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DIFFRACTIVE PRODUCTION ON NUCLEI AND TOTAL CROSS-SECTIONS
OF UNSTABLE PARTICLES

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A B S T R A C T

We remind that the Glauber multiple scattering approach, widely used in order to extract the value of the total cross-sections of unstable particles on nucleons σ_{a*} is quite open to criticism.

We discuss the rôle of non-planar diagrams and show that their presence can affect strongly the determination of σ_{a*} .

From the present data we obtain, in a specific scheme, a value of σ_{a*} much larger than the one obtained using the Glauber method.

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1. - INTRODUCTION

Diffraction production on nuclei can provide us with a measurement of the cross-section of unstable particles, since a resonance can re-interact in the nucleus during its short lifetime. It is, however, necessary to choose a model of multiple scattering in order to be able to extract the resonance nucleon cross-section from the data on diffractive production.

A popular choice is represented by a straightforward extension of the Glauber model ¹⁾ which is known to work very well for the elastic particle-nucleus amplitude ²⁾. In Section 2 we review the model for completeness, but we argue in Section 3 that this procedure is in fact quite unwarranted in the realm of high energy hadronic interactions. In Section 4 we address ourselves to the problem of explaining how a wrong procedure can give the right answer in the case of elastic scattering, but we show in Section 5 that the same happy circumstance is unlikely to generalize to inelastic diffraction. In particular, analyzing the data in a well-defined (and physically motivated) dynamical model we find results for σ_{a*} which are substantially larger than the ones obtained with the Glauber procedure.

2. - REVIEW OF THE GLAUBER MODEL

The basic assumption of the Glauber model is that the S matrix for the particle (a)- nucleus (A) scattering is given by the product of the scattering S matrices of the particle a on the individual nucleons $N_1 \dots N_A$.

In potential scattering, the S matrix for the $a-N_i$ scattering is in fact just a c number, and it is straightforward to derive from this assumption the standard Glauber multiple scattering expansion for the elastic $a-A$ scattering amplitude. In the realm of strong interactions, however, the S matrix is a very complicated object that contains in an essential way the many-particle channels. Even the elastic element is, we believe, determined by the overlap of multiparticle states, and it looks very dangerous to compute the elastic $a-A$ scattering using as an input only the (imaginary !) $a-N_i$ elastic scattering amplitudes. However, such procedure is known to meet with great phenomenological success ²⁾. It is tempting to extend the model to the diffractive production of resonances, neglecting again multiparticle states,

and repeating the Glauber calculation with S_{a-N_i} given this time by a matrix acting in the space of the projectile a and of its resonances $a_1^* \dots a_n^*$. The diagonal elements of this matrix are obtained by the total cross-sections of the different resonances on nucleon, that it is our aim to determine. The Glauber multiple scattering series is easily reproduced under this assumption, and the i^{th} scattering term is given by an $(n+1)$ by $(n+1)$ matrix obtained taking the i^{th} power of the assumed $a-N$ S matrix. In a physically more transparent way one can first diagonalize the $(n+1) \times (n+1)$ $a-N$ S matrix, and expand the incoming a state and the outgoing a^* state in terms of the eigenstates a_k

$$|a\rangle = \sum_k c_k |a_k\rangle \quad |a^*\rangle = \sum_k c_k^* |a_k\rangle$$

The multiple scattering problem is thus reduced to a set of $(n+1)$ one-dimensional Glauber formulae for the propagation of the eigenstates $|a_k\rangle$ in the nucleus,

$$\bar{S}_{kk}^A(b) = \exp \left[- \int \rho(\tilde{b}) \bar{T}_{kk}^P(b - \tilde{b}) d^2 \tilde{b} \right] \quad (1)$$

and

$$S_{aa^*}^A(b) = \sum_k c_k c_k^* \bar{S}_{kk}^A(b) \quad (2)$$

In Eq. (1), b is the impact parameter, and $\rho(b)$ is the nuclear density (correlations between nucleons have been ignored, as usual).

