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RADIATIVE π^+p SCATTERING AND THE MAGNETIC MOMENT OF THE Δ^{++}

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A B S T R A C T

The size of the peak expected in pion-proton bremsstrahlung in the region of the Δ^{++} resonance is calculated using an unambiguous low energy theorem. The only unknown parameter is the magnetic moment of the resonance. From the absence of any resonant structure in the data one obtains for it $\mu_{\Delta} = 3.8 \pm 1.4$ in units $2e\hbar/2M_{\Delta}c$. This is in excellent agreement with the SU(6) prediction $\mu_{\Delta} = 3.66$.

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1. - INTRODUCTION

Ten years ago Kondratyuk and Ponomarev ¹⁾ suggested radiative π^+p scattering as an ideal process for measuring the magnetic moment of the $\Delta^{++}(1232)$ resonance. In fact, ideas for how to measure magnetic moments of strongly unstable particles are badly needed, since conventional methods require much longer times than the lifetimes of the resonances. These authors used a Lagrangian isobar model modified in order to ensure gauge invariance; they showed that when the photon scatters nearly backward with respect to both incoming and outgoing pions a strong cancellation happens in the external charge radiation contribution. Under these circumstances the internal radiation contribution (coming from the Δ^{++}) starts being important and the cross-section turns out to be strongly dependent on the magnetic moment of the Δ^{++} . Another Lagrangian model for this process is due to Baier, Pittner and Urban ²⁾, which includes, apart from the Δ^{++} , the two-pion ρ and σ resonances. Their main concern was the applicability of Low's theorem ³⁾ to resonant pion-nucleon bremsstrahlung, where the energy of the photon is of the order of the half-width of the resonance.

A more phenomenological approach to this process has been followed by Fischer and Minkowski ⁴⁾ using Low's theorem in the Adler-Dothan formulation ⁵⁾. In order to apply this theorem in a hard-photon regime they include off-shell effects on the electromagnetic vertices, they modify the very theorem so that it takes into account the presence of a resonance and they add structure dependent terms (essentially coming from the Δ^{++} magnetic moment). Further generalizations of Low's theorem have been studied by Haddock and Leung ⁶⁾.

The ambiguities in the procedure which is followed for imposing gauge invariance in all the above-mentioned calculations have been repeatedly emphasized by now ⁷⁾⁻⁹⁾. They are unavoidable and non-negligible, as is clear, i.e., from the opposite dependence on the magnetic moment of the Δ^{++} of the models of Refs. 1) and 4) [see Ref. 10]. Even alternative approaches, like Musakhanov's ¹¹⁾ which uses soft-pion limits, are not free from these ambiguities and their results should therefore be taken with care.

The experimental situation, on the other hand, is quickly summarized: the cross-section agrees perfectly with what is obtained with the good low energy Low theorem using the first two terms of the expansion in the photon energy ω up to $\omega = 50$ MeV and then goes on decreasing like ω^{-1} , as is typical for charge bremsstrahlung ¹²⁾. There is no sign of the Δ^{++} resonance, no bump, no interference pattern.

This is for us the most important aspect of the problem, and we will concentrate on it by performing a calculation precisely on (what should be) the peak of the Δ^{++} . This will be done in the narrow width approximation (an approximation which will be relaxed afterwards), so that what will be computed is the residue at the pole. The essential advantage of this approach is that within the above-mentioned approximation the first two orders of a low photon energy expansion can be calculated exactly in terms of the charges and magnetic moments of the π^+ , the p and the Δ^{++} alone, by using the "good" Low theorem. The charge and the magnetic moment of the Δ^{++} , μ_Δ , contribute thus in such a way that they do not require any modification of Low's theorem, which is always a dangerous endeavour. A small or vanishing value of this residue which, let us stress it, depends only on μ_Δ as an unknown parameter, will explain the absence of bumps and interferences in the cross-section in the region of the Δ^{++} resonance, as observed in experiment.

The details of the calculation are given in the next section, together with our notation. The last section is devoted to the comparison with experimental data, to the discussion of our results and to conclusions.

