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SIMPLE PARAMETRIZATIONS OF PARTON DISTRIBUTIONS
WITH Q^2 DEPENDENCE GIVEN BY ASYMPTOTIC FREEDOM

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ABSTRACT

We show that the Q^2 dependence of parton distributions as given by asymptotically free Gauge Theories can be represented by simple analytic expressions. In particular the sea distributions can be read directly from the first two moments. We compare our results with the SLAC ep data and with the recent Fermilab up data. The agreement is good.

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1. INTRODUCTION

Recently there has been a great deal of interest¹⁾⁻¹⁴⁾ in the comparison of the scaling violations predicted by asymptotically free gauge theories (hereafter called ASFT)¹⁵⁾ with the scaling violations observed in the deep inelastic data¹⁶⁾⁻¹⁸⁾. In particular, electroproduction and μp deep inelastic scattering, free of such possible ambiguities as right-handed currents, are good places for testing the theories in question. ASFT seem to survive these tests^{4),7),8)}.

In comparing ASFT with deep inelastic data it would be useful to have explicit expressions for parton distributions with Q^2 dependence given by ASFT. Having such distributions at hand one could use them for quantitative predictions of scaling violations in all processes in which parton ideas (corrected by ASFT effects) could be applied; as for instance neutrino reactions or large $p_{\perp}$ processes^{19,20)}.

Unfortunately ASFT gives only predictions for the Q^2 dependence of the moments of the quark distributions. In order to find the Q^2 dependence of the quark distributions one has to tackle the inversion problem. There exist many techniques²¹⁾ to solve this problem, but since the n dependence of the moments as given by ASFT is complicated, the inversion has to be done numerically. Consequently in most applications of ASFT in the literature the Q^2 dependence of quark distributions has been presented in a numerical form^{4),19)}. However, as has been pointed out in Ref. 12), it is possible to find simple analytic expressions for the Q^2 dependence of quark distributions which, to a very good accuracy, represent the Q^2 dependence predicted by ASFT. In Ref. 12) only the analytic form of valence quark distributions has been discussed. In this paper we extend these considerations to sea and gluon distributions and also make a comparison of the resulting expressions with the experimental data.

Our paper is organized as follows. In Section 2 we present analytic expressions for the valence quark $V(x, Q^2)$, non-charmed sea $S(x, Q^2)$, charmed sea $C(x, Q^2)$, and gluon $G(x, Q^2)$ distributions as well as their numerical evaluation. In Section 3 we compare the expressions in question with the SLAC ep data and μp Fermilab data. We also make predictions for higher values of Q^2 for both en and ep scattering. Finally, in Section 4 we briefly summarize our results.

2. Q^2 DEPENDENT PARTON DISTRIBUTIONS

2.1 Formalism

Following Ref. 2) we decompose parton distributions into valence, SU(3) symmetric non-charmed sea $S(x, Q^2)$ and charm sea^{*}) as follows

$$\begin{aligned} \rho(x, Q^2) &= \rho_v(x, Q^2) + \lambda(x, Q^2) \\ n(x, Q^2) &= n_v(x, Q^2) + \lambda(x, Q^2) \\ \bar{n}(x, Q^2) &= \bar{p}(x, Q^2) = \lambda(x, Q^2) = \bar{\lambda}(x, Q^2) \\ S(x, Q^2) &\equiv 6 \lambda(x, Q^2) \\ C(x, Q^2) &= c(x, Q^2) + \bar{c}(x, Q^2) \end{aligned} \tag{2.1}$$

and introduce a gluon distribution $G(x, Q^2)$ as well as two combinations

$$\begin{aligned} V_8(x, Q^2) &\equiv \rho_v(x, Q^2) + n_v(x, Q^2) \\ \text{and} \\ V_3(x, Q^2) &\equiv \rho_v(x, Q^2) - n_v(x, Q^2). \end{aligned} \tag{2.2}$$

We denote the moments of the quark distributions by

$$\langle q(Q^2) \rangle_n \equiv \int_0^1 dx x^{n-1} q(x, Q^2), \quad n \geq 2 \tag{2.3}$$

with a similar definition for gluons.

