AN ANALYSIS OF PROMPT LEPTON EMISSION AT ISR ENERGIES

M. Block and A. Nakkasyan

Northwestern University, Evanston, Ill., USA^{×)}

J. Irion and F. Navach

Sektion Physik der Universität, Munich, Germany^{xx)}

F. Muller and M. Nussbaum^{xxx)}

CERN, Geneva, Switzerland

ABSTRACT

The e/π ratios at low p_T observed in pp collisions at $\sqrt{s} = 53$ GeV are analysed as a function of R, the ratio of the total number of electrons integrated over electron energies to the total number of pions integrated over pion energies. It is found that R is independent of the decay kinematics and very insensitive to the production dynamics of the parent particles. The total parent cross-section times the branching ratio into an electron is calculated, and the result is interpreted as either the total inclusive cross-section for production of charmed particles, or as the total cross-section for single photons in pp collisions.

Geneva - 29 August 1977

(Submitted to Nuclear Physics B)

×) Supported by the Energy Research and Development Agency, USA.

xx) Supported by the Bundesministerium für Forschung und Technologie, Germany.

xxx) Permanent address: Physics Dept., University of Cincinnati, Cincinnati, Ohio, USA.

Recent experiments^{1,2)} at the ISR have measured the relative yield of electrons to charged pions in a small solid angle (centred near 30°), as a function of p_T , the transverse momentum. They show that the e/π ratio increases dramatically as one goes to low p_T . In this communication, we assume that the electrons are secondary decay products of parent particles produced directly in pp collisions, and attempt to analyse these data, with as little model dependence as possible, in order to extract a reliable estimate of the parent total cross-section times the branching ratio into an electron. We find that the most satisfactory quantity to analyse is the ratio of *integral* fluxes, R , which we define as the total number of charged electrons (for simplicity, we only consider positive electrons, e^+) emitted in a small solid-angle interval $\Delta\Omega$ to the total number of charged pions (π^+) emitted, for the same luminosity, in the same $\Delta\Omega$, i.e.

$$R = \int_0^\infty \frac{d^2\sigma_e}{dp_T d\Omega} dp_T \bigg/ \int_0^\infty \frac{d^2\sigma_\pi}{dp_T d\Omega} dp_T . \quad (1)$$

The advantage of the integral ratio R is that for *massless* particles (we will assume hereafter that the energy release in the decay of the parent to electron is sufficient for us to set $m_e = 0$), this ratio is *completely* independent of the decay spectrum the electron has in the rest frame of the parent particle, if electrons are emitted isotropically in the parent rest frame.

We will parametrize the production spectrum of all particles in the central region by the invariant cross-section

$$E \frac{d^3\sigma}{dp^3} = A e^{-bp_T}, \quad 0 \leq p \leq p_{\max} \quad (2)$$

(A is a normalization constant), which is valid for $p_T \lesssim 1.5$ GeV/c, the region of primary interest for our experiments. At first glance, it appears that our parametrization, being flat in y , differs markedly from the rather complicated parametrization $f(y) = \exp[-5.13(y_{\max} - |y|)^{-0.38}]$ introduced by Bourquin and Gaillard³⁾ to parametrize high-energy pp data. However, it is readily shown by numerical evaluation that over most of the available y region, their parametrization can be well represented by the triangular function $f(y) = \text{const} (1 - |y|/y'_{\max})$, where $y'_{\max}$ is a value slightly smaller than $y_{\max}$. As we will show, most of the contribution to R will arise from parent particles that are emitted in directions close to the daughter electron and, hence, also have rather low values of y , near the y -value for pions. Thus, the flat distribution, convenient for analytical reasons, should not be markedly different in this region from the more complicated function which was numerically fitted to data. The major difference will come in the integration of Eq. (2) to obtain the normalization constant in terms of the total cross-section, where Eq. (2) will overestimate each cross-section by a factor of about two. However, since we will always take ratios of the cross-sections, for electrons and pions, this effect will cancel.

We rewrite Eq. (2) in terms of transverse momentum (p_T) and parallel momentum (p_L) components, and the azimuthal angle ϕ as

$$d^3\sigma = \frac{A e^{-bp_T} p_T dp_T dp_L d\phi}{\sqrt{p_L^2 + p_T^2 + m^2}}, \quad -\sqrt{p_{\max}^2 - p_T^2} \leq p_L \leq \sqrt{p_{\max}^2 - p_T^2}, \quad 0 \leq p_T \leq p_{\max}. \quad (3)$$

We now outline a three-fold program. Firstly, we integrate Eqs. (3) to obtain the normalization constant A in terms of the total cross-section σ , the slope parameter b , the particle mass m , and the maximum momentum $p_{\max}$. Secondly, we calculate in Eq. (1) the numerator which is the cross-section for electrons (the decay products of parent particles) to be detected in $\Delta\Omega$ at θ_0, ϕ_0 . Finally, we calculate in Eq. (1) the denominator which is the cross-section for a charged pion (π^+) to be detected in the same solid angle.

