agree well with phenomenological values (cf. fig. 3).

Without fitting any new parameter to the neutron and proton diffusion coefficients we have studied the question whether we can describe the 'fast' N/Z equilibration by the diffusion in the N-Z plane. Indeed we find (see the discussion below) that our statistical treatment is satisfactory for the description of proton and neutron transfer. No systematic deviations are observed which show evidence for *quantal effects* in the charge equilibration. However, shell-structure effects are seen to be significant (cf. § 2.4).

Let us discuss the calculations in some details.

We solve the Fokker-Planck equation

$$\frac{\partial f}{\partial t} = - \frac{\partial}{\partial N_1} (v_N f) - \frac{\partial}{\partial Z_1} (v_Z f) + D_{NN} \frac{\partial^2 f}{\partial N_1^2} + D_{ZZ} \frac{\partial^2 f}{\partial Z_1^2} \quad (2.8)$$

for the distribution function $f(N_1, Z_1; t)$ analytically by approximating the driving potential $U(N_1, Z_1)$, i.e. liquid-drop energies of the fragments plus proximity interaction at contact plus rotational energy, by a two-dimensional oscillator potential in N_1 and Z_1 . In this case the drift coefficients

$$v_N = - (D_{NN}/T) \partial U / \partial N_1, \quad v_Z = - (D_{ZZ}/T) \partial U / \partial Z_1 \quad (2.9)$$

with T denoting the temperature, are linear in N_1 and Z_1 , respectively. Therefore the distribution function has a Gaussian form which is completely determined by its first and second moments, i.e. the mean values $\langle N_1 \rangle$, $\langle Z_1 \rangle$, the variances σ_{NN}^2 , σ_{ZZ}^2 and the covariance σ_{NZ}^2 . These quantities are determined from a set of five coupled first-order differential equations. Like in the analyses of mass transfer we use the phenomenological model [2,14] where the time dependence of the transport coefficients is re-

placed by an equivalent ℓ dependence. This model also relates the nuclear interaction time $\tau_{int}(\ell)$ to the total kinetic energy loss (TKEL).

We have studied the reactions ^{144}Sm (1000 MeV) + ^{144}Sm , ^{154}Sm (970 MeV) + ^{154}Sm , ^{132}Xe (900 MeV) + ^{197}Au , ^{56}Fe (464 MeV) + ^{56}Fe , ^{165}Ho , ^{238}U ; and ^{129}Xe (775 MeV) + ^{56}Fe where data on the N-Z equilibration became recently available [16-19]. The figs. 5 to 9 show examples of the comparison with the data. Figure 5 shows the N/Z equilibration of the mean values into the valley.

Fig. 5: The calculated drift of the mean values $\langle N_1 \rangle$ and $\langle Z_1 \rangle$ into the valley of the driving potential with increasing TKEL for the system ^{129}Xe (775 MeV) + ^{56}Fe . Here the driving potential is calculated without shell corrections. Including the shell corrections would move the valley close to the experimental results given by the crosses. From ref.[15].

This process is fast because the driving potential is strong and the distance to go is small. There is no necessity to call on quantum effects to explain the fast N/Z equilibration. Figure 6 shows the evolution of the distribution in the N-Z plane for ^{144}Sm (1000 MeV) + ^{144}Sm . With increasing energy loss (interaction time) the distribution rotates into the direction of the valley of the driving potential. A measure for the development of this correlation along the valley is given by the correlation coefficient $\rho_{NZ} = \sigma_{NZ}^2 / (\sigma_{NN} \sigma_{ZZ})$ which is also shown in fig.6.

Fig. 6: Contours of N-Z distributions and the correlation coefficients as functions of total kinetic energy loss (TKEL) for ^{144}Sm (1000 MeV) + ^{144}Sm . From ref. [15].

Conditional variances σ_{AA}^2 for $Z_1 = \text{const}$ and σ_{ZZ}^2 for $A_1 = \text{const}$ are compared in figs. 7 and 8 with experimental data.

Fig. 7: Theoretical curves for $\sigma_{AA}^2|Z$ for ^{56}Fe (464 MeV) + ^{165}Ho and ^{129}Xe (775 MeV) + ^{56}Fe as functions of kinetic energy loss compared with experiment. From ref. [15].

Fig. 8: Theoretical prediction of $\sigma_{ZZ}^2|A_1$ for ^{132}Xe (900 MeV) + ^{197}Au as function of kinetic energy loss compared with the experimental results for the isobars $A_1 = 194, 197$ and 200 . From ref. [15].

Fig. 9: Theoretical curves for σ_{AA}^2 , σ_{ZZ}^2 and $\sigma_{AA}^2/\sigma_{ZZ}^2$ as functions of TKEL for ^{154}Sm (970 MeV) + ^{154}Sm and ^{144}Sm (1000 MeV) + ^{144}Sm compared with experiment. From ref. [15].

Summarizing all these analyses we can say that the pure statistical treatment of the proton

and neutron transfer gives a satisfactory description of experimental data. For a more detailed understanding, however, we need to take shell effects into account. This is clearly demonstrated in fig. 9 for $^{154}\text{Sm} + ^{154}\text{Sm}$ and $^{144}\text{Sm} + ^{144}\text{Sm}$. The observed difference in the behaviour of these very similar heavy-ion system can be understood in terms of the closed neutron shell in ^{144}Sm ($N_1 = 82$). At low kinetic energy losses, where shell effects may be felt, one would expect that, because of the hindrance of the neutron transfer, more protons must be exchanged to obtain the same energy loss and hence larger values of σ^2_{ZZ} are necessary as compared with those in the non-magic isotope ^{154}Sm . A systematic study of shell effects is given in the following subsection.

