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PRECISION MEASUREMENT OF THE MUONIC
5-4 TRANSITIONS IN Pb AND 4-3 TRANSITIONS
IN Ba AS A TEST FOR THE VALIDITY OF QED

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ABSTRACT

The 5-4 and 4-3 transitions in muonic Pb and Ba, respectively, were measured with an accuracy of ~ 10 eV. Agreement with theoretical calculations is established. The vacuum polarization is tested to 3×10^{-3} , and vacuum polarization corrections of higher order in (αZ) to 20-30%.

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1. INTRODUCTION

Quantum electrodynamics (QED) has been successful to a very high degree of accuracy. Experimental tests such as the Lamb shift measurements in hydrogen¹⁾ or the g-2 measurements for the electron²⁾ and muon³⁾ have confirmed it to a precision at the ppm level. However, these experiments involve essentially small momentum transfers or low electric fields (small αZ). Moreover, the effects measured are mainly due to graphs of the self-energy kind. Effects involving closed electron-positron loops (vacuum polarization) are tested by these experiments to a much lower level of accuracy (percent level). Table 1 summarizes these facts.

Consequently there is a need for testing QED, at larger momentum transfers or higher fields, and with a system where the QED effects are mainly due to vacuum polarization. Such a system is the muonic atom. There the vacuum polarization yields the dominant QED correction. The momentum transfers and electric fields depend on the binding energy of the muon and are large only in heavy muonic atoms.

If one selects X-ray transitions between atomic levels for which the energy shifts due to the finite size of the nucleus, the polarization of the nucleus, and the screening of the nuclear charge by the remaining electrons of the atom, are small and thus calculable to a sufficient accuracy, muonic atoms offer an ideal tool for such tests, since they are almost hydrogen-like systems for which the theory is well established. Suitable transitions are, for example, the 5-4 transitions in the Pb region and the 4-3 transition in the Ba region, both having an energy of about 400 keV and showing a vacuum polarization correction of about 2 keV. Thus an absolute accuracy of 20 eV at 400 keV (corresponding to 50 ppm) would correspond to a 1% test of the vacuum polarization.

Measurements on heavy muonic atoms (Pb and Bi) were first done by our group in 1969⁴⁾, leading to results which were not in disagreement with the calculations performed at that time, but having relatively large errors (40-50 eV). Similar but more accurate measurements (≤ 20 eV errors) on a series of elements, ranging from Ca up to Pb, were done one year later by Dixit et al.⁵⁾. These measurements were partly in severe disagreement with theory (by more than 5 standard deviations for one Ba line), leading several authors^{6,7)} to revise the theoretical calculations. Part of the discrepancy could be removed, but a 3-4 standard deviation discrepancy for Pb and Ba remained unexplained. Walter et al.⁸⁾ checked on several heavy elements whether the discrepancy was reproducible. These measurements, having a 25 eV accuracy, were re-evaluated by Engfer et al.⁹⁾ and agreed reasonably with theory (1.5 standard deviation or less). However, the general trend of these data encouraged the authors to interpret them as confirming the discrepancies. Figure 1 shows the differences between the calculated and experimental values of

the various transition energies for the three groups. Except for our old data, the experimental values are systematically larger than the calculated ones, the differences increasing with increasing energies. It should be noticed that the Pb result of Dixit et al.⁵⁾ is in serious disagreement with our old value (2.7 standard deviations), but the older results of ours, having larger errors, had almost no weight in the subsequent discussions of the discrepancies by theoreticians and experimentalists.

Many theoreticians were stimulated by the apparent discrepancies to further improve on the QED calculations and to invent mechanisms for removing them. A review of the theoretical efforts is given by Watson and Sundaresan⁷⁾. No satisfactory solution was found, especially since an experiment was performed on the Lamb shift in muonic He^{10a)}, which is in excellent agreement with theory and rules out explanations for the discrepancies other than those which can be formulated within the framework of QED^{10b)}.

Many details, which will be discussed later, indicate that the systematic errors were seriously underestimated. The conventional experimental techniques for calibrating absolutely in the 400 keV region apparently had been pushed to their limits, and systematic uncertainties of unknown origin and magnitude dominated.

In view of this situation we decided to measure the 5-4 transitions in Pb and the 4-3 transitions in Ba, introducing some different techniques. We made use of the fact that the two line patterns are interleaving each other by bombarding a mixed Pb-Ba target, thus getting a relative Pb-Ba measurement with negligible calibration errors.

We used nuclear γ -lines from μ -capture, whose energies were measured separately to high accuracy, and muonic lines of lower energies to eliminate systematic uncertainties in the absolute energy calibration. Gamma-ray lines from calibration sources, recorded as accidental coincidences in the muonic X-ray spectra, as well as the carefully treated e^+e^- annihilation line were also used for calibration.

In this paper we will give details of this experiment. The results were already published in a letter¹¹⁾. They agree with theory and, together with the Lamb-shift measurement on μ -He, they represented the only data in the domain of muonic atoms confirming QED to better than 1%. By now also Dixit et al. with their new measurements¹²⁾ are in agreement with theory, and the Walter et al. results are now interpreted as confirming theory [Vuilleumier et al.¹³⁾]. The weakness of estimating systematic errors is demonstrated by Dixit et al.¹²⁾, who remeasured Pb and Ba with essentially the same calibration technique as before⁵⁾

and could not reproduce their old values, the inconsistency being of the order of 2-3 standard deviations. The authors could not find a criterion for deciding which one of their experiments is the more reliable¹⁴⁾.

2. OUTLINE OF THE EXPERIMENTAL SET-UP

2.1 Experimental set-up

The experimental set-up is shown in Fig. 2. The experiment was performed at the CERN Synchro-cyclotron (SC) muon channel. Muons from μ^- -decay in the backward direction were slowed down in a carbon moderator and stopped in the target. The target was X-shaped, one half of it (one V) being made from natural Ba, the other one from natural Pb. The target thickness in the beam direction was 3.5 g/cm² Ba and 2.8 g/cm² Pb, so that a roughly equal amount of muons stopped in Ba and Pb. The two V's were fixed between two horizontal Al plates of 2 mm thickness. The beam and target cross-section was 10 × 10 cm². A conventional counter telescope, consisting of four plastic scintillation counters (S1 through S4), was used to define a stopping muon. An additional counter (S5) was placed between the two V's of the target, allowing us to differentiate between stops in the first and second half of the target. Typical stop rates were 10⁴ sec⁻¹. During the measurement the target was inverted with respect to the beam direction, so that the Ba half of the target was upstream for about half the measuring time.

Two planar Ge(Li) detectors were used to measure the X-rays, one (Det. I) having a volume of 700 × 7 mm³, the other one (Det. II) of 200 × 5 mm³. For Det. I the in-beam resolution at 412 keV was 1.49 keV, and for Det. II 1.31 keV. The detectors, placed at a distance of 10 cm from the beam axis, were shielded from beam particles by copper bars. Radioactive sources for calibration were placed close to the target and on the detector axes.

Two multichannel pulse-height analysers, each of 4 k memory, were used to record the spectra. The two spectrometer systems were gain-stabilized. In addition, a magnetic tape unit was used in incremental mode, permitting six different routed spectra to be recorded.

A fast timing pulse for each detector was generated through constant fraction timing discrimination. A time resolution of ~ 20 nsec in the telescope-Ge(Li) timing could be achieved without efficiency losses above 40 keV. A typical Ge(Li) telescope coincidence rate was 50 sec⁻¹.

The analog pulses of the detectors were accepted by the multichannel analysers only when there was a Ge(Li) timing signal within a certain time window which was opened by the telescope triggers. The time zero of the time window was defined by those Ge(Li)-telescope coincidences coming from the muonic X-ray cascade and therefore within 10⁻¹⁴ sec after a muon stop event. The time windows were chosen such

that sufficient nuclear γ -rays from the radioactive calibration sources could be recorded as accidentals. They were 105 nsec and 40 nsec for Det. I and Det. II, respectively. In addition, three time windows were set up for Det. I: one pre-prompt, being 100 nsec wide, followed immediately by a prompt time window of 50 nsec width, and subsequent to the prompt a delayed window of 100 nsec. Routing bits corresponding to these three time windows were generated and used for the incremental mode storage. A pattern was formed by setting an additional bit whenever counter 5 was coincident with the telescope stop trigger, thus defining a stop in the downstream half of the target (see Fig. 2).

2.2 Calibration methods

The energy calibration of a Ge(Li) spectrometer under laboratory (off-beam) conditions is straightforward. The only difficulties might arise from different count rates or different geometry for the calibration source and the unknown activity. Complications arise when using the Ge(Li) spectrometer at a particle beam, especially a high-energy muon or pion beam contaminated with electrons, the optics of which are destroyed by the slowing-down and stopping processes. Then the detector pulses are no longer exclusively due to the activities to be measured, but also neutrons and charged particles hit the detector leading to very high pulses. The limited duty cycle of the accelerator worsens the situation, and finally the cabling is much more complicated than in the laboratory, causing cross-talk, earth loops, etc. Thus the conditions for in-beam measurements are very impure, and care has to be taken that they should at least be made as identical as possible for the X-rays of interest and for the radiation used for calibration. Otherwise systematic errors of unknown origin may be introduced, whose magnitude may change from one experiment to the other.

We tried in this experiment to achieve conditions for the calibration radiation as similar as possible to those of the X-rays to be measured. This was done in several ways:

- a) Identical conditions are experienced by the X-rays of interest and by "known" X-rays. We therefore used various muonic X-rays present in the spectrum as calibration standards. They originated in the target itself and in the plastic counter 5. Well-known lines of this kind were only available below 240 keV, however (see discussion below).
- b) Almost identical conditions are met for nuclear γ -rays originating from μ -capture in the target, provided they are coming from short-lived nuclear states. Such a γ -ray was observed with high yield at 455 keV.
- c) The e^+e^- annihilation line is essentially prompt but might have another source geometry than the X-rays.

- d) The γ -rays from radioactive sources were recorded in a prompt time window triggered with stopped muons. Pre-prompt events were avoided.

Using these methods a very reliable absolute energy calibration was ascertained. Additional information was obtained by measuring Pb and Ba simultaneously, the energies of the 5-4 transitions in Pb and of the 4-3 transitions in Ba interleaving each other with energy spacings of about 5 keV. The relative positions of the two transitions could thus be obtained virtually without calibration errors.

3. THE CALIBRATION STANDARDS

3.1 The muonic lines

The muonic transitions of energies below 400 keV may be calculated with very high accuracy. No deviation of such transition energies from calculation has ever been measured. On the contrary, Vuilleumier et al.¹³⁾ have demonstrated that the calculations coincide with experiment for all transitions between 100 keV and 400 keV in the case of Rh, Hg, and Tl. The muonic X-rays used as calibration standards are listed in Table 2.

