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THE INFLUENCE OF PARTON TRANSVERSE MOMENTA ON LARGE $p_{\perp}$ SPECTRA

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ABSTRACT

An approximate analytical method is presented to study the influence of parton transverse momenta on large $p_{\perp}$ jet production. Parton transverse momentum distributions are used with Gaussian, power law and exponential shape. The single jet distribution at large $p_{\perp}$ increases due to the parton transverse momenta $k_{\perp}$ only by a factor of 2 to 3 within a reasonable range of $\langle k_{\perp}^2 \rangle$.

Furthermore, parton transverse momenta are found to be unable at present energies and transverse momenta to modify significantly the $p_{\perp}^{-n}$ power behaviour of the large $p_{\perp}$ distributions.

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As is well known the quark-quark hard scattering model with gluon exchange is not able to explain the experimentally observed behaviour

$$E \frac{d^3\sigma}{d^3p} \sim p_{\perp}^{-2} f(x_{\perp})$$

of single particle spectra at large $p_{\perp}$. Also, the inclusion of quark-gluon and gluon-gluon scattering - besides of quark-quark scattering - in lowest order QCD and together with quark and gluon distribution functions which deviate from scaling according to QCD, can only give an effective exponent not larger than $n_{\text{eff}} \simeq 6$ ¹⁾ in the presently explored $p_{\perp}$ range.

There are indications from experiments on lepton pair production, hadron production at large $p_{\perp}$ as well as from deep inelastic μN reactions, that the average parton transverse momentum inside a hadron might be as big as $\sqrt{\langle k_{\perp}^2 \rangle} \simeq 0.5$ to 1 GeV/c or even bigger ²⁾⁻⁶⁾. Using Monte Carlo calculations, it was found recently ⁷⁾ that data up to $p_{\perp} \simeq 4$ GeV/c could well be represented taking into account the parton transverse momenta $k_{\perp}$. This essentially corresponds to an increase in n_{eff} at the presently explored s and $p_{\perp}$ range. If this is correct one could hope to understand present data on the basis of hard scattering of quarks and gluons and the additional $k_{\perp}$ smearing of the partons inside the hadrons. Others ^{8),9)} do not find such a striking effect.

Our aim is the study of single particle spectra and their dependence on n_{eff} as they are modified by the parton transverse momenta. In fact, we calculate the $p_{\perp}$ dependence of the single jet distribution; the single particle spectra resulting from jet fragmentation are expected to show the same $p_{\perp}^{-n}$ power behaviour as does the single jet distribution ¹⁰⁾⁻¹²⁾. The effect of parton transverse momenta $\langle k_{\perp}^2 \rangle \neq 0$ on correlation data such as the opposite side distribution dN/dx_e and the p_{out} distribution have been considered by various groups ^{7)-9),13)}.

We present here a method to take account of the parton transverse momenta in an approximate analytical way.

In the hard parton-parton collision, two large $p_{\perp}$ hadronic jets are formed having in the parton-parton rest frame the energy squared $\hat{s}$ and the momentum transfer $\hat{t}$. The system of the two jets - which we shall call the "di-jet" - is characterized in the hadron-hadron c.m.s. by the longitudinal momentum $P_{\parallel} = \frac{\sqrt{s}}{2}(x_1 - x_2)$ or the rapidity Y and - if the original partons have transverse momentum - by a transverse momentum $\vec{P}_{\perp} = \vec{k}_{\perp 1} + \vec{k}_{\perp 2}$.

We use asymptotic kinematics such that

$$\hat{s} \approx x_1 x_2 s$$

$$x_1 = e^{y \sqrt{\frac{\hat{s}}{s}}}, \quad x_2 = e^{-y \sqrt{\frac{\hat{s}}{s}}} \quad (1)$$

x_1 and x_2 are the longitudinal momentum fractions of the partons 1 and 2. Our kinematics implies for the parton momenta $k_{\parallel i} \gg k_{\perp i}$.

In terms of the parton distributions $q(x, \vec{k}_\perp) = \frac{1}{x} \tilde{F}(x, \vec{k}_\perp)$ for "di-jet" rapidity $Y \approx 0$ and resulting jets observed at $\vartheta = 90^\circ$ in the c.m.s. the "di-jet" production cross-section behaves like

$$\frac{d\sigma}{d\hat{s} dy d^2P_\perp} \approx \frac{\text{const}}{\hat{s}^{n/2}} \sum_{i,j} \int d^2k_\perp \tilde{F}_i(x_i, \vec{k}_\perp) \tilde{F}_j(x_j, \vec{P}_\perp - \vec{k}_\perp) \quad (2)$$

where we have already used conservation of transverse momentum. We limit our discussion to the kinematic region where $\hat{s} \gg 1 \text{ GeV}^2$ and $\hat{s} \gg P_\perp^2$. In our numeric calculation we use $n=4$ and 8.

