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THE LONG-RANGE NN INTERACTION AND THE AXIAL POLARIZABILITY

R. Tarrach

CERN -- Geneva
and

Department of Theoretical Physics, Barcelona

and

M. Ericson

CERN -- Geneva
and

Institut de Physique Nucléaire, Lyon

A B S T R A C T

The long-range part of the two-pion exchange nucleon-nucleon interaction is discussed with the use of the polarizability concept. The aim of this approach is to give a physical interpretation of this interaction based on the analogy with Van der Waals forces. These forces are determined at large distances by the electromagnetic structure constants of the interacting systems. The analogue of these constants for strong forces between nucleons is the axial polarizability, proportional to the p wave πN scattering volume.

We show that the long-range part of the two-pion exchange NN interaction is indeed described by the axial polarizability of the nucleon. This is done with the usual technique based on dispersion relations. The analytical expression of the long-range potential (for the spin and isospin averaged part) is derived.

The presence of strong interactions between the two exchanged pions is a new aspect, absent in the electromagnetic problem and it could blur the analogy. It is shown that their effects are unimportant for the long-range potential. Finally the range of validity of this simple polarizability potential is discussed.

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1. - INTRODUCTION

The concept of an axial polarizability (response to a pionic field) has been introduced to describe the exchange corrections to the pionic field of the nucleus [see Ref. 1) and references therein]. In the nuclear medium the nucleons scatter the virtual pions emitted by neighbouring nucleons and thus the pion field is no longer the one corresponding to free nucleons. This rescattering process can also be interpreted as a polarization effect : a nucleon polarizes its neighbours through the exchange of a pion hence modifying their mesic radiation field. This phenomenon is the axial (or rather pseudoscalar) analogue of a well-known electromagnetic effect occurring in a molecular system : a molecule polarizes its neighbours creating induced dipoles in the medium which can in turn radiate.

Pursuing the analogy with the molecular problem it seems natural to apply the concept of axial polarizability to the question of the interaction between nucleons. In the molecular case the mutual polarization creates an attractive force ; it is a fluctuation effect, since there are no net charges, which arises from the exchange of two photons between the molecules. This Van der Waals force can be expressed at large distances in terms of the electric and magnetic polarizability ²⁾. The distance below which the description of this force with the polarizabilities ceases to be valid is given by the size of the system or by the inverse of the typical excitation energies.

Similarly we expect that the mutual axial polarization of two nucleons through two-pion exchange leads to an attractive force which at large distances should be expressible in terms of the axial polarizability. This coefficient is known since it is related to p wave pion nucleon scattering ³⁾.

This work is an investigation of this nucleon-nucleon force which results from the axial polarization. This view-point has not been developed before, as far as we know, and has at least two attractive aspects : the simplicity of the input and the intuitive picture that the analogy with electromagnetism gives to the two-pion exchange interaction.

On the other hand much work has been devoted to the study of the two-pion exchange interaction. This will allow us to see, from a comparison with our results, if the concept of polarizability is a useful one

in this context, and if so, what is its domain of validity. As in electromagnetism, the polarizability potential is expected to fail when the nucleons interpenetrate, i.e., for a distance of the order of the inverse pion mass. This is precisely the region where the NN potential is well known, allowing its detailed comparison with a polarizability potential. Such a comparison is not accessible in the electromagnetic case since a detailed knowledge of the molecular force is not available yet.

In the study of the polarizability potential the following differences between the strong and the electromagnetic case turn out to be quite important.

i) The pion has a mass while the photon is massless. This makes that the long range potential cannot be expressed with the physical pion-nucleon amplitude, whereas the electromagnetic polarizabilities are directly measurable quantities. It also gives to the strong interactions their short-range character (exponential fall-off), while the electromagnetic ones are of long range (inverse power behaviour).

ii) Contrary to a molecule a nucleon is not a neutral object (for pionic emission). It radiates a pion field without external stimulation, since its spin represents a permanent axial dipole moment. The strong interaction should contain a dipole-induced dipole part in addition to the induced dipole-induced dipole term, which is the only one present in Van der Waals forces between spinless molecules.

iii) The axial polarizability is linked to the p wave πN amplitude.

But this amplitude also contains an s wave part which plays the rôle of an induced charge. There is, of course, no electromagnetic analogue to this, since electric charges are conserved. Interactions involving induced charges have a larger range than those involving induced dipoles. It then seems that the longest range potential should be linked to the induced charge and not to the induced dipole (polarizability), which would somehow spoil the analogy with Van der Waals forces. However, the low energy theorem implies a small s wave and hence the polarizability still dominates the long range potential as will be shown.

iv) Finally, pions interact strongly between them, which is not the case for photons. This is an essential point, since a strong $\pi\pi$ interaction could completely spoil the simple polarizability picture. This point will be given a special attention.

We have to emphasize that it is not our aim to improve the existing nucleon-nucleon potentials, but that the interest of this work is firstly and mainly epistemological.

In Section 2 the relevant elastic πN and the $\pi\pi \rightarrow N\bar{N}$ amplitudes are reviewed together with the definition of the axial polarizability. In Section 3 we sketch the dispersive approach to NN forces. We also give the expression of the NN potential in terms of the t channel helicity amplitudes corresponding to a two-pion intermediate state. Section 4 deals with the $\pi\pi$ interaction at low energies; the problem of the analytic continuation through its threshold is studied. In Section 5 the analytical expression for the long range forces is given, allowing an instructive analogy with electromagnetism. In addition a comparison of our $\pi\pi \rightarrow N\bar{N}$ polarizability amplitudes with recent available information is performed. From this we infer the domain of validity of our potential.

