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NUCLEAR EFFECTS IN RADIATIVE MUON CAPTURE IN CALCIUM

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A B S T R A C T

The sensitivity of photon spectrum in the theoretical treatment of radiative muon capture in Ca^{40} to the closure approximation is discussed and its explicit calculation, relaxing the above condition, performed in two models. It is shown that the spectrum is substantially lowered with respect to the closure approximation results in the same models and it is now compatible with experimental data and nucleonic values of the weak coupling constants.

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In spite of its small rate, about 10^{-4} times that for ordinary muon capture, the process of radiative muon capture (RMC) is of special interest because the high energy part of the photon spectrum is particularly sensitive to the value of the induced pseudoscalar coupling constant g_p .

Experimental data even if limited to Ca^{40} and subject to the difficulties of discrimination against the contamination of high energy neutrons ^{1),2)} are precise enough to be compared with theoretical expectations which mainly focus on a confirmation of the Goldberger-Treiman relation and eventually on the possibility of gaining information on the values of the weak coupling constants.

The present situation can be roughly summarized by saying that, within the usual theoretical framework, a value of the RMC spectrum is predicted, about twice the experimental one, so that experimental data can be essentially accounted for only by reversing the sign of the canonical value of g_p ²⁾ (see Fig. 1).

Recently a review of different contributions to the total RMC rate has been given ³⁾ in particular taking into account the hindrance effect due to the renormalization of the muon propagator in the nuclear Coulomb field ; however, a large discrepancy persists between theory and experimental data.

Without entering any speculation on the value of the nuclear coupling constant or on other effects, we tackle in the present letter the problem of nuclear structure encountered in any model in the calculation of the photon spectrum, i.e., closure approximation, investigating the sensitivity of the spectrum to the details of nuclear excitation.

Let us briefly recall ⁴⁾ that the spectrum of the process

$$\mu + a \rightarrow b + \nu(\nu) + \gamma(\kappa) \quad (1)$$

where a is the ground state of Ca^{40} and b any final state, is given by

$$N_{ba}(\kappa) d\kappa = \int d\Omega_\kappa \int d\Omega_\nu \frac{e^2}{4(2\pi)^4} \cdot |\phi_{a\nu}|^2 \sum_{\text{SPIN}} |M_{ba}|^2 \nu_{ba}^2 \kappa d\kappa \quad (2)$$

where ϕ_{av} is the averaged muon wave function, ν_{ab} is the allowed neutrino energy in the transition for a given photon energy k and M_{ab} is given, within the standard approximations [SU(4) invariance, neglect of recoil correction, etc.] by

$$|M_{ba}|^2 = G_r(\nu_{ba}, \kappa, \Theta_{\nu\kappa}) \cdot |<E| \sum_i \tau_i^- e^{i \vec{s}_{ab} \cdot \vec{\epsilon}_i} |a\rangle|^2 \quad (3)$$

where G_r is a combination of the weak nucleon coupling constants given in Rood-Tolhoek's paper ⁴⁾, which we will use with the standard values $\vec{s}_{ab} = \vec{\nu}_{ab} + \vec{k}$. The total photon spectrum is then given by

$$N(\kappa) d\kappa = \frac{e^2}{4(2\pi)^4} \int d\Omega_\kappa \int d\Omega_\nu \int dE \int d\nu \cdot \\ \delta(\nu_\mu - \nu - \kappa - E) G_r(\nu, \kappa, \Theta_{\nu\kappa}) \nu^2 \kappa \\ |<E| \sum_i \tau_i^- e^{-i \vec{s} \cdot \vec{\epsilon}_i} |a\rangle|^2 d\kappa \quad (4)$$

where the sum over the states b has been substituted by $\int |<E| dE$ and ν_μ is the maximum available energy in the reaction $\nu_\mu = \mu - E_B - E_C = 112$ MeV, μ being the muon mass, E_B the binding energy and E_C the Coulomb isotope shift.

