AN EXTENDED THREE COMPONENT MODEL
FOR TOTAL CROSS-SECTIONS OF NEW PARTICLES

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ABSTRACT

The three-component model (Lipkin, Joynson and Nicolescu) of total hadronic cross-sections is extended, without any new parameter, to the case of charmed quark constituents [and optionally of quarks with $SU(N)$ flavours beyond charm].

The predictions for total hadronic cross-sections of the new particles on protons are given.

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It was observed by Lipkin\textsuperscript{1)} that the deviations of the additive quark model predictions for total hadronic cross-sections (without charmed quark constituents) from experimental data possess a remarkable regularity. Lipkin expressed this regularity through a universal "third component" (being proportional to the product of total quark number and non-strange quark number) to be added to all hadronic cross-sections which are calculated via the conventional quark model.

Joyson and Nicolescu\textsuperscript{2)} (JN) on the other hand proposed a form of the third component different from Lipkin's. For the third component these authors assume a proportionality to the mean value of the hypercharge $Y = B + S$ squared of the constituents.

Joyson and Nicolescu parametrize hadron-nucleon total cross-sections $\sigma_{HN}$ as follows\textsuperscript{3)}:

$$\sigma_{HN}(p_{lab}) = \frac{s}{2m_{p}p_{lab}} \left( N_{HN}^{(r)} P + N_{HN}^{(R)} R \sqrt{p_{lab}} + N_{HN}^{(3)} X(p_{lab}) \right)$$

where $s = E_{cm}^2$, $m_{p}$ is the mass of the proton. For the first (Pomeron) component $N_{HN}^{(P)}$ is equal to the number of quark constituents of the beam particle, $P$ is the Pomeron pole equal to 14.4 mb.

For the second (Regge) component $N_{HN}^{(R)}$ is the number of duality diagrams which can be drawn for the $H+N$ process and $R = 10.2$ mb.

The third component is proportional to the $N_{HN}^{(3)}$ which stands for the mean value of the hypercharge $Y = B + S$ squared of the constituent quarks:

$$N_{HN}^{(3)} = \frac{1}{p_{HN}} \sum_{k \in H \cap N} Y_k^2$$

with

$$\frac{p_{BN}}{p_{HN}} = \frac{3}{2} \quad ; \quad N_{pp}^{(3)} = 1.$$
a simple Pomeron pole with $P = 1 \frac{1}{4} \text{mb}$ which has to be compared with Lipkin's first component equal to

$$\left[ 6.75 + 0.625 \ln \left( \frac{p_{lab}}{2.0 \text{GeV}} \right) \right] \text{mb}.$$  

Henceforth, in order to match the experimental data for $p_{lab} \lesssim 2000$ GeV, the third component in the JN scheme is taken to be negative, in contrast to the positive third component of Lipkin. The universal function $X(p_{lab})$ appearing in (1) was extracted from experimental data in the region $10 \text{ GeV} \lesssim p_{lab} \lesssim \text{ISR energies}$. It turned out to be negative and to decrease in absolute value for increasing energies — probably going through zero for $p_{lab} \sim 2000$ GeV.

The regularity of total cross-sections as reflected in the schemes of JN and of Lipkin may be considered as purely empirical observations. Nevertheless, these schemes seem to us to be attractive because of their simplicity and their predictive power.

We want to propose in this note an extension of the three-component model adopted to the inclusion of charmed quark constituents (and optionally of quarks with new flavours beyond charm as well). We will closely follow the lines of JN. The main reason for us to choose the JN scheme as our starting point, instead of Lipkin's model, is because, in our opinion, the JN scheme lends itself to a more natural looking generalization to higher flavour SU(n) ($n > 3$) groups than Lipkin's model\(^3\). In the latter, one obtains for $\sigma^{\text{tot}} (\psi p)$ by definition a vanishing third component. On the other hand, one knows from data that $\sigma^{\text{tot}} (\psi J / p) < \sigma^{\text{tot}} (\psi p)$. This implies that a charm contribution to Lipkin's third component would have to be negative. Following JN one will subtract from a large first component a third component, which in absolute value will increase if one passes from non-strange to strange, from strange to charmed, from charmed to $c$ quark (and so on) constituents, thereby lowering the respective total cross-section the more exotic the involved quark flavours are. That is, extending the JN scheme we can handle all flavours on the same footing, whereas a generalization of Lipkin's scheme would mean that we are forced to handle isospin and hypercharge basically different to all other following flavours.

According to what was said above, we write tentatively the following modified version of the JN third component

$$N_{HN}^{(3)} = \frac{A}{p_{HN}} \left( \sum_k Y_k^2 + \alpha_q C_k^2 + \ldots \right),$$  

(3)
where \( \alpha_4 \) is a (so far) unspecified parameter weighting the average charm-quark contribution to the third component. We use as convention that the hypercharge of the charmed quark is zero. The dots in (3) stand for possible contributions from higher flavours.