*) We assume that the sea of quarks heavier than charm is negligible at presently accessible values of Q^2 .

In a simple parton model without ASFT effects all the distributions above are only functions of x . The standard parametrizations of these Q^2 independent distributions found in the literature²²⁾ are in their simplest form^{*)} given by

$$\begin{aligned}
 x V_i(x) &= A_i x^{\eta_1^{(i)}} (1-x)^{\eta_2^{(i)}} & i = 3, 8 \\
 x S(x) &= A_S (1-x)^{\eta_S} \\
 x G(x) &= A_G (1-x)^{\eta_G} \\
 x C(x) &\equiv 0
 \end{aligned} \tag{2.4}$$

with $A_i, A_S, A_G, \eta_1^{(i)}, \eta_2^{(i)}, \eta_S$ and η_G being constant.

In the presence of ASFT effects (gluon bremsstrahlung, pair creation) all the distributions in Eqs. (2.4) acquire Q^2 dependence and consequently also $x C(x, Q^2)$ will differ from zero. In order to find these Q^2 dependent distributions the standard procedure is to find them from the data at a low $Q^2 = Q_0^2$ value and use this result subsequently as a boundary condition to renormalization group equations (or moment sum rules) whose solutions will give the desired distributions for all $Q^2 \geq Q_0^2$.

Although it is very hard if not impossible to solve the equations in question analytically we have found that their numerical solutions can be represented to an excellent accuracy by analytic formulae. These formulae are simple extensions of the parametrizations in Eqs. (2.4). Their general structure is as follows:

$$\begin{aligned}
 x V_i(x, \bar{s}) &= A_i(\bar{s}) x^{\eta_1^{(i)}(\bar{s})} (1-x)^{\eta_2^{(i)}(\bar{s})} \\
 x S(x, \bar{s}) &= A_S(\bar{s}) (1-x)^{\eta_S(\bar{s})} \\
 x G(x, \bar{s}) &= A_G(\bar{s}) (1-x)^{\eta_G(\bar{s})} \\
 x C(x, \bar{s}) &= A_C(\bar{s}) (1-x)^{\eta_C(\bar{s})}
 \end{aligned} \tag{2.5}$$

*) Linear combinations of expressions like (2.4) are also used.

where

$$\bar{s} = \ln \left[\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}} \right] \quad (2.6)$$

$A_i(0) = A_i$, $A_S(0) = A_S$, $A_G(0) = A_G$, $A_C(0) = 0$ with similar boundary conditions for η 's. The parameter Λ which characterizes the strength of scaling violations is to be found from the data. The functions $A(\bar{s})$ and $\eta(\bar{s})$ on the other hand can be found as we now show from ASFT equations.

2.2 Basic result

2.2.1 $S(x, Q^2)$, $C(x, Q^2)$, $G(x, Q^2)$

It is well known that in order to determine a given distribution from its moments^{*)} (see 2.3) in the whole physical region ($0 \leq x \leq 1$) it is necessary to use for this purpose all its moments. However, in the case of distributions like $G(x, Q^2)$, $S(x, Q^2)$ and $C(x, Q^2)$ which are strongly decreasing functions of x and are of phenomenological importance only for $x \leq 0.3$ our detailed study showed that they can be determined from their first two moments.

As a result $C(x, Q^2)$, $S(x, Q^2)$ and $G(x, Q^2)$ as given by ASFT can be represented as follows

$$\begin{aligned} x S(x, Q^2) &= P_S \left(\frac{1}{\langle x \rangle_S} - 1 \right) (1-x)^{\left(\frac{1}{\langle x \rangle_S} - 2 \right)} \\ x C(x, Q^2) &= P_C \left(\frac{1}{\langle x \rangle_C} - 1 \right) (1-x)^{\left(\frac{1}{\langle x \rangle_C} - 2 \right)} \end{aligned} \quad (2.7)$$

and

$$x G(x, Q^2) = P_G \left(\frac{1}{\langle x \rangle_G} - 1 \right) (1-x)^{\left(\frac{1}{\langle x \rangle_G} - 2 \right)}$$

i.e.,

$$A_k(\bar{s}) = P_k \left(\frac{1}{\langle x \rangle_k} - 1 \right)$$

and

$$\eta_k(\bar{s}) = \frac{1}{\langle x \rangle_k} - 2, \quad k = S, G, C \quad (2.8)$$

*) Only these moments are given by ASFT.