To obtain the normalization constant A , we integrate Eqs. (3) first over p_L and ϕ , a straightforward integration by quadratures. We next note that in the integration over p_T , the rapidly-varying exponential function e^{-bp_T} allows us, for $bp_{\max} \gg 1$, to extend the upper limit to infinity with negligible error, obtaining

$$\sigma = \frac{4\pi A}{b^2} \left\{ \ln \left[\frac{\sqrt{p_{\max}^2 + m^2} + p_{\max} \sqrt{1 - 1/(bp_{\max})^2}}{m} \right] + \right. \\ \left. + \text{ci}(mb) [\cos mb + mb \sin mb] + \text{si}(mb) [\sin mb - mb \cos mb] - 1 \right\}, \quad (4)$$

where

$$\text{si}(x) \equiv \frac{-\pi}{2} + \int_0^x \frac{\sin t}{t} dt \quad \text{and} \quad \text{ci}(x) \equiv - \int_x^\infty \frac{\cos t}{t} dt.$$

We can make an excellent approximation (which can be shown by numerical computation to be better than 2%) by replacing Eq. (4) by the expression

$$\sigma = \frac{4\pi A}{b^2} \ln \frac{2p_{\max}}{\sqrt{m^2 + \frac{4}{b^2}}}. \quad (5a)$$

We will use Eq. (5a) for pions, with $b_\pi = 5.4 \text{ (GeV/c)}^{-1}$ and $p_{\max} = 26 \text{ GeV/c}$. For $mb \gg 1$, a condition valid for heavy particles such as charm, Eq. (5a) goes into

$$\sigma = \frac{4\pi A}{b^2} \ln \frac{2p_{\max}}{m}, \quad (5b)$$

i.e. for a given σ , A/b^2 is effectively independent of the slope parameter b , and only slowly (logarithmically) dependent on the mass m , a result which will be of importance later.

We now calculate the cross-section for the production of electrons in the solid angle $\Delta\Omega$ at θ_0, ϕ_0 . For simplicity we choose $\phi_0 = 0$. The electron cross-section is the product of the branching ratio into an electron times the convoluted product of the parent production cross-section with the probability that electrons are emitted in $\Delta\Omega$. Thus, the cross-section for parent P giving birth to an electron emitted in the rest frame in $\Delta\Omega^*$ is

$$\Delta\sigma(P \rightarrow e) = \frac{B \cdot A e^{-bp_T} dp_T d\phi dp_L}{\sqrt{p_T^2 + p_L^2 + m^2}} \times \frac{\Delta\Omega^*}{4\pi}, \quad (6)$$

where all of the variables refer to the parent particle, except $\Delta\Omega^*/4\pi$ which expresses the isotropy of electrons emitted, *in the rest frame of the parent particle*, in the electron solid angle $\Delta\Omega^*$. In Eq. (6), B is the branching ratio for all electron decay modes of the parent particle. We have already integrated in Eq. (6) over the energy distribution of the electron in the parent rest frame. For *massless* particles, the angular solid angle interval $\Delta\Omega^*$ is related to the laboratory solid angle $\Delta\Omega$ for the electrons by

$$\Delta\Omega^* = \frac{\Delta\Omega}{(\gamma - \beta\gamma \cos \theta_e)^2}, \quad (7)$$

where $\gamma = E/m$, $\beta\gamma = p/m$ of the parent particle, and $\cos \theta_e$ is the angle between the electron direction in the laboratory system and the direction of the parent particle. A Lorentz transformation readily proves the validity of Eq. (7) for massless electrons. It is apparent from Eq. (7) that *no* knowledge of the electron energy in the rest frame of the parent particle is needed, and thus, our results (being integrated over electron energy in the lab. system) are completely independent of the decay-energy spectrum of the electrons^{*)}. We next introduce the relation linking θ_e with the angles θ, ϕ, θ_0 , i.e.

$$\cos \theta_e = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \sin \phi, \quad (8)$$

where θ and ϕ are the polar and azimuthal angles of the parent particle and $p_L = p \cos \theta$, $p_T = p \sin \theta$.

