DETERMINATION OF THE EFFECTIVE QUADRUPOLE MOMENT IN ^{181}Ta

WITH PIONIC X-RAYS

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ABSTRACT

From the hyperfine splitting of the $5g \rightarrow 4f$ and the $6g \rightarrow 4f$ pionic X-rays in ^{181}Ta , an effective quadrupole moment of $Q_{\text{eff}} = 3.58 \pm 0.03$ b was determined. The strong interaction monopole shift ϵ_0 and the width Γ_0 of the $4f$ level were measured to be $\epsilon_0 = 540 \pm 100$ eV and $\Gamma_0 = 248 \pm 57$ eV, in good agreement with the values obtained with the standard optical potential description of the pion-nucleus interaction. Estimating the influence of the finite nuclear size, the deformation induced through the strong interaction between the pion and the finite nucleus, and the relative magnitude between the strong and the electromagnetic quadrupole coupling constants, we arrive at a value for the spectroscopic quadrupole moment of $Q = 3.30 \pm 0.06$ b, and for the intrinsic quadrupole moment of $Q_0 = 7.06 \pm 0.12$ b. This result is in good agreement with earlier values and with recent results reported from kaonic and muonic atoms as well as with the value obtained from Nilsson model calculations.

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1. INTRODUCTION

A somewhat unconventional way of determining electromagnetic (e.m.) properties of a heavy nucleus, such as its electric quadrupole moment Q , is to use pionic atoms. Of course, those pionic X-ray hyperfine (hf) patterns which are most sensitive to Q are to some extent distorted by the strong pion-nucleus interaction^{1,2)}. On the other hand, the strong absorption of the pion removes all "parallel" transitions ($n, \ell = n - 2 \rightarrow n - 1, \ell = n - 3$) and thus, in contrast with muonic atoms, finite-size dependent background subtraction is unnecessary.

The last X-ray transition in a pionic cascade, i.e. a transition to an atomic level whose lifetime is entirely determined by strong absorption, is also the one most sensitive to electric hf interaction. This transition therefore exhibits the usual features, namely a strong interaction induced energy shift ε_0 and a line broadening Γ_0 .

For a deformed nucleus the lower lying levels of the pionic atom show a hyperfine splitting (hfs) if the nuclear spin $I \geq 1$. The hfs is mainly caused by the electric quadrupole moment Q of the nucleus. However, the different orientations of the non-spherical pion wave functions, with respect to the symmetry axis of the nucleus, also lead to variations in shifts and widths among individual hf components^{1,2)}. Using this feature one can gain information on the nuclear neutron density distribution and its deformation³⁾.

The quadrupole moment in ^{181}Ta has been determined from an analysis of the hfs of the $5g \rightarrow 4f$ and $6g \rightarrow 4f$ pionic X-rays. This method was first used by Carrigan et al.⁴⁾ and the results were interpreted in terms of the e.m. interaction. Their value was not corrected for the strong quadrupole interaction and has a rather large error of about 7%. Recently, Batty et al.⁵⁾ reported on the quadrupole moment from kaonic tantalum, $Q_0 = 7.39 \pm 0.21$ b. Results from other methods on the quadrupole moment in ^{181}Ta have been compiled by Löbner et al.⁶⁾. These other methods include optical hfs, Coulomb excitation, giant dipole experiments, half-life measurements and the use of muonic atoms. Most of these values have large errors, although some of the Coulomb excitation results have an uncertainty of about 4%. Recently, McLoughlin et al.⁷⁾ reported results from a study

of muonic K and L X-rays. Their final values are, however, somewhat model dependent in the choice of the charge distribution parameters c_0 and t (the half-density radius and surface thickness of the nuclear charge density distribution). With a quoted experimental accuracy of the order of 0.14% they obtain electric quadrupole moments in the range of 7.25 to 7.50 b for different sets of c_0 and t , which all give similar χ^2 values.

By comparing our result with that obtained from muonic data the charge distribution parameters can be extracted. The monopole and quadrupole density of the neutron distribution in the nucleus can be deduced, as suggested by Scheck²⁾ and recently carried out by Ebersold et al.⁸⁾ for the pionic ^{175}Lu and ^{165}Ho atoms.

