CERN-PRE-77-145

CERN LIBRARIES, GENEVA

CM-P00064267

AMPLITUDE AND NATURAL-PARITY-EXCHANGE ANALYSIS
OF $K^{\pm}p \rightarrow (K\pi)^{\pm}p$ DATA AT 10 GeV/c

A.D. Martin and T. Shimada

University of Durham, Durham City, United Kingdom

R. Baldi, T. Böhringer, P.A. Dorsaz, V. Hungerbühler,

M.N. Kienzle-Focacci, M. Martin, A. Mermoud,

C. Nef and P. Siegrist

University of Geneva, Switzerland

ABSTRACT

High statistics data for the reactions $K^{\pm}p \rightarrow K_S^0\pi^{\pm}p$ at 10 GeV/c are analysed. The $K^*(1^-)$, $K^*(2^+)$, and $K^*(3^-)$ resonance parameters and production cross-sections are calculated. The $K\pi$ production amplitudes are determined as a function of t and the produced $K\pi$ mass. Isoscalar natural-parity-exchange (NPE) is dominant. The t dependence of the $K^{\pm}$ NPE amplitudes have a cross-over at $t = -0.3 \text{ (GeV/c)}^2$ for both $K^*(890)$ and $K^*(1420)$ production, being more pronounced for the $K^*(1420)$. Natural-parity-exchange interference effects are isolated. The NPE amplitudes are decomposed into pomeron, f , and ω -exchange contributions. S-wave $K\pi$ production is found to be consistent with the $K\pi$ partial wave analyses of charge-exchange reactions.

Geneva - 4 October 1977

(Submitted to Nuclear Physics B)

2.1 K π mass spectra

Figures 1 and 2 show the K π mass spectra, in the t range accepted by the spectrometer, $0.05 < -t < 1$ (GeV/c)². For comparison we show the spectra both before (histogram) and after (points with error bars) acceptance correction. The dominant feature of the data is the production of the K*(890) and K*(1420) resonances above a very low background level.

We fit the mass spectra in the interval $0.74 < M_{K\pi} < 2$ GeV with three Breit-Wigner shapes plus a background, parametrized by a second-order polynomial weighted by an exponential in K π mass. We employ a relativistic Breit-Wigner form with an energy-dependent width taking into account centrifugal barrier effects [2]:

$$\frac{d\sigma}{dM} \sim \left| \frac{\sqrt{\Gamma}\Gamma_R}{M_R^2 - M^2 - iM_R\Gamma} \right|^2 \quad \text{with} \quad \Gamma = \Gamma_R \frac{M_R}{M} \left(\frac{q}{q_R} \right)^{2L+1} \frac{D_L(q_R R)}{D_L(qR)}, \quad (1)$$

where

M_R, Γ_R are the resonance mass and width,

q, q_R are the centre-of-mass momentum at M and M_R , respectively,

R is an interaction radius, taken to be 3.5 GeV^{-1} , and

D_L are centrifugal barrier factors (explicitly $D_1(z) = 1 + z^2$,

$D_2(z) = 9 + 3z^2 + z^4$, $D_3(z) = 225 + 45z^2 + 6z^4 + z^6$).

The fit is shown in Figs. 1 and 2 and the parameters are summarized in Table 1. The small, but significant, shift in mass between the K⁺ and K⁻ induced reactions can be attributed to a systematic uncertainty in the calibration of the absolute value of the incident momentum. From the analysis of elastic Kp $\rightarrow$ Kp events, we estimate the systematic error of the mass scale to be $\Delta M_{K\pi} = \pm 4 \text{ MeV}$.

In Figs. 1b and 2b we show the distribution of events remaining after subtraction of the fitted K*(890) and K*(1420) resonance shapes from the data. Between the two resonances there remains a broad, featureless distribution with a maximum around 1200 MeV. The effect is much larger in the K⁺ than in the K⁻ data. We have

1. INTRODUCTION

Here we discuss the physical implications of the high statistics data for the reactions $K^{\pm}p \rightarrow K_S^0\pi^{\pm}p$ at 10 GeV/c that were presented in the preceding paper [1]. These data allow a systematic study of the $K\pi$ production mechanisms as a function of mass and t in the $K^*(890)$ and $K^*(1420)$ resonance regions. The production of K^* resonances by these non-charge-exchange reactions is dominated by isoscalar NPE. In particular, the data are especially interesting for studying pomeron exchange, since if it were an SU(3) singlet it would decouple from $K^*(890)$ production, but not from $K^*(1420)$ production. Moreover, a simultaneous study of the K^+ and K^- initiated reactions allows a separation of even and odd signature exchange.

The dominance of NPE in the reactions $K^{\pm}p \rightarrow (K\pi)^{\pm}p$ allows the P-D interference in the NPE amplitudes to be isolated and studied. The data for these processes are complementary to K^* production by charge-exchange (CEX) reactions, such as $K^-p \rightarrow \bar{K}^{*0}n$, which were found to be dominated by (π, B) unnatural-parity exchange.

The outline of the paper is as follows. We begin by discussing the general features of the data. Then in Section 3 we present the results of the amplitude analyses as a function of t in the resonance regions; and a function of $K\pi$ mass, paying particular attention to NPE and the interference effects. We conclude this section by describing S-wave $K\pi$ production. Exchange mechanisms are discussed in Section 4. The amplitudes are decomposed into their exchange contributions so that their main features (such as the cross-over effect and NPE interference effects) are reproduced. Finally in Section 5 we give our conclusions.

2. GENERAL FEATURES OF THE REACTION $K^{\pm}p \rightarrow K_S^0\pi^{\pm}p$ AT 10 GeV/c

Using the double-arm spectrometer described in the preceding paper [1], we have obtained a sample of 46,450 events in the channel $K^+p \rightarrow K_S^0\pi^+p$ and 17,960 events in the $K^-p \rightarrow K_S^0\pi^-p$ reaction. In this section we discuss the qualitative features of the data.