2. - THE πN SYSTEM AND THE AXIAL POLARIZABILITY

According to CGLN ⁴⁾ the invariant πN T matrix element may be written as ^{*)}

$$\begin{aligned} T_{\pi N}(\nu, \nu_B) = \bar{u}(\tau', \lambda') \left\{ A^{(+)}(\nu, \nu_B) + \frac{1}{2} (k+k')^\mu \gamma_\mu B^{(+)}(\nu, \nu_B) \right\} \delta_{a'a} \\ + i \epsilon_{a'a b} \tau_b \left\{ A^{(-)}(\nu, \nu_B) + \frac{1}{2} (k+k')^\mu \gamma_\mu B^{(-)}(\nu, \nu_B) \right\} u(\tau, \lambda) \end{aligned} \quad (1)$$

where k and r are the incoming pion and nucleon four-momenta, a is the incoming pion isospin index and λ the incoming nucleon helicity. The primed quantities correspond to the outgoing particles, τ_b and γ_μ are the Pauli and Dirac matrices. The variables ν and ν_B are defined in terms of the Mandelstam variables $s = (k+r)^2$ and $t = (k-k')^2$ and the pion and nucleon masses μ and M , respectively, as

*) Since our T matrix has the opposite sign to theirs and our metric is $+, -, -, -$ our A and B amplitudes coincide with CGLN's.

$$\begin{aligned} \nu &= \frac{1}{4M} (2(s - M^2 - \mu^2) + t) \\ \nu_B &= \frac{1}{4M} (2\mu^2 - t) \end{aligned} \quad (2)$$

The Born contributions are (in a pseudoscalar theory)

$$\begin{aligned} A^{(+)\pi} &= A^{(-)\pi} = 0 \\ B^{(+)\pi} &= - \frac{g^2}{M} \frac{\nu}{\nu^2 - \nu_B^2} \\ B^{(-)\pi} &= \frac{g^2}{M} \frac{\nu_B}{\nu^2 - \nu_B^2} \end{aligned} \quad (3)$$

with the pion-nucleon coupling constant $(g^2/4\pi) = 14.5$.

We will later need the spin and isospin averaged amplitude which may immediately be obtained from Eq. (1) and which reads

$$\begin{aligned} T_{\pi N}^{av}(\nu, \nu_B) &= 2 \sqrt{M^2 - \frac{t}{4}} \left(A^{(+)}(\nu, \nu_B) + \frac{M^2 \nu}{M^2 - \frac{t}{4}} B^{(+)}(\nu, \nu_B) \right) \\ &= 2 \sqrt{M^2 - \frac{t}{4}} C^{(+)}(\nu, t) \end{aligned} \quad (4)$$

The πN centre-of-mass pion energy ω is given in terms of s by

$$\omega^2 = (s + \mu^2 - M^2)^2 / 4s \quad (5)$$

and $\cos\theta$ will denote the πN centre-of-mass scattering angle.

The s channel threshold is given by the following values of the variables

$$\begin{aligned} s_0 &= (M + \mu)^2, \quad t_0 = 0 \\ \nu_0 &= \mu, \quad \omega_0 = \mu \end{aligned} \quad (6)$$

The static axial polarizability represents the dipole type response of the nucleon to a static pionic field, in close analogy to the electromagnetic polarizabilities. It is proportional to the p wave scattering volume for energyless pions, that is, for $\omega=0$. This condition, which leads to the constraints

$$s = M^2 - \mu^2, \quad \nu = \frac{1}{4M} (t - 4\mu^2) \quad (7)$$

reduces the number of independent variables to one. The coefficient of $\cos\theta$ in a partial wave expansion will hence determine the static axial polarizability. There are several ways of obtaining its value. For instance, in the study of the nuclear pionic field the authors of Ref. 1) have used the CGLN extrapolation of the p wave πN amplitude to the point $\omega=0$. Another possibility is given by the use of an expansion of the non-Born πN amplitudes around the crossing symmetry point $\nu=t=0$ as given in Ref. 5). Since we will have to consider later the iteration of the OPE potential, which is related to the positive energy Born term, it will be convenient to treat the non-Born and the negative energy Born parts together. For the amplitude introduced in Eq. (4) this gives the combination $\bar{C}^{(+)}(\nu, t) = C^{(+)\text{NB}}(\nu, t) - (g^2/M)$ and which has the expansion 5)

$$\bar{C}^{(+)}(\nu, t) = \left[-1.7 + 1.18 \frac{t}{\mu^2} + 0.035 \left(\frac{t}{\mu^2} \right)^2 + 1.13 \frac{\nu^2}{\mu^2} + \dots \right] \mu^{-1} \quad (8)$$

The first coefficient of this expansion is badly known, its numerical value resulting from a huge cancellation between the non-Born s wave and the negative energy Born contribution $-g^2/M$. The third coefficient is also not well known, it might be a 50% off.

For $\omega=0$ ν^2 is very small for all but very large values of t (it is zero in the limit $M \rightarrow \infty$); the contribution of the ν^2 dependent terms may then be safely neglected. The concept of polarizability is useful as long as the remaining expansion in t may be truncated after the linear term. It amounts to neglecting d and higher waves, which is a common approximation in this region, used, e.g., to relate the σ term to the πN amplitude at the point $\nu=0$, $t=4\mu^2$ 6). Since the t^2 coefficient is not zero, linearity is not perfect. But this coefficient is only about a 3% of the t coefficient; it is not accurately known and anyhow in the absence of the t^3 coefficient its inclusion would not be

more justified than its neglect. Furthermore linearity has been shown by Bohannon ⁷⁾ to be consistent with his result at $t = 4\mu^2$. We will use this linear approximation essential for exploiting the concept of polarizability. For $\omega = 0$ an expansion in $t - 2\mu^2 = -2\mu^2 \cos\theta$ separates in the linear approximation s and p waves, giving for Eq. (8)

$$\bar{C}^{(+)}(\nu=0, t) = \left[0.65 + 1.18 \frac{t - 2\mu^2}{\mu^2} \right] \mu^{-1} \quad (9)$$

The constant term is related to the σ commutator and its smallness is due to the Adler consistency condition. The coefficient of $t - 2\mu^2$ is proportional to the axial polarizability. This expression will be taken to be valid in the region $\omega = 0$, $0 \leq t \leq 4\mu^2$, which is nothing but the allowed region for energyless pions, since it corresponds to $-1 \leq \cos\theta \leq 1$.