2. FORMALISM

The formulae for calculating the quadrupole moment from experimental data can be found, for example, in the textbook by Kopferman¹⁾, or in papers by Scheck²⁾, and Ebersold et al.^{8,9)}. In pionic atoms the main part of the hfs is due to the static electric quadrupole interaction. The hfs will cause either a $(2I + 1)$ or a $(2\ell + 1)$ non-degeneracy, depending on whether $I < \ell$ or $\ell < I$, respectively. The hf levels are labelled by the quantum number F , having values between $I + \ell$ and $|I - \ell|$. We can therefore for each hf level represent its electric quadrupole energy, $E(F)$, by the following expression:

$$E(F) = A_2 C(I, \ell, F) , \quad (1)$$

here A_2 is the electric hf constant of the level (n, ℓ) and

$$C(I, \ell, F) = \frac{3\gamma(\gamma + 1) - 4I(I + 1)\ell(\ell + 1)}{2I(2I - 1)\ell(2\ell - 1)} , \quad (2)$$

in which

$$\gamma = F(F + 1) - I(I + 1) - \ell(\ell + 1) . \quad (3)$$

The value of $C(I, \ell, F)$ is equal to unity for the maximum value of F . In the approximation of the nucleus being a point charge with relativistic hydrogen-like pion wave functions, A_2^p (point-nucleus electric hf constant) is determined to be

$$A_2^P = e^2 Q \frac{\ell}{(2\ell + 3)} \frac{\kappa^3}{s(s+1)(2s+1)}, \quad (4)$$

with

$$s = -1/2 + \sqrt{(\ell + 1/2)^2 - (Z\alpha)^2}, \quad (5)$$

where

Z = nuclear charge

α = fine structure constant

e = electric charge

and

$$\kappa = \frac{\sqrt{\bar{m}^2 - E_P^2}}{\hbar c}, \quad (6)$$

where $\bar{m}$ = the reduced π -nucleus mass, and

$$E_P = \frac{\bar{m}}{\sqrt{1 + \left(\frac{Z\alpha}{n - \ell + s} \right)^2}}. \quad (7)$$

Corrections to $E(F)$ [Eq. (1)] are made for the following effects:

- i) first-order vacuum polarization;
 - ii) magnetic dipole interaction;
 - iii) strong quadrupole interaction;
 - iv) finite-size effects.
- i) The vacuum polarization describes the influence of the creation and annihilation of virtual lepton pairs which will increase the Coulomb potential. In this case it is sufficient to use a first-order perturbation⁹⁾

$$\langle I\ell F | V^{VP} | I\ell F \rangle = \frac{\alpha^2 \hbar c}{3\pi} Q \frac{\ell}{2\ell + 3} C(I, \ell, F) R(n, \ell) \quad (8)$$

with, for $\ell = n - 1$,

$$R(n, \ell = n - 1) = \frac{2}{a_B^3 n^4 (n - 1)(2n - 1)} \left(\ln \frac{2\lambda_e}{na_B} - 1 - \frac{1}{2} \dots - \frac{1}{2n - 3} - \frac{5}{6} \right), \quad (9)$$

where a_B is the pion's Bohr radius and λ_e the electron Compton wavelength divided by 2π .

- ii) The main part of the hfs in normal atoms is due to magnetic dipole interaction. In pionic atoms, however, these interactions have a minor effect,

with a magnitude of only 2% of the electric quadrupole interaction. The formula used here is

$$E(F)^m = A_1 \gamma(I, \ell, F) , \quad (10)$$

where A_1 (the magnetic hf constant) is calculated with non-relativistic wave functions

$$A_1 = g_I \alpha^4 Z^3 \frac{m_\pi c^2}{2} \frac{1}{n^3 \ell(\ell+1)(2\ell+1)} \frac{m_\pi}{m_p} , \quad (11)$$

in which

$$g_I = \frac{\mu_I}{I} , \quad (12)$$

and

μ_I = nuclear magnetic moment expressed in units of nuclear magnetons,
 m_p = proton mass.