2.4 Influence of single-particle shell structure on the energy dissipation [20]

The importance of shell effects in the energy dissipation observed in heavy-ion collisions has been demonstrated by Dakowski et al. [21] by analyzing the specific energy loss dE/dN , i.e. the mean energy loss per transferred nucleon. In deriving a theoretical expression for dE/dN we start out from the rate equation

$$\partial F_i(t)/\partial t = \sum_j w_{ij}(t) \{F_j(t)[1-F_i(t)] - F_i(t)[1-F_j(t)]\} \quad (2.10)$$

for the occupation probabilities $F_i(t)$ of the single-particle states i . The first term on the r.h.s. describes the gain of occupation probability of the state i due to transitions from all states j to the state i , whereas the second term represents the total loss of probability from i . The transition probability $w_{ij}(t)$ is symmetric with respect to the exchange of i and j and vanishes whenever i and j have different isospin. This guarantees conservation of the total numbers of protons and neutrons separately. The rate equation (2.10) takes full account of the Pauli exclusion principle. The energy loss-rate

$$\dot{E} = \sum_i \epsilon_i \frac{dF_i(t)}{dt} = \sum_{i,j} (\epsilon_i - \epsilon_j) w_{ij}(t) F_j(t) [1-F_i(t)] \quad (2.11)$$

is obtained from (2.10). Here ϵ_i denote the energies of the single-particle states i . It should be noted that this expression includes both the excitations with and without transfer. For the rate of exchanged nucleons we can readily write

$$\dot{N} = \sum'_{i,j} w_{ij}(t) F_j(t) [1-F_i(t)] \quad (2.12)$$

where the prime indicates that the sum is taken only over states i, j belonging to different fragments. Combining (2.11) and (2.12) the mean energy loss per transferred nucleon is given by

$$\frac{dE}{dN} = \frac{\dot{E}}{\dot{N}} = \frac{\sum_{i,j} (\epsilon_i - \epsilon_j) w_{ij}(t) F_j(t) [1-F_i(t)]}{\sum'_{i,j} w_{ij}(t) F_j(t) [1-F_i(t)]} \quad (2.13)$$

This expression can be discussed qualitatively by referring to fig. 10.

Fig. 10: Schematic picture of possible transitions for two interacting nuclei (a) without and (b) with shell structure. From ref. [20]. Figure 10a shows in a schematic way all types of transitions (excitations with and without transfer) which are taken into account in (2.13). The effect of single-particle shell

structure is most easily understood by regarding two magic fragments (cf. fig. 10b). It is seen that with each transition an *extra* energy roughly equal to the gap energy $\Delta\epsilon_{\text{shell}}$ is needed. Therefore the rate (2.13) for magic fragments should be roughly given by dE/dN for non-magic fragments plus $\Delta\epsilon_{\text{shell}}$.

Fig. 11: Influence of shell structure on dE/dN as shown in the comparison of TKEL vs. σ_A^2 for ^{208}Pb (7.6 MeV/u) + ^{208}Pb and ^{238}U (7.4 MeV/u) + ^{238}U . From ref. [20].

Fig. 12: Same as fig. 11 for ^{144}Sm (1000 MeV) + ^{144}Sm and ^{154}Sm (970 MeV) + ^{154}Sm . From ref. [20].

Fig. 13: Comparison of theoretical values $(dE/dN)_0$ and experimental values $(dE/d\sigma_A^2)_0$ for the initial slopes in the curves of TKEL vs. σ_A^2 . Both values are obtained from an average over the first 25 MeV energy bin of TKEL. The squares represent systems for which the experimental values were extracted using measured mass widths. In all other cases only charge widths have been measured. From ref. [20].

An important feature of dE/dN as given by eq. (2.13) is the *independence* of the overall *strength* of the transition probabilities $w_{ij}(t)$ which strongly depend on the trajectories of relative motion. Therefore dE/dN does not significantly depend on details of the relative motion. The further evaluation of eq. (2.13) is described in ref. [20]. No parameter is adjusted to the experimental quantity.

Calculations of dE/dN were performed for 21 systems for which experimental data on σ_A^2 or σ_Z^2 as functions of TKEL were available. The number of exchanged nucleons is obtained from $N = \sigma_A^2$ which is valid for the independent statistical transfer of single nucleons (random walk) without an A_1 -dependence of the mass-drift coefficient. Two examples are shown in figs. 11 and 12 demonstrating the importance of shell effects on dE/dN . In fig. 13 a compilation of experimental and theoretical values for the initial slope $(dE/dN)_0$ is given. The agreement supports the underlying physical picture.

3. DYNAMICAL ASPECTS

The restriction to local statistical equilibrium limits the applicability of present transport theories essentially to the third stage of the total process (cf. fig. 1). Therefore, the apparent success of transport theories in the interpretation of experimental data may be partially attributed to the short duration of the initial stages of the process as compared to the third stage. Furthermore, the main success has been obtained for quantities which are likely to be determined only by statistical aspects of the colliding nuclei. However, there are indications even in the slow mass-diffusion process for *inconsistencies* of drift and diffusion coefficients within the statistical approach (cf. fig. 2). The reason for the relative smallness of the drift coefficient as compared to the diffusion coefficient may be due to either an incomplete local equilibration

[14] or an additional contribution to the mass-distribution variance from the neck degree of freedom [22].