As we will see in the next section, the kinematical region relevant to the nucleon-nucleon intermediate and long range forces is the pseudophysical t channel region ($4\mu^2 \leq t \leq 4M^2$), and especially the part $t \gtrsim 4\mu^2$ near the two-pion threshold

$$\begin{aligned} \bar{s}_0 &= M^2 - \mu^2, & \bar{t}_0 &= 4\mu^2 \\ \bar{\nu}_0 &= 0, & \bar{\omega}_0 &= 0 \end{aligned} \quad (10)$$

Notice that this point corresponds to a static situation, so that expression (9) is valid there. The polarizability therefore essentially determines the amplitude at the pseudophysical t channel threshold.

The question now is : does the polarizability also dominate beyond $t = 4\mu^2$, as needed for the NN interaction ? The problems arising from the crossing of the $\pi\pi$ threshold, like the appearance of an imaginary part linked to the $\pi\pi$ interaction, will be dealt with in Section 4. But even without taking into account the $\pi\pi$ interaction, the expression of Eq. (9) could not be continued too far, since due to its linear growth it soon becomes unrealistic, eventually violating unitarity bounds. This situation may be improved by taking into account the dynamic origin of the polarizability, which is the existence of a strong πN resonance, the Δ_{33} which is known from the work of Chew and Low ⁸⁾ to dominate the p wave amplitude. We will do so in the narrow width approximation, commonly used in this context ^{4),9)}. We introduce the Δ_{33} propagator in such a way

that the behaviour of the amplitude around $t=0$ is precisely the one given in Eq. (9). This leads to

$$\bar{C}_{\Delta}^{(+)}(\nu=0, t) = \frac{8\pi\mu}{\Delta + \frac{t}{2}} \left[0.8 + 1.5 \frac{t-2\mu^2}{\mu^2} \right] \quad (11)$$

with $\Delta = M_{\Delta}^2 - m^2 - \mu^2 = 31.6 \mu^2$. The only new parameter is the location of the resonance. Its introduction leaves the behaviour of the amplitude in the region $0 \leq t \leq 4\mu^2$ practically unchanged, since Δ has a large value. It should be noticed that the introduction of the Δ propagator gives to our approach some resemblance with a recent chiral dynamic model of πN scattering, which considered only N and Δ intermediate states ¹⁰⁾. The aim of their work is, however, a different one.

We will give now the relationship between the amplitude $C^{(+)}(\nu, t)$ and the well-known Frazer-Fulco $\pi\pi \rightarrow N\bar{N}$ helicity amplitudes ¹¹⁾. In the region $-1 \leq \cos\theta_t \leq 1$ (θ_t being the $\pi\pi \rightarrow N\bar{N}$ centre-of-mass scattering angle) one has, keeping only the lowest waves,

$$C^{(+)}(t, \cos\theta_t) = \frac{4\pi}{M^2 - \frac{t}{4}} \left[f_+^0(t) + 5 P_2(\cos\theta_t) \left(\frac{t}{4} - M^2 \right) \left(\frac{t}{4} - \mu^2 \right) f_+^2(t) \right] \quad (12)$$

The variables ν^2 and $\cos\theta_t$ are related to each other by

$$\nu^2 = \left(\frac{t}{4M^2} - 1 \right) \left(\frac{t}{4} - \mu^2 \right) \cos^2\theta_t \quad (13)$$

The partial wave helicity amplitudes are obtained from $C^{(+)}(t, \cos\theta_t)$ by projection. In the region $0 \leq t \leq 4\mu^2$ our expressions, Eqs. (9) and (11), lead to

$$\bar{f}_+^0(t) = \frac{M^2 - \frac{t}{4}}{4\pi\mu} \left[0.65 + 1.18 \frac{t-2\mu^2}{\mu^2} \right] \quad (14)$$

and

$$\bar{f}_{+\Delta}^0(t) = \frac{M^2 - \frac{t}{4}}{\Delta + \frac{t}{2}} 2\mu \left[0.8 + 1.5 \frac{t-2\mu^2}{\mu^2} \right] \quad (15)$$

respectively, when taken as a ν independent approximation to $C^{(+)}(t, \cos\theta_t)$.

The inclusion of the v^2 term in the expansion of Eq. (8), which has a coefficient almost equal to the one of the t term, leaves nevertheless Eqs. (14) and (15) almost unchanged [the coefficient of $(t - 2\mu^2)$ varies in less than a 10%]. This supports our idea that in the region $0 \leq t \leq 4\mu^2$ the polarizability dominates the s wave helicity amplitude. The comparison of Eqs. (14) and (15) with the by now well-established semi-phenomenological $\pi\pi \rightarrow N\bar{N}$ helicity amplitudes will be done in Section 5, after having studied, in Section 4, how to cross the point $t = 4\mu^2$.