The 4p-1s transitions in C and O, originating from the plastic scintillation counter 5 and the plexiglas light-guide connected to it (see Fig. 2) were used as calibration lines, their energies having been measured earlier with good accuracy by Dubler et al.¹⁵⁾. The 6-5 and 7-5 transitions in Pb and the 5-4, 6-5, and 6-4 transitions in Ba were also used for calibration. Their energies were calculated as described in Section 6.

3.2 The nuclear γ -lines

Muon capture in Ba often leads to excited states in ^{137}Cs which decay via the first excited $5/2^+$ state at 455 keV to the $7/2^+$ ground state. The yield of this line is about 18% per captured muon in Ba¹⁶⁾. The energy of the line has been determined separately to high accuracy¹⁷⁾ and is listed in Table 2. The lifetime of the muon in the 1s state has been determined to be about 95 nsec¹⁶⁾, and the lifetime of the $5/2^+$ state of ^{137}Cs is negligible in comparison. This follows from the systematics of ℓ -forbidden transitions between essentially $\pi d_{5/2}$ and $\pi g_{7/2}$ levels in this nuclide region, which predicts a lifetime of about 0.1 nsec. Furthermore, the feeding of the $5/2^+$ level does not involve any strong and slow γ -ray transitions (the only observed feeding is a 394 keV γ -ray from a $3/2^+$ level at 848.5 keV, having an intensity nearly 100 times lower than that of the 455 keV line). Thus about 40% of the γ -rays are emitted within 50 nsec after a muon was stopped. The high yield and the fairly prompt character of this γ -ray makes it very suitable for calibration in the region around 450 keV, in which we are interested.

3.3 The e^+e^- annihilation line

The e^+e^- annihilation radiation is in principle a very good absolute energy standard; its origin in muonic X-ray spectra, however, is not known in detail. Presumably a large fraction is coming from high-energy X-rays and nuclear γ -rays giving pair production. This is consistent with the fact that at least 95% of the annihilation line showed up in the prompt spectra. Furthermore, the 511 keV photons were initiated as often by particles stopping in the upstream part as by those stopping in the downstream part of the target, as defined by the telescope signal patterns. The intrinsic FWHM of the total 511 keV line was 2.25 ± 0.02 keV (the instrumental width having been subtracted quadratically), supporting our conjecture that the 511 keV photons mostly emanate from Pb and Ba [the intrinsic FWHMs are 2.29 and 2.08 keV, respectively¹⁸⁾]. Only a small part could possibly come from the Cu shielding [intrinsic FWHM = 2.70 keV¹⁸⁾].

Recently a thorough investigation of the problems connected with using the annihilation line as a calibration standard was performed¹⁸⁾. In accordance with that work, the mean energy which we use for annihilation quanta from our target is 510.998 ± 0.005 keV, this value being lower than m_0c^2 owing to the electron and positron binding energies in Pb and Ba.

3.4 The radioactive sources

Two radioactive sources were used for energy calibration, ^{75}Se and ^{198}Au .

Until 1975 the ^{198}Au standard energy 411.794 ± 0.007 keV¹⁹⁾ was generally accepted. Recently the energy was remeasured, however, by Deslattes et al.²⁰⁾, who obtained 411.8062 ± 0.0014 keV. We have adopted this latter value in our evaluation.

The best energies of the ^{75}Se γ -rays published so far [Table 4 of Helmer et al.²¹⁾] are mostly based on the ^{198}Au standard. We made the appropriate corrections when entering them into our calibration (see Table 2).

4. THE EVALUATION OF SPECTRA

The spectrum as obtained with Det. I is shown in Fig. 3.

The spectra were evaluated by means of the least-squares fitting program EVAL, which fits pure Gaussians and Lorentzians folded with Gaussians. Slight asymmetries in the line shapes were taken into account by an exponential tail. The line shape parameters, i.e. the FWHM of the Gaussian and the asymmetry parameter, were obtained from clean radioactive source lines and clean muonic X-ray lines. Figure 4 shows the FWHM as a function of energy for the two detectors. The value of the asymmetry parameter was found to be such that the asymmetry area was 2.7% and 10% of the total line intensity at 412 keV for Det. I and Det. II, respectively.

A crucial point in evaluating the spectra was the assessment of interfering background lines. To this end some clean $\Delta n = -2$ and $\Delta n = -3$ muonic X-rays in Ba and Pb (Table 3) were used to obtain a value for the cascade parameter α (see Section 6.4) which would later allow us to calculate the intensities of weak unresolved X-rays. Corrections for detector efficiency, including its angular dependence, and for self-absorption in the target were applied. The self-absorption was calculated by the computer code TARGET, using as input the measured beam profile and μ -stop distribution, and the photon cross-sections given by Storm and Israel²²⁾. For all reasonable α -values there was a small systematic discrepancy ($\leq 5\%$) between theoretical and experimental intensities of the strong $|\Delta n| = 1$ transitions. This is tentatively ascribed to the neglect of Rayleigh scattering in the TARGET code. We thus used this information to correct the target absorption curve before extracting the $|\Delta n| \geq 2$ X-ray intensities. As a weighted mean we obtained $\alpha = +0.043 \pm 0.015$. Table 3 shows the result of this fit. This value is in remarkable agreement with that which can be extracted from the data of Backe et al.²³⁾ on μ -Bi X-rays by the same method, namely $\alpha = +0.045 \pm 0.009$. A further check, independent of target absorption, is obtained by the ratio between the well-separated $5f_{5/2}-4d_{3/2}$ peak and the other 5-4 transitions in μ -Bi. The results of Ref. 23 here lead to $\alpha = +0.046 \pm 0.009$.

Since we did not observe any difference between the α -values in Ba and Pb we assumed the value for Ba to be equal to that for Pb. We must, however, because of poorer data, state an uncertainty in α_{Ba} of ± 0.025 , while for Pb the good agreement discussed above makes an uncertainty in α_{Pb} of ± 0.01 appear reasonable.

The relative intensities of muonic X-rays from other elements than Ba and Pb, which interfere with lines of interest, could usually be found in the literature (this was the case for C, O, and Al). They were introduced into the fit by relating their intensity to that of clean strong lines.

As regards nuclear γ -rays, their identification was much facilitated by a study of the routed spectra. Earlier investigations on muon capture in ^{138}Ba (Ref. 16) and ^{206}Pb (Ref. 24) were also of great help, but since our targets had a natural isotopic composition, their results had to be used with some care. For nuclear γ -lines not reported in muonic capture works, we sometimes could assess upper limits based on evidence from nuclear spectroscopy. A detailed discussion of all nuclear γ -lines found in the spectra will be given in Section 5.2.

4.1 The angular dependence of the detector response

The pulse height from a Ge(Li) detector depends on the angle θ between the axis of the detector (assumed planar) and the direction of the incoming photon, since the photoelectron partly "inherits" the photon direction²⁵⁾. The photoelectron will be accelerated in the electric field in the detector, and thus the

available electron energy (apparent photon energy) will be changed with respect to the true photon energy by a small amount $\varepsilon \sim \cos \theta R(E)$, where $R(E)$ is the effective range of the electron.

The large angular extension of the muon target as seen from the detector makes this a non-negligible correction in our case, since the radioactive calibration sources were small. In order to assess the magnitude of the effect in the 400 keV region, we performed an off-beam study of the position of the 393 keV line from a small ^{113}Sn source, placed at different angles but at a constant distance from detector I. The gain and bias of the system was monitored by γ -lines from a fixed ^{133}Ba source. A straight line was fitted to the shifts as a function of $\cos \theta$. The apparent energy shift of a photon incident at $\theta = 90^\circ$ with respect to one incident at $\theta = 0^\circ$ was extrapolated to be 40 ± 15 eV for detector I.

By a slight modification of the TARGET program we calculated the mean value of $\cos \theta$ for our target configuration, weighted with the spatial μ -stop distribution and with the angle- and energy-dependent efficiency of the detector. Also the attenuation of the photons in the target material was taken into account by this procedure. The resulting $\overline{\cos \theta}$, however, turned out to be rather insensitive to the photon energy, and was nearly the same for Ba as for Pb. Thus, $(1 - \overline{\cos \theta})$ decreased from 0.115 at 200 keV to 0.095 at 500 keV for both elements, while at 100 keV it was 0.113 for Pb and 0.139 for Ba.

The energy-dependence of the correction thus enters mainly through the range of the photoelectrons in Ge. Since we measured the effect only at 393 keV, we estimated it at lower energies from Fig. 3 of Ref. 25, where we find $\delta E \propto E^{1.6}$. This is a somewhat steeper relation than would be expected from the electron range alone, which goes as $E^{1.41 \pm 0.07}$ in the region of interest here²⁶⁾. The difference is probably due to the fact that the forward-peaking of the photoelectrons is less pronounced at lower photon energies.

For detector I the target solid angle correction was found to be -5 ± 2 eV at 435 keV and -1.4 ± 0.6 eV at 200 keV.

For the small radioactive sources it is the angular size of the detector that matters; for detector I we find $\overline{(\cos \theta)}_{\text{detector}} = 0.981$. The corresponding energy shift is ≤ 1.0 eV.

In this connection special attention must be paid to the 511 keV line, since part of it may originate (see Section 2.1) from annihilation in the Cu shielding of such positrons as were created there by high-energy X-rays and γ -rays emitted from the target (the Al support of the target is of no concern since it does not extend to larger angles than the target itself). However, the intrinsic half-width observed for the 511 keV line (see Section 3.3) indicates that the Cu part must be fairly small, a reasonable estimate being $15 \pm 10\%$ of the 511 keV line.

This would increase the apparent energy of the peak by $(0.15 \pm 0.1) \times 40 = 6 \pm 4$ eV, assuming the copper to be in the mean at 90° from the detector axis. Our neglect of correcting for this effect would lead to energies that were only 0.6 ± 0.4 eV too low, however, in the 430 keV region.

4.2 The lines used for energy calibration

A detailed list of these lines is given in Table 4, where also the various impurity lines which interfere can be found.

