VIRTUAL PHOTOPRODUCTION OF HADRONS AT LARGE $p_T$

AS A PROBE FOR GLUON BREMSSTRAHLUNG

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ABSTRACT

In a SU(3) colour $\times$ SU(3) flavour invariant interaction of quarks and gluons where the colour degree of freedom is locally gauged, we calculate first-order corrections of the naive parton model for the one-hadron inclusive virtual photoproduction. We study the high $p_T$ distribution of hadrons and we determine the kinematical region in which the prediction of the model dominates over the fall-off $\sim \exp (-bp_T^2)$ which seems to be supported by the low $p_T$ available experimental data.
1. **INTRODUCTION**

The success of parton model ideas \(^1\) in high-energy scattering processes, in which the short-distance structure of the nucleons is probed, indicates that it is worth trying to calculate various kinds of physically acceptable corrections wherever these ideas describe, at least, the gross structure. This happens, for instance, in deep inelastic lepton-hadron scattering where the predicted Bjorken scaling \(^2\) is only slightly violated \(^3\).

In this work we shall study the high \(p_T\) distribution of the inclusive virtual photoproduction of hadrons \((\gamma^* + p \rightarrow h + X)\). Recent experimental studies in inclusive virtual photoproduction of hadrons show that the \(p_T\) distribution is governed by a typical hadronic exponential fall-off \(\sim \exp (-b(W) \times p_T^2)\), where \(W\) is the invariant mass of the final state hadronic system. We accept this as a confirmation of the hypothesis \(^4\) of jet-wise fragmentation of quarks into hadrons with small \(p_T\) relative to the jet axis. The same phenomenon is observed in pp and e\(^+\)e\(^-\) interactions \(^5\).

Anticipating that in analogy with pp high \(p_T\) inclusive distribution of hadrons \(^6\) a deviation from the \(\exp (-b \cdot p_T^2)\) distribution will soon be seen also in \(\gamma p\) interactions, we propose the simple-minded modification of the naive parton model \(^4\) (Fig. 1) that the interacting quark which absorbs the virtual photon is sometimes highly virtual, and before it fragments into hadrons emits a hard gluon which also fragments finally into hadrons (Fig. 2).

In order to be systematic we shall calculate first-order corrections to the naive parton model arising from SU(3) colour locally gauge invariant interaction of quarks and gluons, both assumed to be massless. In this model a three-jet structure emerges for sufficiently high energy of the \(\gamma^* p\) system in its c.m. frame (Fig. 3). This picture has already been proposed for the final hadronic state in e\(^+\)e\(^-\) interactions as a probe for gluon bremsstrahlung \(^7\).

The idea that bremsstrahlung of vector particles (vector mesons) may be a source for the deviation from the exponential fall-off of the \(p_T\) inclusive distribution has been applied in pp collisions \(^8\),\(^9\). Also the idea that a possible universal mechanism of hadron production is the fragmentation of gluon bremsstrahlung from spatially separated quark systems has been proposed and discussed recently \(^10\),\(^11\).
Effects of gluon bremsstrahlung have been taken into account in the renormalization group approach \(^{12}\) and in naive parton models \(^{13},^{14}\) for the explanation of the observed violations of Bjorken scaling. These analyses predict the increase, with the mass of the virtual photon, of the mean transverse momentum. This property has been predicted also by the covariant parton model \(^{15}\).

We find that if bremsstrahlung exists its effects can be seen for \(p_T^2 > 9 \text{ GeV}^2/c^2\) and for \(W > 10 \text{ GeV}, \ Q^2 > 4 \text{ GeV}^2/c^2\). In this region the first order corrections dominate over the contribution obtained from the extrapolation of the data \(^{16}\).

In Section 2 we define the kinematics and the model. In Section 3 we determine the kinematical region of the possible relevance of the model.

