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PARTONS OF A ONE-DIMENSIONAL BOX

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ABSTRACT

The valence and sea quark parton distributions are calculated for deep inelastic scattering from a one-dimensional Dirac box. The sea contribution corresponds explicitly to the excitation of quark-antiquark pairs from the "vacuum" within the box. The connection between co-ordinate space wave functions and null-plane parton distributions is explicitly given and we comment on the relevance of our results to those of other recent calculations.

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1. INTRODUCTION

There has recently been much discussion¹⁾⁻⁸⁾ concerning the variation, with respect to the fraction of longitudinal momentum x , of parton transverse momentum. In particular, several of these discussions base their intuition on parton distributions arising from a treatment of deep inelastic scattering within the framework of the MIT Bag. The scaling predictions of this model were first discussed by Jaffe⁹⁾ but his approximations have the disturbing feature that the scaling functions and parton distributions are not positive-definite. Since the quadratic boundary condition of the Bag plays no essential role in the discussion the "static glueless cavity approximation" to the MIT Bag reduces to the problem of massless quarks confined by a Dirac scalar potential¹⁰⁾. Thus to shed light on these questions, it is perhaps useful to consider in more detail the problem of a colourless, flavourless one-dimensional box⁹⁾.

2. STRUCTURE FUNCTIONS

In one dimension, the structure function for a scalar current is defined by

$$W(q, q_0) = \frac{M}{2\pi} \sum_N 2\pi \delta(q^0 + E_T - E_N) \quad (1)$$

$$|\langle N | \int dz e^{iqz} J(z, 0) | T \rangle|^2$$

for a target state $|T\rangle$ of mass M , normalized by $\langle T|T \rangle = 1$. The eigenfunctions of a massless fermion confined to a box of length 2ℓ are⁹⁾

$$u_m(z, t) = \frac{1}{\sqrt{4\ell}} \begin{pmatrix} e^{iE_m z} \\ (-1)^m e^{-iE_m z} \end{pmatrix} e^{-iE_m t} \quad (2)$$

with

$$E_m = \frac{\pi}{2\ell} (m + 1/2) \quad (3)$$

where m is a positive or negative integer, or zero. We may introduce the "quark" field operator

$$q(z, t) = \sum_m b_m u_m(z, t) \quad (4)$$

with

$$u_m(z, t) = u_m(z) e^{-iE_m t} \quad (5)$$

and where b_m is a creation or destruction operator for E_m negative or positive, respectively.

Let $|0\rangle$ denote the "empty" box defined by

$$b_n^+ |0\rangle = 0 \quad \text{if } E_n < 0$$

$$b_p |0\rangle = 0 \quad \text{if } E_p > 0$$

Consider a target obtained by adding one quark in the lowest positive energy level

$$|T\rangle = b_0^+ |0\rangle$$

The scalar current $J(z, t)$

$$J(z, t) = \bar{q}(z, t) q(z, t) \quad (6)$$

has then two possibilities for exciting this state. These correspond to the two terms

$$J(z, 0) \sim \sum_{p,n} (b_p^+ b_0 \bar{u}_p u_0 + b_p^+ b_n \bar{u}_p u_n)$$

where the first piece excites the initial positive energy quark from state 0 to another positive energy state p , and the second excites one of the occupied negative energy levels n to a positive energy state p -- physically this corresponds to quark-antiquark pair creation from the "empty" box. Notice that the positive energy quark excited by this vacuum process may not occupy the state 0 already filled by the "valence" quark. As remarked by Jaffe, these "empty box" contributions would not occur in the absence of the box, for kinematic reasons; but the presence of the box destroys the relevant conservation laws, which depend on translation invariance. Then these contributions no longer vanish. They correspond to the "bubble" diagram of Jaffe, which he explicitly chose to omit. The reasons for this omission are obscure to us. As noted by Jaffe it is his leaving out of this diagram which leads him to negative antiparticle distributions. We do not follow him in this direction.

The structure function W may thus be written in terms of three contributions

$$W = W_1 + W_0 - W_2 \quad (7)$$

where, with summation over positive levels p and negative levels n ,

$$W_1 = \frac{M}{2\pi} \sum_p 2\pi \delta(q^0 + E_0 - E_p) |\langle p | e^{iqz} | 0 \rangle|^2 \quad (8)$$

$$W_2 = \frac{M}{2\pi} \sum_n 2\pi \delta(q^0 - E_0 + E_n) |\langle 0 | e^{iqz} | n \rangle|^2 \quad (9)$$

$$W_0 = \frac{M}{2\pi} \sum_{p,n} 2\pi \delta(q^0 + E_n - E_p) |\langle p | e^{iqz} | n \rangle|^2 \quad (10)$$

The matrix elements are defined by

$$\langle p | e^{iqz} | n \rangle \equiv \int dz \bar{u}_p(z) e^{iqz} u_n(z) \quad (11)$$

The term W_0 is the "empty" box contribution; to correct for the presence of the valence quark we must subtract off the disallowed contribution W_2 . We can now see directly the reason for the non-positivity of the scaling functions in Jaffe's commutator approach. His approximation retains the "Z-graph" contribution W_2 and neglects the larger vacuum contribution W_0 .