This is indeed expected if the production mechanism is mainly isoscalar NPE (pomeron, ω , f). The $K^{*+}(890)$ cannot be fitted by an exponent linear in t' only, but requires a quadratic term as well. The fitted parameters are given in Table 2.

The striking feature of these data is the difference between K^+ and K^- induced reactions: both the $K^*(890)$ and $K^*(1420)$ show a cross-over effect at the same value of t , $t_c = -0.30 \pm 0.06$ (GeV/c)² (systematic normalization error included). In both cases, the K^- induced reaction has a larger cross-section at small $|t|$ and a steeper slope. The effect is more pronounced in the case of the $K^*(1420)$ than for the $K^*(890)$. A difference in slope is present also in the intermediate region between the two resonances.

3. $K^\pm p \rightarrow (K\pi)^\pm p$ AMPLITUDES

The explicit relations between the $K\pi$ production amplitudes and the moments of the $K\pi$ angular distribution are given elsewhere [4]. We use L_0 , $L_\pm$ to denote the amplitudes for spin L $K\pi$ production, where L_0 describes helicity-zero production and, to leading order in the energy, $L_\pm \equiv (L_{\lambda=1} \pm L_{\lambda=-1})/\sqrt{2}$ describe helicity-one production by natural- and unnatural-parity exchange (NPE and UPE), respectively. Since the t -channel moments with $M = 3, 4$ are consistent with zero we assume that t -channel helicity-two production is negligible. There is also no evidence for higher than D-waves in the $K\pi$ mass region up to 1.5 GeV, as the $J > 4$ moments are compatible with zero.

The data allow the amplitudes to be determined both as a function of t and as a function of M^2 , where M is the mass of the produced $K\pi$ system. Both the $K^\pm$ reactions are dominated by NPE and a particularly informative observable is the interference effect between the P_+ and D_+ amplitudes. This is found to have a significantly different behaviour in the K^- and K^+ initiated reactions, both as a function of t and of M^2 .

3.1 t dependence

Amplitude analyses have been performed [4] in both the $K^*(890)$ mass region ($0.84 < M < 0.94$ GeV) and the $K^*(1420)$ mass region ($1.34 < M < 1.5$ GeV). We may

checked that the difference is not due to a kinematic reflection of baryon resonance production at the proton vertex, or to contamination by other reactions.

In the higher mass region, there is a clear $K^*(1780)$ signal. Mass and width of the $K^*(1780)$ are compatible with the values obtained in a previous analysis [3], by fitting a Breit-Wigner form to the $\langle Y_6^2 \rangle$ moment, which is free of background from lower partial waves. No other significant structures are evident above the $K^*(1780)$ in the present data.

To estimate the production cross-sections of the K^* resonances, we use the fitted number of events in the resonance peaks. We correct for the fraction of events outside the t acceptance of the spectrometer, using the differential cross-section parametrizations of Fig. 3. The total cross-sections for the channels

$$K^\pm p \rightarrow K^{*\pm} p$$

$$\quad \quad \quad \downarrow \quad \quad \quad (-)$$

$$\quad \quad \quad \rightarrow K^0 \pi^\pm$$

are given in Table 1. Note that we do not correct, throughout the paper, for the unseen $K^\pm \pi^0$ decay mode. We find equal cross-sections for resonance production by K^+ and K^- beams at 10 GeV/c.

2.2 Momentum-transfer distributions

Figure 3 shows the momentum-transfer distributions $d\sigma/dt$ in the mass regions of the $K^*(890)$ ($0.84 < M < 0.94$ GeV), the $K^*(1420)$ ($1.36 < M < 1.50$ GeV), the $K^*(1780)$ ($1.68 < M < 1.88$ GeV) and in an intermediate region ($1.1 < M < 1.3$ GeV). Except for this last region, the shape of the distributions is more complex than a simple exponential. We have used the parametrization

$$\frac{d\sigma}{dt} = A t'^n \exp (B t' + C t'^2) \quad \text{with} \quad t' = t - t_{\min}$$

to fit the data. The factor of t' is needed to account for the observed forward dip. It indicates that one unit of net helicity flip is present in the reaction.

below the assumptions made in different regions; we shall see these mainly concern the UPE sector. We should add that, since the NPE amplitudes dominate, the results are relatively insensitive to the specific forms assumed for the UPE amplitudes.

Region I. In the $K^*(1420)$ region we use the observed moments, $\langle Y_J^M \rangle$ with $J \leq 4$, $M \leq 2$, to determine the magnitudes of the five amplitudes (D_+ , P_+ ; D_0 , D_- , S), the PD coherence for NPE, $(\xi \cos \phi)_{P_+D_+}$, and the SD coherence for UPE, $(\xi \cos \phi)_{SD}$. The coherences are defined such that

$$\text{Re } (P_+ D_+^*) = |P_+| |D_+| (\xi \cos \phi)_{P_+D_+} \quad (3)$$

$$\text{Re } (SD_0^*) = |S| |D_0| (\xi \cos \phi)_{SD} \quad (4)$$

$$\text{Re } (SD_-^*) = -|S| |D_-| (\xi \cos \phi)_{SD}$$

To obtain the last result we assume that D_0 and D_- satisfy Eq. (2). We neglect the amplitudes P_0 and P_- and do not fit to the $\langle Y_1^1 \rangle$ and $\langle Y_3^1 \rangle$ moments. The same assumptions were made in the t -dependent amplitude analysis in the $K^*(1420)$ region [4].