4.2.1 Muonic lines

The lowest energy calibration line used in this investigation, μ -C 4-1, appears quite clean. In order to obtain the precise energy of the Ba 6h-5g transition, the three fine-structure components with well-known statistical relative intensities were included, as well as the parallel Ba 6g-5f transition. In addition, the 12-9 and 10-8 transitions in Pb interfered, as well as the Ba 8-6 X-ray and a nuclear γ -line from $^{19}\text{F}(n,n')$, coming probably from teflon supports inside the detector housing. Table 4 gives the total of the pattern fitted in the 100 keV region. All energy spacings were kept fixed, as well as the relative intensities of the Ba X-ray components. The intensities of the Pb 10-8 and 12-9 peaks were fixed absolutely. The μ -O 3-1 and 4-1 transitions were both disturbed by unknown impurity lines, which were only partly resolved. Accordingly they were fitted with their channel number difference fixed from a preliminary calibration, and only the 4-1 transition was used for calibration. The Ba 5g-4f transition again was fitted including one parallel transition and the Pb 10-7 transition. Nuclear γ -lines from $^{19}\text{F}(n,n')$ and ^{205}Tl , as well as a source line from ^{75}Se and an unidentified line (probably Cs γ -ray) were included in the fit. The Se intensity was fixed, as well as the Pb 10-7 intensity; the others were fitted in intensity, the unidentified γ -line also in energy (see Table 4). The Pb 6h-5g transition was fitted including one parallel transition. Furthermore, the 11-7 and 8-6 transitions in Pb were accounted for. One nuclear γ -line from Cs and two unidentified γ -lines were included too. The Ba 6g-4f transition was fitted including one parallel transition. A ^{75}Se line and the π -Pb 6-5 line were introduced with fixed positions and intensities, whereas one nuclear γ -line was fitted. The Pb 7h-5g transition also was fitted including one parallel transition. It is intermixed with the Ba 7g-4f transition, which was fitted in the same manner independently of the Pb group. Pb 11-6 was introduced with fixed energy and intensity.

4.2.2 Source lines

The source lines were partly contaminated by nuclear γ -lines and X-ray lines.

The 121 keV ^{75}Se line was contaminated by the pionic Pb 8-7 transition, whose intensity was calculated from the CASCADE program based on the clean π -Pb 7-6 X-ray.

The $136\text{ keV }^{75}\text{Se}$ line was clean. The $265\text{ keV }^{75}\text{Se}$ line had two contaminants, a pionic 5-4 transition in Ba, which was related by a cascade calculation to the $\pi\text{-Ba } 6\text{-}5$ line, and a γ -line from ^{206}Tl . The intensity of the Se line was fixed from its known ratio to the 136 keV Se line.

The ^{198}Au line at 412 keV was contaminated by a known ^{75}Se line, a ^{203}Tl line, a pionic 6-4 Ba line, two weak unidentified nuclear γ -lines, a ^{135}Cs line, the muonic 3-1 transition in Al, and a ^{205}Tl line. The energies of these contaminants were known, except for the two unidentified lines. The intensities were fitted, except for the $\mu\text{-Al}$ line, the intensity of which was obtained from that of the $\mu\text{-Al } 2\text{-}1$ line at 347 keV and the relative intensities given in Ref. 23, and except for the Se and $\pi\text{-Ba}$ lines, which were fixed as described above.

4.2.3 Nuclear γ -line

The ^{137}Cs line at 455 keV also was slightly contaminated with $\mu\text{-Al}$ lines, namely the 6-1 and 7-1 transitions. For the 6-1 line we used the experimental intensity given in Ref. 23, and for the 7-1 line an intensity that was calculated from the optimal cascade parameters given in that work. The energies were calculated. An almost resolved line at 453 keV was ascribed to γ -ray transitions in ^{206}Tl and/or ^{208}Tl .

4.2.4 The 511 keV annihilation line

In principle this line should be the most reliable of all our calibration lines, the electron rest mass energy being known to an absolute accuracy of 2.8 ppm. There are, however, two circumstances which make it hard to fully exploit this high accuracy. One is the fact that electrons and positrons are more or less bound at the moment of annihilation, the electrons usually coming both from the conduction band (in the case of a metal) and from the uppermost atomic level. Whereas the corresponding electron binding energies are well known, this is not always true for the intensity ratio of these two contributions. As regards the positron binding energy, this is only known from theory. Fortunately the binding energies of interest are usually only of the order of 5 eV or less, so the absolute errors should not be very large. The other circumstance is related to the well-known Doppler-broadening of the annihilation peak (almost entirely due to the electron motion), which makes it essential to treat the instrumental peak skewness in a way that has some reasonable theoretical justification.

Both these aspects have been discussed in a recent paper¹⁸⁾, where it was established experimentally that the mean energy of annihilation photons from Pb and Ba is lower than the electron rest mass energy by 5 eV, with an uncertainty of about $\pm 5\text{ eV}$. As a check of our standard method of accounting for peak skewness, we applied the procedure proposed in that paper to our 511 keV peak and to the

412 keV Au peak recorded in the beam-off calibration runs. The positions of both these lines differed by the same amount when treated by the two methods, and thus we conclude that we can use our annihilation peak position with confidence.

4.3 The 430 keV region

The energy region contains the Pb 5-4 and Ba 4-3 transitions to be measured. Figure 5 shows the spectrum and the lines introduced into the fit. In particular, it was assumed again that the energy spacings between parallel transitions could be calculated accurately and thus be plugged into the fit as known parameters. The $5g_{7/2}-4f_{7/2}$ and $5g_{7/2}-4f_{5/2}$ transitions in Pb and the $4f_{7/2}-3d_{5/2}$ and $4f_{5/2}-3d_{3/2}$ transitions in Ba were varied freely as regards energy, but the ratio of the intensities within each doublet was kept fixed to their statistical values. Their Lorentz widths were kept fixed at the values (115 eV and 118 eV, respectively) calculated theoretically. (Leaving also the Lorentz widths free -- but common for all four lines -- they came out as 136 ± 16 eV.) The Gaussian widths were kept fixed at values extrapolated from the curve shown in Fig. 4.

The corresponding spin-flip transitions $5g_{7/2}-4f_{7/2}$ in Pb and $4f_{5/2}-3d_{5/2}$ in Ba were introduced with fixed energy spacings to the lower-energy line of the corresponding $\Delta j = -1$ transition, their branching ratios being fixed from statistics. The parallel 5f-4d transitions in Pb were introduced as well, with fixed energy spacings to the $5g_{7/2}-4f_{5/2}$ transition, and with their intensity ratios to that transition fixed from the cascade calculation. Their Lorentz widths were calculated and kept fixed. The parallel 4d-3p transitions in Ba were treated equivalently.

In addition, various nuclear γ -lines had to be considered, whose energies are usually not known to better than 0.2 keV. These were the ^{208}Tl line at 433.5 keV, the ^{138}Cs line at 434.5 keV, a possible ^{204}Hg line at 436.7 keV, a ^{204}Tl line at 437.7 keV, and a ^{202}Hg line at 439.5 keV (see Section 5.2).

We did not include the $\mu\text{-Pb}$ 14-6 line group, since it would anyhow have shifted the $\mu\text{-Pb}$ $5g_{7/2}-4f_{7/2}$ peak by less than 1 eV.

The $\mu\text{-Al}$ 4-1 transition was introduced with an intensity obtained in the same way as discussed for $\mu\text{-Al}$ 3-1 in Section 4.1.2. The energy of both these lines was derived theoretically, using our experimental $\mu\text{-Al}$ 2-1 energy (centroid of doublet: 346.84 keV) for obtaining the nuclear finite size correction of the 1s level.

5. EXPERIMENTAL RESULTS

5.1 The calibration polynomials

The non-linearity of a solid-state detector system has a shape which is in general not well suited for a polynomial approximation, since the strongest non-linearities occur in the first 10-20% of the energy range. In the present case,

however, our interest is limited to the region 100-500 keV, and thus a polynomial is expected to be applicable. The positions of the calibration lines are listed in Table 2, col. 4. The errors are statistical errors plus (when necessary) a systematic error due to the intensity uncertainty of the satellites. This systematic error was in no case larger than 0.02 channels. Figure 6 shows the position of the calibration lines from Det. I with respect to a linear calibration function. The integral linearity in the upper 80% of the converter range is clearly better than $\pm 0.2\%$. As is evident from the data points in Fig. 6, the degree of the polynomial must be at least three. It turns out that $\chi^2/(N-P)$, where N is the number of points and P the number of free parameters, is slightly ($\sim 20\%$) better when a fourth-order polynomial is used, whereas it is again worse (by $\sim 15\%$) for a fifth-order polynomial. Both the third- and the fourth-order polynomials are drawn in Fig. 6; it is seen that they differ very little in the region of interest, 430-440 keV.

A very important feature of the calibration points is that there is no significant systematic difference between muonic X-rays and source γ -lines at energies below 300 keV, a fact that adds greatly to our confidence in the ^{198}Au calibration line. We therefore used all the calibration points of Table 2 for fitting the polynomial.

5.2 The nuclear γ -lines

Whereas the intensities of the muonic and pionic X-rays from the target or from impurities in or just outside the target are reasonably well calculable, as soon as one strong line from them has been identified, this is not the case for the nuclear γ -rays, which are emitted after the muon has been captured by a nucleus, or even during the latest steps of the muonic cascade.

For the nuclei involved in the present work, resonance excitation is not expected to play a role below 500 keV. At first sight one could expect that the $1/2^+$ level in ^{135}Ba at 218 keV might be excited by the μ -Ba 5-4 X-ray at 199 keV. However, the $3/2^+$ ground level is essentially a $2d_{3/2}$ shell-model state, while the $1/2^+$ level is mostly $3s_{1/2}$, so the M1 transition is ℓ -forbidden and proceeds only via level impurities. Furthermore, the natural abundance of ^{135}Ba is only 6.6%. No indication of a 218 keV γ -ray is seen in our spectra, and we conclude that the Ba 5-4 transitions will not be affected by nuclear resonance.

In Table 5^{8,16,17,24,27-40)} we list energies and yields of those nuclear γ -ray transitions which were observed in the present work. In cases of nuclear γ -rays known to occur in the nuclei produced after muon capture in Ba and Pb, but too weak or masked by stronger lines in our spectra, we list energies and yields from previous studies of muon capture γ -rays, combined with known nuclear level schemes.

The assignments of the initial and final levels are given whenever known. When two references are cited for the γ -ray energy, the second is usually a muon capture experiment. The agreement between our energy values and those given in the literature is good and supports the identifications made. The yields were normalized to the theoretical muonic X-ray yields obtained from the CASCADE program (71.7 per 100 μ -stops for Pb 5-4, and 71.6 per 100 μ -stops for Ba 4-3). The time window for Detector I being about 105 nsec, about 33% of the Cs-lines ($\tau = 95$ nsec, see Ref. 17) and 24% of the Tl- (and Hg-) lines ($\tau \approx 72$ nsec, see Ref. 24) were missing in our spectra. The yields given in Table 5 are corrected for these losses.

In the following we discuss the implications of γ -rays on our energy calibration and background subtraction.