Here

$$P_q \equiv \langle q(Q^2) \rangle_2 \quad (2.9)$$

is the momentum fraction carried by quarks q and

$$\langle x \rangle_q \equiv \frac{\langle q(Q^2) \rangle_3}{P_q} \quad (2.10)$$

is the corresponding average value of x .

$\langle q(Q^2) \rangle_2$ and $\langle q(Q^2) \rangle_3$ are given in an SU(3) gauge theory with 4 flavours as follows^{*)}

$$\langle S(Q^2) \rangle_n = \frac{3}{4} \mathfrak{D}_2^{(n)}(Q^2) + \frac{1}{4} \mathfrak{D}_1^{(n)}(Q^2) \quad (2.11)$$

$$\langle C(Q^2) \rangle_n = \frac{1}{4} (\mathfrak{D}_2^{(n)}(Q^2) - \mathfrak{D}_1^{(n)}(Q^2))$$

where

$$\mathfrak{D}_1^{(n)}(Q^2) \equiv \langle S(Q_0^2) \rangle_n \exp[-\gamma^n \bar{s}] \quad (2.12)$$

$$\begin{aligned} \mathfrak{D}_2^{(n)}(Q^2) \equiv & \left\{ (1-\alpha_n) \langle q^S(Q_0^2) \rangle_n - \beta_n \langle G(Q_0^2) \rangle_n \right\} \exp[-\gamma_+^n \bar{s}] \\ & + \left\{ \alpha_n \langle q^S(Q_0^2) \rangle_n + \beta_n \langle G(Q_0^2) \rangle_n \right\} \exp[-\gamma_-^n \bar{s}] \\ & - \langle V_8(Q_0^2) \rangle_n \exp[-\gamma^n \bar{s}] \end{aligned} \quad (2.13)$$

Here

$$\langle q^S(Q_0^2) \rangle_n \equiv \langle S(Q_0^2) \rangle_n + \langle V_8(Q_0^2) \rangle_n \quad (2.14)$$

*) Equations (2.11)-(2.14) can be, for instance, derived from Eqs. (2.3)-(2.8) of Ref. 12).

The parameters $\alpha_n, \beta_n, \gamma_{\pm}^n, \gamma_n$ are given in Table 1. Explicit formulae for these parameters as functions of n are given, e.g., in Refs. 15) and 12). Only $n = 2$ and $n = 3$ are used here.

For the gluon distribution we have

$$\begin{aligned} \langle G(Q^2) \rangle_n = & \left\{ \alpha_n \langle G(Q_0^2) \rangle_n - \frac{(1-\alpha_n)\alpha_n}{\beta_n} \langle q^S(Q_0^2) \rangle_n \right\} \exp[-\gamma_+^n \bar{s}] \\ & + \left\{ (1-\alpha_n) \langle G(Q_0^2) \rangle_n + \frac{(1-\alpha_n)\alpha_n}{\beta_n} \langle q^S(Q_0^2) \rangle_n \right\} \exp[-\gamma_-^n \bar{s}] \end{aligned} \quad (2.15)$$

This completes the description of $S(x, Q^2)$, $C(x, Q^2)$ and $G(x, Q^2)$. We have checked that the distributions in Eqs. (2.7) with P_q and $\langle x \rangle_q$ given by (2.11) to (2.15) are very good representations of ASFT for $0 \leq x \leq 0.3$ and any $\bar{s} \geq 0$. The numerical estimates of these distributions are given in subsection 2.3.