2. INCLUSIVE HADRON PRODUCTION FROM VIRTUAL PHOTON-PROTON SCATTERING

The process of interest is

\[ \gamma^* + p \rightarrow h + X \]  \hspace{1cm} (1)

which, if we assume single photon exchange, is studied in experiments of the type

\[ e^+p \rightarrow e^+h + X \quad \text{or} \quad \mu^+p \rightarrow \mu^+h + X \]  \hspace{1cm} (2)

In the infinite momentum frame, which we take to be the centre-of-mass frame of the virtual photon and the proton, the four momenta of the \(p, \gamma^*\) and \(h\) are (we put \(m_{\text{proton}} = m_{\text{hadron}} = 0\))

\[ p = \omega (1,0,0,1), \quad q = \omega (1-2x,0,0,-1), \quad p_h = \omega x_h (1,n_h) \]  \hspace{1cm} (3)

where

\[ \omega = \frac{Q}{2 \sqrt{x(1-x)}}, \quad x = \frac{Q^2}{2 p \cdot q}, \quad Q = \sqrt{-q^2} \]  \hspace{1cm} (3a)

and \(n_h\) is the unit vector for the space direction of \(p_h\). Let

\[ W^2 = (p+q)^2 \]  \hspace{1cm} (4)

be the square of the c.m. energy of the system \(\gamma^*, p, h\), then
\[ x = \frac{Q^2}{Q^2 + W^2} \]  

and the phase space for \( h \) is determined by the relation

\[ 0 \leq x_h \leq 1-x \quad \text{or} \quad \left| \vec{p}_h \right| \leq W/2. \]

The variables \( Q, W \) and the incoming lepton energy in the lab. frame, \( E_L \), determine completely the kinematics of the reactions (2).

If we choose, as the plane of the incoming and outgoing leptons, the \( z-x \) plane in the \( \gamma^*-p \) c.m. system, then their four momenta are \( 17) \): 

\textbf{Outgoing lepton} : \( e = (E', \sin \theta', 0, \cos \theta') \)  

\textbf{Incoming lepton} : \( e = (E + \omega (1-2x), E' \sin \theta', 0, E' \cos \theta' - \omega) \),

where

\[ E' = \frac{(2E_L x - Q^2 (1-x))}{2Q \sqrt{x(1-x)}} \]  

\[ \cos \theta' = \frac{(Q^2 (1-x) - 2E_L x (1-x))}{(2E_L x - Q^2 (1-x))} \]  

The differential cross-section for the processes (2) is given by the expression

\[ 4E' E_h \frac{d\sigma}{d^3 e_d^3 p_h} = \frac{2\alpha^2}{(a\cdot p)} \cdot \frac{1}{\alpha^2} Q_{\mu\nu} W^{\mu\nu}, \]

where

\[ \xi_{\mu\nu} = 2 (e'_{\mu} e_v + e_{\mu} e'_{v} - g_{\mu\nu} e' e) \]
\[
W_{\mu \nu} = \frac{1}{2} \sum_{\nu_{\text{pin}}} \sum_{\chi} \int \frac{dx}{4\pi} \, e^{ix} \langle p \mid J_{\mu}^{+}(x) \mid p_{h} \rangle \langle p_{h} \mid J_{\nu}^{+}(x) \mid p \rangle
\]

is the e.m. current tensor of the proton for the process $\gamma^* + p \rightarrow h + X$.