- iii) We assume the neutron distribution in the nucleus to be proportional to the proton distribution. We further assume the nucleus to have mainly a quadrupole deformation. Consequently, the multipole expansion of the mass density will have the following shape²⁾

$$\rho = \rho_0 + \sqrt{\frac{5}{16\pi}} \rho_2(r) Y_{20}(r, \theta, \phi) . \quad (13)$$

In the following we proceed according to Scheck²⁾. The monopole term ρ_0 will give rise to a shift ϵ_0 of the undisturbed level energy and the strong interaction effect can be written as

$$\epsilon = \epsilon_0 + \epsilon_2 C(I, \ell, F) , \quad (14)$$

in which ϵ_2 is the strong quadrupole constant and $C(I, \ell, F)$ is defined in formula (2). A good approximation to the ratio ϵ_2/ϵ_0 is

$$\frac{\epsilon_2}{\epsilon_0} = - \frac{5}{8\pi} \frac{I(2I-1)}{(I+1)(2I+3)} \frac{(\ell-1)}{(2\ell+1)} \frac{\int_0^\infty r^{2\ell} \rho_2(r) dr}{\int_0^\infty r^{2\ell} \rho_0(r) dr} . \quad (15)$$

We used a three-parameter Fermi distribution

$$\rho_0(r) = f(r) + \frac{1}{8\pi} \beta^2 c^2 \frac{d^2 f}{dr^2} \quad (16)$$

$$\rho_2(r) = \sqrt{\frac{16\pi}{5}} \beta c \left(-\frac{df}{dr} + \beta c \frac{\sqrt{5}}{7\sqrt{4\pi}} \frac{d^2 f}{dr^2} \right), \quad (17)$$

in which

$$f(r) = \frac{N_{\text{def}}}{1 + \exp \left[\frac{4 \ln 3}{t} (r - c) \right]} \quad (18)$$

$$N_{\text{def}} = \frac{3A}{4\pi} \left[c^3 \left(1 + \frac{3\beta^2}{4\pi} \right) + c \left(\frac{t}{4 \ln 3} \right)^2 \right]^{-1} \quad (19)$$

and A is the atomic mass number. The three parameters used were (see Table 1)

c = the radius parameter,

t = the skin thickness parameter,

β = the deformation parameter.

iv) The finite size of the nucleus will cause the following effects⁹⁾:

- a) Inside the charge distribution the pion feels a part of the monopole and quadrupole Coulomb fields only. This will cause a decrease of the electric quadrupole interaction (correction term δ_1).
- b) Owing to the strong interaction, the pion wave function will be changed even at larger distances from the nucleus. This will cause an increase of the long-range electric quadrupole interaction (correction term δ_2).

When we take these four corrections into account the energy difference E(F) becomes

$$E(F) = (A_2 - \epsilon_2) C(I, \ell, F) + A_1 \gamma(I, \ell, F), \quad (20)$$

with

$$A_2 = A_2^P (1 + \delta_1 + \delta_2). \quad (21)$$

The factor A_2^P includes the electric quadrupole interaction and the vacuum polarization correction. The effective quadrupole moment Q_{eff} can be defined as follows:

$$E(F) = Q_{\text{eff}} \left(\frac{A_2^P}{Q} \right) C(I, \ell, F). \quad (22)$$

Thus

$$Q_{\text{eff}} = Q \left(1 + \delta_1 + \delta_2 - \frac{\epsilon_2}{A_2^P} + \frac{A_1 \gamma(I, \ell, F)}{A_2 C(I, \ell, F)} \right). \quad (23)$$

The quantities (A_2^P/Q) and $C(I, \ell, F)$ can easily be model-independently computed. Consequently, by measuring the hfs, Q_{eff} can be evaluated model-independently.

The relative intensities I_{rel} of the separate radiative transitions between hf multiplets may be obtained assuming statistical population of the hf levels in the initial states

$$I_{\text{rel}}(F_i - F_f) \approx (2F_i + 1)(2F_f + 1) \left\{ \begin{matrix} \ell_f & F_f & I \\ F_i & \ell_i & 1 \end{matrix} \right\}^2 (\Delta E_{if})^3, \quad (24)$$

where $\{ \}$ denotes a 6-J symbol. In this case ΔE_{if} has an almost constant value owing to the small energy differences between the transitions of the multiplets. The variation in this factor causes only a 0.2% effect.

