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TRANSVERSE MOMENTUM IN DRELL-YAN PROCESSES

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ABSTRACT

We study the transverse momentum distribution of muon pairs from Drell-Yan processes in QCD. In particular the dependence of $\langle k_{\perp}^2 \rangle$ on Q^2 is considered. QCD predicts an approximately linear rise of $\langle k_{\perp}^2 \rangle$ with S or Q^2 only at fixed $\tau = Q^2/S$. The slope as a function of τ is quantitatively studied for $P\bar{P}$ and P nucleus scattering. The most recent data showing a rather flat $\langle k_{\perp}^2 \rangle$ in Q^2 at fixed S are found to be consistent with QCD.

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In this note the transverse momentum distribution of the muon pairs with large mass $\sqrt{Q^2}$ produced in hadronic collisions is studied in the quark-gluon gauge theory (QCD). In the Drell-Yan picture for these processes ¹⁾ the transverse momentum $k_\perp$ of the muon pair is directly related to the sum of the transverse momenta of the annihilating partons from the two incoming hadrons. In our approach to this problem the hard component of $k_\perp$ is attributed to corrections of order $\alpha_s = g^2/4\pi$ (where g is the QCD coupling constant). These same α_s corrections have been shown in Refs. 2) and 3) [see also Ref. 4)] to induce for the parton densities in the Drell-Yan formula a Q^2 dependence which, to first order, is exactly the same as for the densities relevant to electroproduction. Here we push the analysis further and derive detailed predictions for the average transverse momentum $\langle k_\perp^2 \rangle$ in P nucleus and $P\bar{P}$ collisions that can be directly compared with the data. In particular the most recent data for P nucleus collisions showing a rather flat $\langle k_\perp^2 \rangle$ versus Q^2 are well consistent with our results in spite of the naïve expectation of a linear rise with Q^2 of the average transverse momentum. The point is that the slope of $\langle k_\perp^2 \rangle$ in Q^2 is a function of the scaling ratio $\tau = Q^2/S$ and thus, at fixed S , the behaviour of $\langle k_\perp^2 \rangle$ in Q^2 is crucially dependent on the detailed form of this function of τ which is precisely the object of our study.

To order α_s all terms proportional to Q^2 in the $k_\perp$ distribution arise from the diagrams in Fig. 1, referring to the elementary processes: a) $q + \bar{q} \rightarrow G + \mu^+ \mu^-$ (G is a gluon), and b) $q(\bar{q}) + G \rightarrow q(\bar{q}) + \mu^+ \mu^-$.

The cross-section for process a) can be easily obtained from the detailed QED calculations of Refs. 5) and 6) for $e^+e^- \rightarrow \gamma + \mu^+ \mu^-$ with colour factors properly taken into account. In the centre of mass of the quark-antiquark pair we have:

$$\frac{d\sigma_{q\bar{q}}}{dQ^2 d\cos\theta} = \frac{8\alpha^2\alpha_s}{27Q^2} e^2 \frac{s-Q^2}{s^2 \sin^2\theta} \left\{ 1 + \cos^2\theta + 4 \frac{Q^2 s}{(s-Q^2)^2} \right\} \quad (1)$$

where e is the quark charge (in units of the proton charge), $s = x_1 x_2 S$ is the total squared energy of the partons in terms of their fractions of proton momentum x_1 and x_2 and of the total squared energy S of the incoming hadrons and θ is the centre of mass scattering angle.

Similarly, by crossing, we obtain for the cross-section of process b):

$$\frac{d\sigma_{q\bar{q}}}{dQ^2 d\cos\theta} = \frac{\alpha^2 \alpha_s}{18Q^2} e^2 \frac{s-Q^2}{s^2(1+\cos\theta)} \left\{ \frac{2s}{s-Q^2} + \frac{s-Q^2}{2s} (1+\cos\theta)^2 - \frac{2Q^2}{s} (1-\cos\theta) \right\} \quad (2)$$

where θ is the centre of mass angle between the initial gluon and the final quark ^{*}). In both cases $k_{\perp}^2$ for the muon pair is given by:

$$K_{\perp}^2 = \frac{(s-Q^2)^2}{4s} \sin^2\theta \quad (3)$$

From Eqs. (1) to (3) we find

$$\int_{-1}^1 K_{\perp}^2 \frac{d\sigma_{q\bar{q}}}{dQ^2 d\cos\theta} d\cos\theta = \frac{\alpha^2 \alpha_s}{27Q^2} e^2 (1-\tau_{12})^3 \left[\frac{16}{3} + \frac{16\tau_{12}}{(1-\tau_{12})^2} \right] \quad (4)$$

$$\int_{-1}^1 K_{\perp}^2 \frac{d\sigma_{gg}}{dQ^2 d\cos\theta} d\cos\theta = \frac{\alpha^2 \alpha_s}{27Q^2} e^2 (1-\tau_{12})^3 \left[\frac{3}{2} \frac{1}{1-\tau_{12}} + \frac{1}{4} (1-\tau_{12}) - 2\tau_{12} \right] \quad (5)$$

where $\tau_{12} \equiv \tau/x_1 x_2$. At sufficiently large S we can neglect the $k_{\perp}$ of partons inside the hadrons and identify the total $k_{\perp}$ with its hard component from Eqs. (4) and (5). By convolution with the parton densities in the hadrons h_1 and h_2 , we find:

$$\begin{aligned} \langle K_{\perp}^2 \rangle \left(\frac{Q^2 d\sigma}{dQ^2} \right)_{DY} &= \frac{\alpha \alpha_s (Q^2)}{27} \int_{\tau}^1 dx_1 \int_{\tau/x_1}^1 dx_2 (1-\tau_{12})^3 \cdot \\ &\cdot \left\{ \left[\sum_{\text{flavours}} e_i^2 q_i^{h_1}(x_1, Q^2) \bar{q}_i^{h_2}(x_2, Q^2) + (1 \leftrightarrow 2) \right] \left[\frac{16}{3} + \frac{16\tau_{12}}{(1-\tau_{12})^2} \right] + \right. \\ &\left. + \left[\sum_{\text{flavours}} e_i^2 \left(q_i^{h_1}(x_1, Q^2) + \bar{q}_i^{h_1}(x_1, Q^2) \right) G_i^{h_2}(x_2, Q^2) + (1 \leftrightarrow 2) \right] \left[\frac{3}{2} \frac{1}{1-\tau_{12}} + \frac{1}{4} (1-\tau_{12}) - 2\tau_{12} \right] \right\} \quad (6) \end{aligned}$$

^{*}) Eqs. (1) and (2) do not agree with the corresponding results of Refs. 2) and 3).

where α_s has been replaced by the running coupling constant of QCD; q_i^{ha} ($\bar{q}_i^{ha}$) and G^{ha} are the densities of the quark (antiquark) of flavour i and of the gluons in the hadron ha , respectively. Finally the Drell-Yan cross-section is given by:

$$\left(Q^2 \frac{d\sigma}{dQ^2} \right)_{DY} = \frac{4\pi\alpha^2}{9S} \int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - \tau) \left[\sum_{\text{flavours}} e_i^2 q_i^h(x_1, Q^2) \bar{q}_i^h(x_2, Q^2) + (1 \leftrightarrow 2) \right] \quad (7)$$

In Eqs. (6) and (7) the parton densities are the same functions of Q^2 [actually of $\alpha_s(Q^2)$] as in electroproduction as suggested by the results of Refs. 2) and 3) *). From the above analysis it follows that

$$\langle K_1^2 \rangle = \alpha_s(Q^2) \int f(\tau, \alpha_s(Q^2)) + \dots \quad (8)$$

where the dots stand for terms without the S factor arising from the neglected transverse momentum of partons inside the hadrons. The dependence of f on $\alpha_s(Q^2)$ is introduced by the scaling violations in the parton densities. $f(\tau, \alpha_s)$ is a positive function of τ that vanishes both at $\tau=1$ [at least as $(1-\tau)^2$ as implied by Eq. (3)] and at $\tau=0$ [as $\tau \ln \tau$ if, at $x \rightarrow 0$, $xq_1(x)$, $xG(x) \rightarrow \text{constant}$].