To study the influence of parton transverse momenta on single jet production, the $P_\perp$ dependence of Eq. (2) is more essential than the Y or $P_\parallel$ dependence. Modifying the $P_\parallel$ dependence, we would still expect to find the essential effects caused by the parton $k_\perp$. Especially, if the "di-jet" distribution (2) would be isotropic, the kinematics of the "di-jet" decay into single jets can be treated exactly ¹⁴⁾.

For technical reasons in our treatment we approximate the "di-jet" cross-section (2) by an isotropic cross-section

$$\left(E \frac{d^4\sigma}{d\hat{s} d^3P} \right)_{\text{isotropic}}$$

which at $\vartheta = 90^\circ$ ($Y=0$) agrees with (2).

The energy distribution $d\sigma/d\hat{s}dE$ of the "di-jet" is easily obtained from an isotropic "di-jet" distribution. If the "di-jet" decays isotropically in its rest frame into two jets i and j with momenta $|\vec{p}_i^*| = |\vec{p}_j^*| = \sqrt{\hat{s}}/2$ and polar angles ϑ_i and $\vartheta_j = \vartheta_i + \pi$, the energy distribution of the single jet i in the collision c.m.s. is given by ¹⁴⁾

$$\frac{d\sigma}{d\varepsilon^*} = \int_{E_0}^{E_1} dE \, d\hat{s} \, \frac{d\sigma}{d\hat{s} dE} \, \frac{1}{P} \quad \begin{aligned} E_0 &= \varepsilon + \frac{\sqrt{\hat{s}}}{4\varepsilon} \\ E_1 &= \frac{\sqrt{\hat{s}}}{2} \end{aligned} \quad (3)$$

We use Eq. (3) only for jets produced at $\vartheta = 90^\circ$ in the collision c.m.s.

From Eq. (3) the invariant single jet distribution at $\vartheta = 90^\circ$ is

$$\varepsilon \left. \frac{d\sigma}{d^3 p_i} \right|_{\substack{\vartheta=90^\circ \\ p_i = p_{iL}}} = \frac{1}{2\pi p_i} \frac{d\sigma}{d\varepsilon_i} \quad (4)$$

We perform calculations for factorized structure functions

$$\tilde{F}(x, \vec{k}_\perp) = F(x) g(k_\perp) \quad (5)$$

In general the distribution $g(k_\perp)$ could also depend on x ¹⁵⁾. Different functional forms for the $k_\perp$ distributions have been proposed in the past ¹⁵⁾. We explore in the following the effect of parton transverse momenta using the following distributions :

i) Gaussian distribution

$$g(k_\perp^2) = \frac{1}{2\pi\sigma^2} e^{-k_\perp^2/2\sigma^2} \quad (6)$$

with

$$\sqrt{\langle k_\perp^2 \rangle} = \sqrt{2} \sigma$$

ii) power law

$$g(k_\perp^2) = \frac{N}{\pi} (m^2)^{N-1} \frac{1}{(k_\perp^2 + m^2)^N} \quad (7)$$

which leads, for $N \geq 3$, to

$$\sqrt{\langle k_\perp^2 \rangle} = \left(\frac{N}{N^2 - 3N + 2} \right)^{1/2} \cdot m$$

iii) exponential distribution

$$q(k_{\perp}^2) = \frac{a^2}{2\pi} e^{-a k_{\perp}} \quad (8)$$

with

$$\sqrt{\langle k_{\perp}^2 \rangle} = \frac{6}{a^2}$$

For the Gaussian $k_{\perp}$ distribution (6) all integrals but one can be performed analytically and we obtain for the single jet spectrum at $\theta = 90^\circ$

$$\begin{aligned} \varepsilon_i \frac{d^3\sigma}{d^3p_i} \Big|_{\theta=90^\circ} &= \frac{1}{2\pi p_{i,\perp}} \frac{\text{const}}{2\pi} \int \frac{d\hat{s}}{\hat{s}^{1/2}} \cdot \\ &\cdot \exp \left[-\frac{1}{4\sigma^2} \left(p_{i,\perp} - \frac{\hat{s}}{4p_{i,\perp}} \right)^2 \right] \frac{1}{\sqrt{4\pi}\sigma} F_1 \left(\sqrt{\frac{\hat{s}}{s}} \right) F_2 \left(\sqrt{\frac{\hat{s}}{s}} \right) \end{aligned} \quad (9)$$

An asymptotic estimate of this integral using $F_1(\sqrt{\hat{s}/s}) = \text{const}$ leads to

$$\varepsilon \frac{d^3\sigma}{d^3p} \sim \frac{1}{p_{i,\perp}^n}$$

Thus, asymptotically no change of the power n can be expected.

Figures 1a and 1b, obtained by numerical integration of Eq. (9), show the change of the single jet spectra obtained in pp collisions for various $p_{i,\perp}$ values and the effective power of $\sim p_{i,\perp}^{-n_{\text{eff}}}$ as function of the mean parton transverse momentum $\sqrt{\langle k_{\perp}^2 \rangle}$. In the actual calculation we used $F(x)$ distributions which scale à la McElhaney and Tuan¹⁶⁾. The use of different parametrizations is not expected to change the result significantly.