We introduce Eqs. (7) and (8) into Eq. (6), and integrate first over ϕ and then over p_L by quadratures. We next set the upper limit for p_T to infinity, and replace p_T in slowly varying terms (in contrast to the rapidly varying $p_T e^{-bp_T}$ dependence) by its most likely value $1/b$. In the domain $bp_{\max} \sin \theta_0 \gg 1$ ^{**)}, we find

*) An inspection of the structure of Eq. (7) shows that most of the electrons are emitted with $\cos \theta_e \approx 1$, i.e. along the parent direction, as mentioned earlier.

**) This condition is well satisfied for $\theta_0 \gtrsim 0.7^\circ$, if we use $b \approx 3 \text{ (GeV/c)}^{-1}$ and $p_{\max} \approx 25 \text{ GeV/c}$. The experimental value of θ_0 is $\sim 30^\circ$.

$$\frac{\Delta\sigma}{\Delta\Omega_0} = \frac{BA}{b^2} \frac{(p_{\max}/E_{\max}) \sqrt{1 - 1/(bp_{\max})^2}}{1 - (p_{\max}/E_{\max})^2 [1 - 1/(bp_{\max})^2] \cos^2 \theta_0} \quad (9a)$$

We can further simplify our results by noting that Eq. (9a) reduces to the simple expression, in the domain $bp_{\max} \sin \theta_0 \gg \sqrt{(bm)^2 + 1}$:

$$\frac{\Delta\sigma}{\Delta\Omega_0} = \frac{BA}{b^2 \sin^2 \theta_0} \approx \frac{B\sigma_P}{4\pi \sin^2 \theta_0} \frac{1}{\ln(2p_{\max}/m)}, \quad (9b)$$

where, in the last step, we have used Eq. (5b) to evaluate the normalization constant. Thus, for $bm \gg 1$, the condition of validity for Eq. (9b) is $\sin \theta_0 \gg m/p_{\max}$, or $\theta_0 \gg 4^\circ$ if we take $m \sim 2$ GeV, $b \approx 3$ (GeV/c) $^{-1}$, which is well satisfied for $\theta_0 \approx 30^\circ$, our experimental condition. We note that the simple result of Eq. (9b) is only slowly varying in the parent mass m (the logarithmic term) and is completely independent of b , the slope parameter of the parent. Thus, to high accuracy, we can replace $B\sigma_P$ by $\sum B_i \sigma_{P_i}$, where we now assume that there are several channels for parents, each with its own production cross-section times branching ratio into electrons. As an example, if the parents were charmed particles, the channels would be $D\bar{D}$, $F\bar{F}$, charmed baryon-anti-meson, etc. If we define $\sigma_{\text{tot}}(B) = \sum \sigma_{P_i} B_i$, where $\langle B \rangle$ is the average branching ratio into electrons, and if m in Eq. (9) is interpreted as a mean mass of the parent charmed particles, we see that Eq. (9b) enables us to measure the *total inclusive cross-section* for all charm states produced in pp collisions, if we know the average branching ratio into electrons.

The total numbers of charged pions emitted in $\Delta\Omega$ is readily evaluated by re-writing Eq. (3) in terms of the variables p , θ , ϕ , rather than p_L , p_T , ϕ , and introducing the dimensionless variable $z = b_{\pi} p \sin \theta_0$. We get

$$\frac{\Delta\sigma_{\pi}}{\Delta\Omega_0} = \frac{A_{\pi}}{b_{\pi}^2 \sin^2 \theta_0} \int_0^{z_{\max}} \frac{e^{-z} z dz}{\sqrt{z^2 + (b_{\pi} m_{\pi} \sin \theta_0)^2}}, \quad (10)$$

where $z_{\max} = b_{\pi} p_{\max} \sin \theta_0$. If we restrict ourselves to the domain $b_{\pi} p_{\max} \sin \theta_0 \gg 1$ (we need $\theta_0 \gg 0.4^\circ$ for $b_{\pi} = 5.4$ (GeV/c) $^{-1}$, $p_{\max} = 25$ GeV/c), we can extend the upper limit to infinity and replace z in the slowly varying square root by its most likely value of unity and integrate, to obtain

$$\frac{\Delta\sigma_{\pi}}{\Delta\Omega_0} = \frac{\sigma_{\pi}}{4\pi \sin^2 \theta_0} \frac{1}{\sqrt{1 + (b_{\pi} m_{\pi} \sin \theta_0)^2} \ln \left(2p_{\max} / \sqrt{m_{\pi}^2 + \frac{4}{b_{\pi}^2}} \right)}, \quad (11)$$

where we replaced the normalization constant A_π by its value from Eq. (5a). This result can be shown by numerical means to be accurate to $\sim 1/2\%$. Thus, we can now write as our final answer:

$$R = \frac{B\sigma_P}{\sigma_\pi} \frac{\ln \left(2p_{\max} / \sqrt{m_\pi^2 + \frac{4}{b_\pi^2}} \right)}{\ln (2p_{\max}/m)} \sqrt{1 + (b_\pi m_\pi \sin \theta_0)^2} . \quad (12)$$