2. - CALCULATION

The scattering of positive pions on protons is dominated at low energies by the $\Delta^{++}(1232)$ resonance. There are three kinds of diagrams which contribute to the corresponding radiative process : (i) external emission, corresponding to those in which the photon is attached to one of the external particles ; (ii) internal emission, when the photon is coupled to the charge or magnetic moment of the Δ^{++} ; and (iii) non-resonant background which involves diagrams with $\Delta^{++}\Delta^+\pi^+$ and $\Delta^+p\gamma$ vertices. The contribution of these last diagrams has been shown ^{1),4)} to be negligible. Out of the remaining diagrams we keep those in which the position of the Δ^{++} pole depends on the photon energy and we neglect the final state particle radiation diagrams for which the position of the Δ^{++} pole is fixed by s , the total centre-of-mass energy squared. The residue is thus given by the product of the $\pi^+p \rightarrow \gamma\Delta^{++}$ amplitude (for an on-mass shell Δ^{++}) times the $\Delta^{++} \rightarrow p\pi^+$ vertex, as shown in Fig. 1. The $\pi^+p \rightarrow \gamma\Delta^{++}$ amplitude is computed by means of a low energy theorem according to Fig. 2 and since

the radiationless process is just a vertex, both dominant ω^{-1} and ω^0 terms are fully determined by the charges and magnetic moments of the external particles. Let us give the relevant expressions now.

Our notation is that of Bjorken and Drell ¹³⁾ but the normalization of the Dirac spinors which, for us is $2M$, M being the mass. The momenta of the incoming pion and proton are p_1 and p_2 and λ_2 is the helicity of the proton. The primed quantities correspond to the outgoing pion and proton. The photon and the delta resonance have momenta k and $p = p_1' + p_2'$ and helicities μ and λ , respectively. The pion, proton and delta masses are denoted by m , M and M_Δ . The $\Delta^{++} p \pi^+$ vertex is given by

$$T_{\Delta p \pi} = g \bar{u}(p_2', \lambda_2') p_{1\alpha}' u^\alpha(p, \lambda) \quad (1)$$

where g is the coupling constant and $u^\alpha(p, \lambda)$ the spin $\frac{3}{2}$ Rarita-Schwinger spinor with no lower spin components. Equation (1) leads to a width

$$\Gamma = \frac{g^2}{192 \pi M_\Delta^5} \lambda^{3/2}(M_\Delta^2, M^2, m^2) [(M_\Delta + M)^2 - m^2] \quad (2)$$

where $\lambda(M_\Delta^2, M^2, m^2)$ is Källen's function. With $G = gM$ a width of $\Gamma = 115$ MeV leads to $G^2/4\pi = 16.8$, which corresponds exactly to the value $G_{\Delta N \pi}^2 = 14.9$ given in Ref. 14).

The electromagnetic vertex of the Δ^{++} is given by

$$T_{\Delta \Delta \gamma} = \frac{e}{M_\Delta} \bar{u}_\alpha(p', \lambda') [-(p + p')^\nu + i \mu_\Delta \sigma^{\nu\mu} k_\mu] u^\alpha(p, \lambda) \epsilon_\nu^*(k, \mu) \quad (3)$$

where higher multipoles have been neglected since we will not need them and the magnetic moment is μ_Δ in units $e\hbar/M_\Delta c$. According to $SU(6)$ the magnetic moment of the Δ^{++} is twice the one of the proton ¹⁵⁾ which, taking into account that this last one is given in units $e\hbar/2Mc$, leads to the relation $\mu_\Delta = (M_\Delta/M)\mu_p$.

We will finally need the spin $\frac{3}{2}$ propagator which is (for p going from ν to μ)

$$\frac{-i d_{\mu\nu}(p)}{p^2 - M_\Delta^2 + i\epsilon} \quad (4)$$

with

$$d_{\mu\nu}(p) = P_{\mu\nu}(p) + \frac{2}{3M_\Delta^2} (p^2 - M_\Delta^2) [\gamma_\mu p_\nu - \gamma_\nu p_\mu + (\not{p} + M_\Delta) \gamma_\mu \gamma_\nu] \quad (5)$$

and where $P_{\mu\nu}(p)$ is the spin $\frac{3}{2}$ projector

$$P_{\mu\nu}(p) = (\not{p} + M_\Delta) \left[g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3M_\Delta} (\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3M_\Delta^2} p_\mu p_\nu \right] \quad (6)$$

We would like to point out here that many authors ^{2),4),7)} use a truncated propagator with $d_{\mu\nu} = P_{\mu\nu}$ and at the same time a $\Delta^{++} p \pi^+$ vertex of the form of Eq. (1), which is known to be incompatible with each other ¹⁶⁾. The simultaneous use of this truncated propagator and Eq. (1) has therefore, if not justified, to be considered incorrect.