Following Good and Walker ³⁾ and Feinberg and Pomeranchuk ⁴⁾, one can interpret Eq. (2) by saying that diffraction dissociation arises because of the difference between the cross-sections of the eigenchannels [if all $\bar{S}_{kk}$ were equal, the orthogonality of a and a^* would cause Eq. (2) to vanish]. For the scattering on a large nucleus at small impact parameter the Glauber formalism predicts that all $\bar{S}_{kk}$ become essentially 0, and Eq. (2) vanishes. Hence one expects from (2) that only the edge of the heavy nuclei can contribute substantially to $\sigma_{a \rightarrow a^*}^A$.

3. - GLAUBER, PLANARITY AND AFS

It is apparent from the Glauber formula that multiple scattering is computed exclusively in terms of the cross-sections of on-shell one-particle states. At high energy, when the S matrix for the a nucleon scattering is dominated by Pomeron exchange, the physical picture corresponding to this formula is the same that leads to the Amati-Fubini-Stanghellini (AFS) Reggeon-Reggeon cuts, as already observed a long time ago ⁵⁾, and the Glauber formula, e.g., for double scattering, can be obtained approximating the integral over the mass M^2 in the P - a elastic scattering amplitude by the a particle pole. Unfortunately the remaining discontinuities, i.e., the cut starting at $(m_a + m_p)^2$ (where m_p is the smallest mass that can be diffractively produced with a) receives a contribution from the planar diagrams that is known to cancel exactly the pole contribution in the limit $s \rightarrow \infty$ ⁵⁾. The fact is that the structure of double scattering in the Glauber formalism is of the Reggeon-cut type, and is connected with the existence of non-planar diagrams with third spectral function. But the a particle pole does not contribute to the third spectral function, hence it is exactly cancelled by the remaining planar contributions.

Since the pole is cancelled in the dispersion relation for the P - a elastic scattering amplitude by the cut contribution that corresponds to larger masses, a very violent t dependence of the P - A coupling can, at finite energy, suppress strongly the cut contribution and leave an uncanceled AFS term. Hence there is a logic for the use of Glauber formula at small energy, large A , but not in the limit $s \rightarrow \infty$ at fixed A .

Some time ago Koplik and Mueller ⁶⁾ came to the same conclusion using a nice space-time analysis of the Glauber term corresponding to multiple Regge pole exchange in old-fashioned perturbation theory. Unfortunately such a picture is quite unfounded ⁷⁾ and we carefully refrain from using this argument here. A good guess for the analytical form that describes the energy dependence of the planar contribution (pole = Glauber + cut) to the double scattering is of the form $T_{\text{Glauber}}^{(2)} (1 - e^{-t_G R^2})$ where $t_G = (m_p^2 - m^2)^2 m^2 / s^2$ and m_p represents an average mass of the planar cut discontinuity.

4. - HOW CAN GLAUBER WORK FOR ELASTIC SCATTERING ?

We are left now with the task of evaluating the contribution of non-planar diagrams to multiple scattering. These diagrams contribute bona fide Regge cuts at infinite energy, and, for instance, the Mandelstam diagram asymptotically contributes a term of the same structure of the AFS contribution. However, in the dispersive integral for the P-a scattering amplitude the Mandelstam diagram does not contribute to the pole at $\bar{s}=m^2$, but only to the cut, and it is suppressed by the nuclear form factor very much in the same way as the planar cut. Hence the contribution of the Mandelstam diagram to nuclear scattering can be written as

$$T_M^{(2)} = T_G^{(2)} e^{-t_M R^2} \quad \text{with} \quad t_M = \frac{(m_{NP}^2 - m^2) m^2}{s^2}$$

If $m_{NP} = m_p$, as it is quite reasonable to assume, independently of s and A , the non-planar cut cancels the planar cut contribution, leaving the Glauber term as a net result for the elastic a-A amplitude : however, this happens only because of a very delicate cancellation that, as we will see in the next paragraph, is unlikely to generalize to diffractive production.