The data in regions II and III do not allow a determination of $|D_+|$. On the other hand, we will need $|D_+|$ to estimate the NPE coherence $(\xi \cos \phi)_{P_+D_+}$ of Eq. (3). We therefore fit $|D_+|$ in region I to a Breit-Wigner form and extrapolate this form into regions II and III.

Region II. In each 20 MeV bin in the $K^*(890)$ region we use the moments $\langle Y_0^0 \rangle$, $\langle Y_1^0 \rangle$, $\langle Y_2^{2,1,0} \rangle$, and $\langle Y_3^{2,0} \rangle$ to determine the magnitudes of P_+ , P_0 and P_- , and the NPE coherence $(\xi \cos \phi)_{P_+D_+}$. We include S_0 and D_0 by assuming π exchange is appropriate and by relating them to P_0

$$S_0:P_0:D_0 = (\sin \delta_S^1 e^{i\delta_S^1} - \sin \delta_S^3 e^{i\delta_S^3}) : \sqrt{3} \sin \delta_P^1 e^{i\delta_P^1} : \sqrt{5} \epsilon \sin \delta_D^1 e^{i\delta_D^1}, \quad (5)$$

where δ_L^{2I} are the $K\pi$ phase shifts for spin L and isospin I . The S-wave phases are taken from the parametric forms of Estabrooks et al. [6] and the P- and D-wave phases are given by Breit-Wigner forms for the $K^*(890)$ and $K^*(1420)$. The elasticity ϵ of the $K^*(1420)$ is taken to be 0.5.

summarize the results as follows. For both the $K^\pm p \rightarrow (K^*)^\pm p$ reactions NPE dominates; that is P_+ is the dominant $K^*(890)$ production amplitude and D_+ is dominant for $K^*(1420)$ production^{*)}. The t dependence of the NPE amplitudes is shown in Fig. 4. We note the $K^\pm$ cross-over at $t \approx -0.3 \text{ GeV}^2$ for both $K^*(890)$ and $K^*(1420)$ production, the K^- amplitude being larger than the K^+ near the forward direction. The interference between the resonant D_+ and non-resonant P_+ in the $K^*(1420)$ region is also shown in Fig. 4.

The amplitudes in the UPE sector are given in Ref. 4. Here it suffices to say that the magnitude and t structure of the UPE amplitudes (L_0, L_-) are in agreement with that expected from π and B exchange as given by an analysis [5] of data for the CEX reaction $K^- p \rightarrow (K^- \pi^+) n$. This is true for both the $K^*(890)$ and $K^*(1420)$ production. In particular, the data are consistent with L_0 , and L_- coherence found for such exchanges, namely

$$\text{Re} (L_0 L_-^*) = - |L_0| |L_-| . \quad (2)$$

3.2 $K\pi$ mass dependence

Here we determine the $Kp \rightarrow (K\pi)p$ amplitudes as a function of mass using data in 20 MeV intervals in the range $0.78 < M < 1.52 \text{ GeV}$. The data are shown in Figs. 10 and 11 of Ref. 1 and cover the t interval $0.1 < -t < 0.4 \text{ (GeV/c)}^2$. We see the dominance of P_+ and D_+ in the $K^*(890)$ and the $K^*(1420)$ mass regions, respectively. Further the mass dependence of the $P_+ - D_+$ interference manifest in the data (in particular in the $\langle Y_3^0 \rangle$ and $\langle Y_3^2 \rangle$ moments) is seen to be different for the K^+ and K^- reactions. To carry out the amplitude analysis we divide the mass interval into three regions:

- i) $1.30 < M < 1.52 \text{ GeV}$: the $K^*(1420)$ mass region;
- ii) $0.78 < M < 1.0 \text{ GeV}$: the $K^*(890)$ mass region;
- iii) $1.0 < M < 1.30 \text{ GeV}$: the region between the resonances.

We then use the experimental moments to determine the dominant amplitudes in each region, inputting prescribed forms for the smaller amplitudes. We discuss

^{*)} D_0 is comparable to D_+ for $K^*(1420)$ production for $-t \lesssim 0.1 \text{ GeV}^2$.

One interesting feature is the significantly different mass dependence of $|P_+|$ for the K^+ and K^- reactions. This can be seen by comparing the values of $P_+(K^+)$ of Fig. 5 with the dashed line which represents the average of the values of P_+ for the K^- reaction (Fig. 6). At the $K^*(890)$ resonance $P_+(K^-)$ is larger than $P_+(K^+)$, for the t interval $0.1 < -t < 0.4$ (GeV/c)², whereas when one reaches $M = 1.2$ GeV, $P_+(K^-)$ is less than half $P_+(K^+)$. The different behaviour found for the background to the $K^*(890)$ and $K^*(1420)$, as given in Figs. 1 and 2, can be mainly attributed to this P-wave effect.

The values obtained for the spin phase coherence of the NPE amplitudes, $(\xi \cos \phi)_{P_+D_+}$ of Eq. (3), are also shown in Figs. 5 and 6. In the $K^*(1420)$ region (region I) the data are able to determine $|P_+|$, $|D_+|$, and $(\xi \cos \phi)_{P_+D_+}$. The range of values of $\xi \cos \phi$ indicates that the nucleon spin coherence ξ is compatible with one. This is to be expected as the dominant^{*} NPEs (the pomeron, ω , and f) each couple predominantly to the single helicity-flip amplitudes, with non-flip at the nucleon vertex.

As mentioned previously, in regions II and III the data cannot determine $|D_+|$ and $(\xi \cos \phi)_{P_+D_+}$ separately, but only the product. The value of $(\xi \cos \phi)_{P_+D_+}$ that is plotted in these mass regions ($M < 1.3$ GeV) corresponds to inputting $|D_+|$ as given by the low-energy tail of the $K^*(1420)$ resonance form of Table 3.