The lack of knowledge concerning the energy and intensity of the ^{204}Tl γ -ray at 140 keV made it impossible for us to use the μ -Pb 7-6 lines (centroid position 140.4 keV) for energy calibration. Our yield estimate is based on the excess strength of the line group compared to that calculated from the other muonic Pb lines by means of the TARGET and CASCADE programs. A possible ^{203}Tl line at 158.5 keV was masked in our spectra by the μ -O 3-1 line at 158.4 keV. There is also a ^{135}Cs line at 158.2 keV in this region. The peak we observe at 203.7 keV is a close doublet; we estimated the Cs part of this doublet from our yield of the 240 keV Cs γ -ray and the intensity ratio given by Lucas et al.¹⁶⁾, assuming it to be produced chiefly by the $^{138}\text{Ba}(\mu, \nu 3n)$ reaction. We could derive reasonably good values for the intensity of the 265.8 keV transition in ^{206}Tl (mixed up with the 264.7 keV Se calibration line), because we knew the intensity ratio between the 264.7 keV and 136.0 keV Se lines precisely from a calibration run without beam. Two possible Tl γ -rays were intermixed with the 279.5 keV Se-line, which could thus not be used for calibration. The 373.1 keV $1/2^+ \rightarrow 5/2_2^+$ transition in ^{135}Cs could, in principle, lower the position of the intermixed muonic lines Pb 7-5 and Ba 7-4. Since, however, Lucas et al.¹⁶⁾ do not see either of the two γ -rays de-exciting the analogous $1/2^+$ level at 1490 keV in ^{137}Cs , we did not include it in our fit of these muonic lines. Both these two and the $1/2^+ \rightarrow 5/2_2^+$ γ -ray in ^{135}Cs are outside the energy window of our experiment.

There are a number of γ -rays known in the region 430-440 keV which could affect the muonic X-rays of primary interest here. Of these the γ -rays from ^{138}Cs and ^{208}Tl should be very weak [no neutron emitted, see Backenstoss et al.⁴¹⁾ and Evans⁴²⁾]. For the listed 433 keV line in ^{208}Tl we could put a limit based on our non-observation of a 473 keV line and the known branching ratio between these transitions. For the 434 keV line in ^{138}Cs we list the detectability limit claimed by Lucas et al.¹⁶⁾. The ^{204}Tl line at 437 keV is also excited by neutron capture on ^{203}Tl , where it is found that it is the only one feeding a level that

in turn is only de-excited by the 158.5 keV γ -ray discussed above. During the long (two months) running time quoted in Ref. 24, an equilibrium amount of 12.2 d ^{202}Tl , which primarily (94%) decays to the 439 keV level of ^{202}Hg , should be formed. The detection of this activity only in accidental coincidence would, however, have resulted in a reduction of the intensity by a factor of 10^3 . Thus we conclude that ^{202}Hg is formed directly by the $^{206}\text{Pb}(\mu, \nu p 3n)$ reaction. We have introduced the presumed ^{202}Hg line in our fit with an intensity which is derived from Ref. 24 by taking the different isotope ratios into account, as well as a factor of 1.5 emanating from our different yield normalizations.

A priori one could now expect the reaction $^{208}\text{Pb}(\mu, \nu p 3n)^{204}\text{Hg}$ to take place with a comparable yield. However, the bump which is clearly seen in our spectra at about the energy of the ^{204}Hg 2^+-0^+ transition can be fully accounted for by the $\mu\text{-Al}$ 4-1 X-ray, and it was in fact impossible to force a ^{204}Hg peak into the fit with an intensity twice as large as that of the ^{202}Hg peak, as would be demanded by the isotope ratios if the cross-sections were the same. A reasonable explanation for the different behaviour of the two Pb isotopes could, of course, be connected with the double-magic character of ^{208}Pb .

For the Cs line at 449.4 keV observed in Ref. 16, we scaled the yield in proportion to the abundance (72%) of ^{138}Ba in our target.

5.3 The $\mu\text{-Ba}$ 4-3 and $\mu\text{-Pb}$ 5-4 energies; error discussion

The results of our measurements are presented in Table 6. Here are included also the data from the smaller Detector II. Their statistical accuracy is half as good as for Detector I, and they thus get a weight factor of only $1/4$ when computing the mean. It is, however, gratifying to note that the results from the two detectors differ by even less than the standard deviation for each peak.

In Table 7 we have collected estimates of all the additional errors which may influence the positions of the four main peaks and their centroid. Definitions of the various items are given in the comments to the table. A more comprehensive discussion of some of them will be given below.

The calibration polynomial is adjusted such as to fit the statistically dominating Au peak at 412 keV. The calibration error is thus mainly due to the 4 eV statistical uncertainty of that peak (see Table 2).

There is of course no *a priori* reason why the non-linearity curve should fit neither a third-order nor a fourth-order polynomial, only that it should be a smooth, single-valued function of energy. However, we judge the values given in line 3 of Table 7 to give a reasonable estimate of the uncertainty originating in the unknown functional form of the integral non-linearity (see Fig. 6).

The differential non-linearity claimed by the manufacturer ($< 0.75\%$) will give errors < 1 eV for a single peak at the dispersion used.

Our estimated uncertainty in the peak asymmetry parameter would give a shift of the position obtained for the Au peak corresponding to ± 5.6 eV. Since, however, the asymmetry parameter varies very slowly with energy, an isolated peak at, for example, 435 keV would shift by almost the same amount, and thus the relative positions usually change very little, as can be seen from line 4a of Table 7.

Because of the overlap between the Ba 4-3 and Pb 5-4 line systems, all energies are affected by a change of the satellite intensity of one of the elements. These intensities depend only on the cascade parameter α -- provided that the initial population can be adequately described by n_{init} and α . The errors given in lines 5 and 6 of Table 7 are thus probable errors.

It is reasonable to assume that the three fine-structure (fs) components of each transition have the intensity ratios given by statistical rules. The fairly small position changes obtained when relaxing this restriction for the two main fs components of both elements thus rather gives a check on the consistency of the fit.

The uncertainty stated in Ref. 24 for the yield of the 439.5 keV line refers to the absolute yield. The relative yield error is certainly considerably smaller, so the deviations in line 9 of Table 7 are conservative upper limits.

Lines 10-15 in the table concern γ -rays which to our knowledge have not yet been observed after μ -capture in Ba or Pb. The deviations given in the table refer to the detection limits of Refs. 16 and 24 except for the ^{208}Tl and ^{204}Hg lines, for which upper limits were obtained in the present work. For ^{204}Tl we have assumed^{41,42)} that its yield from ^{208}Pb and ^{207}Pb is considerably smaller than from ^{206}Pb . It is evident from the table that the energy uncertainty of both the ^{204}Tl and the ^{208}Tl line is quite important if the lines have their maximum possible strength.

The influence of the μ -Al 3-1 X-ray on the fitted position of the Au 412 keV line has not been included in the calibration error, since it can in principle be corrected for when improved μ -Al data become available.

Whereas many of the entries of Table 7 have a considerable effect on the position of one or two peaks in the quartet, the centroid remains fixed within ± 2 eV, except for errors due to statistics and calibration. Such systematic errors will thus have but a small effect on the validity of the conclusions drawn from this experiment.

6. THE CALCULATIONS

The calculation of energy levels in muonic atoms was recently described in detail by Watson and Sundaresan⁷⁾. We describe our calculations at those points where they differ from the ones mentioned in Ref. 7, or where more recent methods were developed. A summary of the calculations is given in Table 8.

6.1 The Dirac equation

The Dirac equation is supposed to describe the motion of a muon and a nucleus of charge Z , forming in a good approximation a hydrogen-like system. The solutions, energy eigenvalues, and corresponding wave functions, are well known for point-like field sources⁴³⁾. In muonic atoms the finite extension of the nuclear charge leads to a modification of the $1/r$ -potential⁴⁴⁾. Analytic solutions of the resulting equation of motion are no longer possible and numerical integration has to be performed to obtain the energy eigenvalues and the wave functions.

We applied two different integration procedures.

- i) Numerical integration of the coupled first-order equations is done from $r = 0$ up to $r = r_m$ according to Hamming [Ralston and Wilf⁴⁵⁾]. The radius r_m is chosen such that the $1/r$ -potential and the modified, "true" Coulomb potential are "identical" at this point. From $r = r_m$ on, Whittaker-functions are used. The method of energy variation and matching is described by Acker et al.⁴⁴⁾.
- ii) The Dirac equations are transformed into a second-order equation⁴⁶⁾. The numerical integration is performed from $r = 0$ to $r = r_m$ and from $r \approx \infty$ to $r = r_m$, i.e. the total wave function is obtained numerically. The Numerov⁴⁷⁾ integration method was used. The energy variation was done according to Blatt⁴⁸⁾. The advantage of the second method consists in the fact that no use is made of a $1/r$ -behaviour of the potential, and any potential form may be used.

The two methods were checked and found to yield the same results within 10^{-9} . Thus the main uncertainties arise from the natural constants used:

$$\alpha^{-1} = 137.035982$$

$$m_\mu = 105.65948 \text{ MeV}$$

and from the uncertainty in the charge distributions. We used a two-parameter Fermi distribution for Ba and Pb putting for Ba $c = 5.7414 \text{ fm}$ and $t = 2.3 \text{ fm}$ ⁴⁹⁾, and for Pb $c = 6.650 \text{ fm}$ and $t = 2.311 \text{ fm}$ ⁵⁰⁾.

6.2 Corrections to the Dirac equation

6.2.1 "Recoil correction"

The Dirac equation when separated for centre-of-mass motion contains the reduced mass, but relativistic effects are not included. Such effects are arising

from retardation, induced by the motion of nucleus and muon (Breit-interaction) and from a relativistic mass correction. The effect was calculated by various authors^{43, 51-54)}. We adopted the procedure of Barrett⁵³⁾ for our calculations. The contributions are small and the uncertainties are negligible.

6.2.2 The anomalous magnetic moment of the muon

The lowest-order vertex correction leading essentially to the anomalous magnetic moment was calculated in two ways:

- i) The Dirac equation was extended by a term representing the interaction of the anomalous magnetic moment of the muon with the electromagnetic field in the atom, and then integrated numerically.
- ii) The anomalous magnetic moment contribution was calculated in first-order perturbation⁵⁵⁾ from the induced potential:

$$V(r) = -\frac{\alpha}{2\pi} \lambda_{\mu} \frac{dV_{\text{Coulomb}}}{dr},$$

λ_{μ} being the muon Compton wavelength.

Both methods yield the same result for the levels in question. The error in calculating this contribution is negligible [see also Borie⁵⁶⁾].