For the higher order terms in the multiple scattering series for elastic scattering, the completely planar Glauber term, obtained by taking the pole residues in all dispersive integrals for the P-a amplitudes, is multiply suppressed by the cancelling planar cuts, leaving a net result of this form

$$T_{\text{planar}}^{(i)} = T_G^{(i)} (1 - e^{-t_G R^2})^i$$

The completely nested Mandelstam diagram has a cut that starts at $(m_a + im_b)^2$, and its contribution is of the form

$$T_M^{(i)} \cong T_G^{(i)} e^{-it_M R^2}$$

However, there is a whole hierarchy of partly planar, partly nested diagrams. If $t_G \simeq t_M$ they can merge in such a way that the total elastic amplitude is :

$$T^{(i)} = T_G^{(i)} \sum_{k=0}^i \binom{i}{k} (1 - e^{-t_G R^2})^{i-k} e^{-k t_M R^2} \approx T_G^{(i)} \quad (3)$$

Let us stress here that we are far from claiming that the above considerations "prove" Glauber formula for hadronic elastic scattering : we have only found a conceivable mechanism that could cure the most obvious disease of such an approach. In the next paragraphs we will see that no mechanism can be found along the same lines, in order to justify the extension of Glauber's formula to diffractive production.

5. - DIFFRACTION DISSOCIATION

We take here the attitude that diffraction dissociation, as well as elastic scattering, is produced by the shadow of multiparticle production. The first question that one would like to answer is why, in such a picture, the transition $a \rightarrow a^*$ is much smaller than the elastic amplitude $a \rightarrow a$ (and presumably $a^* \rightarrow a^*$). The only possible answer in this framework is that the a initiated reactions and the a^* initiated ones lead to multiparticle final states that are almost orthogonal. Because of factorization, and since the Regge pole built by the overlap function is the same P pole in all cases, the lack of overlap can only happen in a finite region of rapidity in the projectile fragmentation. Taking the multiperipheral model as an example, a possible mechanism for the suppression of off-diagonal elements is depicted in Fig. 1, where we have assumed for singularity that the leading particle belongs to the same family of a or a^* . This effect can be due to the fact that, even if the coupling constants g_{aa} , g_{aa^*} and $g_{a^*a^*}$ are all of the same order of magnitude, the different vertices couple preferentially to different exchanges and this produces a mismatch of quantum numbers in the top particles of the chain (we will refer to this possibility as model I in the following). Even if this does not happen, the rapidity gap mismatch exhibited in Fig. 1 is enough to produce a strong suppression of the off-diagonal elements.

In fact in the case of no mass difference at the top vertex the preferred rapidity gap Δ^0 is determined by maximizing

$$A = (M^2)^{\alpha_P} (s/M^2)^{2\alpha_E-1} e^{-2a \frac{M^4 m^2}{s^2}} \quad (4)$$

where the exchange of the first link has been parametrized as $(s_1)^{\alpha_E} e^{at_1}$ and $\Delta^0 \simeq \ln s_1 \simeq \ln s/M^2$.

From (4) we obtain

$$\frac{M^4}{s^2} = \frac{\alpha_P + 1 - 2\alpha_E}{4am^2}$$

and with reasonable parameters ($\alpha_E = \frac{1}{2}$, $a = 5$, $m^2 = 1$) $\Delta_0 \simeq 1.5$. (This value checks well with the known value of $\bar{n}$ if the produced objects are clusters of multiplicity around 4.)

For the case of $a \rightarrow a^*$ transition we obtain the preferred gap Δ^+ to be given by

$$\ln \left(\frac{\alpha_P + 1 - 2\alpha_E}{2a\delta^2} \right)$$

where $\delta^2 = m_{a^*}^2 - m_a^2$. With the same values used previously, for the typical case of a transition $p \rightarrow N^*(1680)$ we obtain $\Delta^+ = 3$.

In the third case $a^* \rightarrow a$ with a decrease of mass, $t_{\min}$ is positive and increasing with M^2 , hence the gap Δ^- will try to be as small as kinematics allows, and both mismatches $\Delta^0 - \Delta^+$ and $\Delta^0 - \Delta^-$ are of the order of 1.5 and can produce a suppression of the required order of magnitude. We will refer as model II to the assumption that only the gap mismatch mechanism is operative in producing the suppression of diffractive dissociation.