As for the positive energy Born helicity amplitudes, they are real for $t > 4\mu^2(1 - (\mu^2/4M^2))$ (below this limit, when performing the partial wave projection, one integrates over the pole of the Born term, so that there is an imaginary part). Its s wave amplitude in this region is

$$f_+^{0P}(t) = \frac{g^2}{4\pi} \left[M h \operatorname{arc} \operatorname{tg} (h^{-1}) - \frac{t}{4M} \right] \quad (16)$$

with

$$h = \frac{t - 2\mu^2}{\sqrt{(t - 4\mu^2)(4M^2 - t)}} \quad (17)$$

Finally we recall that most of the difficulties encountered in this section were due to the mass of the pions which splits the different relevant points: s channel threshold, Eq. (6), t channel threshold, Eq. (10), crossing symmetry point and $s = M^2 - \mu^2$, $t = 2\mu^2$ where the axial polarizability is defined. This is not so for photons, where all these points collapse into one (taking $\mu = 0$), making the relevance of the polarizability to both s and t channels almost trivial.

3. - THE NN POTENTIAL

The dispersive approach to the two-pion exchange nucleon forces shows clearly their connection to the pion-nucleon interactions, and is the most suited one in order to study the relevance of the nucleon axial polarizability. Let us shortly review it. The use of the Mandelstam representation in its simplified Cini-Fubini version, that is, taking into account

correctly only the contributions coming from the singularities lying nearest to the low energy physical region, is known since a long time ¹²⁾. These authors also properly considered spin and isospin. The next step, done by Cottingham and Vinh Mau ¹³⁾, was the extraction of nucleon-nucleon potentials from the amplitudes obtained by the previous approach. They studied at the same time the delicate problem of the energy dependence of these potentials and compared them to the existing phenomenological potentials (Yale and Hamada-Johnston). It became clear already at that time that a reasonable knowledge of the $NN \rightarrow \pi\pi$ amplitudes in the pseudophysical region (above the $\pi\pi$ and below the NN threshold) was essential for the determination of the nucleon-nucleon potentials. The energy dependence of these potentials was seen to come mainly from the box diagrams ($NN\pi\pi$ loop) and was thus eliminated as soon as Partovi and Lomon ¹⁴⁾ gave the correct description for subtracting the iterated OPE so as to obtain the two-pion exchange potential. This, together with the advent of a slowly improving picture of the πN and the $\pi\pi$ interactions, resulted in more recent times in a series of very good half-phenomenological half-theoretical intermediate range nucleon-nucleon potentials ¹⁵⁾⁻¹⁸⁾ among which the more recent and complete one is the Paris potential ¹⁸⁾.

Our aim is different : we do not intend to compete with these potentials, but to give a physical picture of the two-pion exchange forces. This will be done by using only the nucleonic polarizability concept which allows to simplify drastically the phenomenological input. It has a clear meaning and a clarifying electromagnetic analogue. We will try to answer the following questions : how good a nucleon-nucleon potential are we able to obtain within the polarizability framework ? and if the description is valid at large distances, where does it cease to be so ?

The relevant expressions for obtaining the low energy two-pion exchange nucleon-nucleon potential are given below. We only deal with the spin and isospin independent forces, that is we start from the spin and isospin averaged amplitudes (for each nucleon). The reason for this choice is that the averaging over isospin leaves only isoscalar exchange in the t channel and hence only even waves. For these ones, no strong low energy $\pi\pi$ resonances exist, contrary to what happens in the $J=1$ $\pi\pi$ channel, where the ρ resonance dominates. The existence of a strong $\pi\pi$ resonance near threshold would make a one-boson exchange approach preferable. It is precisely its absence which makes a description in terms of the polarizability plausible. Indeed the importance of the Δ resonance for this

channel is well known¹⁹⁾. As for the spin averaging it is just a matter of simplification, since anyhow we do not aim to give the full NN potential.

Denoting by $T(s, t)$ the NN spin and isospin averaged amplitudes, we write a fixed s dispersion relation for them, which reads

$$T(s, t) = \frac{1}{\pi} \left[\int_{4\mu^2}^{\infty} - \int_{-\infty}^{-s} \right] dt' \frac{\text{Abs } T(s, t')}{t' - t} \quad (18)$$

The pion pole does not contribute to this particular amplitude. The limits of integration are given by the lowest mass branching points, i.e., the two-pion threshold for the t channel and the elastic one for the u channel. We do not consider subtractions since they do not alter the behaviour of the potential at intermediate or long ranges (but only at short ones as subtractions are due to the high energy behaviour of the amplitude). For low energies, the first integral in Eq. (18) dominates at all but very short distances, since $4\mu^2 \ll s \simeq 4M^2$ *). The potential is then given by the Fourier transform of the first term of Eq. (18) for which the dispersive and Fourier integrations might be exchanged giving

$$V(r) = \frac{1}{(4\pi)^2} \frac{1}{M^2 r} \int_{4\mu^2}^{\infty} dt \text{Abs } T(s, t) e^{-\sqrt{t} r} \quad (19)$$

The exponential in this expression shows that at very large distances only the region around $t = 4\mu^2$ is relevant. This may be understood using a physical picture for nucleons far apart. Then each one feels the static asymptotic pion field of the other one which implies for the momentum of the exchanged pion $k^0 = 0$, $\vec{k} = i\mu\vec{n}$ (where $\vec{n}$ is a unit vector). Since the pion travels back and forth between the two nucleons the momentum transfer is $t = 4\mu^2$.

As mentioned previously, the part of $T(s, t)$ which depends strongly on s is the one coming from the iteration of the OPE potential. We do not take it into account, so that a soft dependence in s is ensured.

*) Notice that s in Eq. (18) refers to the NN system and not to the πN system as in the previous section.

