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WEAK INTERACTION MIXING IN THE SIX-QUARK THEORY

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A B S T R A C T

The weak interaction mixing angles are calculated for the six-quark theory in terms of quark mass ratios. The Cabibbo universality of the currents $(\bar{u}d)$, $(\bar{u}s)$ is violated only to a very small extent, while substantial violations of it are found for the $(\bar{c}s)$, $(\bar{c}d)$ currents. One of the weak mixing angles is expected to be larger than the Cabibbo angle.

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At present there exist indications from various experiments that the number of quark flavors and lepton flavors is larger than four.^{1,2,3)} In particular a six quark and six lepton model based on left-handed weak currents has been considered by several authors^{4,5)} (for recent reviews see e.g. ref. (6,7,8)) :

$$\begin{pmatrix} u & c & t \\ d' & s' & b' \end{pmatrix}_L \quad \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e^- & \mu^- & \tau^- \end{pmatrix}_L \quad (1)$$

(L: left-handed).

Here d', s', b' denote the eigenstates of the weak interactions, which are related to the eigenstates of the quark mass matrix by a unitary transformation:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2)$$

(U: 3 x 3 unitary matrix).

One choice of U is given by

$$\begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & -s_2 s_3 e^{i\delta} + c_1 c_2 c_3 & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_2 s_3 e^{i\delta} - c_1 s_2 c_3 & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}, \quad (3)$$

where $c_1 = \cos \alpha_1$, $s_1 = \sin \alpha_1$ etc., $\alpha_1, \alpha_2, \alpha_3$ are weak mixing angles, and δ is a phase parameter. In order to arrive at eq.(3), one has to adjust the phases of the various quark fields appropriately.⁴⁾ We add the following comments about the scheme (1):

- a) α_1 is essentially the Cabbibo angle.
- b) The phase δ introduces CP violation.⁴⁾ The observed magnitude of CP violation requires $10^{-3} \sim s_2 s_3 \sin \delta$.⁵⁾
- c) In case $\alpha_3 \neq 0$ the Cabibbo universality of the charged weak currents is violated; α_3 must be small⁶⁾ ($s_3 \leq 0.25$).
- d) The angle α_2 is not constrained to be small,^{7,8)} it can be as large as 30° .

Recently there have been several attempts to calculate the Cabbibo angle in terms of quark masses.^{9,10,11,12)} By considering a gauge group $SU_2 \times SU_2 \times U_1$ and requiring the interaction of quarks with Higgs scalars to obey specific discrete symmetries one arrives at

the relation⁹⁾

$$\theta_c = \arctan \sqrt{\frac{m_d}{m_s}} - \arctan \sqrt{\frac{m_u}{m_c}}. \quad (4)$$

In this paper we should like to generalize the considerations made in refs. (9,10) to the case of six quarks. Our aim is to answer the following questions:

- I. Can one calculate the three mixing angles $\alpha_1, \alpha_2, \alpha_3$ in terms of the quark masses?
- II. Do those angles obey the constraints imposed by experiments?
- III. Is the smallness of CP violation natural?

In order to see which kind of relations for the weak mixing angles could be obtained, we recall the prescription used in order to derive eq.(4). In terms of weak interaction eigenstates the mass term of the quarks of equal charge has the simple form

$$\bar{q}_{OL} \mathcal{M} q_{OR} + \text{h.c.} \quad \mathcal{M} = \begin{pmatrix} 0 & a \\ a^* & b \end{pmatrix} \quad (5)$$

(a complex, b real; $q = \begin{pmatrix} u_O \\ c_O \end{pmatrix}$ or $\begin{pmatrix} d_O \\ s_O \end{pmatrix}$ respectively, q_O : eigenstates of weak interaction). The masses of the light quarks u,d are generated by the weak interaction mixing.