6. - DOUBLE SCATTERING

We are going to consider here the planar and non-planar double scattering terms. In the planar contribution of Fig. 2 we can choose ② to be a or a^* (at first order in the non-diagonal transitions) and obtain the Glauber contribution, proportional to $\sigma_{a \rightarrow a^*}(\sigma_a + \sigma_{a^*})$. Again this term is cancelled by the planar cut contribution, and $T_{pl}^{(2)} \rightarrow 0$ when $s \rightarrow \infty$. The non-planar contribution of Fig. 3 provides a non-vanishing asymptotic term, but this time the Glauber result is not fully recovered ^{*}).

In fact in Fig. 3 there are eight possible contributions to examine, corresponding to the choices of ①, ②, ③ to be a or a^* . In the equivalent planar diagram (Fig. 2) all the eight configurations are allowed, but here we see that this is not the case. For instance, choosing ① = a^* and ③ = a leads to a terrible mismatch of the rapidity gaps in the external ladder (Δ^+ vs Δ^-) whereas the internal ladder is still of the order of $\sigma_{a \rightarrow a^*}$. Hence in the dynamic of model II only six contributions are operative. Two more contributions (the choices ① = a , ③ = a^* with ② = a or a^*) are allowed by the dynamic of model II, since there is only one gap mismatch in the internal ladder, but not by the dynamic of model I, since two a generated ladders have to overlap with two a^* generated ones ^{**}). In fact in model I the double scattering contribution turns out to be $\sigma_a T_{aa^*}^a + \sigma_{a^*} T_{aa^*}^{a^*}$, where $T_{aa^*}^a$ and $T_{aa^*}^{a^*}$ are the two contributions of Fig. 4a,b since $T_{aa^*}^a$ should be comparable to $T_{aa^*}^{a^*}$, both are given by $\frac{1}{2}\sigma_{a \rightarrow a^*}$, and the non-planar double scattering term in model I turns out to be one half of the Glauber prediction. In model II the double scattering term can be written, assuming factorization, as

$$\frac{\sigma_{a \rightarrow a^*}}{2} (\sigma_a + \sigma_{a^*} + \sqrt{\sigma_a \sigma_{a^*}})$$

^{*}) If we kept all the Mandelstam-type diagrams, the two-eigenvalue situation discussed for the Glauber model would be recovered, with g_{aa} , g_{aa^*} , g_{a^*a} replacing the scattering amplitudes.

^{**}) In a previous short summary of our results presented at the XII Moriond meeting, Flaine 1977 (CERN Preprint TH. 2299, to be published in the Proceedings, Ed. Tran Thanh Van), we had considered only model I.

It is obvious that in both cases, analyzing high energy double scattering terms in the Glauber framework leads to a determination of σ_{a^*} much smaller than the actual one. Say that the result obtained in the Glauber framework for a p^*p scattering at very large s is of the order of 20 mb : this means that the coefficient of σ_{aa^*} is $(\sigma_p + \sigma_{p^*}^G) = 60$ mb. But in fact this number represents $\frac{1}{2}(\sigma_p + \sigma_{p^*})$ in model I (hence $\sigma_{p^*} = 80$ mb!) or $(\sigma_p + \sigma_p + \sqrt{\sigma_p \sigma_{p^*}})/2$ in model II (hence $\sigma_{p^*} = 40$ mb) *).

7. - MULTIPLE SCATTERING

The considerations of the previous paragraph can be generalized to the case of multiple scattering. In the case of model I, the first particle and the last one have to be equal, and the coefficient of the n^{th} term in the multiple scattering series is $\sigma_a^{n-1} T_{aa^*}^a + \sigma_{a^*}^{n-1} T_{aa^*}^{a^*} \simeq \frac{1}{2} \sigma_{a \rightarrow a^*} (\sigma_a^{n-1} + \sigma_{a^*}^{n-1})$ instead of the Glauber coefficient

$$\sigma_{a \rightarrow a^*} \sum_{m=0}^{n-1} \sigma_a^m \sigma_{a^*}^{n-1-m}$$

Hence in model I the diffractive production is simply related to the total a - A cross-section, as follows

$$\frac{\sigma_{a \rightarrow a^*}^A}{\sigma_{aA}} = \frac{1}{2} \left(\frac{\sigma_{a \rightarrow a^*}}{\sigma_a} + \frac{\sigma_{a \rightarrow a^*}}{\sigma_{a^*}} \right) \quad (5)$$