6.3 The nuclear polarization

For atomic states, where the orbiting particle has little or no overlap with the nucleus, the nuclear E1 polarization was estimated by Ericson and Hufner⁵⁷⁾. The nuclear polarizability α_N was related by these authors to the total photonuclear cross-section σ_Y :

$$\alpha_N = \frac{1}{2\pi^2} \sigma_Y.$$

We used the σ -values from Berman⁵⁸⁾. In addition we applied the relativistic correction to α_N as given by Ref. 57. The correction was calculated in first-order perturbation from the resulting potential

$$V_{NP}(r) = -\frac{1}{2} e^2 \alpha_N / r^4.$$

Inside the nucleus this potential was replaced by a finite (at the origin) one, with an r -dependence of the form $(a - br^2)$ ⁵⁹⁾. Matching to the $1/r^4$ potential was done at the nuclear surface.

The accuracy is given by the uncertainties of α_N and those of the underlying static model. An error estimate is 10%.

6.4 The electron screening

The electron screening correction is due to the potential created by the electron cloud of the muonic atom. Self-consistent calculations for the screening potential⁶⁰⁻⁶²⁾ agree within a few percent with our calculations, which use unperturbed relativistic wave functions for the K- and L-electrons. The screening of the electrons by the orbiting muon was taken into account by an effective nuclear charge $Z-1$ for the K-electrons. The influence of the K-electrons on the L-electrons was considered by reducing the effective nuclear charge by the number of K-electrons present. Table 9 shows the comparison of the results of this simple method with the results of Refs. 60, 61, and 62. The agreement with Vogel's values⁶²⁾ and with the values of Fricke et al.⁶¹⁾ is astonishing. The accuracy is probably of the order of 5%. The dynamic polarization of the electron cloud by the orbiting muon was calculated by Ericson and Hufner⁵⁷⁾ to weaken the screening shift by $\sim 2\%$. This effect is included in the final screening values.

However, owing to the Auger effect during the cascade of the muon, the electronic shells are partly emptied. Cascade calculations, which take into account the Auger effect and the refilling of the electronic K-, L-, and M-shells during the cascade, were performed to obtain the mean populations of these shells when the transition in question occurs. Vogel's cascade calculations⁶²⁾ are in agreement with these calculations as far as the K-shell population is concerned. The main uncertainty of the electron screening seems not to be introduced by the electron density distribution, but rather by the unknown starting conditions of the muon cascade and by the refilling of emptied electronic states.

In our calculations the starting conditions were chosen such as to reproduce the measured relative intensities of the X-ray spectrum. The initial population of the muonic ℓ -substates is assumed to be

$$P(\ell)_{n=17} = (2\ell + 1) e^{\alpha\ell}.$$

The parameter α was adjusted as described in Section 4. The refilling was calculated using tabulated fluorescent yields⁶³⁾. In order to estimate the uncertainty in the number of K- and L-electrons, we also calculated this number for the case where all electrons were present for the transition preceding the one in question.

6.5 The QED corrections

6.5.1 e^+e^- vacuum polarization of order $\alpha(\alpha Z)$

The vacuum polarization of order $\alpha(\alpha Z)$ results in the potentials⁶⁴⁾

$$V = \frac{2\alpha}{3\pi} \frac{\alpha Z}{r} Z_0(r) \quad \text{for large } r,$$

and

$$V = \frac{4\alpha}{3} \frac{\lambda_e}{2} \int_0^\infty \rho(r') \frac{r'}{r} dr' \{Z_1(|r - r'|) - Z_1(r + r')\} \quad \text{for small } r.$$

The functions Z_0 and Z_1 are defined⁶⁴⁾ as

$$Z_n(|r|) = \int_1^\infty e^{-2|r/\lambda_e|\xi} \left(1 + \frac{1}{2\xi^2}\right) (\xi^2 - 1)^{\frac{1}{2}} / \xi^{n+2} d\xi,$$

with λ_e being the electron Compton wavelength. Various approximations exist for Z_0 and Z_1 ⁶⁴⁻⁶⁷⁾. In our case they were calculated numerically with an accuracy of better than 10^{-7} . Ladder graphs were included by adding the vacuum polarization potential to the Coulomb potential before solving the equations of motion.

6.5.2 Vacuum polarization of order $\alpha^2(\alpha Z)$

The vacuum polarization of order $\alpha^2(\alpha Z)$ is taken from Ref. 6. The integral f_2 , i.e. in the notation of Ref. 6

$$f_2 = \frac{V_{21}(r)}{\left(\frac{\alpha^2(\alpha Z)}{\pi^2}\right)},$$

was evaluated numerically up to $r = 3\lambda_e$ with an accuracy of better than 10^{-4} . For small r the power-series expansion of Ref. 6 was used, with the modification that, in order to take the finite extension of the nucleus into account, the Coulomb term $\alpha Z/r$, which was extracted from the series, was replaced by $\rho(r')/|\vec{r} - \vec{r}'|$ and integration was performed over d^3r' . The expansion fails from $\sim 0.5\lambda_e$ on.

This contribution was evaluated in perturbation. The uncertainties are supposed to be negligible.

6.5.3 Vacuum polarization of order $\alpha(\alpha Z)^n$

These corrections correspond to n photon lines between an e^+e^- loop and the nuclear charge. For $n = 3$ this term was evaluated by Wichmann and Kroll⁶⁸⁾ for a point-like nucleus. The corresponding potential was constructed by Blomquist⁶⁾. However, the corrections due to the finite size of the nuclear charge are large owing to the strong coupling to the nucleus even in atomic states with small nuclear overlaps. The calculations of the $\alpha(\alpha Z)^n$ terms are thus quite involved, and only recently did the different authors arrive at similar results^{66, 69-72)}. We adopt the values of Rinker and Willets⁶⁶⁾ and assume an uncertainty of 5% corresponding to the scattering of the different calculations.

6.5.4 Vacuum polarization of order $\alpha^2(\alpha Z)^2$

This correction, corresponding to virtual Delbrück scattering, was calculated by several authors⁷³⁻⁷⁵⁾. The calculations are again involved, for reasons that are similar to those for the $\alpha(\alpha Z)^n$ corrections, and because of the intermediate muon excitation. We adopt the results of Borie⁷⁵⁾.

6.5.5 Muon self-energy correction

The "Lamb shift" consists of the anomalous magnetic moment correction, discussed in Section 6.2.2, and the self-energy, expressed in terms of the "Bethe sum". The Bethe sum for large n and ℓ states with point nucleus wave functions was evaluated by Klarsfeld and Maquet⁷⁶⁾.

6.5.6 Small corrections

All corrections involving $\mu^+\mu^-$ or hadron loops are negligible⁶⁴⁾. Also higher-order corrections to the Lamb shift are negligible⁵⁶⁾. The recoil of the nucleus due to the X-ray emission is accounted for.

7. DISCUSSION

The measurement described in this paper has determined energies of muonic X-rays of 430-440 keV with an absolute accuracy of around 10 eV (see Table 8). These errors are essentially statistical errors. Owing to the use of a novel technique it was possible to almost eliminate systematical shifts.

This measurement also solved the problem of the alleged discrepancies between theory and experiment. Our experimental values are in fair agreement with theory. The theory is checked by this experiment to an experimental accuracy of in the average ± 8 eV for Pb and ± 6 eV for Ba, corresponding to a test of QED of 3.8‰ and 2.5‰, respectively, or, in total, of 2.1‰. The uncertainties in the calculations shift these values to 5‰ for Pb and 2.9‰ for Ba, or, in total 2.8‰. Moreover, the higher-order vacuum polarization corrections involving higher powers of (αZ) are established to within 20% for Pb and 30% for Ba from the experiment above. Again the theoretical uncertainties reduce the quality of the test to 26% for Pb and 36% for Ba, or, in total 24%. These contributions cannot be tested in light muonic atoms.

Table 1

Experimental tests of the vacuum polarization

Experiment	Total effect (theory)	Vacuum pol. (theory)	Exp. value	Accuracy of Vac. pol. test a)
g-2/electron	$115,965.25 \times 10^{-8}$	9.4×10^{-8}	$(115,965.67 \pm 0.35) \times 10^{-8}$	3.7%
g-2/muon	$116,592.1 \times 10^{-8}$	585.6×10^{-8}	$(116,592.2 \pm 0.9) \times 10^{-8}$	0.15%
Lamb shift (H) b)	1,057.916 MHz	27.323 MHz	$1,057.893 \pm 0.020$ MHz	0.07%
Lamb shift (μ He) c)	1.5274 eV	1.6774 eV	1.5274 ± 0.0009 eV	0.05%
μ -Pb $5g_{7/2}^{-4f_{5/2}}$ d)	2,172 eV	2,163 eV	$2,053 \pm 14$ eV	0.65%

a) If the errors were due only to experiment.

b) Various calculations are inconsistent.

c) The badly known radius of ${}^4\text{He}$ reduces the value of this experiment somewhat.

d) Only the QED corrections are considered. Various other effects limit the precision.

Table 2

X-rays and γ -rays used for the energy calibration

Line	Energy (keV)		Channel No.
	Calculated or tabulated	Fitted	(Detector I)
C 4-1	94.095 (15)	94.088 (7)	760.231 (50)
Ba 6-5	108.280 (5)	108.281 (8)	871.108 (62)
O 4-1	167.114 (15)	167.111 (17)	1330.917 (133)
Ba 5-4	199.909 (5)	199.913 (8)	1587.459 (60)
Pb 6-5	233.196 (5)	233.194 (11)	1847.842 (93)
Ba 6-4	307.876 (5)	307.925 (31)	2432.733 (241)
Pb 7-5	372.714 (5)	372.686 (39)	2939.625 (303)
^{75}Se	121.115 (3)	121.125 (8)	971.457 (65)
^{75}Se	136.004 (4)	135.999 (5)	1087.699 (36)
^{75}Se	264.659 (6)	264.657 (10)	2094.070 (80)
^{198}Au	411.8062 (14)	411.806 (4)	3245.689 (31)
^{137}Cs	455.503 (3)	455.502 (18)	3587.316 (141)
annih. rad.	510.998 (5)	510.998 (7)	4020.644 (54)

The energies given for the μ -Ba and μ -Pb lines are those of the strongest component of the fine-structure multiplet.

The μ -C 4-1 and μ -O 4-1 energies are taken from Ref. 15.

The γ -ray energies from the ^{75}Se source (Ref. 21) and from the ^{137}Cs nucleus (Ref. 17) are converted to the new ^{198}Au standard (Ref. 20).