In our opinion, an important question to be posed is whether total cross-sections with charmed constituents can display (and extend) the same type of universality as has been found for "old" physics. That is, the question whether the variation in the Pomeron-like part of the cross-section can be described by the product of a universal function (the same for non-charmed as well as for charmed quarks) times a weighted average over quark quantum numbers. If it is so, then one should be able to fit total cross-sections with charmed constituents to an Ansatz of the form (3) and obtain results of approximately the same precision as has been achieved by \( \bar{C}N \) in their original program.

We use an argument with some group theoretical appeal to fix the parameter \( \alpha_4 \), appearing in (3). We do not claim that beyond some aestheticism the argument has any understood theoretical basis. Our main point is to show what kind of tendency the total cross-sections have to obey in order that the third component universality extends at all to charm physics. If this will be the case then \( \alpha_4 \) cannot be too far from the value given below. We observe that \( Y^2 \) for SU(3) quarks can be expressed in terms of the diagonal SU(3) generator matrices \( \lambda_3, \lambda_8 \) and quark wave functions as follows

\[
Y_u^2 = \frac{1}{3} \bar{u} \lambda_3 Q_u, \\
Y_d^2 = \frac{1}{3} \bar{d} \lambda_3 Q_d, \\
Y_s^2 = \frac{1}{3} \bar{s} \lambda_8 Q_s \eta_s^{-2},
\]

where \( \eta_s \) stands for the over-all normalization factor of \( \lambda_8 : \eta_s = 1/\sqrt{3} \). An immediate formal generalization of (4a)-(4c) to SU(n) flavour is to postulate that the relative weight \( \alpha_n \) of the SU(n) flavour quark \( q_n \) is given by

\[
\frac{1}{3} \bar{q}_n \lambda_{n^2-1} Q_n \eta_{n^2-1}^{-2} = \alpha_n
\]
where the notation is chosen in analogy to that in (4). Equation (5) amounts to putting

\[ \alpha_n = \frac{4}{9} \left( n - 1 \right)^2 \]

and especially \( \alpha_n = 1 \). We present in the following predictions of the extended JN model taking this value for \( \alpha_n \).

Threshold effects could be accommodated in this model only by an ad hoc introduction of threshold functions. We want to avoid that and restrict ourselves therefore to give predictions for "sufficiently large energies" (far away from thresholds).

The JN analysis shows that the universal fit for (non charmed constituents) hadron nucleon cross-sections works for \( p_{lab} \geq 10 \text{ GeV} \). Especially for \( \sigma_{tot}(pp) \) this lower bound of validity of the model is identical to \( (s/2 m_p p_{lab}) \leq 1.1 \).

We take this bound in the following as the criterion for "sufficiently large energies". So we make predictions for

\[
\begin{align*}
\sigma(\psi/J p) & \quad \text{for} \quad p_{lab} \geq 60 \text{ GeV} \\
\sigma(F_p) & \quad \gg 27 \text{ GeV} \\
\sigma(D_p) & \quad \gg 24 \text{ GeV} \\
\sigma(C_p) & \quad \gg 33 \text{ GeV} \\
\sigma(\gamma p) & \quad \gg 500 \text{ GeV}
\end{align*}
\]

Let us point out that for higher energies the described three-component model is in conflict with the \( f \) coupled Pomeron model\(^4\). The latter model predicts\(^5\) (under the assumptions of exchange degeneracy and equal slopes of the vector meson trajectories) a constant ratio for total cross-sections

\[
\frac{\sigma_{\gamma p}}{\sigma_{\gamma' p}} = \frac{m_{\gamma'}}{m_{\gamma}}
\]
whereas in our model these ratios are not constant. The larger the factor \( N_{\nu \bar{p}}^{(3)} \) of our third component becomes, the faster the corresponding total cross-section rises. In particular it is predicted that \( \sigma_{\text{tot}}(\psi/\bar{p}) \) has the fastest rise from all considered cross-sections [leaving aside \( \sigma_{\text{tot}}((b\bar{b})\bar{p}) \) and \( \sigma_{\text{tot}}((t\bar{t})\bar{p}) \), where \( b(t) \) is the bottom (top) quark].

For moderate energies and "old" physics the discrepancies between the two models are not dramatic and both fit the experimental data reasonably well. For example, for \( 10 \text{ GeV} \leq p_{\text{lab}} \leq 30 \text{ GeV} \) one finds from data\(^6\) a nearly constant value of 0.8 (predicted by the \( f \) coupled Pomeron model) for the ratio \( \sigma_{\text{tot}}(K\bar{p})/\sigma_{\text{tot}}(\nu\bar{p}) \).

For higher energies, however, the ratios rise. The experimental values, with errors smaller than 1%, are: 0.83 (100 GeV), 0.86 (240 GeV) -- they are reproduced by the \( JN \) scheme\(^7\).