3. EXPERIMENTAL ARRANGEMENT AND DATA HANDLING

The experiment was carried out using one of the π^- beams of the CERN 600 MeV Synchro-cyclotron. The set-up of the present experiment has already been described elsewhere¹⁰⁾. Two γ -ray spectra were recorded simultaneously in coincidence with a stopped pion in a 2.49 g/cm² thick tantalum target. One spectrum was measured with a 2%, 15 cm³ semiplanar Ge(Li) diode (Princeton Gammatech) and the other with a 10% closed-end Ge(Li) detector (Philips) mounted in a Compton-suppression system^{11,12)}. In each of the spectra the split 5g $\rightarrow$ 4f and 6g $\rightarrow$ 4f transitions were visible. The 6g $\rightarrow$ 4f transition in the large volume detector was contaminated by a strong background line and therefore impossible to use for the Q evaluation. Consequently, two independent spectra were analysed for the 5g $\rightarrow$ 4f complex and only one for the 6g $\rightarrow$ 4f, implying less statistics for the latter.

The 5g and 6g levels of ¹⁸¹Ta are eightfold split and the 4f level sevenfold, as in this case $I = 7/2$ and $\ell = 4$ and 3, respectively. Taking the selection rules into account for transitions between both hf complexes, the resulting X-ray pattern consists of twenty members (Figs. 1 and 2).

The shape and position of the hf pattern as it appears in the energy spectrum are in a first approximation determined by the following parameters:

- i) The energy of the transition.
- ii) The effective quadrupole moment Q_{eff} of the nucleus.
- iii) The relative intensities of the members of the multiplet.
- iv) The width of the 4f level.
- v) The characteristics of the measuring instrument.

The actual parameters which were used for fitting are:

- a) The energy of the $F = {}^{15/2} \rightarrow {}^{13/2}$ transition.
- b) The energy difference ΔE between the ${}^{15/2} \rightarrow {}^{13/2}$ and ${}^{13/2} \rightarrow {}^{11/2}$ transitions.
- c) The intensity of the ${}^{15/2} \rightarrow {}^{13/2}$ transition which has a fixed relation to the other members of the multiplet¹³⁾. The intensities and energy difference of the individual members of the multiplet relative to the ${}^{15/2} \rightarrow {}^{13/2}$ one are given in Table 2, and shown in Fig. 3.

The energy difference ΔE has been determined to be 2.54(3) keV (see Table 3) and 2.58(3) keV in the spectra from the large volume Compton-suppressed and the smaller semiplanar Ge(Li) diodes, respectively, resulting in an effective quadrupole moment $Q_{\text{eff}} = 3.58 \pm 0.03$ b (Table 3). The strong interaction monopole width Γ_0 of the 4f level was measured by comparing the fitted line shape of the $5g \rightarrow 4f$ and $6g \rightarrow 4f$ X-ray complexes with the instrumental one, obtained from a quadratic interpolation of the line width of nuclear γ -ray transitions also present in the spectrum (see Fig. 4). In the data analysis of the hfs the experimental Gaussian line shape is folded with a Lorentzian one. Parameters which were kept fixed were those describing the instrumental line shape and the energies and intensities of neighbouring γ -ray transitions. The 453.49 keV ${}^{179}\text{Hf}$ transition situated in the middle of the $5g \rightarrow 4f$ pionic tantalum hf complex represents about 5% of the total intensity of this complex and had to be taken into account very carefully. The relative intensity of this γ -ray was calculated from other ${}^{179}\text{Hf}$ transitions belonging to the same γ -cascade. The same procedure was followed for a much weaker 456.6 keV ${}^{168}\text{Hf}$ γ -ray transition. To evaluate possible systematic errors of different parameters, like FWHM, intensity, and peak positions of the nuclear γ -rays present in the complex, their influence on χ^2 of the fit to the hf complex was

($c = 1.122 \times A^{1/3}$ fm and $t = 2.18$ fm) in his analysis of the muonic ^{175}Lu data are in good agreement only with those in lines 8 and 9 of Table 1. These Q_0 values also give the best agreement with our pionic atom data. Including the statistical error of 0.06 b and a systematical error of 0.10 b for the uncertainty in our strong interaction correction term, we arrive at a final value for the intrinsic quadrupole moment, $Q_0 = 7.06 \pm 0.12$ b, or for the spectroscopic quadrupole moment, $Q = 3.30 \pm 0.06$ b.