We see from Eq. (12) that R is expected to increase slowly (about 17%) when one goes from 30° to 90° at the ISR, and that it is only slowly dependent on the assumed parent mass m , as well as being completely independent of the electron decay-energy spectrum and essentially independent of the parent slope parameter b . This is a consequence of our choosing to examine the ratios of the total fluxes of electrons and pions, *integrated* over the energy spectrum of electrons and pions, rather than looking at their ratios as a function of p_T . The approach of using R , the ratio of the total numbers of electrons to pions at a given angle, allows us to extract the total inclusive $\sigma_{\text{tot}}(B)$ with *no* assumptions needed, either for the fraction of the various production modes or for the kinematics of the decay schemes. This approach is complementary to the more usual one of fitting the e/π ratios as a function of p_T to a particular production mode and decay scheme, for example, pair production of $D\bar{D}$, with subsequent decays into $K\pi$, $K^*\pi$, $K^{**}\pi$, $K(n\pi)\pi$, etc.

We now examine the experimental situation, and compare with Eq. (12) to extract $B\sigma_P$. What is actually measured in the experiment is $e^\pm/\pi = r^\pm(p_T)$, where $\pi = 1/2(\pi^+ + \pi^-)$, i.e. the ratio, in various p_T intervals, of the number of electrons of a given sign in this p_T interval to the mean number of charged pions of the same sign in this interval of p_T . The ratios are found to be the same (within errors) for both signs, i.e. $r^+(p_T) = r^-(p_T) \equiv r(p_T)$. The experiment measures $r(p_T)$ down to the interval $0.2 \leq p_T \leq 0.3$ GeV/c and thus no knowledge is available for $p_T < 0.2$ GeV/c. Empirically, the data are well represented by the expression $r(p_T) = r(0.2)(0.2/p_T)$, with p_T in GeV/c, i.e. a $1/p_T$ fit for $p_T \lesssim 1$ GeV/c. This enables us (knowing that the pion spectrum experimentally is of the form $dN_\pi = N_\pi b_\pi^2 p_T e^{-b p_T} dp_T$ for $p_T \lesssim 1$ GeV/c, which is the region from which almost all the electrons and pions that contribute to R came from), to calculate the total number of electrons down to $p_T = 0.2$ GeV/c. Since our definition of R in Eq. (1) assumes that we integrate the electron flux down to $p_T = 0$, we must make an extrapolation. Several extrapolations are possible and are, of course, completely arbitrary. The obvious choice of extending the $1/p_T$ law to zero p_T (we do not get an infinity because of the $p_T e^{-b p_T}$ factor in the pion spectrum) probably overestimates the number of electrons. A second, more middle-of-the-road choice, is to assume that $r(p_T) = \text{constant} = r(0.2)$ for the p_T region 0-0.2 GeV/c. A third

choice, which is likely to be an underestimate, is to assume that $r(p_T) = (p_T/0.2)r(0.2)$, for $0 \leq p_T < 0.2$, i.e. a linear rise from 0 at $p_T = 0$ to the value of $r(0.2)$ at $p_T = 0.2$ GeV/c. Using the first as an upper limit, the second as a type of median value, and the third as a lower limit, we obtain

$$R = 0.7 \begin{matrix} + 0.3 \\ - 0.15 \end{matrix} r(0.2) = \left(5.5 \begin{matrix} + 2.3 \\ - 1.2 \end{matrix} \right) \times 10^{-4}, \quad (13)$$

where we used the best-fit value of $r(0.2) = 7.8 \times 10^{-4}$ obtained by fitting the experimental data²⁾ at $\sqrt{s} = 53$ GeV to a $1/p_T$ law. Using a $(\pi^+ + \pi^-)/2$ cross-section of 200 mb, we obtain from Eqs. (12) and (13), with $m_{\text{charm}} = 1.8$ GeV,

$$\sigma_{\text{charm}} \times \langle B \rangle = 68 \begin{matrix} + 27 \\ - 16 \end{matrix} \mu\text{b}. \quad (14)$$