The sum over $X$ sums over all possible states which couple to the proton through $J_{\mu}^{+}(x)$. This current tensor can be analyzed according to its Lorentz structure,

\[
W_{\mu \nu} = g_{\mu \nu} W_1 + \frac{p_{\mu} p_{\nu}}{Q^2} W_2 + \frac{p_{\mu} p_{\nu}}{Q^2} W_3 + \frac{p_{\mu} p_{\nu}}{Q^2} W_4,
\]

where $W_i$, $i = 1, 2, 3, 4$ are Lorentz invariant functions of $q, p$ and $p_h$. We have omitted terms $\sim q^\mu$ because they contribute to the cross-section by terms proportional to the mass of the lepton. Our Lagrangian, which we assume to describe the interaction of the photon, quarks and gluons, is

\[
\mathcal{L} = -\frac{1}{4} F_{\mu \nu}^{\text{em}} F^{\mu \nu} - \frac{1}{4} F_{\mu \nu}^{A} F^{\mu \nu} + \bar{q} (i \gamma \partial \gamma) q,
\]

where

\[
F_{\mu \nu}^{\text{em}} = \epsilon_{\nu A} - \epsilon_{A \mu}^{\nu},
\]

\[
F_{\mu \nu}^{A} = \epsilon_{\nu A} A_{\mu}^{A} - \epsilon_{\mu A}^{\nu} A_{\nu}^{A} + i g f_{abc} A_{\mu}^{B} A_{\nu}^{C},
\]

\[
D_{\mu} = \epsilon_{\nu A} \frac{\partial}{\partial x^\nu} A_{\mu}^{A} - i e Q_c A_{\mu}^{h}
\]

and $A^\mu$ is the photon field, $A^\mu_a$ the gluon field, $\lambda^a/2$, $f_{abc}$ the generators and structure constants of SU(3) colour, $q$ the quark three vector in colour and three vector in flavour space. We denote by $Q_c$ the $3 \times 3$ electromagnetic charge matrix of quarks in flavour space.

Therefore we take a massless colour octet of electromagnetic neutral gluons to interact with three triplets of coloured massless quarks as the strong interaction part of the Lagrangian. We propose to estimate the
production of large $p_T$ hadrons in the $\gamma^* - p$ c.m. frame by calculating corrections to the naive proton model to first order in $g$. Therefore we have to calculate the contribution of the diagrams in Fig. 4.

We see immediately that the contribution from diagrams 4e, 4f and 4g vanishes because they are kinematically forbidden by the assumption that constituents of the proton are on their mass shell.

We denote by $p_1$ the four momenta of the initial interacting quark or gluon which fragments into the detected hadron (Fig. 2) and we separate the contribution of diagrams 4a, 4b and 4c, 4d to $W^{\mu\nu}$

$$W^{\mu\nu}_{q,g} = g^{\mu\nu} W_1 + \frac{p_1^\mu p_1^\nu}{a^2} W_2 + \frac{p_2^\mu p_2^\nu}{a^2} W_3 + \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{a^2} W_4$$

where $W_1^q, W_2^q$ are the invariant structure functions which correspond to the diagrams 4a, 4b and 4c, 4d, respectively. The application of Feynman rules for the diagrams 4a, 4b, 4c and 4d gives the following expressions for $W_1^q, W_2^q$ (we do not include the $\delta$ function for the total four momentum conservation).

$$W_1^q = -\kappa q^2 a_g \frac{(p_1 q)^2 + (2 p_1 p_2 - p_2 q - p_1 q)^2}{(p_1 p_2)(p_1 p_2 + q p_2)}$$

$$W_2^q = -2 W_3^q = -2 W_4^q = -4 \kappa a_g q^2 (p_1 q - p_2 p_2 - p_2 q) (p_1 p_2) (p_1 p_2 + q p_2)$$

$$W_1^g = -2 \kappa q^2 a_g \frac{(p_1 q)^2 + (p_1 q + 2 p_2 p_2 - p_1 q)^2}{(p_1 p_2)(p_1 p_2 - p_1 q)}$$

$$W_2^g = \frac{1}{2} W_3^g = -W_4^g = -4 \kappa a_g q^2 (p_1 q - p_2 p_2 - p_1 q) (p_1 p_2)(p_1 q - p_1 p_2)$$

