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LEPTON PAIR PRODUCTION AND THE DRELL-YAN FORMULA IN QCD

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A B S T R A C T

The Drell-Yan formula is found to be valid in low order perturbation theory calculations in QCD. The formula is only reproduced when quark-antiquark annihilation and bremsstrahlung diagrams are all taken into account. These calculations suggest that when the experimentally extracted quark and antiquark distribution functions (which slowly violate Bjorken scaling) are used in the Drell-Yan formula, subprocesses such as those involving the bremsstrahlung of the massive lepton pair, or ones with gluons in the initial state, should not be calculated independently, as they are already included in the formula.

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Several years ago Drell and Yan ¹⁾ suggested that the mechanism which is predominantly responsible for the production of a massive lepton-pair in high energy hadronic collisions is the annihilation of a quark from one initial hadron with an antiquark from the other. Thus for large S, M^2 (where M^2 is the invariant mass of the lepton pair), with the ratio $(M^2/S) \equiv \tau$ a finite number between 0 and 1 we have

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3n M^4} \sum_{\substack{\text{quark} \\ \text{flavours} \\ a}} Q_a^2 \int_0^1 dx_1 dx_2 x_1 x_2 \left[G_{q/h_1}^a(x_1, M^2) G_{\bar{q}/h_2}^a(x_2, M^2) + G_{\bar{q}/h_1}^a(x_1, M^2) G_{q/h_2}^a(x_2, M^2) \right] \delta(x_1 x_2 \tau) \quad (1)$$

where n is the number of colours. In the parton model picture of Drell and Yan, $G_{q/h_1}^a(x, M^2)$ is independent of M^2 , and is defined to be the probability of finding a quark of flavour a in hadron 1 with fraction x of the momentum of the hadron (defined in the infinite momentum frame). When G functions extracted from deep inelastic scattering data are substituted into the right-hand side of (1), a reasonable fit to the data is obtained [for a recent review, see Ref. 2) and references therein]. It is not clear, however, why other subprocesses (for example ones which involve the bremsstrahlung of the massive lepton pair ³⁾ and for which it is possible to use valence quarks only) should be subdominant, indeed it has been shown ⁴⁾ that for all basic subprocesses which involve only (scale invariant) interactions between quarks and gluons, i.e., do not involve composite particles, the same asymptotic scaling law

$$\frac{d\sigma}{dM^2} \underset{\tau \text{ fixed}}{\sim}_{M^2, S \rightarrow \infty} \frac{1}{M^4} F(\tau) \quad (2)$$

holds. It is not possible, however, to study the relative importance of these or the other subprocesses studied in Ref. 4) since their normalisation is in general not calculable.

So far attempts to derive formula (1), or any similar formula in quantum chromodynamics (QCD) have been unsuccessful ^{*)}. The renormalization group and operator product expansion techniques developed in the study of deep

^{*)} See, however, the derivation of the Drell-Yan formula in a two-dimensional Yang-Mills theory in the $1/N$ expansion ⁵⁾.

inelastic scattering are not directly applicable to massive lepton pair production. In the absence of a "renormalization group improved" perturbation expansion, it may prove instructive to study this process in ordinary perturbation theory, where problems such as gauge invariance, mixing of antiquarks and gluons, etc., are encountered and are well understood. Models which incorporate parton ideas for the hadronic wave functions, and QCD perturbation theory for the interactions of quarks and gluons can then be formulated if desired. We now turn to a study of lepton pair production in perturbation theory.

For simplicity of presentation we take the flavour group to be SU(2), the generalization to more realistic cases is straightforward. Then we define the G functions by the relations

$$G_{u/h}(x, q^2) = \frac{1}{2} F_1^{\bar{\nu}h}(x, q^2) - \frac{1}{4} F_3^{\bar{\nu}h}(x, q^2) \quad (3a)$$

$$G_{\bar{u}/h}(x, q^2) = \frac{1}{2} F_1^{\nu h}(x, q^2) + \frac{1}{4} F_3^{\nu h}(x, q^2) \quad (3b)$$

$$G_{d/h}(x, q^2) = \frac{1}{2} F_1^{\nu h}(x, q^2) - \frac{1}{4} F_3^{\nu h}(x, q^2) \quad (3c)$$

$$G_{\bar{d}/h}(x, q^2) = \frac{1}{2} F_1^{\bar{\nu}h}(x, q^2) + \frac{1}{4} F_3^{\bar{\nu}h}(x, q^2) \quad (3d)$$

where the F 's are the usual structure functions measured in deep inelastic neutrino and antineutrino scattering.