To study Bjorken scaling we put

$$\left. \begin{aligned} q^0 &= q + Mx \\ q &\rightarrow \infty, \text{ with } x \text{ fixed} \end{aligned} \right\} \quad (12)$$

and obtain

$$W(q, q_0) \rightarrow F(x)$$

The scaling function also separates into three identifiable contributions

$$F(x) = F_1(x) - F_2(x) + F_0(x) \quad (13)$$

where

$$F_1(x) = \frac{Ml}{2\pi} \left| \frac{\cos Mx l}{2E_0 l - Mx l} \right|^2 \quad (14)$$

$$F_2(x) = \frac{Ml}{2\pi} \left| \frac{\cos Mxl}{2E_0l + Mxl} \right|^2 \quad (15)$$

$$F_0(x) = \frac{Ml}{2\pi} 2 \sum_{p=0}^{\infty} \left| \frac{\cos Mxl}{2E_pl + Mxl} \right|^2 \quad (16)$$

The empty box contribution ($W_0 \rightarrow F_0$) is finite and scales. Half of the first term in F_0 goes simply to cancel the negative $-F_2$, so that F is positive. Note by the way that $F_0(x)$ is a tabulated function, a special case of the generalized Riemann Zeta function or trigamma function¹¹⁾.

3. PARTON DISTRIBUTIONS

Suppose that a fast hadron can be regarded as an assembly of free massless partons and antipartons, with distributions

$$f(x) dx, \quad \bar{f}(x) dx$$

respectively, where x is the longitudinal momentum fraction. Then straightforward calculation for the structure function of a scalar current gives

$$F(x) = \frac{1}{2} f(x) + \frac{1}{2} \bar{f}(x) \quad (17)$$

We will verify this for the box with f and $\bar{f}$ obtained in a conventional way from the null-plane state (see, for example, Hughes⁷⁾ or Bell and Ruegg¹²⁾).

Since the box only describes a hadron at rest it is necessary to go to the null-plane to obtain the correct parton identification. The distribution of partons defined on an equal-time surface in a frame in which the hadron is moving with infinite momentum is equivalent to the null-plane free particle expansion of the state vector in the rest frame of the hadron.

To put the field operator (4) on the null-plane set $t = -z$ and drop "bad" components

$$\hat{q}(z) = \sum_m b_m \hat{u}_m(z) e^{iE_m z} \quad (18)$$

where the 1-spinor $\hat{u}_m$ is obtained from the Dirac 2-spinor u_m by dropping the $\alpha_3 = -1$ components in a representation with α_3 diagonal.

To identify the parton distributions this must be equated to the null-plane plane-wave expansion

$$\hat{q}(z) = \int_{-\infty}^{\infty} \frac{d\eta}{2\pi|z\eta|} a(\eta) \hat{u}(\eta) e^{i\eta z} \quad (19)$$

where $a(\eta)$ is a destruction or creation operator for η positive or negative, respectively, and the plane wave 1-spinors are chosen to have the normalization

$$\hat{u}^*(\eta) \hat{u}(\eta) = |\eta| \quad (20)$$

Taking (18) to be zero outside the box gives

$$a(\eta) = 2 \sum_m b_m \int dz \hat{u}^*(\eta) \hat{u}_m(z) e^{i(E_m - \eta)z} \quad (21)$$

So

$$\begin{aligned} \langle T | a^\dagger(\eta) a(\eta) | T \rangle &= \sum_n \left| \int dz 2 \hat{u}^*(\eta) \hat{u}_n(z) e^{i(E_n - \eta)z} \right|^2 \\ &+ \left| \int dz 2 \hat{u}^*(\eta) \hat{u}_0(z) e^{i(E_0 - \eta)z} \right|^2 \end{aligned} \quad (22)$$

and

$$\left. \begin{aligned} \langle \tau | a(\eta) a^\dagger(-\eta) | \tau \rangle = \\ \sum_p \left| \int d\bar{z} \, 2 \hat{u}^*(\eta) \hat{u}_p(\bar{z}) e^{i(E_p + \eta)\bar{z}} \right|^2 \\ - \left| \int d\bar{z} \, 2 \hat{u}^*(\eta) \hat{u}_0(\bar{z}) e^{i(E_0 + \eta)\bar{z}} \right|^2 \end{aligned} \right\} \quad (23)$$

With the covariant norm

$$\{a(\eta), a^\dagger(\eta')\} = 2\pi |2\eta| \delta(\eta' - \eta) \quad (24)$$

the parton and antiparton number operators are, respectively,

$$\left. \begin{aligned} \int_0^\infty \frac{d\eta}{2\pi |2\eta|} a^\dagger(\eta) a(\eta) \\ \int_0^\infty \frac{d\eta}{2\pi |2\eta|} a(-\eta) a^\dagger(-\eta) \end{aligned} \right\} \quad (25)$$