5. DISCUSSION

Our result is in good agreement with previously reported results compiled by Löbner et al.⁶⁾. Typical spectroscopic measurements yield values ranging from 8.36 ± 0.86 b to 5.8 ± 0.6 b. The giant dipole method also covers a rather large range of values from 5.2 ± 0.5 b up to 7.1 ± 0.8 b. With monochromatic γ -rays a value of 6.71 ± 0.74 b was reported. In Table 7 a selection of the most accurate values from Coulomb excitation, muonic atoms, and half-life measurements, together with the more recently reported experiments, is presented. There is in fact good agreement between the kaonic atom data reported by Batty et al.⁵⁾, the muonic atom data reported by McLoughlin et al.⁷⁾, and the present pionic atom data.

Assuming that the muonic atom data are best represented by the values (Nos. 8 and 9 in Table 1) 7.251 ± 0.008 b or 7.272 ± 0.008 b, one could then, from our $(Q_0)_{\text{eff}} = 7.73 \pm 0.06$ b, calculate the strong-interaction quadrupole constant ϵ_2 of the 4f state in ^{181}Ta from Eq. (24), using a value of $A_2^P = 495.8$ eV/b for the point-nucleus electric hf constant (see Table 5) and $(\epsilon_2/\epsilon_0)^{\text{th}} = -0.162$ (see Table 1). In a recent paper by Powers et al.²⁴⁾ an intrinsic quadrupole moment $Q_0 = 6.82 \pm 0.06$ b is reported. This value was obtained from the measurement of the energies of the $4f \rightarrow 3d$, $3d \rightarrow 2p$, and $2p \rightarrow 1s$ atomic transitions in muonic ^{181}Ta as well as from the hfs of the 2p and 3d states. In Table 8 the resulting ϵ_2 values from these calculations are shown. The obtained results can then be used to evaluate the monopole shift ϵ_0 . The calculated values presented in Table 8 agree reasonably well with the corresponding values shown in Table 1 and the experimentally determined value $\epsilon_0 = 540 \pm 100$ eV.

The difference between the earlier muon data⁷⁾ and those of Powers et al.²⁴⁾ may find its explanation in the model dependence of the analysis and possibly in nuclear excitation²⁵⁾, resulting in mixing of atomic and nuclear states.

An additional model-independent determination of the quadrupole moment in ^{181}Ta from an experiment with a stopped muon beam, where one measures, for example, the $5g \rightarrow 4f$ transition, would make it possible to determine the strong interaction quadrupole constant more accurately.

The measured monopole shift $\epsilon_0(\text{Ta}) = 540 \pm 100$ eV and the strong interaction width $\Gamma_0(\text{Ta}) = 248 \pm 57$ eV fit well into the existing systematics²⁶⁾ of these shifts and widths for the 4f level. Ebersold et al.⁸⁾ recently reported on strong interaction widths and monopole shifts of the 4f level in ^{175}Lu and ^{165}Ho . Their results -- $\Gamma_0(\text{Lu}) = 230 \pm 70$ eV, $\Gamma_0(\text{Ho}) = 210 \pm 40$ eV, $\epsilon_0(\text{Lu}) = 607 \pm 70$ eV, and $\epsilon_0(\text{Ho}) = 295 \pm 80$ eV -- are in good agreement with the present results for tantalum. Our value for Γ_0 and ϵ_0 also compares well with the old value for $\Gamma_0 = 500 \pm 200$ eV and $\epsilon_0 = 630 \pm 150$ eV reported by Schmitt et al.²⁷⁾. In Figs. 6a and b we show earlier obtained values and our present results for ϵ_0 and Γ_0 as a function of Z . In these graphs we also include our results¹⁰⁾ on Bi.

Although the above calculations of the strong interaction correction term are rather inaccurate, it is demonstrated that using stopped pions, quadrupole moments can be measured to an accuracy of a few per cent. The model-dependent corrections are small and, consequently, the value of Q is an almost model-independent quantity of the nucleus.