It is now straightforward to calculate, according to the diagrams of Fig. 2 and using a low energy theorem, the gauge invariant $\pi^+ p \rightarrow \Delta^{++} \gamma$ amplitude up to terms linear in ω excluded :

$$\begin{aligned} T_{\pi p \rightarrow \Delta \gamma} = & g e \bar{u}_\alpha(p'_1 + p'_2, \lambda) \left\{ \frac{(p_1 - k)^\alpha p_{1\mu}}{p_1 \cdot k} + \frac{p_1^\alpha}{2 p_2 \cdot k} (\not{p}_2 - \not{k} + M) \left(\gamma_\mu - \frac{i}{2M} \epsilon_{\mu\nu\rho\sigma} p_\nu k^\rho \right) \right. \\ & + \frac{1}{M_\Delta} \left[(p_1 + p_2 + p'_1 + p'_2)_\mu - i \epsilon_{\mu\nu\rho\sigma} p_\nu k^\rho \right] \frac{d^{\alpha\beta}(p_1 + p_2)}{2 k \cdot (p'_1 + p'_2)} p_{\lambda\beta} \\ & \left. + \left[1 - \frac{2}{M_\Delta^2} (m^2 + p_1 \cdot p_2) \right] \gamma^\alpha_\mu + \frac{1}{M_\Delta} p_1^\alpha \gamma_\mu \right\} u(p_2, \lambda_2) \epsilon^{\mu*}(k, \lambda) \quad (7) \end{aligned}$$

The residue is now obtained by multiplying the amplitudes of Eqs. (1) and (7) and by summing then over the helicities of the Δ , that is

$$R = \sum_{\lambda} T_{\Delta p n} T_{n p \rightarrow \Delta \gamma} \quad (8)$$

We will take into account at this point that the resonance is not narrow by multiplying R by the denominator of the propagator of Eq. (4), after substituting M_{Δ} by $M_{\Delta} - i/2 \Gamma$ and putting then $p^2 = M_{\Delta}^2$. Adding the part of the amplitude which we have not considered and which will give the background, one may write the complete T matrix element as

$$T = \frac{R}{i M_{\Delta} \Gamma + \frac{1}{4} \Gamma^2} + T_B \quad (9)$$

The measured quantity is

$$\frac{d^5 \sigma}{d\Omega_n d\Omega_{\gamma} d\omega} = \frac{1}{(4\pi)^5} \frac{I}{\lambda^{1/2}(s, m^2, M^2)} \sum_{\lambda_2} \sum_{\lambda_2'} \sum_{\mu} |T|^2 \quad (10)$$

where I is a phase space integral which for coplanar kinematics (where only three variables are independent) reads in the laboratory system

$$I = \frac{\omega (E_1'^2 - m^2)}{E_2' |\vec{p}_1'| + E_1' |\vec{p}_2'| - |\vec{p}_1| \cos \theta_n + \omega \cos (\theta_{\gamma} - \theta_n)} \quad (11)$$

Since the calculation has been performed on the Δ pole there is a further constraint relating ω and $\cos \theta_{\gamma}$ leaving us with only two independent variables. These will be chosen to be the outgoing pion and photon angles θ_{π} and θ_{γ} .

In Eq. (10) the T matrix element squared is needed. It is seen from Eq. (9) that there will be three contributions to it. The resonant term squared which would give a bump in the cross-section at the position of the resonance. The background term squared which should show no structure at all in the region of the resonance and the crossed term which should give rise to a characteristic interference pattern.

Our approach allows one to calculate only the first contribution and furthermore only at precisely the position of the resonance, but the experimental data are such that this is enough for drawing some interesting conclusions. This contribution to the cross-section may be written as a function of the magnetic moment of the Δ as

$$\left. \frac{d^5\sigma}{d\Omega_\pi d\Omega_\gamma d\omega} \right|_\Delta = a + b\mu_\Delta + c\mu_\Delta^2 \quad (12)$$

where a , b and c are known functions of $\cos\theta_\pi$ and $\cos\theta_\gamma$. The term in μ_Δ^2 comes entirely from terms of order ω^0 in Eq. (7), which a priori should not be kept when squared, since there are unknown contributions of the same order coming from the known order ω^{-1} term times the unknown order ω term. In fact what happens is that the charge radiation terms of order ω^{-1} cancel each other for almost backward photons in an even much higher degree than what is known to occur when the whole diagram is considered, where the photon is emitted by the outgoing pion and proton and not by the Δ resonance as in our case. The ω^{-1} term is thus negligible and the ω^0 term is the dominant one, so that it will give, squared, the most important contribution to the resonant cross-section.