In addition to the assumption of local statistical equilibrium, current transport theories suffer from the occurrence of the *strong-coupling limit* in the collision kernel. This strong-coupling limit is treated with by assuming all transitions to be on shell and thus employing an effective weak-coupling limit (cf. § 2.1).

A way out of these difficulties is supplied by the observations that (i) the occurrence of the strong-coupling limit is essentially due to the treatment of specific *manybody* quantities [23] and (ii) that the major part of the coupling should be treated coherently and *not statistically* [24]. In § 3.1 the mean free path is considered as the critical quantity for the applicability of local-equilibrium transport theories or time-dependent Hartree-Fock methods. In the following an approach is discussed which is based on treating *single-particle quantities* within an appropriate *dynamical single-particle basis*. A general outline of the new concept which allows for the explicit treatment of local equilibration is given in § 3.2. In § 3.3 a particularly interesting limit, the *dissipative diabatic dynamics (DDD)* is discussed which is complementary to the classical one-body dissipation of Swiatecki and coworkers [25]. The dissipative diabatic dynamics is treated explicitly within an interacting Fermi gas model in § 3.4. A simplified model for the numerical calculation of observable effects from these new DD dynamics is presented in § 3.5 together with a few results.

3.1 Transport theories vs. TDHF and the mean free path of nucleons

As discussed in some detail in § 2.1 current transport theories make use of an assumed separation of slow collective variables from the (assumed) fast equilibrating intrinsic degrees of freedom. These approaches are

confronted by one-body theories [25], like the self-consistent time-dependent Hartree-Fock (TDHF) method [26]. Whereas in the transport theories the residual interaction is the major agent for the assumed fast equilibration within the intrinsic degrees of freedom, residual interactions are completely ignored in the one-body theories.

The *mean free path* of a nucleon (a hole for $\epsilon < \epsilon_F$) with energy ϵ close to the Fermi energy ϵ_F and for low nuclear temperatures $T \ll \epsilon_F$, is approximately given by [27]

$$\lambda = \alpha \frac{1 + \exp(-|\epsilon - \epsilon_F|/T)}{(\epsilon - \epsilon_F)^2 + \pi^2 T^2} . \quad (3.1)$$

The factor α is $\approx 540 \text{ fm MeV}^2$ when estimated from the Fermi gas model [28] and $\approx 850 \text{ fm MeV}^2$ when estimated from the imaginary part of the optical potential (cf. fig. 23 of ref. [29]). With an average value of $\alpha \approx 700 \text{ fm MeV}^2$ we expect (cf. fig. 14) the mean free path λ to vary from

Fig. 14: Mean free paths of nucleons and local equilibration times as calculated from eqs. (3.1) and (3.2.). Shaded area indicates the mean free path during a heavy-ion collision.

large values at the initial stage of a heavy-ion collision to $\approx 15 \text{ fm}$ at the completely damped stage ($T \approx 2.5 \text{ MeV}$).

Current transport theories are restricted to the completely damped stage of heavy-ion collisions. This becomes apparent from the local equilibration times which follow from the mean free path (3.1). Assuming *three* subsequent collisions to be sufficient for establishing equilibrium for fixed values of the collective variables (*local* equilibrium), we have

$$\tau_{loc} \approx 3\lambda/v_F , \quad (3.2)$$

for the *local* equilibration time (v_F = Fermi velocity). For typical temperatures around 2.5 MeV in the completely damped stage we find $\tau_{loc} \approx 0.5 \cdot 10^{-21} \text{ s}$.

In the completely damped stage of the process the collective velocities might be small enough such that the system is always close to local (i.e. for fixed values of the collective variables) equilibrium and hence, that current transport theories are applicable. However, at the initial stage of heavy-ion collisions and for fission at small and intermediate energies the local equilibration time is considerably larger (of the order of a few 10^{-21} s). Therefore we expect large *memory effects* in the approach phase of heavy-ion collisions (duration about $0.3 \cdot 10^{-21} \text{ s}$).

Such memory effects in the initial stage are of course treated in TDHF. However, the mean free path is not large enough to justify TDHF at the later stages of the collision process. First steps to include two-body collisions in TDHF have been undertaken [30], but because of the numerical difficulties, these attempts are rudimental. We therefore follow a more schematic approach, starting from a single-particle model.

3.2 Single-particle transport theory [31]

It has been realized that the strong-coupling limit for the collision term of current transport theories has two major reasons:

- (i) the specification of many-body quantities (cf. ref. [23]) and
- (ii) the statistical treatment of the strong coupling (random coupling

matrix) between the reaction channels (cf. refs. [2, 32]).

In order to avoid the *strong-coupling* limit with its apparent drawback due to large off-shell contributions, we have formulated in [31] a new and rather general transport theory which starts out from a *dynamical* single-particle basis and considers *single-particle* quantities (if necessary also two-particle correlations). However, because of the small single-particle level density, statistical assumptions can hardly be justified for the one- or two-body interactions. Therefore we have replaced the statistical assumptions on the coupling matrix elements (cf. § 2.1) by a suitable *time averaging* in order to obtain from the Schrödinger equation (time irreversible) transport equations and to justify the Markov approximation for the basic collision terms. A similar time-smoothing has been used by Balian and Veneroni [33] in the extension of TDHF. In the following the main steps of the formulation are outlined.