So we identify parton and antiparton distributions

$$\left. \begin{aligned} f(x) dx &= \frac{d(Mx)}{2\pi 2Mx} \langle T | a^+(Mx) a(Mx) | T \rangle \\ \bar{f}(x) dx &= \frac{d(Mx)}{2\pi 2Mx} \langle T | a(-Mx) a^+(-Mx) | T \rangle \end{aligned} \right\} \quad (26)$$

Taking account of (20), and $|n| = Mx$, and expressing the result in terms of Dirac spinors by the explicit projection $(1 + \alpha_3)/2$ onto good components^{*)}

$$\left. \begin{aligned} f(x) &= \frac{M}{\pi} \left| \int dz \left(\frac{1+\alpha_3}{2} \right) u_0(z) e^{i(E_0-Mx)z} \right|^2 \\ &\quad + \frac{M}{\pi} \sum_n \left| \int dz \left(\frac{1+\alpha_3}{2} \right) u_n(z) e^{i(E_n-Mx)z} \right|^2 \\ \bar{f}(x) &= -\frac{M}{\pi} \left| \int dz \left(\frac{1+\alpha_3}{2} \right) u_0(z) e^{i(E_0+Mx)z} \right|^2 \\ &\quad + \frac{M}{\pi} \sum_p \left| \int dz \left(\frac{1+\alpha_3}{2} \right) u_p(z) e^{i(E_p+Mx)z} \right|^2 \end{aligned} \right\} \quad (27)$$

Evaluation of the integrals with the wave functions (2), gives finally

*) The first term of (27) is essentially that given (in three dimensions) by R. Hughes⁷⁾. Agreeing with Hughes, we disagree with Davis and Squires³⁾.

$$\left. \begin{aligned} f &= \frac{Ml}{\pi} \left| \frac{\cos Mxl}{2E_0l - Mxl} \right|^2 + \sum_n \frac{Ml}{\pi} \left| \frac{\cos Mxl}{2E_nl - Mxl} \right|^2 \\ \bar{f} &= - \frac{Ml}{\pi} \left| \frac{\cos Mxl}{2E_0l + Mxl} \right|^2 + \sum_p \frac{Ml}{\pi} \left| \frac{\cos Mxl}{2E_pl + Mxl} \right|^2 \end{aligned} \right\} \quad (28)$$

in agreement with (18) and (13).

The first ("valence") terms in (28) and the remaining ("sea") sums are plotted separately in the Figure with $Ml = 3\pi/2$ as proposed by Jaffe for a three-valence-quark model of the proton. The cut-off at $x = 1$ in the Figure is quite arbitrary here since the model lacks translation invariance and does not have decent kinematics. Note that the negative $\bar{f}$ (valence) contribution is in fact a correction to the $\bar{f}$ (sea) contribution due to the presence of the valence quark in the target. Colour and flavour have not yet been allowed for in the Figure. With N colours the sea contributions have to be scaled up, for each flavour, by a factor N . For the d (and $\bar{d}$) quarks in the "proton" the valence contributions are equal to those shown: the u (and $\bar{u}$) distributions are twice as large.

4. CONCLUDING REMARKS

A merit of the model is manifestly to respect symmetry (reflection symmetry only, in the one-dimensional case) in the centre of mass system. Several authors have conjectured that the symmetry of a hadron in the rest system should have some simple consequences for the parton distributions. As far as we can see from our results this is not so. The factor $(1 + \alpha_3)$ in (27), and the variation from

term to term of E in the exponent, seem to obscure completely the symmetry of the wave functions u . The first term in (28) does indeed have a symmetry point at

$$x = 2E_0 / M$$

- but the symmetry points of others are different. Nor does (28), with the u 's appropriate to non-zero quark mass^{*)} m , show the symmetry between x and $m^2(M^2x)^{-1}$ which would correspond to a centre-of-mass-symmetric distribution of on-mass-shell free quarks^{14),15)}.

A demerit of the model is the lack of translational invariance, and the consequent failure of momentum conservation. Unfortunately, this failure is quite obtrusive in our results. With correct kinematics the momentum sum rule

$$\sum_i \int_0^1 dx \, x (f_i(x) + \bar{f}_i(x)) \leq 1$$

should hold, where the summation is over all colours and flavours. In agreement with Jaffe we find that the valence contributions alone almost saturate this sum rule, contributing

$$3 \times (0.31 - 0.02) = 0.86$$

for the "proton". However, the additional sea contributions, in the case of N' flavours and N colours, amount to

$$N \times N' \times (0.04 + 0.04) = N \times N' \times 0.08$$

Clearly, even for quite small N and N' the sum rule is greatly oversaturated. It seems essential therefore to deal with the problem of translational invariance in such a model. We conjecture that this will be so also in the 3-dimensional box. Note, however, that even apart from the lack of translational invariance two other features of the model may have exaggerated the "sea" contribution; these are the extreme relativistic character of massless quarks, and the verticality of the confining walls.

*) Jaffe's box model with non-zero quark mass has been studied by D.J. Broadhurst¹³⁾.

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parton distributions f and
antiparton distributions $\bar{f}$
for a 'proton', ignoring
colour and flavour factors