We now compare our final results in Eq. (14) with the complementary approach, i.e. theoretical attempts to fit in detail $r(p_T)$, fits in which various decay models are assumed. One such attempt³⁾ considered the production of $D\bar{D}$, parametrized by the B-G model⁴⁾, with the decay scheme $D \rightarrow K^* e \nu$, and obtained $\text{Br}[D(1.8) \rightarrow K^* e \nu] = 27 \mu\text{b}$. The theoretical prediction was considerably below the experimental points at low p_T (where the bulk of the electron events lie) and any attempt at over-all normalization would clearly require a factor of at least two greater cross-section. Ferro Fontán and Odorico⁵⁾ have used a model which assumes that many π 's are created statistically in the decay of D particles, and hence, that the electron spectrum is highly peaked in the p_T region *below* 0.2 GeV/c, a situation which we called an *upper limit*. They get a fit with $\sigma \sim 90 \mu\text{b}$, which is near our upper limit. Bourquin and Gaillard⁴⁾ attempt a fit using $D \rightarrow K^* e \nu$ also, and get a value of $\sim 150 \mu\text{b}$, somewhat higher than our limit.

If we assume the value of $B \approx 0.1$ for the electron branching ratio, we conclude that at $\sqrt{s} = 53$ GeV, the total inclusive production cross-section for charmed states in all channels in pp collisions is

$$\sigma_{\text{charm}} = 0.68 \begin{matrix} + 0.27 \\ - 0.16 \end{matrix} \text{mb}, \quad (15)$$

if we interpret all of the electron signal as being due to charm production and subsequent electron decay. This cross-section, although rather large, is not in contradiction with ISR measurements⁶⁾ of the *upper limit* at this energy of the production cross-section times branching ratio of $\sim 9 \mu\text{b}$ of $D^0 \rightarrow K^\pm \pi^\mp$, since only about 25% of the 0.68 mb given by Eq. (15) is probably due to $D^0 \rightarrow K^\pm K^\mp$, whose branching ratio is quite small ($\sim 3\text{--}5\%$), and thus would contribute $\sim 4\text{--}6 \mu\text{b}$ to the $9 \mu\text{b}$ upper limit of Ref. 6. However, it is getting very close to the upper limit. Further, the equality of e^+/π and e^-/π is more readily suggestive of an electromagnetic process than of charm decay. A possible process is the production

of single γ -rays in pp collisions, suggested by Ferrar and Frautschi⁷⁾, which could result in virtual γ -rays being converted into massive e^+e^- pairs (Dalitz-like pairs), whose opening angles could be large enough for one of them to escape the veto system, and hence, be registered in our apparatus as a "prompt" single electron. In order to estimate the number of single photons arising from this source, we would have to interpret the branching ratio in Eq. (14) as the product of the probability for pair production (Dalitz-decay probability) times the acceptance efficiency of our apparatus for measuring single electrons when a pair is produced. An estimate using an energy release of 700 MeV, and taking into account the experimental geometry, indicates that the Dalitz factor for a single γ -ray is $\sim 1/100$ (it varies very slowly with the energy release) and that the acceptance for high-mass pairs is $\sim 1/5$, giving an effective branching ratio of $\sim 1/500$. Thus, the cross-section estimate for single photons is about 30 mb, giving a ratio of the total cross-sections of $\gamma/\pi^0 \approx 0.15$, where the bulk of the γ flux arises from electron p_T values below 0.5 GeV/c.

In summary, we have given estimates of the cross-section times branching ratio necessary to explain our electron data if we assume that it is completely due to *either* charm production *or* single photon emission in pp collisions at $\sqrt{s} = 53$ GeV. However, we require a rather high charm production, ~ 0.7 mb. Moreover, the equality of e^+/π and e^-/π for all p_T values is suggestive of an electromagnetic process. If we assume that single-photon emission in pp collisions is the source of our electron signal, we conclude it must have a strength of $\sim 15\%$ of π^0 production.

We would like to thank Dr. E. Berger of Argonne National Laboratory for many fruitful discussions.

REFERENCES

- 1) L. Baum et al., Phys. Letters 60B (1976) 485.
- 2) M.M. Block et al., Nuclear Phys. B, same issue.
- 3) I. Hinchliffe and C.H. Llewellyn Smith, Phys. Letters 61B (1976) 472;
I. Hinchliffe and C.H. Llewellyn Smith, Nuclear Phys. B114 (1976) 45.
- 4) M. Bourquin and J.-M. Gaillard, Nuclear Phys. B114 (1976) 334.
- 5) C. Ferro Fontán and R. Odorico, A common (charm) origin for pp direct electrons and e^+e^- anomalous electron events, Trieste, March 1977.
- 6) J.C. Alder et al., Phys. Letters 66B (1977) 401.
- 7) G.R. Ferrar and S. Frautschi, Phys. Rev. Letters 36 (1976) 1617.