A generalization of this method to six quarks should preserve eq.(4) at least in some good approximation. One might expect that this is the case if the mass term of e.g. the (u_O, c_O, t_O) system takes the form

$$\overline{(u_O \ c_O \ t_O)}_L \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix} \begin{pmatrix} u_O \\ c_O \\ t_O \end{pmatrix}_R + L \leftrightarrow R \quad (6)$$

(a,b complex, c real). An analogous relation is supposed to be valid for the (d_O, s_O, b_O) -system (with different parameters a,b,c). An intuitive interpretation of this mass matrix would be as follows: The masses of the very heavy quarks t,b are introduced initially; the masses of the "lighter" quarks u,d;c,s are generated by weak interaction mixing. This mass generation proceeds like a cascade: First $m_c(m_s)$ is generated via the mixing described by the parameters b, subsequently $m_u(m_d)$ is generated via the mixing given by a. Before we analyse mass matrices of the type eq. (6) in more detail, we discuss under which circumstances such matrices are obtained naturally.

As in ref.(9,10) we base ourselves on the gauge group $SU_2^L \times SU_2^R \times U_1$; (L: lefthanded; R: righthanded); this group has been discussed by several authors.⁽¹³⁾ The only scalar fields which can contribute to the generation of quark masses via the spontaneous symmetry breaking are the ones transforming like $(\frac{1}{2}, \frac{1}{2})$. They can be represented by 2×2 matrices. Suppose the interaction of these scalar fields with the quarks is as follows:

$$\sum_{i,j} (g_{i,j} \bar{q}_{iL}^0 M q_{jR}^0 + h_{i,j} \bar{q}_{iL}^0 N q_{jR}^0) + h.c., \quad (7)$$

where M and N are two 2×2 matrices, $q_1 = \begin{pmatrix} u \\ d \end{pmatrix}$, $q_2 = \begin{pmatrix} c \\ s \end{pmatrix}$, $q_3 = \begin{pmatrix} t \\ b \end{pmatrix}$, and g,h are coupling constants. We require this interaction to be invariant under the following reflection symmetry R:

$$\begin{aligned} \begin{pmatrix} u_o \\ d_o \end{pmatrix}_L &\rightarrow -i \begin{pmatrix} u_o \\ d_o \end{pmatrix}_L ; & \begin{pmatrix} u_o \\ d_o \end{pmatrix}_R &\rightarrow +i \begin{pmatrix} u_o \\ d_o \end{pmatrix}_R \\ \begin{pmatrix} c_o \\ s_o \end{pmatrix}_L &\rightarrow +i \begin{pmatrix} c_o \\ s_o \end{pmatrix}_L ; & \begin{pmatrix} c_o \\ s_o \end{pmatrix}_R &\rightarrow -i \begin{pmatrix} c_o \\ s_o \end{pmatrix}_R \\ \begin{pmatrix} t_o \\ b_o \end{pmatrix}_{L,R} &\rightarrow \begin{pmatrix} t_o \\ b_o \end{pmatrix}_{L,R} \end{aligned} \quad (8)$$

$$M \rightarrow M, \quad N \rightarrow iN.$$

The only terms in eq.(7) which are allowed by the R symmetry are as follows:

$$\begin{aligned} \overline{(c_o s_o)}_L N \begin{pmatrix} t_o \\ b_o \end{pmatrix}_R + h.c. , & \quad \overline{(t_o b_o)}_L N \begin{pmatrix} c_o \\ s_o \end{pmatrix}_R + h.c. , \\ \overline{(c_o s_o)}_L M \begin{pmatrix} u_o \\ d_o \end{pmatrix}_R + h.c. , & \quad \overline{(u_o d_o)}_L M \begin{pmatrix} c_o \\ s_o \end{pmatrix}_R + h.c. , \\ \overline{(t_o b_o)}_L M \begin{pmatrix} t_o \\ b_o \end{pmatrix}_R + h.c.. \end{aligned} \quad (9)$$

Furthermore we require the interaction (7) to be parity conserving (for a justification of this requirement see ref.(9)). It is easy to see that this constraint, combined with eq.(9), leads to mass matrices of the type given in eq.(6).

