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AN IMPROVEMENT TO ITERATIVE TRACKING FOR
MOMENTUM DETERMINATION

H. Wind

CERN, Geneva, Switzerland

ABSTRACT

Iterative tracking is a common method of determining the momentum of a particle when some track coordinates are observed before and after known magnetic fields. For this the momentum is somehow estimated, and a track computed and iteratively adjusted to fit the observations.

In the present method the momentum, as well as the four other unknown track parameters, is evaluated from a set of five linear equations.

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The method we propose resembles the "Quintic Spline Method"¹⁾. The difference is that the present method also allows detectors outside the magnetic fields, whilst the earlier method is only valid for all detectors inside the field.

As shown in Ref. 1, it follows directly from the equations of motion that

$$\begin{aligned} F_y(x) \equiv py'' &= (1 + y'^2 + z'^2)^{1/2} \{B_x z' + B_y y' z' - B_z (1 + y'^2)\} \\ F_z(x) \equiv pz'' &= (1 + y'^2 + z'^2)^{1/2} \{-B_x y' - B_z y' z' + B_y (1 + z'^2)\} , \end{aligned} \quad (1)$$

where y'' is d^2y/dx^2 , etc., and p is the momentum in normalized units. Although it is not indicated here, one should bear in mind that y' , z' , y'' , and z'' are all functions of x and that the components of the magnetic field, B_x , B_y , and B_z are in general functions of x , y , and z .

We assume detectors at x_i perpendicular to the x -axis and separated by magnetic field regions.

Let us now assume for a moment that the right-hand sides of Eqs. (1) are known throughout the magnetic fields. This assumption is justified if the field does not vary too sharply and $y'(x)$ as well as $z'(x)$ are small. The functions $F(x)$ can then be determined and would be pulse-like functions (see Fig. 1 for qualitative illustration). The function $F_y(x)$ can now be double integrated, starting from the first detector giving $Y(x)$, where at $x = x_1$, $Y(x) = 0$ and $(d/dx) Y(x) = 0$.

Similarly, we define $Z(x)$ for the double integrated function $F_z(x)$.

Let $y(x)$ and $z(x)$ be the, as yet unknown, track. Let us further denote the four unknown track parameters by a_1 , a_2 , b_1 , and b_2 , where a_1 and a_2 are the y and y' at the first detector and b_1 and b_2 similarly for z . The fifth unknown track parameter is, of course, the momentum p .

We then find

$$\begin{aligned} y(x) &= a_1 + a_2 x + \frac{1}{p} Y(x) \\ z(x) &= b_1 + b_2 x + \frac{1}{p} Z(x) . \end{aligned} \quad (2)$$

We distinguish between y_i and $y(x_i)$, the first being the observed y -value at the i^{th} detector and the second the fitted value. Initially $y(x_i)$ is put equal to y_i , and the same is done for z .

If indeed we have good estimates of the right-hand sides of Eq. (1) and consequently of $Y(x)$ and $Z(x)$ at the N detectors we can make a least-squares fit to Eqs. (2), the left-hand sides being the observed track coordinates y_i and z_i for $i = 1$ to N .

To make this fit we have to solve a set of five linear equations with a_1 , a_2 , $1/p$, b_1 , and b_2 as the unknowns:

$$\begin{aligned}
 a_1 N + a_2 x_i + \frac{1}{p} Y_i &= y_i \\
 a_1 x_i + a_2 x_i^2 + \frac{1}{p} (x_i Y_i) &= x_i y_i \\
 a_1 Y_i + a_2 (x_i Y_i) + \frac{1}{p} (Y_i^2 + Z_i^2) + b_1 Z_i + b_2 (x_i Z_i) &= (y_i Y_i) + (z_i Z_i) \quad (3) \\
 \frac{1}{p} Z_i + b_1 N + b_2 x_i &= z_i \\
 \frac{1}{p} (x_i Z_i) + b_1 x_i + b_2 x_i^2 &= x_i z_i
 \end{aligned}$$

where $Y_i \equiv Y(x_i)$ and $Z_i \equiv Z(x_i)$. Every term containing a subscript i is to be summed for $i = 1$ to N .

The solutions can readily be obtained when we first define

$$\begin{aligned}
 T_1 &= (x_i^2)(y_i) - (x_i)(x_i y_i) \\
 T_2 &= (Y_i)(x_i^2) - (x_i Y_i)(x_i) \\
 T_3 &= N(x_i^2) - (x_i)^2 \\
 T_4 &= - (x_i)(y_i) + N(x_i y_i) \\
 T_5 &= - (Y_i)(x_i) + N(x_i Y_i) \\
 T_6 &= (x_i^2)(z_i) - (x_i)(x_i z_i) \\
 T_7 &= (x_i^2)(Z_i) - (x_i)(x_i Z_i) \\
 T_8 &= - (x_i)(z_i) + N(x_i z_i) \\
 T_9 &= - (Z_i)(x_i) + N(x_i Z_i)
 \end{aligned}$$

Before every left-hand bracket we have again suppressed the $\sum_{i=1}^N$ sign.