For simplicity we consider only one collective variable q and treat it as a classical variable $q(t)$. A *dynamical* single-particle basis $\phi_\alpha(q)$ is defined by

$$H_0(q) \phi_\alpha(q) = \epsilon_\alpha(q) \phi_\alpha(q). \quad (3.3)$$

Here $H_0(q) \equiv H(q) - H'(q)$ depends on the dynamical situation which is encountered in the problem. In the *adiabatic* limit $H' = 0$ and hence $H_0(q) = H(q)$ the total single-particle Hamiltonian. *Diabatic* states can be constructed from adiabatic states by a maximum-overlap condition or by projecting out of $H(q)$ the $H_0(q)$ which exhibits higher symmetries.

We can now consider the equation of motion for the single-particle density matrix,

$$i\hbar \frac{\partial}{\partial t} \rho_{\alpha\beta}(t) = [V(t), \rho(t)]_{\alpha\beta} = \sum_{\gamma\delta} L_{\alpha\beta\gamma\delta}(t) \rho_{\gamma\delta}(t) \quad (3.4)$$

where the Liouville operator is defined by

$$L_{\alpha\beta\gamma\delta}(t) = V_{\alpha\gamma}(t) \delta_{\beta\delta} - V_{\delta\beta}(t) \delta_{\alpha\gamma} \quad (3.5)$$

from the coupling matrix elements

$$V_{\alpha\beta}(t) = \{-i\hbar \dot{q} \langle \phi_\alpha | \frac{\partial}{\partial q} | \phi_\beta \rangle + \langle \phi_\alpha | H' | \phi_\beta \rangle\} \cdot \exp \left\{ -\frac{i}{\hbar} \int_{t_0}^t dt' [\epsilon_\beta(t') - \epsilon_\alpha(t')] \right\}. \quad (3.6)$$

At this point we do not consider the residual interaction. The modification due to residual interaction is discussed below.

The formal transformation of the equation of motion for the density matrix $\rho_{\alpha\beta}(t)$ is similar to that of refs. [2,4]. The equation for the diagonal part becomes

$$\frac{\partial}{\partial t} \rho_{\alpha\alpha}(t) = -2 \sum_{\beta} \int_0^{t-t_0} d\tau M_{\alpha\beta}(t, \tau) \rho_{\beta\beta}(t-\tau) \quad (3.7)$$

where the memory kernel $M_{\alpha\beta}(t, \tau)$ is determined by the coupling matrix elements $V_{\alpha\beta}$. In writing down eq. (3.7) we have assumed that initially, i.e. before the collision at $t=t_0$, the non-diagonal elements of ρ vanish. Apart from this assumption (i.e. the neglect of the initial ground-state correlations of the approaching nuclei) eq. (3.7) is exact. We arrive at a time-irreversible equation by *averaging* over t . The essential condition on the averaging interval Δt is that it should be small compared to the decay time of a single-particle state

$$\chi_\alpha(t) = \phi_\alpha(q(t)) \exp \left\{ -\frac{i}{\hbar} \int_{t_0}^t dt' \epsilon_\alpha(t') \right\}. \quad (3.8)$$

The time averaging leads to the vanishing of fluctuating parts in the memory kernel. This results finally in a fast decay of the memory kernel in τ and hence justifies the *Markov* approximation

$$\frac{\partial}{\partial t} \rho_{\alpha\alpha}(t) = \sum_{\beta} W_{\alpha\beta}(t) [\rho_{\beta\beta}(t) - \rho_{\alpha\alpha}(t)] \quad (3.9)$$

with the transition probability given by

$$W_{\alpha\beta}(t) = 2\hbar^{-2} \text{Re} \int_0^\infty d\tau \{ V_{\alpha\beta}(t) V_{\beta\alpha}(t-\tau) \}_{\Delta t} \quad (3.10)$$

where $\{...\}_{\Delta t}$ denotes the average over Δt around t . The residual interactions $V_{\alpha\beta\gamma\delta}(t)$ modify the master equation (3.9) by an additional two-body collision term

$$K_{\alpha\alpha}(t) = 2\pi^{-2} \sum_{\beta\gamma\delta} \int_0^{\infty} \text{Re} \int d\tau \{ V_{\alpha\beta\gamma\delta}(t) V_{\gamma\delta\alpha\beta}(t-\tau) \} \Delta t$$

$$\cdot [(1-\rho_{\alpha\alpha}(t)) (1-\rho_{\beta\beta}(t)) \rho_{\gamma\gamma}(t) \rho_{\delta\delta}(t) - \rho_{\alpha\alpha}(t) \rho_{\beta\beta}(t) (1-\rho_{\gamma\gamma}(t)) (1-\rho_{\delta\delta}(t))]$$
(3.11)

which differs from usual collision terms in the time average of the correlation function.

The equation of motion for the collective variable $q(t)$ is obtained from the conservation of the total energy by considering

$$\frac{d}{dt} \langle H \rangle = \dot{q} \left(\frac{\partial U}{\partial q} + \xi \dot{q} + M \ddot{q} \right) = 0. \quad (3.12)$$