The phase of each of the $Kp \rightarrow (K\pi)p$ amplitudes can be written as the sum of the $K\pi$ production (Regge) phase, $\theta(L_\lambda)$, and the decay phase, δ_L . Thus

$$\phi_{P_+D_+} = \delta_P - \delta_D + \theta(P_+) - \theta(D_+) \quad (7)$$

should manifest Breit-Wigner phase behaviour, arising from the appropriate δ_R , over each resonance region. If we assume the only mass dependence of $\phi_{P_+D_+}$ comes from this resonant phase then we may fit $(\xi \cos \phi)_{P_+D_+}$ to the form $\alpha \cos(\delta_R - \Delta)$. The curves for $\xi \cos \phi$ on Figs. 5 and 6 are the result of such fits to the coherence; the values of the parameters α and Δ are given in Table 4. We set $\alpha = 1$ in the

^{*}) The isovector NPEs, ρ , and A_2 , are known⁴⁾ to be relatively small in $K^\pm p \rightarrow (K^*)^\pm p$.

Region III. In the region between the resonances we use the moments $\langle Y_{3,2,1,0}^0 \rangle$, $\langle Y_{3,2}^2 \rangle$ to determine the magnitudes of S and P_+ , and the NPE coherence $(\xi \cos \phi)_{P_+D_+}$. For the magnitudes of the UPE amplitudes we use the ratios^{*)} determined in the respective resonance regions

$$\begin{aligned} |P_0| &= 0.22|P_+|, \quad |P_-| = 0.11|P_+| \\ |D_0| &= 0.35|D_+|, \quad |D_-| = 0.17|D_+|, \end{aligned} \quad (6)$$

and the relative phases in the UPE sector are determined by Eqs. (2) and (5).

The main results of these amplitude analyses as a function of mass are shown in Figs. 5, 6 and 7. The consistency of the values of the amplitudes in going from one mass region to another suggests that the input assumptions are reasonable. We discuss the results in the NPE and UPE sectors in turn.

3.3 NPE amplitudes as a function of mass

The NPE amplitudes (P_+ and D_+) for the K^+ and K^- initiated reactions are shown in Figs. 5 and 6 respectively. The $K^*(890)$ and $K^*(1420)$ resonances are clearly evident and, for each reaction, we fit $|P_+|$ and $|D_+|$ to the Breit-Wigner resonance form of Eq. (1)^{**)} . The masses and widths of the resonances are given in Table 3. A slight difference between these values and those of Table 1 is expected because of the different treatment of non-resonant effects. In particular, allowance for the structure in the S-wave in the $K^*(1420)$ region tends to decrease $\Gamma_R(1420)$ and increase $M_R(1420)$. As mentioned in Section 2, the shift in resonance mass between the K^+ and K^- initiated reactions can be attributed to uncertainties in incident momentum calibration. If the numerator of the Breit-Wigner form of Eq. (1) were taken to be Γ , a form appropriate to K^* production by π exchange at small t , then the resonant masses M_R are decreased by about 3 MeV, whereas the widths Γ_R remain essentially unchanged.

*) This is a reasonable assumption since the amplitudes are most important at the end of the interval nearest the respective resonance.

**) In these fits the interaction radius was taken to be 5 GeV^{-1} and a factor M_R/M was omitted from Γ .

We have normalized $|S_0|$ in this region by fitting $|D_0|$ to a $K^*(1420)$ resonance form of elasticity $\epsilon = 0.5$, see Eq. (5). The decrease of $|S_0|$ as the mass increases towards 1.5 GeV is in agreement with the $\kappa(1450)$ state proposed by Ref. 7. It is due to cancellation of the $I = 1/2$ and $I = 3/2$ amplitudes.

The UPE amplitudes are not so well determined for the K^- initiated reaction. The mass structure is compatible with that shown in Fig. 7, but the amplitude magnitudes are on an average somewhat decreased.

4. EXCHANGE MECHANISMS

In Ref. 4 we showed that $K^*(890)$ and $K^*(1420)$ production were dominated by NPE. The amplitudes in the UPE sector (L_0, L_-) were found to have magnitudes and t structure expected from π -B exchange as given by an analysis of CEX K^* production data. On the other hand, we concluded NPE was dominated by t -channel $I = 0$ exchanges (pomeron, f , ω), since comparison with the CEX analysis shows the $I = 1$ exchanges (ρ, A_2) to be relatively small. Such NPE exchanges couple dominantly to the single-helicity-flip amplitudes and give rise to the observed $\sqrt{-t}$ behaviour.

In this section we study the decomposition of the NPE amplitudes, P_+ and D_+ , both as a function of t and as a function of the produced $K\pi$ mass. To avoid confusion with the charge, we shall omit the NPE subscript in this section. For example, we will use $P(K^\pm)$ to denote the amplitudes describing P-wave $(K\pi)^\pm$ production by NPE.

4.1 The cross-over effect

The t structure of the NPE amplitudes for the $K^\pm$ initiated reactions is shown in Fig. 4. For both $K^*(890)$ and $K^*(1420)$ production there is a cross-over at $-t \approx 0.3 \text{ GeV}^2$, the cross-over being more enhanced for $K^*(1420)$ production.

Consider first $K^*(890)$ production. We seek to decompose the NPE amplitudes for the reactions $K^\pm p \rightarrow K^*(890)^\pm p$ into their exchange contributions^{*)}

$$P(K^\pm) = P + f \mp \omega . \quad (8)$$

) An additional over-all $\mp$ sign is often included in Eq. (8), see for example Michael [9]. However, this is cancelled by a $\mp$ sign arising from the $K^(890) \rightarrow (K\pi)^\pm$ decay. For clarity we omit both sets of $\mp$ signs. This has no observable effect.