For $P\bar{P}$ scattering the function $f_{P\bar{P}}(\tau, \alpha_s)$ is dominated by the rather well-known valence quark densities and can be calculated quite reliably. Note, in fact, that in Eq. (6) the quark gluon term multiplies a much smaller weight function than the quark-antiquark term and therefore $f_{P\bar{P}}$ does not depend much on the badly known gluon density. Neglecting the Q^2 dependence of densities as a first approximation (and dropping charmed quarks) we take the simple fit of Ref. 7) to the parton densities and set

$$x G(x) = A_n (1-x)^n \quad (9)$$

*) There is some ambiguity in this case on whether α_s and the parton densities are to be taken as functions of Q^2 or S or some other related variable, the difference being, however, formally of order α^2 .

with $n=4$ or 6 , A_n being fixed by momentum conservation. We obtain the results in Fig. 2 for $f_{P\bar{P}}$. The results are almost independent of $G(x)$ as expected. We see that $f_{P\bar{P}}(\tau)$ is an increasing function of τ only for small τ ($\tau \lesssim 0.1$) and decreases for larger values of τ . Note that being $f_{P\bar{P}}(\tau)$ at most of order 10^{-3} , it is only for S or order 10^3 GeV^2 that the hard component of $\langle k_{\perp}^2 \rangle_{P\bar{P}}$ is about 1 GeV^2 .

The precise determination of $f_{PN}(\tau)$ for P nucleus collisions is more difficult because the valence gluon terms in Eq. (6) are important with respect to the valence sea terms and actually become dominant at large τ for a gluon density that is more spread out in x than that of sea quarks. In particular, in order to obtain a reasonable $f_{PN}(\tau)$, it is necessary to explicitly take into account first order corrections to $(Q^2 d\sigma/dQ^2)_{DY}$. For this purpose we integrate Eq. (2) over $\cos\theta$, with a suitable regularization of the infra-red logarithmic singularity²⁾. To leading logarithms we find:

$$\frac{Q^2 d\sigma_{qq}}{dQ^2} = \frac{4\pi\alpha^2}{g_S^2} e^2 \left[\frac{\alpha_s(Q^2)}{2\pi} \frac{\tau_{12}^2 + (1-\tau_{12})^2}{2x_1 x_2} \ln Q^2/\mu^2 \right] \quad (10)$$

with μ^2 a convenient normalization mass. The modified Drell-Yan cross-section then becomes:

$$\begin{aligned} (Q^2 \frac{d\sigma}{dQ^2})_{DY} = & \frac{4\pi\alpha^2}{g_S^2} \int_0^1 dx_1 \int_0^1 dx_2 \left\{ \left[\sum_{\text{flavours}} e_i^2 q_i^{h_1}(x_1) \bar{q}_i^{h_2}(x_2) + (1 \leftrightarrow 2) \right] \delta(x_1 x_2 - \tau) \right. \\ & \left. + \left[\sum_{\text{flavours}} e_i^2 (q_i^{h_1}(x_1) + \bar{q}_i^{h_1}(x_1)) G^{h_2}(x_2) + (1 \leftrightarrow 2) \right] \theta(x_1 x_2 - \tau) \frac{\alpha_s(Q^2)}{2\pi} \frac{\tau_{12}^2 + (1-\tau_{12})^2}{2x_1 x_2} \ln Q^2/\mu^2 \right\} \quad (11) \end{aligned}$$

It is remarkable that this equation, to first order, can also be recast in the same form of Eq. (7) provided we make on quarks and antiquarks the replacement:

$$q_i(x) \rightarrow q_i(x) + \frac{\alpha_s}{2\pi} \ln Q^2/\mu^2 \int_x^1 \frac{dy}{y} G(y) \frac{1}{2} \left[\left(1 - \frac{x}{y}\right)^2 + \left(\frac{x}{y}\right)^2 \right] \quad (12)$$

which is the same first order corrections to quark or antiquark densities from gluons as in electroproduction [see Ref. 8)]. The additional correction from quarks can be computed in the same way from $d\sigma_{q\bar{q}}$ in Eq. (1),

but in this case contributions from wave function and vertex corrections to the lowest order process $q + q^- \rightarrow \mu^+ \mu^-$ must also be taken into account ²⁾.