where

$$\kappa = \frac{e}{\pi^2} \quad \text{and} \quad a_g = q^2/4\pi$$

In order to give the expression of our model for the full $W^{\mu\nu}$ structure tensor we need the usual assumptions of the naive parton model 4).
The nucleon is composed of nearly on-mass shell partons which are identified with the quarks, antiquarks and gluons having longitudinal momentum distributions $f_{q/p}(s)$, $f_{\bar{q}/p}(s)$, $f_{g/p}(s)$,

where: $\mathcal{P} = p$, $0 \leq s \leq 1$ \hspace{1cm} (fig. 2)

and $p$ is the momentum of the proton. The quarks, antiquarks and gluons fragment into hadrons after the interaction with fragmentation functions $f_{h/q}(s)$, $f_{h/\bar{q}}(s)$, $f_{h/g}(s)$,

where: $\mathcal{P} = p$, and $0 \leq s \leq 1$. \hspace{1cm} (fig. 2)

From these assumptions it is obvious that, to first order in $g$, the corrections to the naive parton model give contributions to $W_{\mu\nu}$ of the form:

\[
W_{\mu\nu}^{\text{corr}} = W_{\mu\nu} - W_{\mu\nu}^{\text{parton}} = \int_{\mu}^1 \frac{d\xi}{\xi} \int_{\xi_{\text{min}}}^1 \frac{d\bar{s}}{\bar{s}} \left[ \frac{1}{\xi} \sum_{i=1} Q_i^2 f_{q_i/P}(\xi) W_{q_i}^{\mu\nu} \cdot f_{h/q_i}(s) \delta_{+}(\mathcal{P} + q - \mathcal{P}) \right]
\]

where $\xi_{\text{min}} = \left( \frac{p_i}{p_h} \right)_{\text{max}}$ and $Q_i$ is the charge of the $i^{\text{th}}$ quarks or antiquarks in units of e.

At this stage we do not want to study quantum number effects and to complicate the assumptions about the distributions $f_{q/P}, f_{h/q}, \ldots$. Therefore we take a global distribution for quarks and antiquarks $f_{q/P}(s) = \sum_{i} Q_i^2 f_{q_i/P}(s)$, which we identify with $1/\xi [w_{2}(s)]$ as measured in $e^+e^-\rightarrow h+X$. Also we make the simplifying hypothesis that all quarks and gluons have the same fragmentation function into hadrons $h$ which we take to be pions. So we accept the following relation,

\[
f_{h/q_i}(s) = f_{h/q}(s)
\]

and we identify this function with $1/\xi [w_{2}(s)]$ as measured in $e^+e^-\rightarrow h+X$. 

\[\tag{12}
\]
Finally, carrying out the $\xi$ integration in (12), we obtain

$$W_1 - W_{1,PM} = W_{1,c} = \left[ \frac{1}{2} \int_{s_{\text{min}}}^{s} \left[ f_{q/p}(\xi_o) W_q^9 + f_g(\xi_o) W_g^9 \right] f_{h/q}(\xi) \right] / (2 p_1 \cdot q - s q)$$  \hspace{1cm} (13)$$

$$W_2 - W_{2,PM} = W_{2,c} = \left[ \frac{1}{2} \int_{s_{\text{min}}}^{s} \left[ f_{q/p}(\xi_o) W_q^9 + f_g(\xi_o) W_g^9 \right] f_{h/q}(\xi) \right] / (2 p_1 \cdot q - s q)$$  \hspace{1cm} (13a)$$

$$W_3 - W_{3,PM} = W_{3,c} = \left[ \frac{1}{2} \int_{s_{\text{min}}}^{s} \left[ f_{q/p}(\xi_o) W_q^9 + f_g(\xi_o) W_g^9 \right] f_{h/q}(\xi) \right] / (2 p_1 \cdot q - s q)$$  \hspace{1cm} (13b)$$