We proceed to analyse these mass matrices in more detail. We emphasize that the eigenvalues of the quark mass matrix are usually chosen to be positive. However, the sign of the mass parameter of a spin 1/2 field has no physical meaning. Thus there is no problem in dealing with mass matrices which have negative eigenvalues. It is

easy to see that the eigenvalues of the matrices eq.(5) or eq.(6) are not positive definite. Throughout this paper we shall write all expressions in terms of the positive values of the various quark mass parameters.

Solving the characteristic equation $\det(\mathcal{M}-\lambda) = 0$ ($\mathcal{M}$: mass matrix) and requiring that the eigenvalues of $\mathcal{M}$ are $m_u, -m_c, m_t$, one obtains:

$$\begin{aligned} |a| &= \left(\frac{m_u \ m_c \ m_t}{m_t - m_c + m_u} \right)^{1/2} \\ |b| &= \left(m_t \ m_c + m_u \ m_c - m_t \ m_u - \frac{m_u m_c m_t}{m_t - m_c + m_u} \right)^{1/2} \\ c &= m_t - m_c + m_u. \end{aligned} \quad (10)$$

The eigenvectors $\vec{e}_i$ of the mass matrix are obtained by solving the equation $(\mathcal{M}-\lambda_i) \cdot \vec{e}_i = 0$. ($\lambda_i = m_u, -m_c, m_t$). One finds

$$\vec{e}_i = \frac{1}{\sqrt{N}} \left(1, \frac{\lambda_i}{a}, \frac{\lambda_i b^*}{a(\lambda_i - c)} \right), \quad (11)$$

where N is a normalization factor such that $|\vec{e}_i|^2 = 1$.

The eigenvectors $\vec{e}_i$ denote the weak interaction eigenstates in terms of the mass eigenstates. They are determined uniquely in terms of the mass eigenvalues, if we neglect the phases of a and b (see later). The weak currents are given by the doublets

$$\left(\begin{array}{c} \vec{e}(m_u) \cdot \overrightarrow{(u,c,t)} \\ \vec{e}(m_d) \cdot \overrightarrow{(d,s,b)} \end{array} \middle| \begin{array}{c} \vec{e}(-m_c) \cdot \overrightarrow{(u,c,t)} \\ \vec{e}(-m_s) \cdot \overrightarrow{(d,s,b)} \end{array} \middle| \begin{array}{c} \vec{e}(m_t) \cdot \overrightarrow{(u,c,t)} \\ \vec{e}(m_b) \cdot \overrightarrow{(d,s,b)} \end{array} \right). \quad (12)$$

This solves our general problem. The rest of this paper consists of rewriting eq.(12) in a form suitable for phenomenological applications. We shall use $m_s = 150$ MeV, $m_c = 1300$ MeV (see e.g. ref. (9)), and the phenomenological constraint $m_t \gg m_c, m_b \gg m_s$. In view of the existence of $T(9.5)$ we shall assume $m_t, m_b \geq 5$ GeV.

First we notice an encouraging result. In the limit $m_t, m_b \rightarrow \infty$ we obtain our previous result (4). This can be seen as follows. For $m_b \rightarrow \infty$ one finds $|a| = \sqrt{m_d m_s}$, $|b| \rightarrow \sqrt{m_b(m_s - m_d)}$, $c \rightarrow m_b$, i.e.

$$\vec{e}(m_d) = \left(\sqrt{\frac{m_s}{m_d + m_s}}, \sqrt{\frac{m_d}{m_s + m_d}}, 0 \right)$$

$$\begin{aligned}\vec{e}(m_s) &= \left(-\sqrt{\frac{m_d}{m_s+m_d}}, \sqrt{\frac{m_s}{m_s+m_d}}, 0 \right) \\ \vec{e}(m_b) &= (0, 0, 1).\end{aligned}\quad (13)$$