By ordering $d\langle H \rangle/dt$ according to terms proportional to $\dot{q}$, $\dot{q}^2$ and $\dot{q} \ddot{q}$ we find the following leading terms for the time-dependent potential $U(q,t)$, the friction coefficient $\xi(q,t)$ and the mass parameter $M(q,t)$,

$$U(q,t) \approx \sum_{\alpha} \epsilon_{\alpha}(q) \rho_{\alpha\alpha}(t) + \frac{1}{2} \sum_{\alpha} (T-H(q))_{\alpha\alpha} \rho_{\alpha\alpha}(t) \quad (3.13)$$

$$\xi(q,t) \approx \sum_{\alpha\beta} [\rho_{\alpha\alpha}(t) - \rho_{\beta\beta}(t)] \cdot \int_0^{\infty} d\tau \{ \langle \phi_{\alpha} | \frac{\partial H_0}{\partial q} | \phi_{\beta} \rangle_t \langle \phi_{\beta} | \frac{\partial}{\partial q} | \phi_{\alpha} \rangle_{t-\tau} g_{\alpha\beta}(t,\tau) \} \Delta t \quad (3.14)$$

$$M(q,t) \approx \sum_{\alpha\beta} [\rho_{\beta\beta}(t) - \rho_{\alpha\alpha}(t)] \cdot \int_0^{\infty} d\tau \tau \{ \langle \phi_{\alpha} | \frac{\partial H_0}{\partial q} | \phi_{\beta} \rangle_t \langle \phi_{\beta} | \frac{\partial}{\partial q} | \phi_{\alpha} \rangle_{t-\tau} g_{\alpha\beta}(t,\tau) \} \Delta t \quad (3.15)$$

with $\hbar\omega_{\alpha\beta} = \epsilon_{\alpha} - \epsilon_{\beta}$

$$g_{\alpha\beta}(t,\tau) = \exp \left[-i \int_{t-\tau}^t \omega_{\alpha\beta}(t') dt' \right]. \quad (3.16)$$

Regarding the matrix elements $\langle \phi_{\alpha} | \partial H_0 / \partial q | \phi_{\beta} \rangle$ in eqs. (3.14) and (3.15) to be essentially constant over the averaging interval Δt we can easily perform the average with a Lorentz weight function and obtain in this limit

$$\xi(q,t) \approx 2\pi \sum_{\alpha\beta} \rho_{\alpha\alpha}(t) \left| \langle \phi_{\alpha} | \frac{\partial H_0}{\partial q} | \phi_{\beta} \rangle \right|^2$$
(3.17)

$$\hbar^{-1} \frac{2 \Gamma_{\alpha\beta} \omega_{\alpha\beta}}{\pi(\omega_{\alpha\beta}^2 + \Gamma_{\alpha\beta}^2)^2}$$

$$M(q,t) \approx 2 \sum_{\alpha\beta} \rho_{\alpha\alpha}(t) \left| \langle \phi_{\alpha} | \frac{\partial H_0(q)}{\partial q} | \phi_{\beta} \rangle \right|^2$$

$$\hbar^{-1} \frac{\omega_{\alpha\beta}(\omega_{\alpha\beta}^2 - 3\Gamma_{\alpha\beta}^2)}{(\omega_{\alpha\beta}^2 + \Gamma_{\alpha\beta}^2)^3} \quad (3.18)$$

where $\Gamma_{\alpha\beta} = T |\dot{\omega}_{\alpha\beta}|$. This width vanishes for $\dot{\omega}_{\alpha\beta} = 0$ and hence, $\xi(q,t) = 0$ and $M(q)$ becomes the familiar cranking expression in the limit of uniform translation or rotation.

In the following subsection we consider the limit of diabatic single-particle motion. In this limit we expect the friction coefficient $\xi(q,t)$ to be small. Therefore, the energy dissipation should occur in a very special way, namely by the time-dependent potential $U(q,t)$.

3.3 Basic concept of dissipative diabatic dynamics

We still consider *slow collective motions* such that the Fermi velocity v_F is large compared to the collective velocity v_{coll} . In this limit the individual nucleons can adjust to the collective motion in a time-reversible manner. A particular problem is encountered in the quasi-crossing of two adiabatic single-particle levels as functions of the collective variable (Landau-Zener crossing, cf. fig. 15).

Fig. 15: Illustration of the basic concepts of dissipative diabatic dynamics. From ref. [38].

Due to some (usually small) part H' of the single-particle Hamiltonian, a crossing of adiabatic levels is forbidden. The neglect of this coupling leads to the definition of corresponding *diabatic* levels which cross each other. If the collective velocity is not too small, a nucleon occupying the lower level before the crossing of an unoccupied level will stay in the diabatic level and finds itself in the upper level after the crossing. Thus, this diabaticity leads to the excitation of a particle-hole state. The jump probability (i.e. the degree of diabaticity) is determined by

$$J = \exp(-\pi G^2) \quad (3.19)$$

$$G^2 = \frac{2|H'|^2}{\hbar|\eta|v_{\text{coll}}} \quad (3.20)$$

where η is the difference in the slopes of the diabatic levels [34].

The condition of more than 90% *diabaticity* in a Landau-Zener crossing leads to the condition $v_{\text{coll}} \geq 25|H'|^2 R / \hbar \epsilon_F$ where R and H' denote the radius of the nucleus and the interaction matrix element between the diabatic levels. With a typical value of 0.5 MeV for H' we obtain $v_{\text{coll}} \approx v_F/40$ for the minimum collective velocity. Combining this with $v_{\text{coll}} \ll v_F$ we find

$$(1/40) \text{ MeV} \leq E_{\text{coll}}/A \ll 40 \text{ MeV} \quad (3.21)$$

for the range of the collective kinetic energy per nucleon where the diabatic approximation is applicable. These estimates are well supported by numerical studies on fission and by TDHF calculations on heavy-ion collisions [35]. In general, the definition of diabatic single-particle states appropriate for the solution of a time-dependent problem is not straightforward. A simple example is given below for the single-particle motion in a box with moving walls.

On the basis of the diabatic single-particle motion the mechanism of energy dissipation is two-fold (cf. fig. 15):

(i) Due to the direct coupling of the collective motion to the diabatic shift of single-particle levels, the energy is transformed in a *time-reversible* fashion to the single-particle motion. In this way the collective kinetic energy is temporarily stored as a conservative potential which results from the diabatic excitation of particle-hole states.