$$W_4 - W_{4,PM} = W_{4,c} = \left[ \frac{1}{2} \int_{s_{\text{min}}}^{s} \left[ f_{q/p}(\xi_o) W_q^9 + f_g(\xi_o) W_g^9 \right] f_{h/q}(\xi) \right] / (2 p_1 \cdot q - s q)$$  \hspace{1cm} (13c)$$

where

$$\xi_o = \left( \frac{2 p_1 \cdot q - s q}{2 s (p q - p_1 p) \xi} \right)$$  \hspace{1cm} (13d)$$

and $W_q^9$, $W_g^9$ are given by the relations (11) - (11c) with $p_1 = \xi_0 p$ and $p_2 = p_{1/q}$. In the naive parton model \textsuperscript{20} only two of the $q, p, p_{1/q}$ vectors are linearly independent. We take these to be $q, p$ and we omit terms proportional to $q_{\mu}$ because when contracted with $\xi_{\mu\nu}$ they give contributions proportional to the mass of the lepton which we assume to be zero. So finally,

$$W_{PM}^{h\nu} = (- q_{\mu\nu} W_{1,PM} + p_{1\mu} p_{1\nu} W_{2,PM} ) f^{c} (p_{T}^2, W),$$  \hspace{1cm} (14)$$

where we have neglected the proton mass and put in by hand a factorized $p_T$ dependence.
Here

\[ W_{3,p,m} = \frac{1}{x} \int f_{q/p}(x) \int f_{h/q}(s) \]

(14a)

\[ W_{4,p,m} = x \int f_{q/p}(x) \int f_{h/q}(s) \]

(14b)

where

\[ x = \frac{-q^2/2p \cdot q}{s} \quad \tau = \left| \frac{p_{h,m}^2}{p_{h,m}^2} \right| = 2 \left| \frac{p_{h,m}^2}{\Lambda} \right. \]

(14c)

From expressions (14a)-(14b) we see that we have again oversimplified the dependence on the quantum numbers. Also the \( W_3, W_4 \) terms vanish

\[ W_{3,p,m} = W_{4,p,m} = 0 \]

We shall discuss the function \( f(p_x^2, \Lambda) \) in Section 3.

Having made the necessary definitions and assumptions, we can now turn to the determination of the kinematical region in which we believe that single gluon bremsstrahlung may have some relevance.

3. **Large \( p_T \) Hadrons and Gluon Bremsstrahlung**

In order to obtain some quantitative comparison with experiment we have to specify the distribution functions \( f_{q/p}, f_{g/p} \) and \( f_{h/g} \).

As we already mentioned in Section 2 we identify \( f_{q/p} \) with a fit to \( 1/\xi[\omega_2(\xi)] \) which is measured in ep experiments \( ^{18} \). Therefore we take \( f_{q/p} \) to be

\[ \xi f_{q/p}(\xi) = (1-\xi)^3 \left[ 1.274 + 0.985 (1-\xi) - 1.675 (1-\xi)^2 \right] \]

(15)

For the fragmentation function \( f_{h/g} \) we take the form adopted in Ref. 7) which is an enveloping curve of the \( e^+e^- \) data for \( 1/\xi[\omega_2(\xi)] \) \( ^{19} \),
\[ S_{hl}(\xi) = 2.2 (1-\xi)^3 + 0.25 (1-\xi). \]  

(16)

For the distribution of gluons inside the proton, nothing is known experimentally except from the missing momentum constraint 21)

\[ \int_0^1 d\xi \xi f_{g/p}(\xi) = 0.48. \]  

(17)

There are some theoretical arguments that gluons must be related to the sea quarks and so their longitudinal momentum distribution must be concentrated around \( \xi \sim 0 \) like \( \sim c/\xi \), and it must fall faster than the valence quarks distribution for \( \xi \sim 1 \) 14).