Analogously we can write down $\vec{e}(m_u)$..., replacing $m_d \rightarrow m_u, m_s \rightarrow m_c, m_b \rightarrow m_t$. From eq.(13) follows immediately relation (4). Taking into account m_b, m_t explicitly, one will in general obtain a violation of the relation (4) and of the Cabbibo universality, which is of order $\frac{m_s}{2m_b} \ll 0.02$ (in the charge - 1/3 system) and $\frac{m_c}{2m_t} \ll 0.12$ (in the charge 2/3 system). The effect can be calculated for each mass configuration exactly according to eq.(12). For phenomenological purposes it is sufficient to expand the various mixing parameters in powers of $m_s/2m_b$ or $m_c/2m_t$ and to disregard all terms of order $(m_s/2m_b)^n$ or $(m_c/2m_t)^n$ ($n \geq 2$). In this case one finds

$$\begin{aligned}|a| &= \sqrt{m_d m_s} \sqrt{1 + \frac{m_s - m_d}{m_b}}, \quad |b| = \sqrt{m_b (m_s - m_d)} \\ \vec{e}(m_d) &= \left(\sqrt{\frac{m_s}{m_s+m_d}}, \sqrt{\frac{m_d}{m_s+m_d}} \sqrt{1 - \frac{m_s - m_d}{m_b}}, -\sqrt{\frac{m_d}{m_s+m_d}} \sqrt{\frac{m_s - m_d}{m_b}} \right) \\ \vec{e}(m_s) &= \left(-\sqrt{\frac{m_d}{m_s+m_d}}, \sqrt{\frac{m_s}{m_s+m_d}} \sqrt{1 - \frac{m_s - m_d}{m_b}}, -\sqrt{\frac{m_s}{m_s+m_d}} \sqrt{\frac{m_s - m_d}{m_b}} \right) \\ \vec{e}(m_b) &= \left(0, \sqrt{\frac{m_s - m_d}{m_b}}, \sqrt{1 - \frac{m_s - m_d}{m_b}} \right).\end{aligned}\quad (14)$$

The vectors $\vec{e}(m_u)$, $\vec{e}(m_c)$, and $\vec{e}(m_t)$ are calculated analogously.

Introducing the angles

$$\theta_2 = \arcsin \sqrt{\frac{m_d}{m_s+m_d}}, \quad \phi_2 = -\arcsin \sqrt{\frac{m_s - m_d}{m_b}}, \quad (15)$$

we can rewrite eq. (14) as follows:

$$\begin{aligned}\vec{e}(m_d) &= (\cos \theta_2, \sin \theta_2 \cos \phi_2, \sin \theta_2 \sin \phi_2) \\ \vec{e}(m_s) &= (-\sin \theta_2, \cos \theta_2 \cos \phi_2, \cos \theta_2 \sin \phi_2) \\ \vec{e}(m_b) &= (0, -\sin \phi_2, \cos \phi_2).\end{aligned}\quad (16)$$

Analogously we calculate the vectors $\vec{e}(m_u)$, $\vec{e}(m_c)$ and $\vec{e}(m_t)$, introducing the angles $\theta_1 = \sin^{-1} \sqrt{\frac{m_u}{m_c + m_u}}$ and $\phi_1 = -\arcsin \sqrt{\frac{m_c - m_u}{m_t}}$. If we

define the linear combinations

$\tilde{c} = c \cos \phi_1 + t \sin \phi_1$, $\tilde{t} = t$, $\tilde{s} = s \cos \phi_2 + b \sin \phi_2$, $\tilde{b} = b$, as well as $u' = u \cos \theta_1 + \tilde{c} \sin \theta_1$, $\tilde{c}' = \tilde{c}$, $d' = d \cos \theta_2 + \tilde{s} \sin \theta_2$, $\tilde{s}' = \tilde{s}$, we can write the weak doublets as follows:

$$\begin{pmatrix} u' & \tilde{c}' & \tilde{t} \\ d' & \tilde{s}' & \tilde{s} \end{pmatrix}_L, \quad (17)$$

or:

$$\begin{pmatrix} u \\ \cos(\theta_2 - \theta_1) d + \sin(\theta_2 - \theta_1) (\cos \phi_2 s + \sin \phi_2 b) \\ -\sin(\theta_2 - \theta_1) \cos \phi_1 d + \cos(\theta_2 - \theta_1) \cos \phi_1 (\cos \phi_2 s + \sin \phi_2 b) - \sin \phi_1 (-\sin \phi_2 s + \cos \phi_2 b) \\ t \\ -\sin(\theta_2 - \theta_1) \sin \phi_1 d + \cos(\theta_2 - \theta_1) \sin \phi_1 (\cos \phi_2 s + \sin \phi_2 b) + \cos \phi_1 (-\sin \phi_2 s + \cos \phi_2 b) \end{pmatrix} \quad (18)$$

We compare eq.(18) to the weak coupling matrix discussed earlier (eq.(3)). It is evident that the matrix describing the couplings denoted in eq. (18) is identical to the one of eq.(3) if we set $\delta = 0$ and make the identification

$$\theta_2 - \theta_1 = \alpha_1 \quad \phi_1 = -\alpha_2, \quad \phi_2 = \alpha_3. \quad (19)$$

Thus we arrive at the following relations:

$$s_1 = \sin \alpha_1 = \frac{\sqrt{m_d m_c} - \sqrt{m_u m_s}}{\sqrt{(m_c + m_u)(m_s + m_d)}} \quad (20)$$

$$s_2 = \sin \alpha_2 = -\sin \phi_1 = \sqrt{\frac{m_c - m_u}{m_t}}$$

$$s_3 = \sin \alpha_3 = \sin \phi_2 = -\sqrt{\frac{m_s - m_d}{m_b}}.$$

We add the following comments:

- a) The parameters a , b in the matrix (6) are complex. Thus one might expect a violation of CP. However, it is easy to see that the phases of a and b can be absorbed in the yet unspecified phases of the quark fields. We conclude: If we start from a mass matrix as described in eq.(6) CP is conserved automatically. Thus the

small violation of CP observed in nature cannot be attributed to the quark mass matrix. However, it is possible that our assumptions which led to eq.(6) are not exactly fulfilled, but only approximately (e.g. violated by radiative corrections), in which case CP violation could result via such a violation.

- b) The constraint from universality mentioned in the introduction ($|s_3| \leq 0.25$) implies according to eq. (19) $m_b \geq 16 (m_s - m_d)$, i.e. $m_b > 2$ GeV for $m_s - m_d = 140$ MeV. It is almost certain that $m_b \geq 5$ GeV, in which case one finds $s_3 \leq 0.17$, $c_3 \geq 0.985$. Thus the violation of the Cabibbo type universality is tiny; the parameter $\cos\theta_c$ changes by less than $1^\circ/100$, if we change the interpretation of the parameter $\sin\theta_c$ measured in strange particle decay: $\sin\theta_c \rightarrow s_1 c_3$. This implies that eq.(4) is a very good approximation. A violation of eq.(4) is expected to be less than $3^\circ/100$.
- c) An interesting aspect of the mixing scheme discussed here is that the angle $\alpha_2 = \arcsin \sqrt{\frac{m_c - m_u}{m_t}}$ is potentially rather big. The lower limit of m_t given by experiment is of the order of 5 GeV. Using $m_c - m_u = 1.3$ GeV and taking the values $m_t = 5$ GeV; 10 GeV; 15 GeV, one finds respectively $\alpha_2 = 31^\circ$ ($s_2 = 0.52$); 21° ($s_2 = 0.36$); 17° ($s_2 = 0.29$). Thus α_2 is likely to be larger than the Cabibbo angle. Only if $m_t > 30$ GeV, one has $\alpha_2 < \theta_c \approx 13^\circ$.