3. - COMPARISON WITH EXPERIMENT AND CONCLUSIONS

As said in the Introduction the most interesting feature of the experimental data ¹²⁾ is the total absence of both bumps or interference patterns in the resonance region. This means that the experimental value should be almost entirely given by the background contribution alone, which we have not calculated ! Since the other two contributions, resonant and crossed one, cannot cancel each other over the whole resonant region one has to conclude that both have to be small. Since we have calculated the resonant one in a complete model independent way with μ_Δ as only unknown parameter, its smallness will allow us to extract, within certain errors, the value of μ_Δ .

The experiment has been done with incident pions of a kinetic energy $T = 298$ MeV. The scattered pion was detected in a spectrometer centered at an angle of $\theta_\pi = -50.5^\circ$. For coplanar kinematics the photon was

observed at angles $\theta_Y = 160^\circ, 140^\circ$ and 103° . The photon energies on the resonance are $\omega = 57, 59$ and 67 MeV, respectively. The coefficients, a , b and c of Eq. (12) are shown for these three kinematics in the Table, together with the contribution which is due only to the charge radiation term ω^{-1} in Eq. (7). Our result at $\theta_Y = 103^\circ$ is much less reliable than at the other two angles. This is due to two different reasons: first, the higher photon energy makes an expansion in this variable less accurate, and second, the now non-negligible charge contribution to the coefficient a makes the unknown order ω^{-1} times order ω terms important and hence our result uncertain. We therefore will compare our expression with the experimental data only for the two larger photon scattering angles.

It is seen from Fig. 2 of Ref. 12) that the upper limit for bumps at the resonant region is of the order of the experimental error bars. This gives 1.2 and $2 \text{ nb/sr}^2 \text{ MeV}$ for $\theta_Y = 160^\circ$ and 140° , respectively. This implies immediately from Eq. (12) and the Table

$$\mu_\Delta = 3.8 \pm 1.4 \quad (\theta_Y = 160^\circ) \quad (13)$$

and

$$\mu_\Delta = 4.0 \pm 1.9 \quad (\theta_Y = 140^\circ) \quad (14)$$

in good agreement with each other. Since the experimental data are more stringent and the theory is more accurate for $\theta_Y = 160^\circ$ and anyhow the value of μ_Δ obtained for this angle lies within the errors of the one obtained for $\theta_Y = 140^\circ$, we consider $\mu_\Delta = 3.8 \pm 1.4$ as our final result. This value is in perfect agreement with the $SU(6)$ prediction $\mu_\Delta = (M_\Delta/M)\mu_p = 3.66$, and is compatible with Musakhanov's determination ¹¹⁾, which in our units is $\mu_\Delta = 2.4 \pm 1.3$.

A beautiful confirmation of our result, Eq. (13), would be obtained if one could find a kinematics in which this value of the magnetic moment would give a bump in the cross-section. This could then be checked experimentally. We have not succeeded in finding it. Changes of $\pm 20^\circ$ in the angle of emission of the outgoing pion leave the value of the magnetic moment for which the resonant contribution is minimal almost unchanged. On the other hand, as long as the photon is almost backward ($\theta_Y \simeq 160^\circ$), a

necessary condition for having the essential charge radiation suppression, non-coplanar kinematics will never be far from a coplanar one (for $\theta_Y = 180^\circ$ any kinematics is coplanar), and one cannot expect important quantitative changes. This is confirmed by other theoretical calculations as shown in Fig. 2 of Ref. 12), where it is seen that a non-coplanarity of 20° does not affect at all the dependence on μ_Δ of the cross-section. We therefore believe that it will be difficult to observe any signal of the Δ^{++} resonance in radiative π^+p scattering if the value of the magnetic moment is as given in Eq. (13). The non observation leads thus to a determination of this important electromagnetic parameter of the Δ^{++} resonance.

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TABLE

θ_Y	ω	a	b	c	a_{ch}
160°	57	6.75	-3.34	0.44	0.008
140°	59	8.38	-4.03	0.50	0.094
103°	67	13.67	-5.45	0.60	2.30

The coefficients a, b and c of Eq. (12) are given in nb/sr² MeV for three different photon emission angles, the first one also in the charge radiation approximation. The photon energy ω on the resonance is given in MeV.

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FIG. 1

FIG. 2