We adopt, in this work, the trial form

\[ \xi f_{g/p}(\xi) = 2.88 (1-\xi)^5, \]  

(18)

which satisfies the above criteria and the constraint (17). We shall compare our predictions with the ep experiment of the Cornell-Harvard Collaboration 16a), 22) performed at Cornell because of the more precise characterization of their kinematical region, although other experiments 23)-25) agree quite well with this one. In this experiment, among other things, inclusive virtual photoproduction of pions is observed in the plane transverse to the direction of the photon momentum in the \( ^+ \gamma - p \) c.m. frame, and their \( p_T \) distribution is measured. Giving an over-all parametrization of the normalized cross-section in the form

\[ F_{ex} = \frac{2E_n}{\sigma_T} \frac{d^3\sigma}{d^3p_n} = \frac{\varphi(x_T)}{Q^n W^n}, \]  

(19)

in order to check predictions of dimensional quark counting rules 26), they find that \( n \) turns out to be dependent on \( x_T \) and \( m \approx 0 \). In the region \( 1.2 < Q^2 < 3.6 \) and \( 2.2 < W < 3.1 \) they give the following expression for \( n \),

\[ n(x_T) = 13.41 x_T^{1.71}. \]  

(20)
where

\[ x_T = \frac{2p_T}{W}. \]

Unfortunately, they do not give the function \( f(x_T) \), but we can extract this from their exponential fit of the data at \( W = 3.1, Q^2 = 1.2, \)

\[ F_{ex}(W = 3.1, Q^2 = 1.2) = e^{-5.3 \cdot p_T^2} \quad \text{for} \quad p_T^2 > 2.2 \ \text{GeV}^2/c^2 \]  

(21)

Finally, we obtain

\[ F_{ex}(x_T, W) = e^{-n(x_T) \log(W/3.1) - 7.2 x_T^2} \]  

(22)

Now our \( F \) is defined as

\[ F_{TH} = 4 E'_E E'_h \frac{\alpha_s^2}{\delta^3 e'_d} \int \frac{\delta^3 \sigma}{\delta^3 e'_d} \]  

(23)

where the index \( \phi \) means that \( \phi \) integration, for the azimuthal angle of the hadron \( h \), has been carried out and \( \int \delta^3 \sigma / \delta^3 e' \) is the total cross-section for finding a hadron in the observation plane. This cross-section is given by:

\[ 2E' \frac{\delta^3 \sigma}{\delta^3 e'} = \frac{2\alpha^2}{(ep)} \frac{1}{Q^4} \int \frac{\delta^3 \sigma}{\delta^3 e'_d} W_{\nu\nu}^{\nu} = \sigma_{TOT} \]  

(24)

\( W_{\nu\nu}^{\nu} \) is given in our model as

\[ W_{\nu\nu}^{\nu} = \int \frac{\delta^3 p_h}{2p_h} \mathcal{S} (P_{hV}) (W_{hM}^{\nu} + W_{hC}^{\nu}). \]  

(25)

In relation (25) we put \( p_{hV} \) of the gluon equal to zero, corresponding to 90° in the \( \gamma^* - p \) c.m. system. Because of the vanishing of the masses of the quarks and gluons in this integration, we have a kinematical infrared singularity. We avoid this difficulty by choosing a cut-off for \( p_T \) \( (p_T)_{\text{min}} = 0.7 \ \text{GeV}/c, \) we do not in any case believe that our model has any
meaning below this value. From our numerical estimations we find that the normalized cross-section $F$ is not strongly dependent on this cut-off.

Now we separate the contribution from the parton model and the corrections to it:

$$F_{P,M}^{\text{un}} \equiv \int_{0}^{\pi_{0}} d\phi_{\perp} \, e_{k} \, W_{P,M}^{\nu} / \sigma_{\text{tot}}$$

$$F_{\text{cor}}^{\text{un}} \equiv \int_{0}^{\pi_{0}} d\phi_{\perp} \, e_{k} \, W_{\text{cor}}^{\nu} / \sigma_{\text{tot}}$$

Moreover, we assume that the $f(F_{P,M}^{2}, W)$ function in relation (14) is the same as $f(x_{0})$ in relation (22). We do this in order to adjust a correct normalization in the region of present experimental data, and as we said this is in accordance with the jet-wise fragmentation of quarks. Also we can have, in this way, estimates of the unnormalized cross-sections.