The discrepancies become larger if one considers \( \sigma_{\text{tot}}(\psi/\bar{p})/\sigma_{\text{tot}}(\phi\bar{p}) \). For \( p_{\text{lab}} = 20 \text{ GeV} \) the experimental value may be approximately 0.11 and our predictions of this ratio for higher energies are: 0.18 (100 GeV); 0.39 (200 GeV); 0.50 (300 GeV). To see whether these ratios really change so drastically will be a crucial test of the extended three component model.

The coefficients \( N_{\nu \bar{p}}^{(3)} \) according to (3) and (5) are the following:

\begin{enumerate}
  \item[a)] for meson-proton processes:
  \begin{align*}
    & \frac{5}{4} \left( \pi \bar{p} \right); \quad \frac{2}{3} \left( K \bar{p} \right); \quad \frac{11}{4} \left( \phi \bar{p} \right); \\
    & \frac{13}{4} \left( D \bar{p} \right); \quad \frac{16}{3} \left( F \bar{p} \right); \quad \frac{21}{4} \left( \psi/\bar{p} \right); \\
    & \frac{35}{4} \left( (b\bar{b}) \bar{p} \right); \quad \frac{53}{4} \left( (t\bar{t}) \bar{p} \right);
  \end{align*}

  \item[b)] for baryon-proton processes:
  \begin{align*}
    & 1 \left( \Lambda \bar{p} \right); \quad \frac{3}{2} \left( \Lambda(\Sigma) \bar{p} \right); \quad 2 \left( \Xi \bar{p} \right); \\
    & \frac{5}{2} \left( \Omega \bar{p} \right); \quad \frac{7}{3} \left( C \bar{p} \right); \quad \frac{13}{6} \left( \Sigma(A) \bar{p} \right); \\
    & \frac{10}{3} \left( T \bar{p} \right); \quad \frac{11}{3} \left( X_{u,d} \bar{p} \right); \quad \frac{25}{6} \left( X_{s} \bar{p} \right); \\
    & 5 \left( \Omega_c \bar{p} \right); \quad (\Omega_c \text{ denotes a (ccc) resonance}).
  \end{align*}
\end{enumerate}
In the Table we give predictions for total cross-sections: \( \sigma_{\text{tot}} (\psi/Jp) \), \( \sigma_{\text{tot}} (Fp) \), \( \sigma_{\text{tot}} (D^0p) \) and \( \sigma_{\text{tot}} (Op) \) using the universal function \( X(p_{\text{lab}}) \) from Ref. 2). In addition, we give a prediction for \( \sigma((b\bar{b})p) \), which might become an interesting issue in view of the recently discovered \( T \) particle\(^8\).

At present only data for \( \sigma_{\text{tot}} (\psi/Jp) \) exist. They are extracted on the one hand from the forward photoproduction amplitude assuming vector dominance and using the optical theorem\(^9\),\(^10\). This procedure renders for \( \sigma_{\text{tot}} (\psi/Jp) \) a value of approximately 1 mb for \( 20 \text{ GeV} \leq p_{\text{lab}} \leq 100 \text{ GeV} \). On the other hand, data obtained from the measurement of the \( \Lambda \) dependence of \( \psi \) photoproduction\(^11\) give a higher value:

\[
\sigma_{\text{tot}} (\psi/Jp)_{20 \text{ GeV}} = 3.5 \pm 1.3 \text{ mb}
\]

The few existing data are consistent with predictions of the presented model.

Our aim was to propose a plausible and as simple as possible an extension of the \( JN \) scheme to charm physics, which has to be confronted with future experimental data. The model renders predictions for charm as well as higher-flavour cross-sections. Within this framework no new parameters for higher flavours are introduced.

We consider the simplicity of the scheme as its main virtue. If it will turn out to be consistent with data then it will challenge a deeper theoretical understanding of the third component regularity in "old" as well as in "new" physics.

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<table>
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<th>$P_{lab}$ (GeV)</th>
<th>$\sigma(\bar{u}p)$ (mb)</th>
<th>$\sigma(Fp)$ (mb)</th>
<th>$\sigma(\psi p)$ (mb)</th>
<th>$\sigma(Cp)$ (mb)</th>
<th>$\sigma((b\bar{b})p)$ (mb)</th>
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References and Footnotes

3) It seems also that the JN parametrization of hadron-proton cross-sections fits experimental data more exactly than Lipkin's one [see Ref. 2] for approximate tests.
7) There exists still other evidence for violation of the $f$ coupled Pomeron model. The ratio

$$\frac{\sigma(\pi^+p) + \sigma(\pi^-p)}{\sigma(pp) + \sigma(pp) + \sigma(pn) + \sigma(p\bar{n})}$$

should be independent of energy according to this model. However, data from $E_{Lab} = 5$ GeV up to FNAL energies clearly show rising (see V. Barger, Lecture given at the McGill Institute, June 1975).