The quantity plotted in Fig. 1a is

$$R(\sqrt{\langle k_{\perp}^2 \rangle}) = \frac{\varepsilon \frac{d^3\sigma}{d^3p_i} \Big|_{\langle k_{\perp}^2 \rangle \neq 0}}{\varepsilon \frac{d^3\sigma}{d^3p_i} \Big|_{\langle k_{\perp}^2 \rangle = 0}} \quad (10)$$

In Figures 1 to 3 we present the results of our calculations only with input powers $n=4$. Corresponding calculations performed for input $n=8$ led to similar behaviour.

For power and exponential parton $k_{\perp}$ distributions (7) and (8) one and two integrations over parton transverse momenta, respectively, remain for numerical evaluation.

Figures 2 and 3 show, for the power behaved ($N=4$ for Fig. 2a, $N=4,6,8$ for Fig. 2b) and for the exponential case, respectively, the ratios R , Eq. (10) for various values $p_{\perp}$ and the effective power n_{eff} , both as functions of $\sqrt{\langle k_{\perp}^2 \rangle}$.

As the curves show, we find an increase of large $p_{\perp}$ single jet distributions by a factor of 2 to 3, depending on $p_{\perp}$, the collision energy $\sqrt{s}$ and the actual value of the average parton transverse momentum $\sqrt{\langle k_{\perp}^2 \rangle}$. According to earlier results ¹⁰⁾⁻¹²⁾ we expect the single particle spectrum to be modified by the same factor. This is in accord with some of the earlier results ^{8),9)} for special parton distributions [exponential in Ref. 8), Gaussian in Ref. 6)]. For all three parton $k_{\perp}$ distributions considered the increase of the effective power n_{eff} of the large $p_{\perp}$ cross-section in the presently explored $p_{\perp}$ range is marginal.

We summarize : we have presented a method for taking account of parton transverse momenta in the particular situation where the "di-jet" rapidity $Y \simeq 0$ and the jets are observed under $\vartheta = 90^\circ$. In this kinematical situation $\hat{t}$ is big and our large $p_{\perp}$ results do not depend sensitively on a cut-off mass in the propagator of the hard subcollision.

We remark here that in general jets at $\vartheta = 90^\circ$ might also be produced by a "di-jet" with $Y \neq 0$. This case was not treated here; it might receive contributions from smaller $\hat{t}$ and therefore be more sensitive to a mass cut-off.

In the case treated here, at present energies and transverse momenta at large $p_{\perp}$ triggers, parton transverse momenta superimposed on a quark and gluon hard scattering cross-section do not make the large $p_{\perp}$ spectra as steep as they are observed experimentally.

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FIGURE CAPTIONS

Figure 1 : The effect of the Gaussian distribution of parton transverse momenta (6) as function of $\sqrt{\langle k_T^2 \rangle}$ using a hard scattering cross-section $d\sigma/d\hat{t} \sim \hat{s}^{-2}$.

- a) the function $R(\sqrt{\langle k_T^2 \rangle})$, Eq. (10), for $\sqrt{s}=20$ and 50 GeV and $p_{\perp}=3$ and 6 GeV/c;
- b) the effective power in the large $p_{\perp}$ cross-section $\sim p_{\perp}^{-n_{\text{eff}}}$ for $p_{\perp}=5$ GeV/c and $\sqrt{s}=30$ and 50 GeV.

Figure 2 : The effect of the power behaved parton transverse momentum distribution (7) as function of $\sqrt{\langle k_T^2 \rangle}$.

- a) the function $R(\sqrt{\langle k_T^2 \rangle})$, Eq. (10) for $N=4$ [cf., Eq.(7)] at $\sqrt{s}=50$ GeV and $p_{\perp}=3, 6$ and 9 GeV/c;
- b) the effective power in the large $p_{\perp}$ cross-section $\sim p_{\perp}^{-n_{\text{eff}}}$ for $p_{\perp}=5$ GeV/c at $\sqrt{s}=50$ GeV and the power $N=4, 6$ and 8 in $g(k_{\perp})$, Eq. (7).

Figure 3 : The effect of the exponential parton transverse momentum distribution Eq. (8) as function of $\sqrt{\langle k_T^2 \rangle}$.

- a) the function $R(\langle k_T^2 \rangle)$ at $\sqrt{s}=50$ GeV and $p_{\perp}=3$ and 6 GeV/c;
- b) the effective power n_{eff} in the large $p_{\perp}$ cross-section $\sim p_{\perp}^{-n_{\text{eff}}}$ at $\sqrt{s}=50$ GeV and $p_{\perp}=5$ GeV/c.

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FIG. 1b

FIG. 1a

$$g(k_1^2) = \frac{N}{\pi} (m^2)^{N-1} \frac{1}{(k_1^2 + m^2)^N}$$

FIG. 2b

FIG. 2a

FIG. 3b

FIG. 3a

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