The solution for the inverse momentum is then (with the same summation convention)

$$\frac{1}{p} = \frac{T_3[(y_i Y_i) + (z_i Z_i)] - T_1(Y_i) - T_4(x_i Y_i) - T_6(Z_i) - T_8(x_i Z_i)}{T_3[(Y_i^2) + (Z_i^2)] - T_2(Y_i) - T_5(x_i Y_i) - T_7(Z_i) - T_9(x_i Z_i)}$$

and with $1/p$ known we get

$$a_1 = (T_1 - T_2/p)/T_3$$

$$a_2 = (T_4 - T_5/p)/T_3$$

$$b_1 = (T_6 - T_7/p)/T_3$$

$$b_2 = (T_8 - T_9/p)/T_3$$

This may seem very long-Wind-ed, but in actual fact there are only some $9N + 36$ multiplications and three divisions to do.

Now that we have estimates of a_1 , a_2 , b_1 , b_2 , and p , we also have estimates of the slopes at the detectors:

$$y'_i = a_2 + \frac{1}{p} Y'_i$$

$$z'_i = b_2 + \frac{1}{p} Z'_i .$$

These we can use to get better estimates of the right-hand sides of Eqs. (1) and also (for some detector types) get more precise values of the original observations y_i and z_i .

If necessary we can also include weight factors w_i . This is indicated if we have reason to believe that some observations are more reliable than others. This can be done by adding in the above equations $\sum_{i=1}^N w_i$ instead of just $\sum_{i=1}^N$.

After the first fitting, improvement can be made by determining more precisely the left-hand sides of Eqs. (1), because we know now better where the track is and what slopes it has. We can then do the fitting again. So in that sense we still need iteration, but only to get better values of the integrated fields, i.e. the double integral of the right-hand sides of Eqs. (1). This gives a very rapid convergence. In simulated error-free tracks we found a reduc-

tion in the sum of squares of residuals

$$\sum_{i=1}^N \left\{ \left[y_i - (a_1 + a_2 x_i + \frac{1}{p} Y_i) \right]^2 + \left[z_i - (b_1 + b_2 x_i + \frac{1}{p} Z_i) \right]^2 \right\} .$$

by 10 orders of magnitudes in 5 iterations. The way the detector positions, fields and momenta were chosen is indicated in Table 1. The simulated observations were obtained by conventional Runge-Kutta tracking. These tests were made to establish that there are no errors in the FORTRAN program applying the present method. The momentum, as well as the other four track parameters, came out correctly to 10 significant digits after five iterations.

Determining the double integrals in Eqs. (1) is unconventional. If for each magnet it is known, for example from parametrization²⁾, how the incoming track parameters

$$y_{in}, z_{in}, y'_{in} \quad \text{and} \quad z'_{in}$$

are changed for given momentum p , we can use this rather than doing the double integration. We will now show how this can be done.

Let $Y'(x)$ be the integral of the right-hand side of the first equation (1) with $Y'(x_1) = 0$. We then find $y'(x) = a_2 + (1/p) Y'(x)$ so that the change in Y' as it passes the magnet field region is

$$\Delta Y' = (y'_{out} - y'_{in})p ,$$

where y'_{out} is dy/dx as it leaves the magnetic field region.

The change of $Y(x)$ in between the magnetic regions can be obtained by integrating the constant values of Y' . The change of $Y(x)$ in the magnetic region can be obtained from Eqs. (2) and is

$$\Delta Y = (y_{out} - y_{in} - a_2 X_0)p ,$$

where X_0 is the length in the x -direction of the magnetic field region.

All this applies, of course, in full analogy for z .

Carrying it out this way is mathematically speaking identical with actually doing the double integration. For the initial estimate of the slopes and positions

in the fields we assumed straight segments between the observations. We also need an initial estimate of the momentum to apply the above formulae, but this estimate is not very relevant. In fact, for our numerical tests we initially took p to be 10^{20} , but after the first fit it came out to be a reasonable value fifteen orders of magnitude lower. As mentioned above, after five iterations the five track parameters came out correctly to 10 significant digits.

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REFERENCES

- 1) H. Wind, Nuclear Instrum. Methods 115 (1974), 431.
- 2) H. Wind, Function parametrization, *in* Proc. CERN Computing and Data Processing School, Pertisau, 1972, CERN 72-21 (1972), p. 51.

Table 1

The way in which the track parameters and
detector positions were chosen for the numerical tests

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P = 1.E + 05 * RNDM(BB)
DYDX = -1. + 2. * RNDM(BB)
DZDX = -1. + 2. * RNDM(BB)
Y(1) = -1. + 2. * RNDM(BB)
Z(1) = -1. + 2. * RNDM(BB)

X(1) = RNDM(BB)
N = 20
DO 7 I = 1,N
PD(I) = X(I) + RNDM(BB)
QD(I) = PD(I) + RNDM(BB)
7 X(I + 1) = QD(I) + RNDM(BB)

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where

$X(I)$ are the supposed detector positions,

$[PD(I), QD(I)]$ are the supposed magnetic field regions

P is the supposed momentum

$DYDX$ and $DZDX$ are the initial slopes y'_1 and z'_1

$Y(1)$ and $Z(1)$ are the initial positions y_1 and z_1

$RNDM(BB)$ is a random number in the interval $[0,1]$

The magnetic field was assumed to be

$$B_y = 1000z/(y^2 + z^2)$$

$$B_z = 1000y/(y^2 + z^2)$$

$$B_x = 0.$$

The tracking stopped when $|y'|$ or $|z'|$ became larger than one or when
21 detectors had been passed.

Fig. 1 A qualitative illustration of the meaning of some functions