Table 3

Muonic X-ray lines used for the CASCADE fit

Line	X-rays per 100 μ -captures	
	Y (calc., $\alpha = +0.043$)	Y (exp)
Ba 8-5	2.31	2.23 ± 0.14
6-5	46.1	44.7 ± 0.8
7-4	3.17	3.40 ± 0.77
6-4	9.47	9.44 ± 0.52
5-4	51.1	52.8 ± 0.5
4-3	71.7	69.3 ± 0.7
Pb 9-7	8.57	8.32 ± 0.66
9-6	2.74	impure
8-6	7.28	impure
7-6	49.0	impure
8-5	2.35	2.59 ± 0.47
7-5	8.86	8.00 ± 1.11
6-5	61.0	63.0 ± 1.1
5-4	71.6	69.3 ± 1.0

Table 4

Line groups used in the fits of the calibration lines

Identification	Energy (keV)	Relative intensity
μ -C 4-1	94.10	100
μ -Ba 6h-5g	108.28	100
μ -Ba 6g-5f	108.69 F	16.4 F
unidentified	102.5	4.5
μ -Pb 12-9	102.45 F	3.8 FA
μ -Pb 10-8	106.23 F	8.3 FA
μ -Ba 8-6	107.23 F	12.3 F
$^{19}\text{F}(n,n')$	109.89 F	7.6
μ -O 4-1	167.11	100
unidentified	168.87	11
μ -Ba 5g-4f	199.91	100
μ -Ba 5f-4d	201.37 F	8.2 F
μ -Pb 10-7	196.53 F	2.2 FA
$^{19}\text{F}(n,n')$	197.15 F	1.6
unidentified	198.11	3.2
^{75}Se	198.60 F	1.4 FA
$^{205}\text{Tl} + \text{Cs}$	203.73	8.8
μ -Pb 6h-5g	233.20	100
μ -Pb 6g-5f	235.02 F	10.7 F
μ -Pb 11-7	229.05 F	3.0 F
unidentified	229.36	3.8
μ -Pb 8-6	230.48 F	14.0 F
unidentified	231.70	3.5
Cs	240.01	12.2
μ -Ba 6g-4f	307.88	100
μ -Ba 6f-4d	309.64 F	17 F
^{75}Se	303.92 F	5.0 FA
^{206}Tl	304.92	26
π -Pb 6-5	309.30 F	24 F
μ -Pb 7h-5g	372.71	100
μ -Pb 7g-5f	374.98 F	22 F
μ -Ba 7g-4f	372.95 F	53
μ -Ba 7f-4d	374.82 F	13 F ¹⁾
μ -Pb 11-6	368.91 F	12 F
^{75}Se	121.12	100
π -Pb 8-7	120.13 F	8.3 FA

1) Fixed relative to the μ -Ba 7g-4f area.

Table 4
(Cont.)

Identification	Energy (keV)	Relative intensity
^{75}Se	136.00	100
^{75}Se $\pi\text{-Ba } 5-4$ ^{206}Tl	264.66 265.02 F 265.83	100 FA 23 FA 29
^{198}Au ^{75}Se ^{203}Tl unidentified " $\pi\text{-Ba } 6-4$ ^{135}Cs $\mu\text{-Al } 3-1$ ^{205}Tl	411.81 400.66 F 401.31 F 403.5 405.5 407.97 F 408.18 412.89 F 415.71	100 3.8 FA 0.9 0.7 1.7 0.9 FA 1.7 1.17 FA 3.4
^{137}Cs $\mu\text{-Al } 6-1$ ^{206}Tl and ^{208}Tl $\mu\text{-Al } 7-1$	455.50 452.45 F 452.99 455.94 F	100 1.8 FA 19.6 0.6 FA
Annih. rad. $^{205}\text{Tl } \frac{9}{2}^+ \rightarrow \frac{7}{2}^+$	511.00 506.0	100 1.3

For each fine-structure triplet of $\Delta n = -1$ type we list only the energy of the $(n, \ell, \ell + \frac{1}{2}) \rightarrow (n-1, \ell-1, \ell - \frac{1}{2})$ transition, whereas the intensity refers to the whole triplet.

Multiplets for which $|\Delta n| \geq 2$ are represented by their strongest component as far as energies are concerned; their intensities include one or two parallel transitions depending on Δn .

An F in the energy column means that the line position was fixed with respect to the main line of the group, using an approximate dispersion factor.

An F in the intensity column indicates the line area was kept fixed with respect to that of the main line during the peak-fitting procedure; FA means that it was fixed absolutely.

Table 5
Nuclear γ -lines

Identification	E_{γ}^* (keV) Literature	Ref.	E_{γ}^* (keV) This work	Yield **)	Note
^{204}Tl $0^- \rightarrow 2^-$ (g.s.)	140.6 (1)	a, a'		~ 10	1
^{135}Cs $3/2^+ \rightarrow 5/2^+$	158.197 (9)	b		2.3 ± 0.3	
^{205}Tl $3/2^+ \rightarrow 1/2^+$ (g.s.)	203.74 (4)	c, d	} 203.7 (1)	12.3 ± 0.8	2
Cs	203.8 (8)	e		1.1 ± 0.3	
$(^{136})\text{Cs}$	240.3 (5)	e		5.3 ± 0.4	
^{135}Cs $5/2^+ \rightarrow 7/2^+$ (g.s.)	249.794 (15)	b, e	249.77 (3)	7.5 ± 0.5	
^{138}Cs $(1,2)^- \rightarrow 3^-$ (g.s.)	258.31 (5)	f, e	258.44 (16)	1.4 ± 0.4	
^{206}Tl $2^- \rightarrow 0^-$ (g.s.)	265.7 (2)	g, d	265.83 (8)	13.7 ± 1.7	
^{206}Tl	278.2	g	} masked by ^{75}Se line		
^{203}Tl $3/2^+ \rightarrow 1/2^+$ (g.s.)	279.1897 (49)	h, d			
^{206}Tl $1^- \rightarrow 0^-$ (g.s.)	304.8 (3)	g	304.92 (8)	4.1 ± 0.5	
^{204}Tl $? \rightarrow 2^-$ (g.s.)	318.4 (1)	a, a'	319.0 (1)	3.1 ± 0.6	
^{207}Tl $3/2^+ \rightarrow 1/2^+$ (g.s.)	351.0 (1)	i	351.03 (3)	12.7 ± 0.6	
^{135}Cs $1/2^+ \rightarrow (5/2^+)_2$	373.13 (10)	b	-	masked	3
^{137}Cs $3/2^+ \rightarrow 5/2^+$	393.35 (5)	j, e	393.47 (16)	1.6 ± 0.4	
^{138}Cs $(0,1)^- \rightarrow 1^-$	396.43 (5)	f	396.18 (26)	1.0 ± 0.3	
^{203}Tl $5/2^+ \rightarrow 3/2^+$	401.31 (6)	k	-	1.5 ± 0.6	
^{135}Cs $3/2^+ \rightarrow 7/2^+$ (g.s.)	407.99 (1)	b	408.18 (12)	2.3 ± 0.3	
^{205}Tl $5/2^+ \rightarrow 3/2^+$	415.71 (9)	c, d	415.69 (7)	6.5 ± 0.6	
^{208}Tl $4_2^+ \rightarrow 4^+$	433.5 (5)	l		< 0.4	4
^{138}Cs $(0,1)^- \rightarrow 1^-$	434.49 (5)	f		< 0.5	5
^{204}Hg $2^+ \rightarrow 0^+$ (g.s.)	436.70 (19)	m		< 1.3	
^{204}Tl	437.7 (2)	a, a'		≤ 0.4	6
^{202}Hg $2^+ \rightarrow 0^+$ (g.s.)	439.544 (15)	n, d	fixed	1.3	7
Cs	449.4 (6)	e	449.3 (3)	1.7	8
^{208}Tl $3^+ \rightarrow 4^+$	452.83 (10)	l	} 452.99 (10)	4.3 ± 0.5	
^{206}Tl $(5^+) \rightarrow 4^-$	453.2 (5)	o			
^{137}Cs $5/2^+ \rightarrow 7/2^+$ (g.s.)	455.490 (3)	p, e		18.1 ± 0.4	
^{208}Tl $4_2^+ \rightarrow 5^+$ (g.s.)	473.5 (2)	l	fixed	< 1.0	
^{205}Tl $9/2^+ \rightarrow 7/2^+$	506.2 (2)	q	506.0 (1)	4.0 ± 0.5	

*) For practical reasons the old ^{198}Au energy standard is used in this table.

**) Corrected for the width of our time window, and based on our theoretical cascade yields (see Table 2).

Notes

- 1) Estimate based on the apparent excess intensity of the Pb 7-6 line group. The line was not reported in Ref. d, but it would not easily have been observed in that work [see Fig. 8 in Ref. d].
- 2) Yield fixed from the intensity ratio given in Ref. e and our yield value of the 240 keV Cs γ -ray.
- 3) No transition from the analogous level in ^{137}Cs was observed in Ref. e; if present it would lower the position of the (Pb 7-5) + (Ba 7-4) multiplet.
- 4) Based on our limit for the 473.5 keV line and the branching ratio in Ref. l).
- 5) Detection limit of Lucas et al. [Ref. e].
- 6) Tentatively estimated from our 319.0 keV intensity and the ratio of (n, γ) intensities given in (a) and (a'); also consistent with the detection limit of Anderson et al. [Ref. d].
- 7) From the yield given in Ref. d, increased by 50% (see Section 5.2), and corrected for the ^{206}Pb content of our natural Pb target. We assume that Hg isotopes are produced only via the (μ, ν $_{\text{p}}$ 3n) reaction.
- 8) From Ref. e, tentatively adjusted to the 72% ^{138}Ba content in our target.

References for nuclear γ -rays (Table 5)

- a) Fubini et al., Nuovo Cimento 18 A, 711 (1973).
- a') Nuclear Data Sheets 5 (1971), p. 604 and 609.
- b) Nuclear Data Sheets 14 (1975), p. 198 and 213.
- c) Nuclear Data Sheets 6 (1971), p. 427 and 432.
- d) Anderson et al., Phys. Rev. 187, 1565 (1969).
- e) Lucas et al., Phys. Rev. C 7, 1678 (1973).
- f) Carlson et al., Phys. Rev. C 9, 283 (1974).
- g) Nuclear Data Sheets 7 (1972), p. 164 and 177.
- h) Borchert et al., Z. Naturforsch. 30a, 274 (1975).
- i) Nuclear Data Sheets 5 (1971), p. 208 and 215.
- j) Nuclear Data Sheets 15 (1975), p. 338 and 349.
- k) Nuclear Data Sheets 5 (1971), p. 532 and 541.
- l) Nuclear Data Sheets 8 (1972), p. 174.
- m) Backe et al., Nuclear Phys. A189, 472 (1972).
- n) Walter et al., Phys Letters 40 B, 197 (1972).
- o) Bergström et al., Z. Phys. 278, 257 (1976).
- p) Fransson et al., Nuclear Instrum. Methods 131, 511 (1975).
- q) Bergström et al., Research Institute of Physics, Stockholm, Ann. Rep. 1976, p. 99.