$K^*(1420)$ region, since we expect the NPE amplitudes to be essentially nucleon spin coherent ($\xi = 1$). The inequality $\alpha \neq 1$ in the $K^*(890)$ region is a measure of the departure of $|D_+|$ from the extrapolated Breit-Wigner form of the $K^*(1420)$ resonance. We discuss the implications of the observed $P_+ - D_+$ interference in Section 4.

3.4 UPE amplitudes as a function of mass

We have seen that K^* production is dominated by NPE [4], and consequently the amplitudes in the UPE sector are, in general, not so well determined by the data. In Fig. 7 we show the magnitudes of the helicity-zero $K\pi$ production amplitudes as a function of mass for the K^+ initiated reaction. Helicity-one $K\pi$ production by UPE is even smaller; the values of $|P_-|$ and $|D_-|$ found in the respective resonance regions are given by Eqs. (6), namely $|L_-| \approx 0.5|L_0|$.

We have previously shown [4] that, in the $K^*(890)$ and $K^*(1420)$ resonance regions, the UPE amplitudes are in agreement, both in magnitude and t structure, with the expectations of π -B exchange. It is therefore possible to use S_0 , P_0 and D_0 to extract information concerning $K\pi$ phase shifts, see Eq. (5). The S-wave is of particular interest since it is sizeable throughout the mass region under study.

The main source of information about $K\pi$ scattering is the one-pion-exchange reactions $K^\pm p \rightarrow K^\pm \pi^+ n$ (or $K^\pm p \rightarrow K^\pm \pi^- \Delta^{++}$). In the recent analysis [6,7] of the high statistics 13 GeV/c SLAC data, the $I = \frac{1}{2}$ S-wave was found to rise monotonically to 90° in the region of $M = 1.3$ GeV and then to increase rapidly*) in the 1.4 to 1.5 GeV region. It was thus concluded that this was evidence of an S-wave resonance with a mass of 1.4-1.45 GeV and width 200-300 MeV, which we denote by $\kappa(1450)$. It is very elastic in all but one of the possible partial wave solutions. The $I = \frac{3}{2}$ $K\pi$ S-wave was found negative and to decrease monotonically to about -30° at 1.5 GeV.

The magnitude and mass dependence of $|S_0|$ of Fig. 7 are consistent with this behaviour of the $K\pi$ S-waves. This can be seen explicitly by the continuous curve through the values of $|S_0|$, which is obtained by taking the $K\pi$ S-waves shown in Fig. 8. Figure 8 assumes that the S-waves are elastic with phases found in Ref. 7.

*) See also Bowler et al. [8].

The decomposition is similar for $K^*(1420)$ production except that the pomeron contribution, P is larger. If we increase P in Fig. 10 we see that this will enhance the cross-over effect, as observed in the data.

4.2 Interference effects between NPE amplitudes

The above exchange decomposition has allowed us to make predictions of the phases of the P_+ amplitudes describing $K^*(890)$ production, and of the D_+ amplitudes describing $K^*(1420)$ production by NPE. On the other hand, we have seen in Section 3 that the data allow P_+-D_+ interference effects to be determined. In particular, the data determine the behaviour of $\cos \phi_{P_+D_+}$ of Eq. (3) as a function of the produced $K\pi$ mass for the t interval $0.1 < -t < 0.4 \text{ GeV}^2$. We assume $\xi = 1$ for the reasons given in Section 3. To interpret this observed behaviour of the relative phase of the NPE P- and D-wave amplitudes we write [cf. Eq. (7)]

$$\phi_{PD} \equiv \phi_P - \phi_D = \theta_P + \delta_P - \theta_D - \delta_D ,$$

where θ is the production phase and δ the decay phase of the $K\pi$ system. For clarity we are again omitting the NPE subscript. From the discussion in Section 4.1 we have expectations for the production phases, θ_P and θ_D , but only in the respective resonance regions. To predict the behaviour of $\cos \phi_{PD}$ in the $K^*(890)$ region requires knowledge of θ_D well below the $K^*(1420)$ resonance. The simplest assumption is to suppose the phase is essentially unchanged. Then for $-t \sim 0.2 \text{ GeV}^2$ the phases of the P- and D-wave amplitudes (corresponding to the above exchange picture) are shown in Fig. 11. For the K^- reaction we see that P and D are approximately perpendicular at the $K^*(890)$ resonance, $\delta_P = 90^\circ$, and that as the $K\pi$ mass is increased through the resonance the observed behaviour of $\cos \phi_{PD}$ is reproduced (cf. Fig. 6). Similarly, we see that the essential features of the observed behaviour of $\cos \phi_{PD}$ for the K^+ reaction are also reproduced.

To repeat the above procedure in the $K^*(1420)$ region would require knowledge of δ_P and θ_P in this mass range. Now we have seen that the relative magnitude of $P(K^\pm)$ has a striking mass dependence (cf. Fig. 5). Therefore rather than assume the θ_P are unchanged in going from the $K^*(890)$ to the $K^*(1420)$, we use the data to

Data at a single energy cannot separate the pomeron (P) and f contributions. For the qualitative discussion we present below, we use the couplings obtained from the f and f' dominated pomeron scheme [10]. This scheme^{*)} is compatible with a large class of dynamical models for the pomeron, and with much elastic and inclusive data. It leads to a pomeron contribution, relative to the f, which is about four times larger for $K^*(1420)$ as compared to $K^*(890)$ production [13]. Recall that if the pomeron were a pure SU(3) singlet, generalized C parity would forbid it in $K^*(890)$ production, but allow it in $K^*(1420)$ production.