Assuming the nucleus to be a homogeneously charged body with a sharp surface and a shape corresponding to that of an equipotential surface, the intrinsic quadrupole moment calculated for the Nilsson state $[404]_{\frac{1}{2}}^+$ in ^{181}Ta at equilibrium deformation, $\epsilon_2 = 0.238$ and $\epsilon_4 = 0.063$ or $\beta_2 = 0.263$ and $\beta_4 = -0.052$, is shown to be $Q_0 = 6.97 \text{ b}^{28)}$, in good agreement with our experimental value.

Conversely, one calculates a deformation parameter $\beta_2 = 0.265 \pm 0.003$ from our value of the quadrupole moment using a value²⁸⁾ of $\beta_4 = -0.052$.

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Table 1

Strong interaction correction to $Q_{\text{eff}} = 3.58 \pm 0.03$ b for ^{181}Ta using different sets of charge distribution and deformation parameters. A comparison between the present results and the model-dependent data⁷⁾ from K and L X-rays in muonic atoms is given. The value c_0 is related to c by $c = c_0 A^{1/3}$. To calculate ϵ_2 from the theoretically calculated ratio ϵ_2/ϵ_0 , our experimentally determined value for $\epsilon_0 = 540 \pm 100$ eV was used.

No.	c (fm)	t (fm)	$(\epsilon_2/\epsilon_0)^{\text{th}}$	ϵ_2 (eV)	β	Q_0^{calc} (b)	$Q_0^{\mu\text{-exp}}$ (b)	$Q_0^{\pi\text{-exp}}$ (b)
1	6.51 a)	1.50 a)	-0.144	-78 ± 14	0.25	6.29		7.11 ± 0.06 d)
2	6.228	2.4479	-0.166 -0.158	-90 ± 17 -86 ± 16	0.319 0.303	7.455 7.06 ± 0.07	7.452 ± 0.008 c)	7.05 ± 0.06 d) 7.07 ± 0.07 d)
3	6.228	2.4413	-0.167 -0.159	-90 ± 17 -86 ± 16	0.321 0.303	7.505 7.06 ± 0.07	7.503 ± 0.002 c)	7.05 ± 0.06 d) 7.07 ± 0.06 d)
4	6.253	2.3895	-0.165 -0.159	-89 ± 16 -86 ± 16	0.314 0.301	7.390 7.07 ± 0.07	7.387 ± 0.005 c)	7.05 ± 0.07 d) 7.07 ± 0.07 d)
5	6.253	2.3767	-0.167 -0.159	-90 ± 17 -86 ± 16	0.318 0.301	7.490 7.07 ± 0.07	7.480 ± 0.002 c)	7.05 ± 0.07 d) 7.07 ± 0.06 d)
6	6.307	2.3049	-0.162 -0.158	-87 ± 16 -86 ± 16	0.306 0.296	7.315 7.07 ± 0.07	7.316 ± 0.011 c)	7.06 ± 0.07 d) 7.07 ± 0.07 d)
7	6.307	2.2095	-0.166 -0.159	-90 ± 17 -86 ± 16	0.310 0.296	7.415 7.06 ± 0.07	7.427 ± 0.007 c)	7.05 ± 0.07 d) 7.07 ± 0.07 d)
8	6.347	2.1416	-0.162 -0.159	-87 ± 16 -86 ± 16	0.300 0.293	7.253 7.07 ± 0.07	7.251 ± 0.008 c)	7.06 ± 0.07 d) 7.07 ± 0.07 d)
9	6.347	2.1550	-0.162 -0.158	-87 ± 16 -86 ± 16	0.301 0.293	7.278 7.07 ± 0.07	7.272 ± 0.008 c)	7.06 ± 0.07 d) 7.07 ± 0.07 d)
10	6.409	2.01	-0.154	-83	0.279	6.848	6.82 ± 0.06 e)	7.09 ± 0.06 d)

a) Values reported by de Wit et al.¹⁷⁾.

b) A value of $t = 1.5$ fm gave an unacceptable χ^2 in the analysis of the muonic data⁷⁾.

c) Data from McLoughlin et al.⁷⁾.

d) In this error no systematic error is included.

e) Muonic data reported by Powers et al.²⁴⁾.