Halliday ⁶⁾ has studied the process $e^+e^- \rightarrow \mu^+\mu^-X$ in massive QED, in the leading logarithm approximation and found that formula (1) indeed holds for this theory, the G functions now correspond to the Bjorken scale breaking structure functions previously calculated by Gribov and Lipatov ⁷⁾. This implies that up to order αg^2 , formula (1) is also true for the process quark + antiquark $\rightarrow \mu^+\mu^-X$ in the leading logarithm approximation. This last result has recently been explicitly demonstrated by Politzer ⁸⁾. In this letter we study the three processes $qg \rightarrow \mu^+\mu^-X$, $g g \rightarrow \mu^+\mu^-X$ and $q q \rightarrow \mu^+\mu^-X$ [where q (g) denotes quark (gluon)], in each case to the lowest order in perturbation theory and in the leading logarithm approximation. The qg case is thus calculated to order g^2 and the gg ,

qq cases to order g^4 . We do not attempt here a consistent calculation of all processes (including $q\bar{q}$) to order g^4 , the examples we calculate are nevertheless independent, and provide a necessary condition for the validity of the Drell-Yan formula to this order. We shall neglect the mass of the quark, but keep the initial particles slightly off-mass-shell. None of the conclusions below are changed by the inclusion of a quark mass. Here we present only the results of our calculations, the details will be presented elsewhere.

(A) - $qg \rightarrow \mu^+ \mu^- X$

The structure function of a gluon can be readily calculated and leads to

$$G_{q/g}(x, q^2) = G_{\bar{q}/g}(x, q^2) = \frac{g^2}{(4\pi)^2} (1 - 2x(1-x)) \log q^2/p^2 \quad (4)$$

where p is the momentum of the gluon.

There are two Feynman diagrams in the lowest order for the process $qg \rightarrow \mu^+ \mu^- X$, these are the two diagrams of Fig. 1. We would like to call the diagram of Fig. 1a the $q\bar{q}$ annihilation diagram, and that of Fig. 1b a bremsstrahlung diagram. When the cross-section corresponding to these diagrams is calculated we find that formula (1) is satisfied, the leading logarithm contributions in the Feynman gauge come from the square of the diagram of Fig. 1a and the interference of the diagrams of Figs. 1a and 1b with relative weights of 1 and $-2\tau(1-\tau)$, respectively. Thus even in the leading logarithm approximation one needs the bremsstrahlung diagram to obtain the Drell-Yan formula ! It is, of course, only the sum of both the diagrams of Fig. 1 which is gauge invariant.

(B) - $gg \rightarrow \mu^+ \mu^- X$

This process is perhaps more interesting from the point of view of hadronic collisions, in that even in the lowest order the lepton pair can be produced throughout the rapidity range, whereas in the $qg \rightarrow \mu^+ \mu^- X$ case, the lepton pair is produced always with roughly the same rapidity. The lowest order Feynman diagrams for this process are shown in Fig. 2. It is found the contributions to the cross-section which have the wrong group theory structure to reproduce the Drell-Yan formula do not have a leading logarithmic behaviour ! We are thus left with the "QED-like" diagrams. Let us denote by

σ_{ij} the contribution to the cross-section from the interference of the diagram of Fig. 2i with that of Fig. 2j, in the Feynman gauge. Then defining K_{ij} by the relation

$$\frac{d\sigma_{ij}}{dM^2} = \frac{4\pi\alpha^2}{3nM^4} \sum_a Q_a^2 \int_0^1 \frac{g^4}{(4\pi)^4} dx_1 dx_2 x_1 x_2 S(x_1 x_2 - \tau) \times \log\left(\frac{M^2}{-p_1^2}\right) \log\left(\frac{M^2}{-p_2^2}\right) K_{ij} \quad (5)$$

We find $K_{aa} = K_{dd} = 1$, $K_{ab} + K_{ba} = K_{de} + K_{ed} = -2x_1(1-x_1)$, $K_{ac} + K_{ca} = K_{df} + K_{fd} = -2x_2(1-x_2)$, $K_{ad} = K_{da} = K_{bc} + K_{cb} = K_{ef} + K_{fe} = 2x_1 x_2 (1-x_1)(1-x_2)$. The other contributions to the cross-section do not have a doubly logarithmic behaviour. Formula (1) is thus again reproduced.