Table 6

The muonic 5-4 and 4-3 transition in Pb and Ba, respectively.
(in eV)

	Det. I	Det. II	Weighted mean
Pb $5g_{9/2} - 4f_{7/2}$	431,357 (13)	431,372 (25)	431,360 (11)
Pb $5g_{7/2} - 4f_{5/2}$	437,753 (15)	437,728 (28)	437,748 (12)
Ba $4f_{7/2} - 3d_{5/2}$	433,925 (10)	433,930 (18)	433,926 (8)
Ba $4f_{5/2} - 3d_{3/2}$	441,379 (12)	441,355 (23)	441,374 (9)
$E(\text{Pb } 5g_{7/2} - 4f_{5/2}) - E(\text{Pb } 5g_{9/2} - 4f_{7/2})$			6,388 (16)
$E(\text{Ba } 4f_{5/2} - 3d_{3/2}) - E(\text{Ba } 4f_{7/2} - 3d_{5/2})$			7,448 (12)

The errors contain statistical errors of the lines, quadratically compounded with the calibration error. The latter is negligible for the fine-structure splittings. The correction for the angular dependence of the detector response (Section 4.1) is applied.

Table 7

Error summary for Detector I
(in eV)

	Error source	Transition ^{*)}				Weighted mean
		Pb↑	Ba↑	Pb↓	Ba↓	
1	Statistical	±11.5	±9.0	±14.1	±10.2	
2	Parameters of calibration polynomial	±4.8	±4.9	±5.2	±5.4	±5.1
3	Order " " "	±2.7	±3.0	±3.4	±3.7	±3.2
4a	Peak asymmetry	±2.6	±0.6	±1.3	±0.5	±0.8
4b	" width	±2.6	±0.2	±2.2	±0.3	
5	Pb satellites	±0.2	±0.3	±2.4	0.0	±0.5
6	Ba "	±5.7	±2.4	±0.1	±0.1	±0.4
7	Relative intensity of f.s. components	+1.1	-0.2	-4.2	+0.6	-0.3
8	Effective angular extension of target	±2	±2	±2	±2	±2
9	Yield of ²⁰² Hg 439.5 keV (Y = 1.3%)	0.0	0.0	±4.1	±3.6	±0.4
10	Entering ²⁰⁸ Tl 433.5 keV (Y = 0.4%)	0.0	+1.9	-0.8	-0.1	+0.5
11	" ²⁰⁹ Tl 433.0 " (Y = 0.4%)	-1.5	+3.5	-0.8	-0.1	+0.8
12	" ¹³⁸ Cs 434.5 " (Y = 0.5%)	-0.8	-4.7	+1.0	0.0	-1.7
13	" ²⁰⁴ Hg 436.7 " (Y = 1.3%)	-1.5	-5.2	+6.2	0.0	-1.3
14	" ²⁰⁴ Tl 437.7 " (Y = 0.4%)	-0.4	-0.4	-0.6	0.0	-0.3
15	" ²⁰⁴ Tl 437.0 " (Y = 0.4%)	-0.3	-0.3	+4.2	+0.1	+0.5
16	Including μ-Pb 14-6	< +1	0.0	0.0	0.0	< +0.2
17	Relative intensity of μ-Al 3-1 line	±0.8	±0.8	±0.8	±0.8	±0.8
18	" " " μ-Al 4-1 "	±0.2	±1.0	±0.9	0.0	±0.1

*) The transitions are listed in increasing energy order; Pb↑ means μ-Pb $5g_{9/2} \rightarrow 4f_{7/2}$ (spin parallel to orbital angular momentum), and so on.

COMMENTS

- 3 A positive shift of this size was produced when changing from a fourth-order polynomial ($\chi^2/F = 0.40$) to a third-order one ($\chi^2/F = 0.50$). As can be seen from Fig. 6, a negative shift might be just as reasonable.
- 4a The upper signs give the shifts produced by increasing, for all relevant peaks, the amount of low-energy tail from 2.5% to 3.0%.
- 4b Shifts produced by increasing either the Gaussian or the Lorentzian half-width by 20 eV (see Section 4.2).
- 5 Shifts produced by increasing the parameter α for Pb by ± 0.01 units (see Section 4).
- 6 Shifts produced by increasing the parameter α for Ba by ± 0.025 units (see Section 4).
- 7 Shifts produced by letting the relative intensity of the two main components in both Ba and Pb adjust themselves freely.
- 8 Accuracy obtained in the off-beam experiment (see Section 4.3).
- 9 Shifts produced by applying a $\pm 50\%$ yield uncertainty (Ref. 24) to the 439.5 keV line ascribed to ^{202}Hg .
- 10,11 For the intensity estimate see Table 5; the lower energy is $E_\gamma - \delta E_\gamma$ as given in Ref. 2 of that table.
- 12 The yield $Y = 0.5\%$ represents the detection limit stated by Lucas et al., Ref. 16.
- 13 See Table 5.
- 14,15 See Table 5; the two energies are from Refs. a and a' of that table.
- 17 Shift produced by increasing the intensity ratio between $\mu\text{-Al 3-1}$ and $\mu\text{-Al 2-1}$ by the error given in Ref. 23 ($\pm 11\%$), thus shifting the Au 412 keV calibration line by 1 eV.
- 18 Shift produced by increasing the intensity ratio between $\mu\text{-Al 4-1}$ and $\mu\text{-Al 2-1}$ by the error given in Ref. 23 ($\pm 12\%$).

Table 8

Calculations and comparison with experimental values
(in eV)

	Pb		Ba	
	$5g_{7/2} \rightarrow 4f_{7/2}$	$5g_{7/2} \rightarrow 4f_{5/2}$	$4f_{7/2} \rightarrow 3d_{5/2}$	$4f_{5/2} \rightarrow 3d_{3/2}$
E_0 a,b)	429,334.1	435,663.7	431,590.2 (1.0)	438,933.9 (3.0)
Relativistic recoil b)	2.2	2.2	3.6	3.7
Nuclear polarization b)	4.5 (0.5)	5.0 (0.5)	7.9 (0.8)	9.5 (1.0)
Electron screening b,c)	-75.7 (7.0) d)	-75.7 (7.0) d)	-12.9 (2.5) e)	-12.9 (2.5) e)
$\alpha(\alpha Z)$ b,c)	2,105.0 (1.0)	2,188.8 (1.0)	2,327.5 (1.0)	2,434.5 (1.0)
$\alpha^2(\alpha Z)$ b)	14.5	15.1	16.2	17.0
$\alpha(\alpha Z)^{n \geq 3}$ b)	-42.0 (2.0)	-42.0 (2.0)	-19.0 (1.0)	-19.0 (1.0)
$\alpha^2(\alpha Z)^2$	1.2	1.2	1.0	1.0
Self-energy	-0.9	-0.9	-1.5	-1.5
Nuclear recoil	-0.5	-0.5	-0.7	-0.7
E^{th}	431,342 $\pm$ 7	437,757 $\pm$ 7	433,912 $\pm$ 3	441,366 $\pm$ 4
E^{exp}	431,360 $\pm$ 11	437,748 $\pm$ 12	433,926 $\pm$ 8	441,374 $\pm$ 9
$E^{th} - E^{exp}$	-17 $\pm$ 13	9 $\pm$ 14	-14 $\pm$ 9	-8 $\pm$ 10
$\langle E^{th} - E^{exp} \rangle$	-4 $\pm$ 11		-11 $\pm$ 7	

a) Contains anomalous magnetic moment.

b) Contains finite size contribution.

c) Contains ladder graphs.

d) 1.8 $\pm$ 0.2 K-electrons, 6 L-electrons

e) 1.4 $\pm$ 0.3 K-electrons, 6 L-electrons

} assumed to be present when the muonic transition takes place.

Table 9

Electron screening calculations for the μ 5g-4f transitions in ^{208}Pb .
(Energies in eV)

	Only 2 K-electrons	2 K- and 8 L-electrons
Unperturbed relativistic electron wave functions	-71	-83
Self-consistent calculations according to Fricke b)	-65	-77 [-80 a)]
Self-consistent calculations according to Vogel c)	-70	-81 a)
Self-consistent calculations including rearrangement energies, Fricke d)	-	-82

a) All electrons present when transition occurs.

b) B. Fricke, Nuovo Cimento Letters 2, 859 (1969).

c) P. Vogel, Phys. Rev. A 7, 63 (1973).

d) B. Fricke, I.T. Waber and V.L. Telegdi, Northwestern University, Evanston, preprint, 1972.

REFERENCES

- 1) S.R. Lundeen and F.M. Pipkin, Phys. Rev. Letters 34, 1368 (1975).
D.A. Andrews and G. Newton, Phys. Rev. Letters 37, 1254 (1976).
- 2) J.C. Wesley and A. Rich, Phys. Rev. A 4, 1341 (1971).
S. Granger and G.W. Ford, Phys. Rev. Letters 28, 1479 (1972).
- 3) J. Bailey, K. Borer, F. Combley, H. Drumm, F.J.M. Farley, J.H. Field, W. Flegel,
P.M. Hattersley, F. Krienen, F. Lange, E. Picasso and W. von Rüden, Phys.
Letters 67B (1975).
For reviews see:
- E. Picasso, *in* "Nova Acta Leopoldina" (ed. E. Schmutzer) (Deutsche Akademie
der Naturforscher Leopoldina, Halle, Saale, 1976).
- S.J. Brodsky, *in* "Atomic Physics" (eds. G.K. Woodgate and P.G.H. Sandars)
(Plenum Press, London, 1971), p.1.
- 4) G. Backenstoss, S. Charalambus, H. Daniel, Ch. von der Malsburg, G. Poelz,
H.P. Povel, H. Schmitt and L. Tauscher, Phys. Letters 31B, 233 (1970).
- 5) M.S. Dixit, H.L. Anderson, C.K. Hargrove, R.J. McKee, D. Kessler, H. Mes and
A.C. Thompson, Phys. Rev. Letters 27, 878 (1971).
- 6) J. Blomqvist, Nuclear Phys. B48, 95 (1972).
- 7) P.J.S. Watson and M.K. Sundaresan, Canad. J. Phys. 52, 2037 (1974), and
references cited therein.
- 8) H. Walter, J.H. Vuilleumier, H. Backe, F. Boehm, R. Engfer, A.H. von Gunten,
R. Link, R. Michaelson, C. Petitjean, L. Schellenberg, H. Schneuwly,
W.V. Schröder and A. Zehnder, Phys. Letters 40B, 197 (1972).
- 9) R. Engfer, J.L. Vuilleumier and E. Borie, Proc. 4th Internat. Conf. on Atomic
Physics, Heidelberg, 1974 (eds. G. zu Pultitz, E.W. Weber and A. Winnacker)
(Plenum, New York, 1975), p. 191.
- 10a) A. Bertin, G. Carboni, A. Placci, E. Zavattini, U. Gastaldi, G. Gorini, G. Neri,
O. Pitzurra, E. Polacco, G. Torelli, A. Vitale, J. Duclos and J. Picard,
Phys. Letters 55B, 411 (1975).
- 10b) R. Barbieri, Phys. Letters 56B, 266 (1975).
- 11) L. Tauscher, G. Backenstoss, K. Fransson, H. Koch, A. Nilsson and J. De Raedt,
Phys. Rev. Letters 35, 410 (1975).
- 12) M.S. Dixit, A.L. Carter, E.P. Hincks, D. Kessler, J.S. Wadden, C.K. Hargrove,
R.J. McKee, H. Mes and H.L. Anderson, Phys. Rev. Letters 35, 1633 (1975).
- 13) J.L. Vuilleumier, W. Dey, R. Engfer, H. Schneuwly, H.K. Walter and A. Zehnder,
Z. Phys. A278, 109 (1976).
- 14) C.K. Hargrove et al., contributed paper at the 5th Internat. Conf. on High-
Energy Physics and Nuclear Structure, Santa Fe, 1975.
- 15) T. Dubler, L. Schellenberg, H. Schneuwly, R. Engfer, J.L. Vuilleumier,
H.K. Walter and A. Zehnder, Nuclear Phys. A219, 29 (1974).