The essential point lies therefore in the evaluation of the excitation spectrum

$$F(E, s) = |<E| \sum_i \tau_i^- e^{-i \vec{s} \cdot \vec{\epsilon}_i} |a\rangle|^2 \quad (5)$$

Some different approaches have been used in the literature as an extension of techniques used for ordinary muon capture : the standard shell model ⁴⁾, modification of it based on empirical information gained from photoabsorption ^{5),6)}, and the Fermi gas model (FGM) ⁷⁾. The main drawback of the former kind of calculations up to the most recent version ⁸⁾ is that, according to the authors themselves, it betrays the familiar shortcomings of the shell model, i.e., it under-estimates excitation energies and over-estimates total rates by a factor two for the ordinary muon capture. It is therefore, to say the least, very optimistic to hope of a better agreement in the case of radiative muon capture.

The second approach corrects the shell model results as far as the dipole part is concerned, taking energy and strength from photoreactions, and carries over the former calculation of the other multipoles, obtaining a satisfactory agreement as far as muon capture is concerned. As a final step, in both calculations, the closure approximation is used, i.e., all intermediate states are concentrated at the same energy $\langle E \rangle$, fixed by the end-point spectrum photon energy, which corresponds to the monopole-quadrupole shell model energy equal to the experimental giant dipole energy.

The same technique is used with Fermi gas model ⁹⁾.

The consequences of such an attitude are, of course, of paramount importance for the photon spectrum, because a maximum photon energy $k_m = \nu_\mu - E$ corresponds to a given excitation energy. If higher excitation bands are present there correspond lower k_m 's, giving a net decrease in the high energy part of the spectrum. In addition, also the phase space, Eq. (4), relatively depresses higher energy contributions.

We will therefore supplement the usually accepted fact that the GDR accounts for about $\frac{1}{2}$ of the total capture rate ⁵⁾ with the information from photoreactions as well as theoretical estimates ⁹⁾, all of which support the presence of a collective isovector quadrupole and monopole excitations at an energy of about twice the GDR's.

A first rough estimate can be done using energy weighted sum rules in order to calculate the total strength. This can easily be evaluated for $T=0$ nuclei both for muon and muon radiative captures, as

$$F(E, x) = \sum_b |\langle b | \sum_i \tau_i^- e^{-i\vec{x} \cdot \vec{r}_i} | \alpha \rangle|^2 =$$

$$= \frac{1}{E} \langle \alpha | \left[[H, \sum_i \tau_i^3 e^{i\vec{x} \cdot \vec{r}_i}] , \sum_j \tau_j^3 e^{-i\vec{x} \cdot \vec{r}_j} \right] | \alpha \rangle = \frac{x^2}{2M^* E} \quad (6)$$

where x is, respectively, either $\vec{v}$ or $\vec{s}$, M^* takes into account all exchange potentials not commuting with the excitation operator and E is the mean energy of the process. This shows that $F(E, x)$ represents the natural quantity which normalizes our results (in particular, M^* is seen to disappear in the ratio of radiative to normal capture) but clarifies also that whereas different spectra around the same mean energy and strength yield the same result for muon capture rates, the same is no longer true for the

photon spectrum in RMC, where closure loses predictivity if one identifies $\nu_\mu - E$ with the energy spectrum end-point.

As a matter of fact, let us now estimate what happens with the very naïve assumption that half the strength is located at the GDR energy, half at a double energy with two δ excitation functions. In such a case it is easy to get, from Eq. (6)

$$\begin{aligned} \int N(k) dk &\simeq \iint k \nu^2 \left[\frac{1}{2} \delta(\nu_\mu - \nu - k - E_1) + \right. \\ &\quad \left. + \frac{1}{2} \delta(\nu_\mu - \nu - k - E_2) \right] s^2 dE dk \\ &\simeq \frac{1}{2} k_{m1}^6 + \frac{1}{2} k_{m2}^6 \end{aligned} \quad (7)$$

where $k_{m1} = \nu_\mu - E_1$, $k_{m2} = \nu_\mu - E_2$. It is then straightforward to see that the rate is hindered almost by a factor two with respect to the case where all the strength is concentrated around the first resonance and that the same thing happens for the photon spectrum in the region of interest ($k \geq 60$ MeV) since the second term in the above expression, contributing below k_{m2} , plays there an irrelevant rôle.