This allows us to calculate $\text{Abs } T(s, t)$ in the region $4M^2 - t \geq s \geq 0$ instead of in the low energy NN region $s \gtrsim 4M^2$. There the absorptive part is the imaginary part of the amplitude and unitarity gives in the two-pion approximation

$$\text{Im } T(s, t) = \frac{-1}{32\pi^2} \left(M^2 - \frac{t}{4}\right) \sqrt{\frac{t-4\mu^2}{t}} \int d\Omega_t C^{(+)*}(t, \cos\theta_t') C^{(+)}(t, \cos\theta_t) \quad (20)$$

$C^{(t)}$ being the spin and isospin averaged πN amplitudes defined in Eq. (4). The dependence in s is contained in $\cos\theta_t'$.

We will only keep the $\pi\pi$ s wave contribution to the integrand of Eq. (20). At low t values, higher even waves (the odd ones do not contribute anyhow) are dominated by their positive energy Born contribution which should not be included. Their non-Born contribution is totally negligible as compared to the s wave. Since for s waves $C^{(t)}$ in Eq. (20) depends only on t , $\text{Im } T(s, t)$ is independent of s , and the long-range potential is energy-independent. Equation (20) together with Eq. (12) restricted to s waves gives

$$V(r) = - \frac{1}{2\pi r M^2} \int_{4\mu^2}^{\infty} \frac{dt}{4M^2 - t} \sqrt{\frac{t-4\mu^2}{t}} |f_+^o(t)|^2 e^{-\sqrt{t}r} \quad (21)$$

Notice that the presence of the damping factor $e^{-\sqrt{t}r}$ makes a precise knowledge of $|f_+^o(t)|$ unnecessary for large values of t .

The expressions for $f_+^o(t)$ given in the previous section will allow the evaluation of the potential from Eq. (21), once they are continued above $t = 4\mu^2$ as will be discussed in the next section. The only problem left here is the subtraction of the iterated OPE potential. This is in general a delicate point but not in our case, where it is sufficient to omit the squared positive energy Born contribution to $|f_+^o(t)|^2$. This procedure gives essentially the same result as the correct one since they differ in relativistic effects and these are unimportant at large distances.

4. - THE TWO-PION THRESHOLD

We have seen in the last section that the obtention of the nucleon-nucleon potential requires the knowledge of $|f_+^0(t)|$ for $t \geq 4\mu^2$. On the other hand, we know from Section 2 that $\bar{f}_+^0(t)$ is reasonably well known for $0 \leq t \leq 4\mu^2$ where it is real and essentially given by the polarizability, and $f_+^{\text{op}}(t)$ is known and real in the whole region of interest to nucleon forces. We will study in this section how the moduli below and above the two-pion threshold are related to each other. This problem has no equivalent in electromagnetism since the exchanged photons do not interact strongly. We will use the K matrix multichannel formalism²⁰⁾ together with the effective range approximation. An alternative approach would be the N/D method, but we choose the first one which seems simpler to us.

We first recall some results from potential scattering. The scattering amplitude $f(p)$ is related to the phase shift $\delta(p)$ by

$$f(p) = \frac{\sin \delta(p) e^{i\delta(p)}}{p} \quad (22)$$

where we have restricted ourselves to s waves, since higher waves do not interest us. The effective range expansion reads

$$\phi(p) = p \cotg \delta(p) = \frac{1}{a} + \frac{r}{2} p^2 + O(p^4) \quad (23)$$

where a is the scattering length and r the effective range. Unitarization of the amplitude is achieved in a simple way in the K matrix formalism by which the amplitude is written in the form

$$f(p) = \frac{1}{p} \frac{\tan \delta(p)}{1 - i \tan \delta(p)} = \frac{1}{\phi(p) - ip} \quad (24)$$

and $\phi(p)$ is real both above and below threshold. Furthermore $\phi(p)$ is a function of p^2 and analytic through the threshold $p=0$. It is thus also analytic in s . Below threshold $f(p)$ is real (since $p=i|p|$) and its modulus reads for small $|p|$

$$|f(p)|_{p=i|p|} \simeq \frac{|a|}{|1 + |p|a + \frac{ra}{2} p^2|} \quad (25)$$

whereas it is complex above threshold and its modulus there is, again for small $|p|$,

$$|f(p)|_{p>0} \simeq \frac{|a|}{\sqrt{1+a(q+r)p^2}} \simeq \frac{|a|}{|1+\frac{a^2+ar}{2}p^2|} \quad (26)$$

It is seen from these last two expressions that the modulus of the amplitude is not analytic at the threshold and that the term responsible for the non-differentiability of $|f(p)|$ with respect to $|p|$ at the threshold $p=0$ is the term $a|p|$ in Eq. (25). The use of one of the expressions, Eqs. (25) or (26) for $|f(p)|$ in the region of validity of the other, although analytically incorrect, is numerically good as long as $a|p| \ll 1$.

We now extend this formalism to a multi-channel one, so as to generalize our previous results to several competing channels. The K matrix method easily allows this extension. It provides the adequate framework for studying the $\pi\pi \rightarrow N\bar{N}$ amplitudes near the $\pi\pi$ threshold, since there the elastic $\pi\pi$ channel has to be taken into account simultaneously (the multipion and $K\bar{K}$ channels are not relevant at the $\pi\pi$ threshold). An example of the inter-relation between both channels is well known from unitarity : the phase of the $\pi\pi \rightarrow N\bar{N}$ amplitude is given by the elastic $\pi\pi$ process (up to π).