Table 2

Relative intensities of the transitions of the $5g \rightarrow 4f$ multiplet¹³⁾ with respect to the $1^5_2 \rightarrow 1^3_2$ transition. The energy differences are given in eV/b relative to the $1^5_2 \rightarrow 1^3_2$ transition.

Transition	Relative intensity	ΔE (eV/b)
$1^1_2 \rightarrow 1^1_2$	9.4	- 449
$1^1_2 \rightarrow 3^1_3$	3.1	- 91
$3^1_2 \rightarrow 1^1_2$	6.7	- 514
$3^1_2 \rightarrow 3^1_2$	14.3	- 156
$3^1_2 \rightarrow 5^1_2$	4.0	345
$5^1_2 \rightarrow 3^1_2$	14.7	- 251
$5^1_2 \rightarrow 5^1_2$	18.9	250
$5^1_2 \rightarrow 7^1_2$	3.8	751
$7^1_2 \rightarrow 5^1_2$	25.3	145
$7^1_2 \rightarrow 7^1_2$	21.8	646
$7^1_2 \rightarrow 9^1_2$	3.0	946
$9^1_2 \rightarrow 7^1_2$	38.7	557
$9^1_2 \rightarrow 9^1_2$	22.0	858
$9^1_2 \rightarrow 1^1_{1/2}$	1.8	701
$1^1_{1/2} \rightarrow 9^1_2$	55.4	822
$1^1_{1/2} \rightarrow 1^1_{1/2}$	18.9	664
$1^1_{1/2} \rightarrow 1^3_{1/2}$	0.7	- 266
$1^3_{1/2} \rightarrow 1^1_{1/2}$	75.7	724
$1^3_{1/2} \rightarrow 1^3_{1/2}$	11.8	- 206
$1^5_2 \rightarrow 1^3_2$	100.0	0

Table 3

Experimental results on the energy difference ΔE of the $(^{15}/_2 \rightarrow ^{13}/_2)$ and $(^{13}/_2 \rightarrow ^{11}/_2)$ transitions and the calculated model-independent effective quadrupole moments

	5g $\rightarrow$ 4f data		6g $\rightarrow$ 4f data	Adopted value
	Compton suppression spectrometer	Semiplanar Ge(Li)	Semiplanar Ge(Li)	
ΔE (keV)	2.55 ± 0.03	2.58 ± 0.03	2.8 ± 0.4	2.58 ± 0.03
Q_{eff} (b)	3.55 ± 0.03	3.60 ± 0.04	3.39 ± 0.12	3.58 ± 0.03
$(Q_0)_{\text{eff}}$ (b)	7.61 ± 0.06	7.70 ± 0.08	7.26 ± 0.26	7.67 ± 0.06

Table 4

Calculated and experimental values for the 5g $\rightarrow$ 4f and 6g $\rightarrow$ 4f transition energies, shifts and widths of the 4f level

Transition	Calculated energies (keV)	Experiment (keV)	Γ_0^{exp} (eV)	ϵ_0^{exp} (eV)	Γ_0^{th} (eV)	ϵ_0^{th} (eV)
5g $\rightarrow$ 4f	453.020 ± 0.002	453.55 ± 0.11	248 ± 57	530 ± 110		
6g $\rightarrow$ 4f	696.330 ± 0.006	696.89 ± 0.23		560 ± 230		
Adopted mean values			248 ± 57	540 ± 100	300 a)	560 a)

a) Calculated perturbatively by Scheck²⁾.

Table 5

Interaction constants

Interaction	4f level	5g level
Electric quadrupole interaction constant A_2^p per barn	495.8 eV	127.1 eV
Vacuum polarization per barn	0.87 eV	0.12 eV
Strong quadrupole constant ϵ_2	-87.5 eV	0
Magnetic dipole hf constant A_1	1.45 eV	0.35 eV
Finite size correction $\delta_1 + \delta_2$	0.8%	0%

Table 6

Effects contributing to the energy displacement with respect to the undisturbed level energies. In the last column of the table are shown the percentages of the different effects on the displacement for the energy difference between the $^{15}_2 \rightarrow ^{13}_2$ and the $^{13}_2 \rightarrow ^{11}_2$ transitions.