We remark that equations (2.12) to (2.15) are only true if $\langle C(Q_0^2) \rangle_n \equiv 0$. If $\langle C(Q_0^2) \rangle_n \neq 0$, $\langle S(Q_0^2) \rangle_n$ in Eq. (2.12) should be replaced by

$$\langle S(Q_0^2) \rangle_n - 3 \langle C(Q_0^2) \rangle_n$$

and $\langle q^S(Q_0^2) \rangle_n$ in Eqs. (2.14) should read

$$\langle q^S(Q_0^2) \rangle_n = \langle S(Q_0^2) \rangle_n + \langle V_8(Q_0^2) \rangle_n + \langle C(Q_0^2) \rangle_n$$

2.2.2 $V_8(x, Q^2)$, $V_3(x, Q^2)$, $p_V(x, Q^2)$ and $n_V(x, Q^2)$

The Q^2 dependence of the moments of these distributions is much simpler than that for the sea

$$\langle V_i(Q^2) \rangle_n = \langle V_i(Q_0^2) \rangle_n \exp[-\bar{s} \gamma^n] \quad i = 3, 8 \quad (2.16)$$

where¹⁵⁾

$$\gamma^n = G \left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=1}^n \frac{1}{j} \right] \quad ; \quad G = \frac{4}{25}^{*)} \quad (2.17)$$

and $\langle V_i(Q_0^2) \rangle_n$ are the corresponding moments at a particular momentum $Q^2 = Q_0^2$. However, contrary to the sea distribution, V_3 and V_8 do not vanish as rapidly for x going to 1 and are therefore more sensitive to higher moments. It nevertheless turned out to be possible to get the Q^2 behaviour of the first 12 moments to within 1% by taking $\eta_1(\bar{s})$ and $\eta_2(\bar{s})$ to be linear functions of $\bar{s}$. $xV_3(x, Q^2)$ and $xV_8(x, Q^2)$ are then given as follows^{**) :}

$$x V_8(x, Q^2) = \frac{3}{B(\eta_1(\bar{s}), 1 + \eta_2(\bar{s}))} x^{\eta_1(\bar{s})} (1-x)^{\eta_2(\bar{s})} \quad (2.18)$$

$$\begin{aligned} x V_8(x, Q^2) - x V_3(x, Q^2) &= \\ &= \frac{2}{B(\eta_3(\bar{s}), 1 + \eta_4(\bar{s}))} x^{\eta_3(\bar{s})} (1-x)^{\eta_4(\bar{s})} \end{aligned} \quad (2.19)$$

where

$$\eta_i(\bar{s}) = \eta_i(0) + \eta'_i G \bar{s} \quad i = 1, 2, 3, 4 \quad (2.20)$$

$\eta_i(0)$ are to be found from the data at $Q^2 = Q_0^2$. The slope's η'_i are obtained by fitting (2.18) and (2.19) to (2.16).

*) The factor $4/25$ corresponds to four flavours. In general, one has $4/(33-2m)$ for m flavours.

**) $B(,)$ is Euler's beta function. Its appearance is necessary if we want to satisfy the known sum rules: $\int dx V_8(x, s) = 3$, $\int dx V_3(x, s) = 1$.

We should like to remark that contrary to Eqs. (2.7), which seem to work for any $\bar{s}$, Eqs. (2.18) and (2.19) are good representations of ASFT only for $0 \leq \bar{s} \leq 1.4$. However, as can be seen in Table 2 this range of $\bar{s}$ is by far larger than that explored by present experiments and experiments to be performed in the near future.

2.2.3 Numerical estimates of quark distributions

In order to complete our discussion of parton distributions, we have to determine various constants appearing in Eqs. (2.7)-(2.15) and (2.18)-(2.20). The accuracy of the data is not good enough to give a unique solution to this problem. Here we present only one solution and comment on other possibilities in Section 3 where a comparison with ep and μp data is made. The numbers presented below have been found on the basis of ep SLAC data^{16),17)} and μp Fermilab data¹⁸⁾. Also the data on $\nu W^{\text{en}}/\nu W^{\text{ep}}$ have been taken into account. The complete list of parameters, which together with Eqs. (2.7)-(2.15), (2.18)-(2.20) and Table 1 fully determine the x and Q^2 dependence of $p_V(x, Q^2)$, $n_V(x, Q^2)$, $S(x, Q^2)$, $G(x, Q^2)$ and $C(x, Q^2)$ is as follows