The generalization of Eq. (24) to two channels a and b is given by

$$\begin{pmatrix} f^a & f^{ab} \\ f^{ab} & f^b \end{pmatrix} = \begin{pmatrix} \phi^a - i\rho_a & \phi^{ab} \\ \phi^{ab} & \phi^b - i\rho_b \end{pmatrix}^{-1} \quad (27)$$

where $p_a = \sqrt{s/4 - \mu^2}$, $p_b = \sqrt{s/4 - M^2}$ and the functions ϕ^a , ϕ^b and ϕ^{ab} are analytic and real through the threshold $s_a = 4\mu^2$. They can therefore be expanded around $s = 4\mu^2$ giving

$$\phi = \alpha + (s - 4\mu^2)\beta + \dots \quad (28)$$

Equations (27) and (28) give the following values of the amplitudes at threshold

$$f^a(s=4\mu^2) = \frac{\alpha^b + \sqrt{M^2 - \mu^2}}{\alpha^a \alpha^b - (\alpha^{ab})^2 + \sqrt{M^2 - \mu^2} \alpha^a} = a^a$$

$$f^{ab}(s=4\mu^2) = \frac{-\alpha^{ab}}{\alpha^a \alpha^b - (\alpha^{ab})^2 + \sqrt{M^2 - \mu^2} \alpha^a} \quad (29)$$

a^a being the $a \rightarrow a$ scattering length. Below and above threshold, Eq. (27) gives, for the modulus of the amplitude f^{ab}

$$|f^{ab}(s < 4\mu^2)| = \frac{|\phi^{ab}|}{|\phi^a \phi^b + |p_a p_b| - (\phi^{ab})^2 + |p_b| \phi^a + |p_a| \phi^b|} \quad (30)$$

and

$$|f^{ab}(s > 4\mu^2)| = \frac{|\phi^{ab}|}{\sqrt{(\phi^a \phi^b - (\phi^{ab})^2 + |p_b| \phi^a)^2 + p_a^2 (|p_b| + \phi^b)^2}} \quad (31)$$

and, using the expansion of Eq. (28) keeping only terms linear in $|p_a|$ one has near threshold

$$|f^{ab}(s = 4\mu^2 - \epsilon)| = \frac{|f^{ab}(s = 4\mu^2)|}{|1 + |p_a| a^a|} \quad (32)$$

and

$$|f^{ab}(s = 4\mu^2 + \epsilon)| = |f^{ab}(s = 4\mu^2)| \quad (33)$$

These two expressions differ only by the term $a^a |p_a|$ in the denominator of the first one, as in the single channel case. For the inelastic amplitude as well, the dominant analyticity breaking term at $s = 4\mu^2$ is proportional to the scattering length of the elastic process.

In our case a and b are the $\pi\pi$ and $N\bar{N}$ channels, respectively. With the recommended value for the $J=I=0$ $\pi\pi$ scattering length⁵⁾

$$a = (0.30 \pm 0.15) \mu^{-1} \quad (34)$$

the term $a|p|$ remains small in the neighbourhood of the two-pion threshold.

We conclude therefore that, due to the smallness of the $\pi\pi$ threshold interaction, the modulus of the $\pi\pi \rightarrow N\bar{N}$ helicity amplitude above the $\pi\pi$ threshold may be approximated at least in the neighbourhood of the threshold, by the same analytic expression which describes the amplitude below it. This analytic continuation, although mathematically incorrect, is expected to be numerically acceptable in the region relevant to the long range force. Indeed, our amplitude obtained by this continuation is well compatible with the ones given in the literature, as will be shown in the next section. This indicates that also the part of the pseudophysical region near $s = 4\mu^2$ (for the modulus of the $J = I = 0$ amplitude) may be described by the axial polarizability ^{*)}.

5. - RESULTS

We will present and discuss our results in this section. This is done on several levels. First we give a compact expression for the long-range potential for which only the behaviour of the amplitude near $t = 4\mu^2$ is needed. The limit $\mu \rightarrow 0$ gives the physical interpretation of each of its terms from the analogy with the two-photon exchange forces.

Having obtained this potential, the next step is to test it, since it is not a priori evident that the concept of polarizability can usefully be applied to this problem, as already emphasized. This test is done in the form of a comparison of our helicity amplitude with the available ones. A good agreement for low values of t implies a good asymptotic potential.

^{*)} The centre-of-mass energy squared of the $\pi\pi \rightarrow N\bar{N}$ channel which has been called s in this section corresponds to t in the others where the s channel is the NN or πN one.

For larger values of t our amplitude is unrealistic, which implies the failure of our potential at shorter distances. We then discuss where this happens.

For the obtention of an analytic expression for the long-range potential, we neglect t compared to $4M^2$. This allows us to write Eq. (14) for $\bar{f}_+^0(t)$ in the form

$$\bar{f}_+^0(t) = M^2 [\alpha + \beta (t - 4\mu^2)] \quad (35)$$

where $\beta = 1.18/4\pi\mu^3$ is the polarizability and $\alpha = (0.65 + 2 \times 1.18)/4\pi\mu$. The positive energy Born amplitude $f_+^{0P}(t)$ of Eqs. (16) and (17) is not analytically integrable and must be further approximated. This requires a careful treatment because of the very rapid variation of h near $t = 4\mu^2$. Indeed, h , which is infinite for $t = 4\mu^2$, takes the value unity already near $t = 4\mu^2 + (\mu^4/M^2)$ and from there on varies smoothly.