For the ω and f exchange contributions a first approximation would be to assume that they are exchange degenerate (EXD). If the pomeron contribution is neglected this would give equality of the magnitudes of $P(K^\pm)$, but if present, it leads to a cross-over at the ω wrong-signature-zero at $-t \approx 0.5 \text{ (GeV/c)}^2$. However, there is evidence^{**)} to suggest that the ω trajectory lies below that for the f. In fact $|\omega|$ can be isolated using information on the SU(3)-related process $\pi^0 p \rightarrow \rho^0 p$. This had been done by taking the combination [15]

$$|\omega|^2 = \frac{1}{8} \left[\frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^+ p) + \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^- p) - \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n) \right].$$

These data show that $|\omega|$ has a minimum at $-t \approx 0.4 \text{ (GeV/c)}^2$. Combined with the $K^*(890)^\pm$ data, we thus conclude that ω has the behaviour shown in the Argand plot of Fig. 9. The curves in Fig. 9 are oriented for $-t$ increasing from 0 to 0.5 GeV^2 . ω passes through B for $-t \approx 0.4 \text{ GeV}^2$, corresponding to the dip in $\pi^0 p \rightarrow \rho^0 p$. The $K^\pm p \rightarrow K^{*\pm} p$ cross-over at $-t \approx 0.3 \text{ GeV}^2$ occurs when ω is perpendicular to the $P + f$ contribution, the point labelled by A on the Argand plot. For ease of comparison with ω -f EXD phase behaviour, $\pm 1 - e^{-i\pi\alpha}$, this sketch does not include the full t structure, in particular the $\sqrt{-t'}$ behaviour of the exchanges.

Figure 10 shows the exchange contributions at two different t values, taken either side of the cross-over. We also show the resulting NPE amplitudes for $K^*(890)$ production, $P(K^\pm)$ of Eq. (8).

*) An alternative scheme, proposed by Chew and Rosenzweig [11], known as pomeron-f identity does not appear to be supported by data [12].

***) A recent determination [14] of $\alpha_\omega(0)$ gives $\alpha_\omega(0) = 0.39$.

- i) The reactions $K^+p \rightarrow K^{*+}p$ are dominated by isoscalar NPE in contrast to the CEX K^* production reactions where UPE (π, B) is dominant.
- ii) The structure of the dominant NPE amplitudes (in particular the cross-over effects) can be described in terms of pomeron-, ω - and f-exchange contributions, with pomeron exchange being stronger in $K^*(1420)$ than in $K^*(890)$ production.
- iii) By comparing $K^*(890)^+$ and $K^*(890)^-$ production, together with the SU(3)-related ρ -production data, we are able to estimate the magnitude and phase of ω exchange.
- iv) The interference effects of the P- and D-wave NPE amplitudes are isolated from the data both as a function of M and t. There is, in general, agreement with a naïve pomeron-, ω - and f-exchange picture, except for the odd-signature exchange contribution to the P-wave in the $K^*(1420)$ region.
- v) The NPE P-wave amplitudes are found to have a significantly different dependence on the mass of the produced $K\pi$ system. The P-wave production in the K^- reaction decreases, relative to that in the K^+ reaction, by about a factor of two as M increases from 890 to 1200 MeV.
- vi) The UPE amplitudes for $K^+p \rightarrow K^{*+}p$ are consistent in magnitude and structure with (π, B) exchange, and increase, relative to NPE, with increasing resonance mass.
- vii) The S-wave $K\pi$ production as a function of mass is consistent, both in magnitude and structure, with the S-waves obtained in the recent $K\pi$ partial wave analyses of data for CEX $K\pi$ production; in particular with the rapid increase in the $I = \frac{1}{2}$ S-wave phase around 1450 MeV.

Finally, we stress that these reactions are a powerful probe of the properties of the pomeron. A study of these processes at higher energies is under way.

predict the θ_p in the $K^*(1420)$ region. The results are shown in Fig. 12. Using the expected phases of $D(K^\pm)$, Fig. 12a, together with the observed behaviour of $\cos \phi_{PD}$, we obtain the phases of $P(K^\pm)$ shown in Fig. 12b. To obtain the production phases θ_p , shown in Fig. 12c, we take the decay phase in the $K^*(1420)$ region to be $\delta_p \sim 170^\circ$. We see that the phase of the even-signatured amplitude, $P(K^+) + P(K^-)$, corresponds to the expectations of $\rho + f$ exchange, but that of the odd-signatured amplitude, $P(K^-) - P(K^+)$ has opposite sign to that expected from ω exchange for this t range. This is another indication of the rapid change of the NPE amplitudes, $P(K^\pm)$, as a function of mass, and it implies that the production mechanisms are more complicated off-resonance.

5. CONCLUSIONS

Here we summarize the main results of our analysis of the reactions $K^\pm p \rightarrow K_S^0 \pi^\pm p$ at 10 GeV/c. We find that:

- i) the $K\pi$ system is dominated by production of the $K^*(890)$, $K^*(1420)$, and $K^*(1780)$ resonances with spins 1^- , 2^+ and 3^- , and that the non-resonant background is extremely low;
- ii) the total cross-section for K^* production is the same for K^{*+} and K^{*-} resonances; in contrast, the differential cross-section is steeper for K^{*-} than K^{*+} , thus producing a cross-over at $-t \sim 0.3 \text{ GeV}^2$ for both $K^*(890)$ and $K^*(1420)$ production;
- iii) although the cross-over point is the same, the difference in cross-section between K^{*+} and K^{*-} is larger for the $K^*(1420)$ than for the $K^*(890)$.

Using the moments of the $K\pi$ angular distribution we perform amplitude analyses, both as a function of t in the resonance regions, and as a function of the mass M of the produced $K\pi$ system. In this way we determine the amount of S-, P-, and D-wave $K\pi$ production, in the various helicity states, as a function of t and M . We summarize our conclusions below.