(ii) Due to two-body collisions which lead to single-particle levels with random slopes, the conservative diabatic potential is destroyed. This *time-irreversible* process thus leads finally to the dissipation of the collective energy.

What are the consequences of this dissipation mechanism on the collective dynamics? In order to study this question we treat an interacting Fermi gas model which allows to describe analytically the main mechanism of energy dissipation as the result of diabatic excitations of particles and holes and subsequent equilibration by two-body collisions. The treatment shows the connection to the classical one-body dissipation mechanism [36] and clarifies the role of giant vibrations [37] in the dissipation mechanism. The analytic study may also guide future developments of more realistic formulations of transport theories which bridge the gap between TDHF and current transport theories.

3.4 Interacting Fermi gas within moving walls [38]

We consider a box located around the origin with the surfaces defined by

$$\bar{x}(t) = \bar{y}(t) = \frac{1}{2} a [1+\gamma(t)]^{-1/2} = -\underline{x}(t) = -\underline{y}(t)$$

and

$$\bar{z}(t) = \frac{1}{2} a [1+\gamma(t)] = -\underline{z}(t)$$

filled with a Fermi gas of A nucleons. The collective variable $\gamma(t)$ defines a collective motion of the system *without* compression. Using periodic boundary conditions (thus neglecting surface effects) we obtain the moving diabatic single-particle wave functions

$$\psi_j(r,t) = a^{-3/2} \exp [i(\vec{k}_j(t) \cdot \vec{r} - \hbar^{-1} \int_0^t \epsilon_j(t') dt' - \hbar^{-1} M \Phi(\vec{r}, t))] \quad (3.22)$$

which satisfy the Schrödinger equation except for terms proportional to $\dot{\gamma}^2$ and $\ddot{\gamma}$. The quantities $\epsilon_j(\gamma)$, M and Φ denote the energy $\hbar^2 k_j^2(\gamma)/(2M)$ of the state j , the nucleon mass and the velocity potential

$$\Phi(\vec{r}, t) = -\frac{1}{2} \dot{\gamma} r^2 P_2(\cos \theta) \quad (3.23)$$

of the incompressible irrotational flow. A 100% diabaticity of the single-particle motion is built into this model by the high degree of symmetry of the box. The motion of the walls determine the diabatic change of the momenta

$$\vec{k}_j(\gamma) = \{k_x^{(j)}(1+\gamma)^{1/2}, k_y^{(j)}(1+\gamma)^{1/2}, k_z^{(j)}(1+\gamma)^{-1}\} \quad (3.24)$$

which leads to a deformation of the momentum distribution (Fermi sphere).

The diabatic deformation of the Fermi sphere gives rise to a restoring force

$$F = -\frac{\partial}{\partial \gamma} \sum_j \epsilon_j(\gamma) n_j = \frac{1}{2} \hbar^2 Q/M \quad (3.25)$$

where n_j denotes the occupation probability of the moving diabatic state ψ_j and

$$Q(t) = \sum_j \{3[k_z^{(j)}(t)]^2 - k_j^2(t)\} n_j \quad (3.26)$$

is the quadrupole moment of the momentum distribution. Thus the classical equation of motion for the collective variable $\gamma(t)$ reads

$$m \ddot{\gamma}(t) = \frac{1}{2} \hbar^2 Q(t)/M \quad (3.27)$$

where the collective mass $m = \frac{1}{8} a^2 AM$ is determined from the moving wave functions (3.22). For simplicity, terms proportional to γQ have been neglected in (3.25) and (3.27).

Up to here we have considered the diabatic approximation where the occupation probabilities n_j do not change in time. Such changes can be induced by *two-body collisions* between the nucleons. We approximate the collision integral

by a relaxation ansatz

$$\partial n_j(t)/\partial t = -\tau_{loc}^{-1} [n_j(t) - \bar{n}_j(\gamma; \mu, T)] \quad (3.28)$$

where $\bar{n}_j$ denotes the equilibrated occupation probability for fixed values of the collective variable γ . The chemical potential $\mu(t)$ and the temperature $T(t)$ are determined by the conservation of the total number of particles and of the total excitation energy. The local equilibration time τ_{loc} introduced in (3.2) is taken as the measure for the time necessary to reach this (local) equilibrium. Evaluating the time derivative of (3.26) with (3.24) and (3.28) we obtain

$$\partial Q(t)/\partial t + 2\langle k^2 \rangle_0 \dot{\gamma} = -Q(t)/\tau_{loc} \quad (3.29)$$

with $\langle k^2 \rangle_0 = \frac{3}{5} A k_F^2$ and k_F the Fermi momentum (equal for protons and neutrons). In deriving (3.29) the momenta $\vec{k}_j$ have been replaced by a continuous variable and the limits $T \ll \epsilon_F$ (Fermi energy) and $Q \ll k_F^2$ have been taken. The equation (3.29) reflects the diabatic motion of the nucleons by the term $2\langle k^2 \rangle_0 \dot{\gamma}$ and the local equilibration due to two-body collisions by $-Q(t)/\tau_{loc}$.