$$\eta_1(\bar{s}) = 0.70 - 1.1 G \bar{s}$$

$$\eta_2(\bar{s}) = 2.60 + 5.0 G \bar{s}$$

$$\eta_3(\bar{s}) = 0.85 - 1.5 G \bar{s}$$

$$\eta_4(\bar{s}) = 3.35 + 5.1 G \bar{s}$$

$$\langle S(Q_0^2) \rangle_2 = 0.110 \quad , \quad \langle S(Q_0^2) \rangle_3 = 0.9167 \cdot 10^{-2} \quad (2.21)$$

$$\langle G(Q_0^2) \rangle_2 = 0.402 \quad , \quad \langle G(Q_0^2) \rangle_3 = 0.335 \cdot 10^{-1}$$

$$\langle V_8(Q_0^2) \rangle_2 = 0.488 \quad , \quad \langle V_8(Q_0^2) \rangle_3 = 0.157$$

$$\Lambda = 0.3 \text{ GeV} \quad , \quad Q_0^2 = 1.8 \text{ GeV}^2$$

These parameters differ slightly from those given in Ref. 12). For large values of x the former give better agreement with the SLAC ep data¹⁶⁾.

Using these numbers and formulae of subsections 2.2.1 and 2.2.2 it is easy to find analytic expression for all quark distributions at any Q^2 . In Table 2 we present numerical values of Q^2 dependent parameters as $A_k(\bar{s})$ and $\eta_k(\bar{s})$ which characterize the quark distributions. The plots of these distributions for various values of $\bar{s}$ are shown in Fig. 1. In Table 2 we also present the parameters corresponding to $G(x, Q_0^2) \sim (1-x)^5$ as contrasted with the case $G(x, Q_0^2) \sim (1-x)^{10}$ used in (2.21)*). Notice that the parameters of the non-charmed sea and in particular those of the charmed sea are strongly dependent on the assumed shape of the gluon distribution which is easily understood. A detailed discussion of the dependence of sea distributions on the assumed shape of gluon distribution has been already presented in Refs. 10) and 12) and we do not repeat it here.

The confrontation of our parametrizations with the existing data is presented in Section 3. Here we want to make a few comments on the numbers appearing in Eqs. (2.21) as well as on the interaction effects which we have neglected in our analysis.

Notice first that for large x the dominant contribution to νW_2 behaves like $(1-x)^{2.6}$ for $Q_0^2 = 1.8 \text{ GeV}^2$ and changes to $(1-x)^{3.1}$ at the highest Q^2 value of the SLAC data. This seems at first sight to be contradictory to the result $(1-x_S)^4$ ¹⁷⁾ found at SLAC. But we should remark that for large x , x_S is always smaller than x and to fit the data in x_S a higher power is needed. In addition, the $(1-x_S)^4$ behaviour has been found for $2MxW_1$ rather than for νW_2 . But since the Callan-Gross relation $2MxW_1 = \nu W_2$ is not exactly satisfied in the data we have to correct it before we can read W_1 from νW_2 . Such corrections to the Callan-Gross relation are indeed predicted by the theory under consideration^{23), 8)}. We have not included them in our analysis because they constitute only a part of the corrections of order $\bar{g}^2$ to the leading order discussed here. For consistency of course all corrections of order $\bar{g}^2$ should be included at the same time. Unfortunately, estimation of some of them are still missing in the literature. Whether these corrections will modify our results visibly is an interesting question. A study in this direction is in progress^{24), 25)}.

Notice also that the $C(x, Q^2)$, $G(x, Q^2)$ and $S(x, Q^2)$ are rapidly decreasing functions of x . In addition, the steeper they are the higher the Q^2 value. Such a steep behaviour is dictated by the Fermilab data as will be discussed in the next section.

*) Notice that $\eta_G(\bar{s})$ increases much faster with $\bar{s}$ than $\eta_S(\bar{s})$.