REFERENCES

- [1] R. Baldi et al., Systematic study of $K\pi$ production in the reaction $K^+p \rightarrow K_S^0 \pi^+ p$: Technique and measurements at 10 GeV/c, preceding paper.
- [2] A. Barbaro-Galtieri, Advances in Particle Physics (Eds. R.L. Cool and R.E. Marshak) (Wiley-Interscience, NY, 1968), Vol. 2, p. 175.
F. von Hippel and C. Quigg, Phys. Rev. D 5 (1972) 624.
- [3] R. Baldi et al., Phys. Letters 63B (1976) 344.
- [4] R. Baldi et al., Amplitudes and exchange mechanisms for K^* resonances produced by the reactions $K^+p \rightarrow K^{*\pm} p$ at 10 GeV/c, submitted to Phys. Letters (1977).
- [5] P. Estabrooks et al., Phys. Letters 60B (1976) 473.
P. Estabrooks and A.D. Martin, Nuclear Phys. B102 (1976) 537.
- [6] P. Estabrooks et al., Study of $K\pi$ scattering using the reactions $K^+p \rightarrow K^+ \pi^+ n$ at 13 GeV/c, submitted to the 18th Internat. Conf. on High-Energy Physics, Tbilisi, 1976, SLAC-PUB-1886.
- [7] P. Estabrooks et al., Study of $K\pi$ scattering using the reactions $K^+p \rightarrow K^+ \pi^+ n$ and $K^+p \rightarrow K^+ \pi^- \Delta^{++}$ at 13 GeV/c, SLAC-PUB-2004, submitted to Nuclear Phys.
- [8] M.G. Bowler et al., Nuclear Phys. B126 (1977) 31.
- [9] C. Michael, Nuclear Phys. B57 (1973) 292.
- [10] R. Carlitz, M.B. Green and A. Zee, Phys. Rev. Letters 26 (1971) 1515; Phys. Rev. D 4 (1971) 3439.
P.G.O. Freund, H. Jones and R.J. Rivers, Phys. Letters 36B (1971) 89.
- [11] G.F. Chew and C. Rosenzweig, Phys. Letters 58B (1975) 93.
- [12] J.C. Romao and P.G.O. Freund, Nuclear Phys. B121 (1977) 413.
D.W. Duke, The pomeron-f identity and vector meson production, to be published in Phys. Letters.
- [13] A.C. Irving, Nuclear Phys. B121 (1977) 1116.
- [14] J. Roehrig et al., Phys. Rev. Letters 38 (1977) 176.
- [15] A. Contogouris, J. Tran Thanh Van and H. Lubatti, Phys. Rev. Letters 19 (1967) 1352.
J. Bartsch et al., Nuclear Phys. B46 (1972) 46.
H.A. Gordon, K.W. Lai and J.M. Scarr, Phys. Rev. D 8 (1973) 779.

Acknowledgements

We gratefully acknowledge the hospitality of CERN, and we thank the British Science Research Council and the Fonds National Suisse for support of this work.

Table 3

Resonance parameters found by fitting
 $|P_+|$ and $|D_+|$ mass distributions

	Mass interval fitted	M_R	Γ_R
$K^{*+}(890)$	0.78-1.0	895.3 ± 0.5	53.2 ± 0.8
$K^{*-}(890)$	0.78-1.0	889.5 ± 0.7	48.6 ± 1.0
$K^{*+}(1420)$	1.3-1.5	1428.0 ± 2.3	96.5 ± 3.8
$K^{*-}(1420)$	1.3-1.5	1423.8 ± 2.2	97.7 ± 4.0

Table 4

Parameters describing P_+ and D_+ coherence

	α	Δ (degrees)
$K^{*+}(890)$	1.15	-83
$K^{*-}(890)$	1.37	-8
$K^{*+}(1420)$	1.0 (input)	-189
$K^{*-}(1420)$	1.0 (input)	-118

Table 1

Parameters of fits of the $K\pi$ -mass distribution
in the momentum-transfer interval $0.05 < |t| < 1 \text{ (GeV/c)}^2$

Resonance	J^P	$M_R^{\text{a)}$ (MeV)	$\Gamma_R^{\text{a)}$ (MeV)	$\sigma^{\text{b)}$ (μb)	$P(\chi^2)$
$K^{*+}(890)$	1^-	894.4 ± 0.3	50.5 ± 0.5	93 ± 10	0.05
$K^{*+}(1420)$	2^+	1425.6 ± 1.0	99 ± 3	32 ± 5	
$K^{*+}(1780)$	3^-	1779 input	135 input	3 ± 1	
$K^{*-}(890)$	1^-	889.5 ± 0.4	50.0 ± 0.5	105 ± 10	0.02
$K^{*-}(1420)$	2^+	1421.5 ± 1.4	106 ± 5	35 ± 5	
$K^{*-}(1780)$	3^-	1774 input	135 input	5 ± 1	

a) Errors are statistical.

b) Systematic error included. Cross-sections have been corrected to include the tails of the Breit-Wigner form and the fraction of events outside the t acceptance of the spectrometer.

Table 2

Parameters of fits of the momentum-transfer distributions,
using the form $d\sigma/dt = At'^n \exp(Bt' + Ct'^2)$

	Mass region [GeV]	t range fitted	n	A [$\mu\text{b}/(\text{GeV/c})^{2+2n}$]	B [$(\text{GeV/c})^{-2}$]	C [$(\text{GeV/c})^{-4}$]	$P(\chi^2)$ [%]
$K_S^0 \pi^+$	0.84-0.94	0.05-1	1	3610	6.97 ± 0.18	1.24 ± 0.24	74
	1.1 -1.3	0.05-1	0	64	4.37 ± 0.12	0	63
	1.36-1.5	0.1 -1	1	900	6.06 ± 0.07	0	2×10^{-3}
	1.68-1.88	0.05-1	1	321	6.85 ± 0.15	0	4
$K_S^0 \pi^-$	0.84-0.94	0.1 -1	1	6880	9.72 ± 0.10	0	18
	1.1 -1.3	0.05-1	0	56	5.38 ± 0.18	0	71
	1.36-1.5	0.05-1	1	1722	8.74 ± 0.13	0	64
	1.68-1.88	0.05-1	1	408	7.22 ± 0.23	0	11

from the NPE amplitudes (Table 3). The curve for $|S_0|$ corresponds to the $K\pi$ S-waves of Fig. 8.