Depending on the distance r this small region ($\Delta t = (\mu^4/M^2) \simeq 0.02 \mu^2$) of rapid variation contributes or not. For very large distances ($\mu r \gg (M^2/\mu^2)$) this small region where h remains large is the relevant one and it is justified to make the approximation

$$f_+^{0P}(t) = \frac{g^2 M}{4\pi} \quad (36)$$

For smaller values of r , on the contrary, the main contribution comes from the region where $h < 1$ and where we can use an expansion in h giving

$$f_+^{0P}(t) = \frac{g^2}{16} \frac{t - 2\mu^2}{t - 4\mu^2} \quad (37)$$

The first approximation fed into expression (21) of the potential gives

$$V(r) = - \frac{\mu}{4\pi r^2} \left[\alpha \left(\alpha + \frac{2}{M} \frac{g^2}{4\pi} \right) + 12 \frac{\mu\beta}{r} \left(\alpha + \frac{1}{M} \frac{g^2}{4\pi} \right) + 60 \frac{\beta^2 \mu^2}{r^2} \right] \sqrt{\frac{\pi}{\mu r}} e^{-2\mu r} \quad (38)$$

where for each combination of α , β and g^2 only the longest range term has been kept. This potential, though it is the true asymptotic one, has only academic interest since it applies only above several hundred fermis.

For smaller distances ($\mu r < (M^2/\mu^2)$) the other approximation Eq. (37) leads to

$$V(r) = -\frac{\mu}{2\pi r^2} \left\{ g^2 \left(\frac{\mu}{M} \right)^2 e^{-2\mu r} \left[\frac{\alpha}{8\mu} \left(1 + \frac{2}{\mu r} + \frac{1}{\mu^2 r^2} \right) + \frac{\beta}{2r} \left(1 + \frac{9}{2\mu r} + \frac{6}{\mu^2 r^2} + \frac{3}{\mu^3 r^3} \right) \right] \right. \\ \left. + \alpha^2 K_1(2\mu r) + 12\alpha\beta\mu^2 \frac{K_2(2\mu r)}{\mu r} + 60\beta^2\mu^4 \frac{K_3(2\mu r)}{\mu^2 r^2} \right\} \quad (39)$$

where K_i are the modified Bessel functions.

For comparison with electromagnetism this last expression is the relevant one, since in the limit $\mu \rightarrow 0$ its region of validity extends to infinity. Taking this limit, one obtains from Eq. (39)

$$V(r) = -\frac{1}{4\pi r^3} \left\{ \frac{g^2}{4M^2 r} \left(\alpha + 12\frac{\beta}{r^2} \right) + \alpha^2 + 12\frac{\alpha\beta}{r^2} + 120\frac{\beta^2}{r^4} \right\} \quad (40)$$

which can be compared to the electromagnetic long-range potentials obtained in Ref. 21). The different terms of expression (40) have different r dependence corresponding to several types of interactions. For instance, the term proportional to β^2 has the characteristic r^{-7} behaviour of an induced dipole-induced dipole interaction. The term in $g^2\beta$ is the dipole-induced dipole interaction with its r^{-6} dependence. The terms dependent on α do not have a direct electromagnetic analogue, since charge conservation does not allow the existence of induced charges. But their r dependences are characteristic for dipole-induced charge ($g^2\alpha$), induced charge-induced charge (α^2) and induced charge-induced dipole ($\alpha\beta$) interactions. Among the different terms of the potential those in α^2 and β^2 are evidently attractive; the character of the others depends on the sign of α and β . Since these are positive the potential is attractive.

The behaviour of the different terms of the potential (39) for $\mu r \gg 1$ is obtained from the asymptotic expression of the modified Bessel functions $K_1(2\mu r) \sim \sqrt{\pi/4\mu r} e^{-2\mu r}$. One then sees that the induced charge terms have a longer range than the corresponding induced dipole ones. Of all terms the one of longest range is the dipole-induced charge one. However, in the region of interest ($\mu r \approx 1$) all terms are of similar magnitude and should be kept.

The occurrence of induced charge terms, which bear a priori no relation to the polarizability, raises doubts about the usefulness of this concept in the context of long-range NN forces. This is not so since the value of α is indeed dominated by the polarizability; the reason is the smallness of the s wave term, which contributes only a 25% to it [see Eq. (35)]. When the s wave is neglected, $\alpha = 2\mu^2 \beta$ and the long-range potential, Eq. (39), is determined by the polarizability, though its r dependence is more complicated than the one of an induced dipole.

We will now discuss the validity of our results. The natural comparison would be between our potential and the available ones. But published values for these extend only up to $r \sim 2$ fm which is insufficient for our purposes.

We have therefore compared our helicity amplitudes with some recent ones, which incorporate accurate πN low energy phase shift data. The Figure shows the modulus of our helicity amplitudes together with that of Bohannon⁷⁾ and that of an unpublished amplitude due to Höhler, Koch and Pietarinen²²⁾. There also exist several determinations by Nielsen and Oades. Among their latest ones [unpublished²³⁾] one set agrees rather well with that of Höhler, Koch and Pietarinen, while the other is quite at variance with all the other amplitudes. If we take Bohannon's amplitude, our simplest polarizability amplitude is about 15% larger up to $t \simeq 16 \mu^2$ and then diverges from it. When a Δ propagator is introduced the disagreement is reduced to less than 10% up to $t \simeq 20 \mu^2$. Above this point it increases although less so than without the propagator. One can thus conclude that the introduction of this propagator improves the agreement with Bohannon's amplitude. As for Höhler et al.'s solution no such clear conclusion can be drawn since the deviations of our two amplitudes from theirs are similar and not more than $\approx 15\%$ in the whole region up to $t \simeq 14 \mu^2$. The error given by Bohannon (supposing no error for the $\pi\pi$ phase shift) is about 10%. Taking this value as representative for the errors of the

helicity amplitudes we can conclude that both our polarizability amplitudes give a reasonable description up to $t \sim 15\mu^2$ whereas above this point it becomes insufficient, the deviation being, however, less pronounced with the Δ propagator. The immediate consequence is that the potential should be well described in terms of the polarizability for distances such that the region above $t = 15\mu^2$ is of little importance.