- 16) G.R. Lucas, Jr., P. Martin, G.H. Miller, R.E. Welsh, D.A. Jenkins, R.J. Powers and A.R. Kunselman, *Phys. Rev. C* 7, 1678 (1973).
- 17) K. Fransson, A. Nilsson, J. De Raedt and L. Tauscher, *Nuclear Instrum. Methods* 131, 511 (1975).
- 18) K. Fransson, A. Nilsson, J. De Raedt and K.G. Rensfelt, *Nuclear Instrum. Methods* 137, 479 (1976).
- 19) G. Murray, R.L. Graham and J.S. Geiger, *Nuclear Phys.* 45, 177 (1963).
O. Piller, W. Beer and J. Kern, *Nuclear Instrum. Methods* 107, 61 (1973).
- 20) R.D. Deslattes, E.G. Kessler, W.C. Sauder and A. Henins, *in* Proc. 5th Internat. Conf. on Atomic Masses and Fundamental Constants, Paris, 1975 (eds. J.H. Sanders and A.H. Wapstra, Plenum Press, New York, 1976), p. 48, and private communication.
- 21) R.G. Helmer, R.C. Greenwood and R.J. Gehrke, *Nuclear Instrum. Methods* 96, 173 (1971).
- 22) E. Storm and H.I. Israel, *Nuclear Data Tables* 7, 565 (1970).
- 23) H. Backe, *Z. Phys.* 241, 435 (1971).
- 24) H.L. Anderson, C.K. Hargrove, E.P. Hincks, J.D. McAndrew, R.J. McKee, R.D. Barton and D. Kessler, *Phys. Rev.* 187, 1565 (1969).
- 25) R.G. Helmer, R.J. Gehrke and R.C. Greenwood, *Nuclear Instrum. Methods* 123, 51 (1975).
- 26) R.D. Birkhoff *in* *Handbuch der Physik* (ed. S. Flügge) (Springer Verlag, Berlin, 1958), Vol. 34, p. 136.
- 27) A. Fubini, R. Alberini and D. Lattanzi, *Nuovo Cimento* 18A, 711 (1973).
- 28) M.J. Martin, *Nuclear Data Sheets* B5, 601 (1971).
- 29) E.A. Henry, *ibid.* 14, 191 (1975).
- 30) M.R. Schmorak, *ibid.* B6, 425 (1971).
- 31) G.H. Carlson, W.L. Talbert, Jr., and J.R. McConnell, *Phys. Rev. C* 9, 283 (1974).
- 32) K.K. Seth, *Nuclear Data Sheets* B7, 161 (1972).
- 33) G.L. Borchert, W. Scheck and K.P. Wieder, *Z. Naturforsch.* 30a, 274 (1975).
- 34) M.R. Schmorak and R.L. Auble, *Nuclear Data Sheets* B5, 207 (1971).
- 35) R.L. Bunting, *ibid.* 15, 335 (1975).
- 36) R.L. Auble, *ibid.* B5, 531 (1971).
- 37) S.C. Pancholi and M.J. Martin, *ibid.* B8, 165 (1972).
- 38) H. Backe, R. Engfer, U. Jahnke, E. Kankeleit, R.M. Pearce, C. Petitjean, L. Schellenberg, H. Schneuwly, W.U. Schröder, H.K. Walter and A. Zehnder, *Nuclear Phys.* A189, 472 (1972).

- 39) I. Bergström, J. Blomqvist, C.J. Herrlander and C.G. Lindén, Z. Phys. A278, 257 (1976).
- 40) I. Bergström, J. Blomqvist, C.G. Lindén, O. Knuuttila and T. Lönnroth, Annual Report 1976, Research Inst. for Physics, Stockholm (contrib. 3.3.13).
- 41) G. Backenstoss, S. Charalambus, H. Daniel, W.D. Hamilton, U. Lynen, Ch. von der Malsburg, G. Poelz and H.P. Povel, Nuclear Phys. A162, 541 (1971).
- 42) H.J. Evans, Nuclear Phys. A207, 379 (1973).
- 43) H.A. Bethe and E.E. Salpeter, *in* Handbuch der Physik (ed. S. Flügge) (Springer Verlag, Berlin, 1957), Vol. 35.
- 44) H.L. Acker, G. Backenstoss, C. Daum, J.C. Sens, S.A. De Wit, Nuclear Phys. 87, 1 (1966).
- 45) A.R. Ralston and H.S. Wilf, Mathematical methods for digital computers (Wiley, New York, 1960), p. 95.
- 46) E.M. Rose, Relativistic electron theory (J. Wilets and Son, New York, 1961).
- 47) D.R. Hartree, Numerical analysis, 2nd edition (Oxford University Press, London, 1958), p. 142.
- 48) J.M. Blatt, J. Comput. Phys. 1, 382 (1967).
- 49) R. Engfer, H. Schneuwly, J.L. Vuilleumier, H.K. Walter and A. Zehnder, Atomic Data and Nuclear Data Tables 14, 509 (1974).
- 50) D. Kessler, H. Mes, A.C. Thompson, H.L. Anderson, M.S. Dixit, C.K. Hargrove and R.J. McKee, Phys. Rev. C 11, 1719 (1975).
- 51) B. Fricke, Phys. Rev. Letters 30, 119 (1973).
- 52) J.L. Friar and J.W. Negele, Phys. Letters B46, 5 (1973).
- 53) R.C. Barrett, D.A. Owen, J. Calmet and H. Grotch, Phys. Letters B47, 297 (1973).
- 54) V.A. Rizov, I.T. Todorov and B.L. Aneva, Nuclear Phys. B98, 447 (1975).
- 55) R.J. McKee, Phys. Rev. 180, 1139 (1969).
- 56) E. Borie, Helv. Phys. Acta 48, 671 (1975).
- 57) T.E.O. Ericson and J. Hüfner, Nuclear Phys. B47, 205 (1972).
- 58) B.L. Berman, Atomic Data and Nuclear Data Tables 15, 319 (1975).
- 59) J. Hüfner, private communication.
- 60) B. Fricke, Nuovo Cimento Letters 2, 859 (1969).
- 61) B. Fricke, I.T. Waber and V.L. Telegdi, Northwestern University, Evanston, preprint, 1972.
- 62) P. Vogel, Phys. Rev. A 7, 63 (1973).
- 63) W. Bambynek, B. Crasemann, R.W. Fink, H.-U. Freund, H. Mark, C.D. Swift, R.E. Price and P.V. Rao, Rev. Mod. Phys. 44, 716 (1972).

- 64) B. Fricke, Z. Phys. 218, 495 (1969).
- 65) J.M. McKinley, unpublished, quoted in Ref. 49.
- 66) G.A. Rinker and L. Wilets, Phys. Rev. A 12, 748 (1975).
- 67) L.W. Fullerton and G.A. Rinker, Phys. Rev. A 13, 1283 (1976).
- 68) E.H. Wichmann and N.M. Kroll, Phys. Rev. 101, 843 (1956).
- 69) I. Arafune, Phys. Rev. Letters 32, 560 (1974).
- 70) L.S. Brown, R.N. Cahn, L.D. McLerran, Phys. Rev. Letters 32, 562 (1974).
- 71) I. Calmet and D.A. Owen, Phys. Letters 53B, 67 (1974).
- 72) M. Guylassy, Phys. Rev. Letters 32, 1393 (1974).
- 73) L. Wilets and G.A. Rinker, Phys. Rev. Letters 34, 339 (1975).
- 74) M.K. Sundaresan and P.J.S. Watson, Phys. Rev. D 11, 230 (1975).
- 75) E. Borie, Nuclear Phys. A267, 485 (1976).
- 76) S. Klarsfeld and A. Maquet, Phys. Letters B43, 201 (1973).

Figure captions

- Fig. 1 : Deviation of experiment from theory in heavy muonic atoms at the time when the present work was started. The black dots are from Walter et al., open circles from Dixit et al., both Ref. 7, and the squares are from our early work (Ref. 4). The dashed line shows the effect of introducing the new Au standard (Ref. 20).
- Fig. 2 : Experimental set-up. (Top view).
- Fig. 3 : Total spectrum recorded by spectrometer I. The channel contents have been added two by two.
- Fig. 4 : Resolution of the two spectrometers as a function of photon energy. The ordinate is the FWHM parameter of the Gaussian after application of optimal foot parameter. Dots refer to detector I.
- Fig. 5 : Part of the spectrum shown in Fig. 3. The vertical bars below the spectrum represent the positions and intensities of the lines entered into the fit.
- Fig. 6 : Non-linearity of spectrometer I. The ordinate is the correction which has to be applied to a linear calibration function in order to obtain true energies. Circles represent muon-induced γ -rays; crosses, γ -rays from the sources and the positron annihilation line. When double error bars occur, the inner ones represent our statistical errors, while the outer ones comprise also the uncertainty of energies given in the literature. The fully drawn line is a fourth-order polynomial, dashed line a third-order one.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6