3. COMPARISON WITH EXPERIMENT

A comparison of ASFT with the ep scattering has been already done in the literature^{4),7),8)}. Here we want to extend the analysis to the Fermilab np data, which give information about the very small x region and compare our parametrizations with these as well as with the SLAC ep data.

In terms of $V_8(x, Q^2)$, $V_3(x, Q^2)$, $S(x, Q^2)$ and $C(x, Q^2)$ we have for $e_n^{(P)}$ and $\mu_n^{(P)}$ collisions

$$\nu W_2^{(P)}(x, Q^2) = x \left\{ \frac{5}{18} V_8(x, Q^2) + \frac{1}{6} V_3(x, Q^2) + \frac{2}{9} S(x, Q^2) + \frac{4}{9} C(x, Q^2) \right\} \quad (3.1)$$

In Fig. 2 we show $\nu W_2^{(P)}(x, Q^2)$ for fixed x as a function of Q^2 , together with the SLAC data^{16,17)}. In Fig. 3 we compare the same $\nu W_2^{(P)}(x, Q^2)$ with the Fermilab data¹⁸⁾, where we plot versus x , keeping Q^2 fixed at the mean values of the Q^2 at which the data points have been given. We observe that the agreement of our formulae with both Fermilab and SLAC data is very good for $0 < x < 0.6$ in the whole available Q^2 range above $Q^2 \approx 2 \text{ GeV}^2$. For $x > 0.6$ we observe a slight discrepancy between our formulae and the SLAC data. Our curves are flatter than the SLAC data for $0.6 < x < 0.75$. For $x = 0.8$ the agreement is good again. We were not able to cure the discrepancy of our fits with the data in the $0.6 < x < 0.75$ range by changing Λ or input $V_3(x, Q_0^2)$, $V_8(x, Q_0^2)$ without destroying the agreement for $0.2 \leq x \leq 0.6$. In Figs. 2 and 3 we demonstrate that the choice $\Lambda = 0.5$ would spoil the agreement of our parametrizations with the data, especially for $x > 0.4$ and $Q^2 > 3 \text{ GeV}^2$. We think that these additional scaling violations at large values of x (of order of a few per cent) could be due to target mass corrections²⁶⁾⁻²⁸⁾, higher order corrections and higher twist contributions which we have not included in our analysis. All these effects should begin to manifest themselves at such high values of x . There is a belief⁸⁾ that higher twist contributions are only of importance for $x > 0.9$. On the other hand target mass corrections and higher order in g^{-2} corrections should show visible effects for $x > 0.6$. Both are presumably equally important and should be included together in a phenomenological analysis. Such an analysis will be possible once all g^{-2} corrections to the leading order discussed here are known.

In Fig. 4 we show all contributions to νW_2^{ep} at two values of Q^2 , $Q^2 = 1.8 \text{ GeV}^2$ and $Q^2 = 22 \text{ GeV}^2$. We observe that the increase of both charmed and non-charmed sea at small values of x is responsible for the increase of νW_2 at these values of x . Figure 5 illustrates the importance of the charm contributions.

A striking feature of the Fermilab data is a two-component structure which begins to show up in νW_2 ; a strongly increasing singlet contribution (sea) for $x < 0.15$ and decreasing non-singlet contributions (valence) for $x > 0.15$. We would like to point out that it is the first time the sea contribution is so clearly seen in the data. If this separation into two components will become better at higher Q^2 values (as predicted by ASFT) our knowledge about the sea will certainly be improved.

We have checked that our parametrizations are in agreement with the existing data for the ratio $\nu W_2^{\text{en}}/\nu W_2^{\text{ep}}$ at low Q^2 values. But since the error bars are large and the variation of this ratio with Q^2 very small we do not show these results here.

In Fig. 6 we show our predictions for νW_2^{ep} at fixed values of x in a very large range of Q^2 . Notice that scaling violations as predicted by ASFT are only strong for Q^2 up to 1000 GeV^2 . For higher values of Q^2 the effects of ASFT are very weak, much weaker than the expected violations due to the Z boson propagator, which will show up at such high values of Q^2 if the Z exists. In Fig. 7 we plot our predictions for νW_2^{en} and νW_2^{ep} for higher Q^2 values.