Fig. 8 : The $I = \frac{1}{2}$ and $I = \frac{3}{2}$ $K\pi$ S-wave amplitudes, assumed to be elastic, with the phases of Ref. 7. For the $K^+p \rightarrow (K^0\pi^+)p$ reaction $|S_0|$ is proportional to $|S_{1/2} - S_{3/2}|$.

Fig. 9 : Argand plot of the ω and f exchanges. The circles correspond to $\pm 1 - \exp(-i\pi\alpha)$ expected from ω - f exchange degeneracy. The curves on the circles run from $-t = 0$ to 0.5 (GeV/c)^2 . The curve passing through A and B is the ω contribution implied by the data.

Fig. 10 : The decomposition of the NPE amplitudes describing $K^\pm p \rightarrow K^*(890)^\pm p$ into their pomeron-, ω -, and f -exchange components. The scale on the $-t = 0.5 \text{ (GeV/c)}^2$ diagram has been expanded by a factor of two relative to $-t = 0.1 \text{ (GeV/c)}^2$ diagram. The P, f separation is that of Ref. 13 with $x_p = 1$.

Fig. 11 : The P- and D-wave NPE amplitudes for $K^\pm p \rightarrow (K\pi)^\pm p$ in the $K^*(890)$ region for $-t \approx 0.2 \text{ (GeV/c)}^2$. The phase of each amplitude is the sum of a production, θ , and a decay, δ , phase. As the $K\pi$ mass is increased through the $K^*(890)$ resonance the amplitudes $P(K^\pm)$ rotate counter-clockwise as indicated.

Fig. 12 : Argand plots of the P- and D-wave NPE amplitudes for $K^\pm p \rightarrow (K\pi)^\pm p$ in the $K^*(1420)$ region for $-t \sim 0.2 \text{ GeV}^2$. Plot (a) shows the D-wave amplitudes at resonance, $\delta_D = 90^\circ$, using the production phases, θ_D , obtained from the P, f and ω decomposition. The phases of the P-wave amplitudes in plot (b) are obtained using plot (a) and the observed values of $\cos \phi_{PD}$. Plot (c) shows the resulting P-wave production phases if the decay phase δ_P is taken to be 170° .

Figure captions

- Fig. 1 : $K_S^0 \pi^+$ effective mass spectrum, for $0.05 < |t| < 1 \text{ (GeV/c)}^2$, with fit as described in the text. The spectrum is shown before (histogram) and after (points with error bars) acceptance correction. The lower figure shows the spectrum remaining after subtraction of the $K^*(890)$ and $K^*(1420)$.
- Fig. 2 : $K_S^0 \pi^-$ effective mass distribution, for $0.05 < |t| < 1 \text{ (GeV/c)}^2$. Fit and subtraction of $K^*(890)$ and $K^*(1420)$ as in Fig. 1.
- Fig. 3 : Momentum-transfer distributions $d\sigma/dt$ for $K^\pm p \rightarrow (K^0 \pi^\pm) p$ in the mass regions of the $K^*(890)$, $K^*(1420)$ and $K^*(1780)$ and in an intermediate region. The full lines are fits as described in the text.
- Fig. 4 : The NPE amplitudes for $K^\pm p \rightarrow (K^0 \pi^\pm) p$ in the $K^*(890)$ and $K^*(1420)$ regions ($0.84 < M < 0.94$ and $1.34 < M < 1.5 \text{ GeV}$, respectively), as determined in the amplitude analyses described in Ref. 4. The amplitudes are normalized so that their moduli squared give the contribution to $d\sigma/dt$. The $P_+ - D_+$ interference in the $K^*(1420)$ region is also shown [cf. Eq. (3)].
- Fig. 5 : The NPE amplitudes and their coherence found by analysing 10 GeV $K^+ p \rightarrow (K^0 \pi^+) p$ data for the interval $0.1 < -t < 0.4 \text{ (GeV/c)}^2$. The curves for $|P_+|$ and $|D_+|$ correspond to the Breit-Wigner fits of Table 3, and for the coherence correspond to the parameters of Table 4. The values of $|P_+|$ and the coherence (normalized to $\alpha = 1$) found by analysing $K^- p \rightarrow (\bar{K}^0 \pi^-) p$ data are shown by dashed curves for comparison.
- Fig. 6 : The same as Fig. 5 but for the $K^- p \rightarrow (\bar{K}^0 \pi^-) p$ reaction.
- Fig. 7 : The magnitudes of the amplitudes describing helicity-zero $K\pi$ production found in the analysis of 10 GeV/c $K^+ p \rightarrow (K^0 \pi^+) p$ data. The curves for $|P_0|$ and $|D_0|$ are drawn using the Breit-Wigner parameters obtained

Fig. 2

Fig. 1

Fig. 4

Fig. 3

$K^- p \rightarrow (\bar{K}^0 \pi^-) p$ NPE amplitudes and coherence

Fig. 6

$K^+p \rightarrow (K^0\pi^+)p$ NPE amplitudes and coherence

Fig. 5

Fig. 8

Fig. 9

Fig. 10

Fig. 7

$K^*(890)$ Mass region

Fig. 11

$K^*(1420)$ Mass region

Fig. 12