For a numerical estimate of $f_{PN}[\tau, \alpha_s(Q^2)]$ in the range of Q^2 values of the data from Ref. 9) at $\sqrt{s} = 27$ GeV, we take the valence densities as in Ref. 7), the gluon density as in Eq. 9). We then fix the sea density in order to fit the Drell-Yan cross-section of Ref. 9) at $\sqrt{s} = 27$ GeV, with the modified expression in Eq. (11) ^{*)} computed for $\mu^2 = 100$ GeV² and $\alpha_s(\mu^2) = 0.25$. This large value of μ^2 is required in order to make the first order corrections in Eq. (11) reliable in the range of the Q^2 values of the experiment. We find that $xs(x) \approx 0.44(1-x)^{10}$ for the sea density gives a good fit. Note that this is not the complete sea density, according to Eq. (12), and in fact at large x it is the gluon correction that dominates.

The resulting $f_{PN}[\tau, \alpha_s(Q^2)]$ is plotted in Fig. 2. The qualitative form and the order of magnitude is the same as in $P\bar{P}$ scattering. However, the sensitivity to the gluon density is very large in this case, particularly at large τ values. In Fig. 3 we compare the results for $\langle k_{\perp}^2 \rangle_{PN}$ given by Eq. (8) to the data ^{**) of Ref. 9)}. The experimental data for $\langle k_{\perp}^2 \rangle$ were transformed into data for $\langle k_{\perp}^2 \rangle$ by using the experimental fact ⁹⁾ that the $k_{\perp}^2$ distribution is approximately of the form $(1 + k_{\perp}^2/a^2)^{-3}$ with $a \approx 1.5$ GeV, thus implying $\langle k_{\perp}^2 \rangle = 16/\pi^2 \langle k_{\perp} \rangle$. The solid curves are predictions that only include the hard component of $\langle k_{\perp}^2 \rangle_{PN}$. We see that QCD prediction is rather flat, contrary to naïve expectations, and that it can only account at most for about one half of the effect. An intrinsic $\langle k_{\perp}^2 \rangle$ per parton of at least 0.5 GeV² is also apparently needed.

In conclusion we stress the following points. QCD predicts an approximate linear rise of $\langle k_{\perp}^2 \rangle$ with S or Q^2 only for fixed τ . A precise determination of the slope $f[\tau, \alpha_s(Q^2)]$ is in general quite involved.

*) With trivial changes to take the fact into account that the data refer to zero rapidity y . However, in the following computation of $\langle k_{\perp}^2 \rangle$ we consider the average over all y for simplicity. Also an equal number of neutrons and protons are assumed in the nucleus.

**) We chose to give theoretical predictions on $\langle k_{\perp}^2 \rangle$ in order to enhance the weight of the hard tail of the $k_{\perp}$ distributions.

Our analysis does not support any simple relation between the longitudinal cross-section in e-P scattering and $\langle k_{\perp}^2 \rangle$ as for example that proposed in Refs. 10). More in general we believe that the $k_{\perp}$ distribution of muon pairs from Drell-Yan processes is not simply related to the transverse momentum of the quark jet in e-P scattering ¹¹⁾.

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FIGURE CAPTIONS

Fig. 1: Lowest order diagrams for: a) quark+antiquark $\rightarrow$ gluon + $\mu^+\mu^-$ and
b) gluon+quark (antiquark) $\rightarrow$ quark (antiquark) + $\mu^+\mu^-$.

Fig. 2: The functions $f(\tau, \alpha_s)$, defined in Eq. (8), are plotted for both $p\bar{p}$ (dashed lines) and pN (full lines). In each case the dependence on the power n of the gluon distribution is shown for $n=4,6$ ($p\bar{p}$) and $n=4,5,6$ (pN).

Fig. 3: The hard component of the $\langle k_1^2 \rangle$ of the muon pair as a function of their invariant mass is compared with the experimental points taken from Ref. 9) for three different powers $n=4,5,6$ of the gluon distribution, following the procedure described in the text.

(a)

(b)

FIG.1

FIG.2

FIG.3