With eqs. (3.27) and (3.29) we have derived a *closed set* of two coupled equations for the collective variable $\gamma(t)$ and the quadrupole moment $Q(t)$ of the momentum distribution. Solving eq. (3.29) formally for $Q(t)$ and inserting this result in eq. (3.27) we find

$$\begin{aligned} \ddot{\gamma}(t) + \int_0^t dt' n(t-t') \dot{\gamma}(t') \\ = \frac{1}{2} [\hbar^2 Q(0)/Mm] \exp(-t/\tau_{loc}). \end{aligned} \quad (3.30)$$

for the collective variable $\gamma(t)$ alone. The friction coefficient of current friction models is replaced here by the friction kernel

$$n(t-t') = \omega_0^2 \exp[-(t-t')/\tau_{loc}] \quad (3.31)$$

Depending on the relative values of ω_0 and τ_{loc}^{-1} we have two interesting limits for the retarded friction term in eq. (3.30).

(i) *Dissipative limit* ($\omega_0 \ll \tau_{loc}^{-1}$): In this limit we can replace $\dot{\gamma}(t')$ by $\dot{\gamma}(t)$ in the integral and obtain a classical friction term $(\xi/m)\dot{\gamma}(t)$ with the friction coefficient given by

$$\xi = m \int_0^\infty d\tau \eta(\tau) = \frac{6}{5} A \epsilon_F \tau_{loc}. \quad (3.32)$$

Since the *friction coefficient* is proportional to the local equilibration time, i.e. proportional to the mean free path (3.1), its temperature dependence becomes proportional to T^{-2} for n_j close to the equilibrium distribution. From the wall formula of one-body dissipation (piston model) [36] we find $\xi_w = (9/8) A \epsilon_F (a/v_F)$ for the friction coefficient. This agrees with expression (3.32) for $\tau_{loc} = (15/16) a/v_F \approx a/v_F$, i.e. for the local equilibration time equal to the time of flight of a nucleon across the box. Such a short equilibration time is expected in the one-body dissipation approach by randomization through reflections from walls with sufficiently large and random distortions, however, not for systems with symmetries which are not (as for the box) or not too strongly perturbed (as necessary for the diabaticity of single-particle motion).

Fig. 16: Time dependence of a quadrupole vibration (collective variable δ) within an oscillator model with $\tau_{loc} = 10^{-21}$ s. The fast giant vibrations are damped on the time scale of $2\tau_{loc}$ whereas the system relaxes very slowly to the equilibrium point ($\delta = 0$). From ref.[40]. (1bs = 10^{-23} s).

Fig. 17: Classical analog of the dissipation mechanism: The local equilibration time is determined by the time the acid needs for dissolving the spring. Whenever the interaction time (between spring and acid) is small compared to τ_{loc} the system moves elastically, whereas it becomes dissipative for $\tau_{loc} \lesssim \tau_{int}$.

(ii) *Elastic limit* ($\omega_0 \gg \tau_{loc}^{-1}$): Starting with a local equilibrium at $t=0$, i.e. $Q(0) = 0$, eq. (3.30) describes a harmonic vibration with energy

$$\hbar\omega_0 = 66 A^{-1/3} \text{ MeV} \quad (3.33)$$

which is readily identified with the isoscalar giant quadrupole vibration [39] (experimental energy $63 A^{-1/3}$ MeV). The direct coupling of the collective motion to the diabatic single-particle motion results in pure giant vibrations. The use of the giant vibrations of the colliding nuclei

by Broglia et al. [37] as agents for the dissipation of energy in heavy-ion collisions can be regarded as an example of the dissipation mechanism described here. However, the general concept goes beyond [37] because it is not restricted to oscillations around the equilibrium shapes of the colliding nuclei.

A somewhat more general discussion of DDD and some numerical results of the dissipation process for a single-particle oscillator model is described in [40]. The qualitative features of the dissipation process are discussed in fig. 16. A simple classical analog of DDD is illustrated in fig. 17. A simplified approach to the DDD is discussed in the following section.

3.5 Decaying diabatic potential model

A crude approximation to the friction kernel in heavy-ion collisions can be considered by assuming that the approach of the nuclei is largely diabatic. This fast approach gives rise to a *diabatic potential* which then decays in time. Since this diabatic potential arises from the diabatic single-particle motion and not from nuclear compression it is conceptually different from other sudden potentials [41].

Such a decaying diabatic potential has been introduced in [24] in the description of heavy-ion collisions (cf. fig. 18) including fusion and a long-living or slow dissipative component (sometimes referred to as fast fission [42]). According to [24] we consider a dynamical potential

$$U_{\text{dyn}}(r, t) = U_{\text{ad}}(r) [1 - \chi(t)] + U_{\text{diab}}(r) \chi(t) \quad (3.34)$$

with the decay factor

$$\chi(t) = \exp \left[- \frac{1}{\tau_{\text{loc}}} \int_{t_0}^t f(r(t')) dt' \right] \quad (3.35)$$

where we regard τ_{loc} as an unknown parameter. The integral over the form factor $f(r)$ (normalized by $f(r_{\text{contact}}) = 1$) smoothly switches on the equilibration. The quantities $r(t)$ and t_0

Fig. 18: Three different trajectories in the (r, α) plane for Ar (400 MeV) + Pb. Here r corresponds to the distance between the fragments and α describes the mass asymmetry. The trajectory with the smallest l -value ($l=51$) leads to the formation of a compound nucleus. The largest l -value ($l=104$) corresponds to a fast process known as dissipative (or deeply inelastic) collision. The trajectory with $l=102$, although similar to that with $l=104$ for small times, is captured. In contrast to the trajectory with $l=51$ it does not lead to a compound nucleus because $l > l(\text{crit}) \approx 70$ (largest l -value for the existence of the compound nucleus). Instead, the system develops towards mass symmetry ($\alpha=0$) and is expected to split into two fragments if deformation of the fragments are allowed. This long-living component of dissipative collisions rotates several times and hence, exhibits essentially a $1/\sin \theta$ angular distribution like the fragments from compound-nucleus fission. From ref. [24].

denote respectively the trajectory and a time well before the collision. As compared to the transport theory formulated so far [1-3] the essential new feature is the use of the explicitly time-dependent dynamical potential, which leads from the diabatic potential U_{diab} to the adiabatic potential U_{ad} (cf. fig. 19). This concept has been applied to heavy-ion collisions by several authors [24, 32, 42-44]. However, in these calculations no attention has been paid to a consistent modification of the transport coefficients. In the following such a more

consistent description [45] is discussed together with some results.