This is investigated in the following way. We split the interval of integration in t into two parts below and above $t = 15\mu^2$. Though our two amplitudes lead to an overestimate of this last contribution ($t > 15\mu^2$) this splitting gives a feeling for the validity of our description. Tables 1 and 2 show the two contributions to the potential for both our amplitudes. The numerical computation of our potential for different distances is done using expressions (14), (15), (16) and (17) for the helicity amplitude [without the approximations which led to Eq. (37) as their only aim was to obtain an analytical expression for the potential]. The term $|f_+^{\text{op}}(t)|^2$ is not included, since it approximates the iterated OPE potential. It is seen from the Tables that above 3 fm the contribution coming from $t \geq 15\mu^2$ is several times smaller than the other one and therefore the polarizability definitely dominates the NN potential. The contrary happens at 1.5 fm where the polarizability is of no relevance anymore.

The comparison with the phenomenological Hamada-Johnston potential ²⁴⁾ given in Table 3 shows that this conclusion is correct. It is not surprising that the dependence in r even above 2 fm is different since they used an a priori form for their potential.

6. - CONCLUSIONS

We have given a description of the two-pion exchange nucleon-nucleon forces based on the notion of the axial polarizability. This work was inspired by the corresponding problem of the molecular forces. Its aim was to get a simple and physical picture by analogy with electromagnetism.

It is not a priori obvious that an analogy with the Van Der Waals forces can be established even at large distances. The finite mass and the mutual strong interaction of the exchanged pions make that the analogy is not guaranteed. The principal conclusion of our investigation is that the electromagnetic analogy holds, in spite of these adverse factors.

We have also studied the range of validity of such a description ; it breaks when the nucleons interpenetrate, i.e., at a distance smaller than twice their radius (≈ 2 fm).

We have given an analytic expression of the long range potential. The physical meaning of its terms appears clearly from the parallelism with electromagnetism in the limit of vanishing pion mass. This shows that the nucleon which possesses a static axial dipole moment (its spin) may induce on its partner not only an axial dipole but also an axial charge. This last effect has no direct electromagnetic analogue due to current conservation. Fortunately the induced axial charge introduces no new parameter since its value is essentially given by the polarizability.

The comparison between our potential and the semi-phenomenological ones has been made, not directly, but in the form of a comparison of the helicity amplitudes which enter as an ingredient in its evaluation. The agreement is rather good up to $t \approx 15 \mu^2$. From this follows that our potential is rather accurate for distances such that the region $t > 15 \mu^2$ does not give a significant contribution ; this distance is ≈ 3 fm.

The agreement for small values of t between the moduli of our polarizability amplitudes which do not incorporate the effect of the $\pi\pi$ interaction and that of the semi-phenomenological ones which do so reflects the absence of a strong effect of the $\pi\pi$ interaction. This condition was needed for the success of the whole polarizability picture.

Below 2 fm the polarizability picture breaks down ; the $\pi\pi$ interaction is now the dominating dynamics. The nucleons are too close to each other ; the forces between them cannot be described with a picture based on their individuality.

The success of the polarizability description is made possible by the approximate realization in our world of the chiral symmetry : it ensures the smallness of the πN and $\pi\pi$ s waves. This should be no

surprise. In a world where the pion mass would vanish (exact chiral symmetry) the soft pions would interact weakly. It is then natural that the interactions that they mediate resemble those mediated by photons.

Note

After having completed this work we have been informed of some lecture notes by Sucher ²⁵⁾ where he points out the relationship between the long-range two-pion exchange forces and the πN amplitude at an unphysical point. In the limit of vanishing pion mass this point becomes physical as happens in the electromagnetic case.

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μr	r (fm)	$4 \leq \frac{t}{\mu} \leq 15$	$\frac{t}{\mu} > 15$
1	1.42	2.7	> 10
1.25	1.77	0.9	> 2
1.5	2.13	0.38	0.39
1.75	2.49	0.18	0.09
2	2.84	0.061	0.028
2.25	3.19	0.027	0.007

TABLE I

μr	r (fm)	$4 \leq \frac{t}{\mu} \leq 15$	$\frac{t}{\mu} > 15$
1	1.42	2.2	~ 5
1.25	1.77	0.8	1.0
1.5	2.13	0.30	0.22
1.75	2.49	0.13	0.06
2	2.84	0.049	0.020
2.25	3.19	0.022	0.005

TABLE II

μr	r (fm)	$-V$ (MeV)	$-V_{\Delta}$ (MeV)	$-V_{HJ}$ (MeV)
1	1.42	> 12	> 7	2.8
1.5	2.13	0.8	0.5	0.5
2	2.84	0.09	0.07	0.10

TABLE III

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FIGURE CAPTION

The modulus of the $\pi\pi \rightarrow N\bar{N}$ helicity amplitude $f_+^0(t)$ is shown.

Dashed line : our amplitude, Eqs. (14), (16) and (17).

Continuous line : our amplitude, Eqs. (15), (16) and (17).

Dotted line : Höhler, Koch and Pietarinen's amplitude ²²⁾.

Dashed-dotted line : Bohannon's amplitude ⁷⁾.

TABLE CAPTIONS

Table I Gives the contribution to $-V(r)$ in MeV for different distances coming from below and above $t = 15 \mu^2$ when calculated with the amplitude, Eqs. (14), (16) and (17).

Table II The same but for the amplitude, Eqs. (15), (16) and (17).

Table III Shows the comparison of our polarizability potentials (without and with Δ propagator) with the Hamada-Johnston potential ²⁴⁾.

FIG. 1