This very rough estimate calls for a recalculation of the total muon capture rate, comparing, in given models, the results obtained using the closure approximation [i.e., disregarding the energy conservation constraint in the evaluation of Eqs. (4), (5)] with the exact evaluation of Eq. (4).

Two excitation models will be used, both incorporating the above-mentioned features, i.e., strength divided between GDR and higher states, and no excitation below 20 MeV (as experimentally observed). We will therefore retain as essential feature of the process the GDR multiplied by its form factor¹⁰⁾ and account for higher multipoles through :

- a) the Fermi gas model response function¹¹⁾, the dependence on s and E being explicitly known in the model ;
- b) an isovector quadrupole-monopole resonance at an energy of 35 MeV with its appropriate form factor (calculated, e.g., in the harmonic oscillator model).

In both cases there is a free parameter which must be fixed by the total capture rate value. So in case a) we use a value of the effective mass of 0.6 M and in case b) a strength of the monopole-quadrupole resonance

equal to the dipole one. This is in agreement with the standard values of the effective mass and with the estimate, through energy weighted sum rules of the quadrupole-monopole strength ^{*)}.

We then compute by numerical integration the photon spectrum with our excitation spectra and plot it with the results of the closure approximation ^{**)} in Fig. 2 (using the physical value of 20 MeV for the GDR energy).

As expected, we find a strong hindrance of about $\frac{1}{2}$ in both models with respect to the closure approximation. Although this correction is of the right order of magnitude and of the right sign, so that the comparison with experimental data looks rather promising, some caution is recommended in the conclusions.

First, there are many effects which have been simply overlooked here, which should be kept under control before direct comparison is possible (e.g., muon propagator in the Coulomb field, recoil terms), some of which are already considered in Ref. 3), and shown to go in the right direction ; further work is in progress along this line.

For these reasons, it follows that, at the present stage, it is probably premature to draw conclusions about the magnitude of g_p in Ca^{40} even if in reconsidering data with our approach, the trend is towards the canonical values. On the other hand, we hope sounder information will be got by comparison of future experiments with a forthcoming systematic treatment of medium heavy nuclei with $(N-Z)/A \simeq 0.1$, where the Fermi gas model, no unwanted excitation being present at low energy, could be directly used.

^{*)} As a matter of fact, it is straightforward from

$$\sum_n E_n | \langle n | \sum_i \tau_i^- (\vec{s} \cdot \vec{e}_i) | \alpha \rangle |^2 = \frac{Z s^2}{2M}$$

and

$$\sum_n E_n | \langle n | \sum_i \tau_i^- \frac{(\vec{s} \cdot \vec{e}_i)^2}{2} | \alpha \rangle |^2 = Z s^2 \frac{\langle (\vec{s} \cdot \vec{e})^2 \rangle}{M}$$

to set a ratio for the unretarded part of dipole and quadrupole of about 1, at $s \simeq 90$ MeV. Form factors further decrease this estimate.

^{**)} It is worth remarking that because of the strong dependence of the spectrum on the photon end-point, the Rood-Tolhoek's plot in Fig. 1 where $k_m = 89$ MeV has been used, somewhat differs from ours where, because of the experimental giant dipole, resonance energy $k_m = 92 \div 94$.

Finally, we want to stress that whereas in muon capture the total rate can be expressed as a function of the mean energy and strength only (or alternatively of E and M^{*12}), radiative muon capture is, in principle, suitable to probe more detailed features of the excitation spectrum. As a conclusion, we would therefore like to regard the present work as an analysis of the sensitivity of RMC to nuclear dynamics as opposed to the g_p value.

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FIGURE CAPTIONS

Figure 1 Photon spectrum in radiative muon capture. Experimental results (crosses) and theoretical prediction within the closure approximation [Ref. 3] with different values of the pseudoscalar coupling constant g_p .

Figure 2 Theoretical predictions of the photon spectrum in the models explained in the text in the closure approximation (full line and according to the exact calculations for the cases a) and b) (dotted lines).

FIG.1

FIG. 2