Until now we have used in our calculations the parametrizations as given in Section 2. Although we were not able to find a better, equally simple, parametrization of V_3 and V_8 , the Fermilab data due to large uncertainties allow for slightly different choices of $G(x, Q_0^2)$ or $S(x, Q_0^2)$ without spoiling the agreement of the theory with the data for $0.2 \leq x \leq 0.6$. For instance by making $S(x, Q_0^2)$ slightly flatter and $G(x, Q_0^2)$ at the same time steeper or vice versa would not change drastically our results. But generally both distributions should be rather steep with both η_S and η_G close to 10. As an illustration we show in Fig. 8 νW_2^{ep} at $Q^2 = 11.5 \text{ GeV}^2$ for the choice $G(x, Q_0^2 = 1.8 \text{ GeV}^2) \sim (1-x)^5$ and the amount of glue as in (2.21). The agreement with the data is now worse than that achieved with $G(x, Q_0^2) \sim (1-x)^{10}$. We see that a strong increase of the sea at small values of x is only possible if there are many gluons at these values of x .

4. SUMMARY

In this paper we have presented analytic expressions for parton distributions with a Q^2 dependence given by ASFT. We hope that these distributions will be of help to both phenomenologists and experimentalists in their study of scaling violations. It should be clear that the parametrizations presented here are not the only possible parametrizations of the data. But our method is very general and everyone who disagrees with our input functions at $Q^2 = Q_0^2$ can easily follow the steps of Section 2 and find the Q^2 dependence of quark distributions for her (his) favourite input.

As we have shown in Section 3 our parametrizations represent very well scaling violations seen in FNAL data as well as in SLAC data for $0 \leq x \leq 0.6$ in the Q^2 range above 2 GeV^2 , where perturbative calculations on which our analysis was based make sense. For $x \geq 0.6$ a slight discrepancy presumably due to target mass effects and higher order corrections in $\bar{g}$ is visible. We have not included target mass effects in our analysis because we think they should be studied together with $\bar{g}^2$ corrections which are not fully known at present. It is of great interest to see how these corrections would effect our distributions and whether they would improve the agreement of the theory with the data for large x . Work in this direction is in progress^{24),25)}.

It is clear that having the Q^2 dependent parton distributions at hand it is a simple matter to estimate asymptotic freedom effects in ν and $\bar{\nu}$ charged and neutral current processes. For charged current processes such an analysis has already been done^{4),12)-14)}. Less is known about asymptotic freedom effects in neutral current processes, although some discussions have already appeared in the literature²⁹⁾. We shall present our analysis of neutral current processes in the subsequent publication³⁰⁾. There is a good chance that also there asymptotic freedom could be consistent (except for possible anomalies presumably due to right-handed currents) with the data. This is indicated by the phenomenological analysis of Ref. 31) which shows that scaling violations seen in $e p$ and μp collisions are similar to those seen in ν and $\bar{\nu}$ processes (except for high y anomalies). A stronger conclusion has been reached in Ref. 32), where it is claimed that even the high y anomalies are consistent with the scaling violations seen in $e p$ and μp data. A detailed analysis of these and related questions will be presented elsewhere³⁰⁾.

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Table 1

Values of parameters $\alpha_n, \beta_n, \gamma_+, \gamma_-$ entering (2.12)-(2.15)

n	γ^n	γ_+^n	γ_-^n	α_n	β_n
2	0.427	0.747	0	0.429	0.429
3	0.667	1.386	0.609	0.925	0.288

Table 2

Parameters characterizing the sea and gluon distributions as defined in Eqs. (2.5) for various values of $\bar{s}$. The values of Q^2 correspond to $\Lambda = 0.3$ GeV. The parameters characterizing valence quark distributions can be read directly from Eqs. (2.21). The numbers in parentheses correspond to $\eta_G(0) = 5$.