(C) - $qq \rightarrow \mu^+ \mu^- X$

Perhaps the most straightforward way to calculate the function $G_{q/q}^-$ is to use the techniques of Gribov and Lipatov ⁷⁾, with the result

$$G_{q/q}^-(x, q^2) = \frac{g^4}{(4\pi)^4} \frac{\log^2 q^2/p^2}{2} \frac{(n^2-1)}{n} \times \int_x^1 \frac{d\beta}{\beta^2} \left[2 \frac{1-\beta}{\beta^2} + 1 \right] \left[(\beta-x)^2 + x^2 \right] \quad (6)$$

The diagrams which contribute to the process $qq \rightarrow \mu^+ \mu^- X$, are shown in Fig. 3. Individual contributions to the cross-section, such as, for example, from the square of Fig. 3a, violate the Drell-Yan scaling law, Eq. (2), by powers (i.e., terms like M^2/p_2^2 appear), and not just by logarithms. When gauge invariant sets of diagrams are summed, however, these power violations disappear and formula (1) is again reproduced.

Of course, in real experiments the initial particles are hadrons, and not quarks and gluons, and so if we are to relate the above results to experiments, some model assumptions have to be made. One attractive possibility is to assume that one can write a soft wave function for the hadrons, and that interactions between hadrons are given by the convolution of these soft wave functions with QCD perturbation theory for the interactions between quarks and gluons. With this assumption the validity of formula (1) for quarks and gluons implies its validity also for hadrons.

To see what lessons can be drawn from the above results, let us recall how lepton pair production cross-sections are usually calculated ²⁾. First quark and antiquark distribution functions are extracted from deep inelastic scattering data using relations such as (3). Of course, in a field theory these do not correspond only to the "handbag" diagram or any other single diagram, but include the contributions from all diagrams. These distribution functions are then substituted into the right-hand side of (1) to obtain the lepton pair cross-section. The results presented above suggest that this indeed is a correct procedure in QCD, but it is now difficult to interpret formula (1) as a $q\bar{q}$ annihilation ^{*)}. Whereas a parton model advocate would worry about what the correction to formula (1) from the other subprocesses would be, the present results suggest that in QCD these subprocesses all contribute to formula (1) and indeed are necessary to reproduce it. This implies that the independent inclusion of other subprocesses (such as bremsstrahlung processes, gluonic processes, etc.) involves double counting and is therefore wrong. This is closely related to the fact that the deep inelastic structure functions themselves indirectly include the effects of gluons as well as quarks and antiquarks. Unfortunately it is now unclear how to calculate the corrections to formula (1). An attractive feature of this formula is that it relates the violations of Bjorken scaling measured in deep inelastic scattering (where q^2 is spacelike), to the violations of Drell-Yan scaling [Eq. (2)] ^{10),11)} in massive lepton pair production (where M^2 is timelike).

There remain many unsolved questions concerning higher orders in perturbation theory (where the non-Abelian nature of the theory becomes more apparent), the behaviour of non-leading logarithms, the transverse momentum of the lepton pair, etc., which are presently being studied, and the conclusions will be presented with the details of the present calculation. It will also be interesting to see whether the higher order terms in the subprocesses for other hard scattering processes (such as the production of particles with large transverse momentum) can also be absorbed into the non-scaling distribution functions of the initial particles. As concerns the question of the significance of the perturbative results, we agree with the philosophy of Politzer ^{7),12)} which we interpret as follows :

*) This does not exclude the possibility of such an interpretation in some other gauge. Cf. Ref. 9) for a probabilistic interpretation of QCD results for deep inelastic structure functions.

if in all orders of perturbation theory all the $\log p^2$ terms in the lepton pair cross-section can be absorbed into the structure functions, then the Drell-Yan formula is valid up to logarithmic corrections (at the very least with quarks and gluons in the initial states). The results presented in Ref. 7) and above suggest that this indeed may be the case.

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FIGURE CAPTIONS

- Figure 1 Lowest order diagrams for the process $\text{quark} + \text{gluon} \rightarrow \mu^+ \mu^- X$.
- Figure 2 Lowest order diagrams for the process $\text{gluon} + \text{gluon} \rightarrow \mu^+ \mu^- X$.
- Figure 3 Lowest order diagrams for the process $\text{quark} + \text{quark} \rightarrow \mu^+ \mu^- X$.

FIG.1

FIG.2

FIG.3