This behaviour differs even qualitatively from Glauber's result, in that in the large A limit not only the edges, but also the centre of the nucleus contribute to diffractive production, in contrast with what we found at the end of Section 2. Physically this result is very transparent, since it is clear that in model I a and a^* behave as eigenstates for what concerns the nuclear propagation and the transition $a \rightarrow a^*$ is only allowed in the first or the last intersection.

*) The Mandelstam diagram could be written in the form of Fig. 5, where the third line is the baryonic one and in this form seems to contain one power of σ_B and one power of σ_M, σ_B (σ_M) being the total cross-section for some baryonic (mesonic) state. In fact we can write it in terms of σ_B^2 using the self-consistency condition of Fig. 6, that is implicit in our scheme.

In conclusion, in this model the diffractive production behaves like $A^{2/3}$ at large A . Let us remark, however, that this result is very model-dependent : for instance in model II, when more intermediate structure is allowed, in the limit $\sigma_a \simeq \sigma_{a^*}$ the diffraction at a given nuclear thickness T behaves like

$$\frac{\sigma_{a \rightarrow a^*}}{\sigma_a} \left(e^{-T \frac{\sigma_a}{2}} - e^{-T \sigma_a} \right)$$

and again vanishes for large T .

8. - CONCLUSIONS

We have pointed out in this paper that the naïve extension of the Glauber model to diffraction dissociation is unwarranted, and leads naturally to an underestimate of the cross-sections of unstable particles. Unfortunately the quantitative description of diffractive production on nuclei depends substantially on the details of the overlap producing diffraction. We have considered two specific examples, and shown that even qualitative features like the large A dependence of the diffractive production depends on the choice of the model : the only thing that emerges in both the schemes that we have considered is that the experimental observation of a double scattering term that would yield $\sigma_{p^*} \simeq 20 \text{ mb}$ ^{*)} when analyzed "à la Glauber" corresponds in fact to $\sigma_{p^*} \geq \sigma_p$. This is most reassuring in view of the constraints on σ_{p^*} obtained in Ref. 8).

*) Let us stress here that we believe that our conclusions apply to the diffractive production of resonances, and not necessarily to the background constituted by the nucleon-pion systems in a given mass bin.

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FIGURE CAPTIONS

- Fig. 1: The overlap producing the elastic amplitude and diffraction dissociation.
- Fig. 2: The planar double scattering diagram.
- Fig. 3: The non-planar double scattering diagram.
- Fig. 4: The contributions to the diffraction dissociation diagram.
- Fig. 5: The P-B elastic amplitude.
- Fig. 6: The relation between the PBB and PVM couplings.

$$\begin{array}{c}
 \left| \begin{array}{c} a \quad a \\ \hline \Delta^0 \\ \hline \vdots \end{array} \right|^2 + \left| \begin{array}{c} a \quad a^* \\ \hline \Delta^+ \\ \hline \vdots \end{array} \right|^2 \approx \left| \begin{array}{c} a^* \quad a^* \\ \hline \Delta^0 \\ \hline \vdots \end{array} \right|^2 + \left| \begin{array}{c} a^* \quad a \\ \hline \Delta^- \\ \hline \vdots \end{array} \right|^2 \\
 \\
 \gg \left| \begin{array}{c} a \quad a \\ \hline \Delta^0 \\ \hline \vdots \end{array} \right| \otimes \left| \begin{array}{c} a \quad a^* \\ \hline \Delta^- \\ \hline \vdots \end{array} \right| + \left| \begin{array}{c} a \quad a^* \\ \hline \Delta^+ \\ \hline \vdots \end{array} \right| \otimes \left| \begin{array}{c} a^* \quad a^* \\ \hline \Delta^0 \\ \hline \vdots \end{array} \right|
 \end{array}$$

FIG.1

FIG.2

FIG. 3

FIG.4

FIG. 5

FIG. 6