$\bar{s}$	0.	0.2	0.4	0.6	0.8	1.0	1.2	1.4
Q^2	1.80	3.49	7.86	21.1	70.8	310.	1879	17×10^3
$A_S(\bar{s})$	1.210	1.476 (1.367)	1.792 (1.582)	2.164 (1.853)	2.598 (2.183)	3.099 (2.574)	3.674 (3.033)	4.330 (3.566)
$\eta_S(\bar{s})$	10.0	11.29 (10.39)	12.76 (11.14)	14.40 (12.19)	16.24 (13.48)	18.29 (15.02)	20.56 (16.80)	23.09 (18.84)
$A_C(\bar{s})$	0.0	.113 (.066)	.247 (.150)	.403 (.253)	.583 (.378)	.790 (.527)	1.025 (.701)	1.291 (.904)
$\eta_C(\bar{s})$	-	16.87 (9.42)	18.76 (10.99)	20.84 (12.72)	23.14 (14.65)	25.66 (16.78)	28.43 (19.13)	31.48 (21.74)
$A_G(\bar{s})$	4.418 (2.410)	5.452 (3.256)	6.619 (4.257)	7.936 (5.427)	9.419 (6.780)	11.09 (8.33)	12.97 (10.11)	15.10 (12.13)
$\eta_G(\bar{s})$	10.0 (5.0)	11.82 (6.66)	13.86 (8.56)	16.14 (10.72)	18.71 (13.18)	21.59 (15.97)	24.84 (19.13)	28.50 (22.70)
P_p	0.325	0.298	0.274	0.251	0.231	0.212	0.195	0.179
P_n	0.163	0.150	0.138	0.127	0.116	0.107	0.098	0.090
P_S	0.110	0.120	0.130	0.140	0.151	0.161	0.170	0.180
P_C	0	0.0063	0.0125	0.0185	0.0242	0.0296	0.0348	0.0398
P_G	0.402	0.425	0.446	0.463	0.478	0.491	0.502	0.512

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Figure captions

- Fig. 1 : The various parton distributions for various values of $\bar{s}$:
- a) p quarks
 - b) n quarks
 - c) non-charmed sea quarks
 - d) charmed sea quarks
 - e) gluons.
- Fig. 2 : The Q^2 behaviour of vW_2^p for various values of x , compared with the SLAC data¹⁶⁾. The solid curves are for $\Lambda = 0.3$ GeV, the dotted lines are for $\Lambda = 0.5$. (For $x = 0.8$, the $\Lambda = 0.5$ curve disappears from the plot. The errors displayed are random errors only and do not include Q^2 dependent systematic errors.
- Fig. 3 : vW_2^p as a function of x for various values of Q^2 , compared with the Fermilab data¹⁸⁾. The solid and dotted curves represent the same Λ values as in Fig. 2. The triangles ($Q^2 = 3.5$ GeV) are SLAC data from Ref. 16).
- Fig. 4 : vW_2^p as a function of x for
- a) $Q^2 = 1.8$ GeV² and
 - b) $Q^2 = 22.5$ GeV²
- together with the contribution from p_V , n_V , non-charmed sea and charmed sea.
- Fig. 5 : vW_2^p as a function of Q^2 for two values of x .
- a) and b) $x = 0.02$; c) and d) $x = 0.05$.
- In curves b) and d) the contribution from the charmed sea is not included.
- Fig. 6 : vW_2 as a function of Q^2 from $Q^2 = 2 - 10.000$ GeV² for various x values.
- Fig. 7 : vW_2^p and vW_2^n as a function of x for
- a) $Q^2 = 1.8$ GeV²
 - b) $Q^2 = 10.000$ GeV²
- Fig. 8 : vW_2^p at $Q^2 = 11.5$ GeV². The solid line represents our parametrization for $\eta_G = 10$, the dashed line for $\eta_G = 5$. The data are from Ref. 18).

FIG. 1a

FIG. 1b

FIG. 1c

FIG.1d

FIG. 1e

FIG. 2

FIG. 2 cont.

FIG. 2 cont.

FIG. 3a

FIG. 3b

FIG. 3c

FIG. 3d

FIG. 4a

FIG. 4b

FIG. 5

FIG.6

FIG. 7a

FIG. 7b

FIG.8