The coefficient of the $(\bar{c}s)$ - current determines the life-time of charmed particles. In case $s_2, s_3 \neq 0$ this coefficient is reduced compared to the "standard" value $\cos\theta_c$. Thus the life-time of charmed particles is expected to be larger than the life-time estimated within the standard four-quark scheme. However, this effect will be very small. The reduction factor is $((c_1 c_2 c_3 - s_2 s_3) / \cos\theta_c)^2$. It varies in the interesting region $m_t, m_b \geq 5$ GeV between 0.8 and 1. The increase of the life-time of charmed particles will not be larger than $\sim 20\%$.

- d) In order to illustrate how big the various mixing angles can be, we consider two special examples, using $m_s - m_d = 0.14$ GeV, $m_c - m_u = 1.3$ GeV. One of the masses m_b or m_t will be set to 5 GeV, in view of an interpretation of T (9.5) as either a $\bar{b}b$ or a $\bar{t}t$ bound state.

Example A: $m_b = 5$ GeV, $m_t = 10$ GeV.

The weak currents are

$$\left(\begin{array}{c|c|c} u & c & t \\ \hline 0.98d+0.22s-0.04b & -0.20d+0.96s+0.20b & 0.08d-0.19s+0.98b \end{array} \right).$$

Example B: $m_b = 10 \text{ GeV}$, $m_t = 5 \text{ GeV}$.

The weak currents are

$$\left(\begin{array}{c|c|c} u & c & t \\ \hline 0.98d + 0.22s - 0.03b & -0.19d + 0.89s + 0.41b & 0.11d - 0.39s + 0.91b \end{array} \right).$$

It is interesting to see the relatively strong communication between t and s , as well as between c and b , exhibited by those examples, especially in example B. At energies sufficiently far above t -threshold one will produce t -flavored particles in neutrino-nucleus scattering at a substantial rate:

$$\begin{aligned} \sigma(t) / \sigma(c) &\approx 0.1 && \text{in example A;} \\ \sigma(t) / \sigma(c) &\approx 0.3 && \text{in example B.} \end{aligned}$$

The production of t -flavored particles takes place both off the $\bar{s}$ - s sea and the d valence quarks. (both processes occur with about comparable rates). Typically it is very difficult to produce b -flavored particles in ν or $\bar{\nu}$ scattering, due to the smallness of the $(\bar{u}b)$ - coupling. In case of $\bar{\nu}$ -scattering one estimates the ratio $\sigma(b)/\sigma_{\text{tot}} \approx 1^\circ/\infty$.

In example A, where $m_b < m_t$, t -flavored particles decay mainly via the chain $t \rightarrow b \rightarrow c \rightarrow s$. This will lead to multilepton events in neutrino nucleus scattering (for a more detailed discussion, especially of trimuon events, see e.g. ref. (7,8)).

In example B, where $m_t < m_b$, t flavored particles decay predominantly ($\sim 80\%$) into strange particles, while b -flavored particles decay predominantly ($\sim 90\%$) via the chain $b \rightarrow t \rightarrow s$.

Conclusions:

In this paper we have calculated the weak interaction mixing angles of the six-quark theory in terms of the quark masses. We based our considerations on the gauge group $SU_2^L \times SU_2^R \times U_1$, although it might be that our results are of wider generality and are true for other gauge groups as well, e.g. even for the "minimal" gauge group $SU_2 \times U_1$. The expression for the Cabibbo angle which was derived previously within the four-quark theory remains essentially unaltered. The Cabibbo universality for the currents $(\bar{u}d)$, $(\bar{u}s)$ is violated only to a tiny extent. Substantial violations of the Cabibbo-type universality in the $(\bar{s})^-$ system are found. Especially a relatively strong mixing between c and t leads to the existence of a weak angle α_2 which is larger than the Cabibbo angle for $m_t < 30\text{GeV}$. The main decay modes of the t- and b-flavored particles involve cascades (e.g. $t \rightarrow b \rightarrow c \rightarrow s$). The production and subsequent decay of t-flavored particles in high energy ν and $\bar{\nu}$ scattering leads to multilepton events.

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