We take $W = 3.1$, $Q^{2} = 1.2$ as the point of normalization, and we find a normalization factor 0.35:

$$F_{P,M} = 0.35 F_{P,M}^{\text{un}}$$

$$F_{\text{cor}} = 0.35 F_{\text{cor}}^{\text{un}}$$

$$F_{\text{TH}} = F_{P,M} + F_{\text{cor}}$$

We plot $F_{\text{cor}}$, $F_{\text{TH}}$ and the extrapolation of $F_{\text{exp}}$ in the kinematical regions (W in GeV, Q in GeV/c), a) $W = 6$, $Q = 2$, b) $W = 6$, $Q = 6$, c) $W = 6$, $Q = 10$, d), e) $W = 10$, $Q = 2, 10$, f), g) $W = 20$, $Q = 10, 20$ (Figs. S5a-5g).

In all cases we choose the value $\epsilon_{\gamma} = 0.95$ for the polarization of the photon,

$$\epsilon_{\gamma} = \left[ 1 + 2 \tan^{2} \theta / (1 + \theta^{2}) \right]^{-1}.$$
We take as the coupling constant of the quarks and gluons the running coupling constant \( \alpha_q \) for SU(3)\text{colour} \times SU(3)\text{flavour},

\[
\alpha_q = \frac{12\pi}{27 \log (Q^2/\Lambda^2)},
\]

(32)

where \( \Lambda = 0.2 \text{ GeV}/c \).

We observe that for \( W > 10 \text{ GeV} \) and \( P_T^2 > 9 \text{ GeV}^2/c^2 \) the dominance of \( P_{\text{cor}} \) over \( P_{\text{exp}} \) (extrapolated) is visible, but the effect, if it exists, is a very small portion \( (10^{-6} - 10^{-5}) \) of the cross-section for \( e^+ e^- \pi \) in the same \( W, Q^2 \) region. Perhaps it will be observable in forthcoming experiments such as the \( \mu \) experiment at the SPS (CERN) where \( W, Q^2 \) reach large values, \( W^2_{\text{max}} \sim 250 \text{ GeV}^2, Q^2_{\text{max}} \sim 250 \text{ GeV}^2/c^2 \).

4. CONCLUSIONS

In this work we have shown that the contribution, from single gluon bremsstrahlung of quarks to the \( P_T \) distribution of inclusive virtual photoproduction of hadrons, becomes important relative to the extrapolation of the present experimental data in the region \( W > 10 \text{ GeV} \) and \( P_T^2 > 9 \text{ GeV}^2/c^2 \).

It would be of interest to find the kinematical region in which the three-jet structure of virtual photoproduction will be clearly visible \(^7\), because only then will we have a real test for the existence of gluons.

This problem, as well as the question of the contribution to scaling violations of the gluon bremsstrahlung mechanism, is under study.

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Fig. 1: Naive parton model picture for $\gamma^* + p \rightarrow h + X$.

Fig. 2: Modified naive parton model picture for $\gamma^* + p \rightarrow h + X$.

Fig. 3: Three-jet picture in $\gamma^* - p$ c.m. system.

Fig. 4: First-order diagrams in $g$ in $\gamma^* - p$ interaction.

Fig. 5: The continuous line represents $F_{\text{COR}}$, the dotted bar line the $F_{\text{TH}}$, and the dotted line the extrapolation of $F_{\text{EXP}}$ at the kinematical points:

$(W = 6, Q = 2, 6)$, $(W = 6, Q = 10)$, $(W = 10, Q = 2, 10)$, $(W = 20, Q = 10, 20)$. 