Interaction	(n, l, F): 5, 4, $^{15}_2$	(n, l, F): 4, 3, $^{13}_2$	$\Delta E(^{15}_2 \rightarrow ^{13}_2 - ^{13}_2 \rightarrow ^{11}_2)$
Electric quadrupole	423.0 eV	1649.3 eV	93%
Vacuum polarization	0.4 eV	2.9 eV	0.3%
Magnetic dipole hf	-9.7 eV	-30.4 eV	-0.6%
Strong hf	0.0 eV	87.5 eV	6%
Finite size	0.0%	0.8%	1%

Table 7

Values for the intrinsic quadrupole moment Q_0 in ^{181}Ta . Present experimental results are compared with previously reported values [see also the compilation by Löbner et al.⁶⁾]

Experimental method	Q_0 (b)	Ref.
Muonic atom [Ge(Li)]		
Coulomb excitation (NaI) ($0 \rightarrow 1$)	7.5 ± 0.4	17
Coulomb excitation (NaI) ($0 \rightarrow 2$)	6.95 ± 0.27	19-22
Coulomb excitation inel. ($0 \rightarrow 1$)	7.09 ± 0.29	19, 20
Coulomb excitation inel. ($0 \rightarrow 2$)	7.08 ± 0.25	23
$T_{1/2}$ ($1 \rightarrow 0$)	6.79 ± 0.34	23
Pionic atom neglecting the strong interaction	6.16 ± 0.46	7
Pionic atom corrected value	7.85 ± 0.5	5
Kaonic atom	7.0 ± 0.5	5 and this paper
	7.39 ± 0.21	6
	$\left\{ \begin{array}{l} 7.316 \pm 0.011 \text{ a)} \\ 7.251 \pm 0.008 \text{ a)} \\ 7.272 \pm 0.008 \text{ a)} \end{array} \right.$	$\left. \begin{array}{l} \\ \\ \end{array} \right\} 8$
Muonic atom (see Table 4)		
Muonic atom	$6.82 \pm 0.06 \text{ a)}$	24
Present result from pionic atom	7.06 ± 0.12	This paper

a) The result depends on the choice of the charge-distribution parameters c_0 and t .

Table 8

The strong interaction monopole shift (ϵ_0) and quadrupole constant (ϵ_2) as derived from the data reported by McLoughlin et al.⁷⁾ and the present result on $(Q_0)_{\text{eff}}$. The calculated values, ϵ_0^{calc} , are obtained from the ϵ_2 values using a ratio $(\epsilon_2/\epsilon_0)^{\text{th}} = -0.162$ and a value of $A_2^p = 495.8$ eV/b.

$(Q_0)_{\text{eff}}$ a) (b)	Q_0 b) (b)	ϵ_2 (eV)	ϵ_0^{calc} (eV)	ϵ_0 a) (eV)
7.67 ± 0.12	7.316 ± 0.011	-42 ± 25	259 ± 154	540 ± 100
	7.272 ± 0.008	-50 ± 25	309 ± 154	
	7.251 ± 0.008	-53 ± 25	327 ± 154	
	6.82 ± 0.06	-132 ± 25	815 ± 154	

a) This paper.

b) McLoughlin et al.⁷⁾ and Powers et al.²⁴⁾.

Figure captions

- Fig. 1 : The split 4f level in pionic tantalum.
- Fig. 2 : Allowed transitions between the eightfold-split 5g and sevenfold-split 4f levels. This hf complex is composed of twenty transitions.
- Fig. 3 : The relative positions and intensities of the transitions between the 5g and 4f multiplets (see also Table 2). The energy scale is given by $\Delta E = 2.58$ keV. The height of the lines is proportional to the intensity of the transitions.
- Fig. 4 : The fitted $5g \rightarrow 4f$ complex at an energy of approximately 453 keV.
a) The hf complex as recorded with the large volume Compton-suppressed Ge(Li) diode.
b) The hf complex as recorded with the small semiplanar detector.
- Fig. 5 : The neutron mass distribution of the nucleus and the 4f wave function of the pionic tantalum.
- Fig. 6 : a) 4f level widths as a function of Z.
b) 4f level shifts as a function of Z.
The filled dots are taken from a review by Tauscher²⁶). The open squares are the present results.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

67534