Fig. 19: Example of adiabatic potential for Ar + Pb used in numerical calculations. For comparison the adiabatic part and also the proximity potential is shown.

We consider six collective variables: position r and momentum p of radial motion, angle θ and angular momentum l of rotation, deformation ϵ and mass asymmetry α . With these variables the Fokker-Planck equation reads

$$\begin{aligned} \partial f(r, p; \theta, l; \epsilon, \alpha; t) / \partial t = & \\ & - \frac{p}{\mu} \frac{\partial f}{\partial r} + \frac{\partial U_{\text{dyn}}}{\partial r} \frac{\partial f}{\partial p} - \frac{\partial}{\partial p} (v_p f) + \frac{\partial^2}{\partial p^2} (D_{pp} f) \\ & - \frac{l}{I_{\text{rel}}} \frac{\partial f}{\partial \theta} - \frac{\partial}{\partial l} (v_l f) + \frac{\partial^2}{\partial l^2} (D_{ll} f) \\ & - \frac{\partial}{\partial \epsilon} (v_\epsilon f) + \frac{\partial^2}{\partial \epsilon^2} (D_{\epsilon\epsilon} f) \\ & - \frac{\partial}{\partial \alpha} (v_\alpha f) + \frac{\partial^2}{\partial \alpha^2} (D_{\alpha\alpha} f). \end{aligned} \quad (3.36)$$

The transport coefficients are taken from the microscopic expressions given in [10] with a strength factor proportional to the square root of the volume overlap. The diffusion coefficients D_{pp} , D_{ll} , $D_{\epsilon\epsilon}$ are multiplied in addition by the factor $[1 - \chi(t)]$ in order to account for the diabaticity at the initial

Fig. 20: Angular distributions for the reaction ^{86}Kr (8.18 MeV/u) + ^{166}Er using three different local equilibration times, $\tau_{\text{loc}} \cdot 10^{21} \text{s} = 2$ (---), 1 (—) and 0.5 (---).

Fig. 21: Energy distributions for the reactions ^{86}Kr (8.18 MeV/u) + ^{166}Er using three different local equilibration times (cf. fig. 20).

stage of the process. For the adiabatic potential we use the model of Möller and Nix [46].

In the numerical treatment of the Fokker-Planck equation we introduce a moment expansion up to second order. Figures 20 and 21 give results for the reaction ^{86}Kr (8.18 MeV/u) + ^{166}Er . Both, the angular distribution and the energy distribution show a remarkable dependence on the size of the local equilibration time. Therefore, we expect that detailed analyses of experimental data will make it possible to decide whether such memory effects are present in heavy-ion collisions or not.

4. CONCLUDING REMARKS

We have seen that transport theories based on the assumption of local equilibrium are remarkably successful in describing (even quantitatively in many cases) various relaxation phenomena like mass transfer, angular-momentum dissipation, equilibration of neutron-to-proton ratio and shell effects in the kinetic energy loss. However, these microscopic theories suffer in principle from two approximations. The strong-coupling limit is treated only in a crude way [2] and the assumption of local equilibrium (local with respect to the collective degrees of freedom) is not justified in the initial stage of the collision.

In order to avoid the strong-coupling limit a new transport theory has been introduced which starts out from a dynamical single-particle basis and considers single-particle quantities. Transport equations are obtained by a suitable *time averaging*. A particular dissipation mechanism is considered in detail which is characterized by the diabatic excitation of particle-hole states and the dissipation via two-body collisions. This *dissipative diabatic dynamics* (DDD) is expected to be effective for collective kinetic energies in the range $(1/40)\text{MeV} \leq E_{\text{coll}}/A \ll 40 \text{ MeV}$ and hence, should be essential for the energy dissipation in heavy-ion collisions. As a result of the large local equilibration times ($0.5 \cdot 10^{-21}\text{s} \leq t_{\text{loc}} \leq 5 \cdot 10^{-21}\text{s}$) the dissipation process is retarded. Thus the nuclear matter responds pure

elastically to fast perturbations ($\tau_{\text{loc}} \gg \tau_{\text{coll}}$) and pure dissipatively to slow perturbations ($\tau_{\text{loc}} \ll \tau_{\text{coll}}$). First calculations of effects from this retardation show that detailed analyses of heavy-ion collisions should reveal more about this dissipation process. The recent observation [47] of a barrier considerably larger than the bare Coulomb barrier for the mutual capture of two nuclei might be an indication for the presence of a repulsive diabatic potential. This diabatic potential in the entrance channel would be an alternative explanation of the *extra push* [48] due to friction forces alone. No extra push, however, is expected within our picture for bombarding energies close to the barrier whenever the collective kinetic energy is smaller than or close to the lower limit in relation (3.21) and hence, the single-particle motion becomes adiabatic. Indeed, no extra push has been observed recently [49] in fusion reactions at the Coulomb barrier.

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