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COVARIANT QUANTIZATION OF SUPERGRAVITY

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ABSTRACT

Recently noted modifications of the quantization rules for supergravity can be understood within the framework of conventional covariant quantization, provided one uses a formulation of the theory in which the supersymmetry algebra closes. Using such a formulation due to Breitenlohner we discuss the quantization, and are naturally led to terms quartic in the ghost fields in the quantum action.

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The quantization of field theories with local gauge invariance is by now a well-understood procedure. It is usually necessary to remove the degrees of freedom that are associated with the local symmetry, and this is most conveniently done by imposing a set of gauge conditions on the fields. However, such a quantization procedure must be carried out with some care, since it is obviously required that gauge-invariant quantities remain independent of the particular set of gauge conditions that has been chosen. In the functional integral approach this is achieved by modifying the integration measure, and this modification can be elegantly expressed by the addition of terms quadratic in so-called Faddeev-Popov ghost fields to the invariant action¹⁾. This method is usually referred to as covariant quantization, and it has been shown for a variety of gauge theories that it leads to a gauge-independent and unitary S-matrix²⁾.

However, it was argued recently by Fradkin and Vasiliev³⁾ based on results obtained in the Hamiltonian formalism that the conventional covariant quantization procedure leads to incorrect results for supergravity. The correctly quantized action must contain terms quadratic and quartic in the ghost fields, whereas the standard quantization only leads to terms quadratic in the ghost fields. The reason for this discrepancy can be traced back to the fact that the gauge-group algebra (in this case containing local supersymmetry, spin rotations, and space-time translations) does not close when acting on the fields unless the field equations are used⁴⁾. Intuitively one expects this defect to lead to serious complications, since the gauge group will then effectively be larger than initially assumed. Indeed, the closure of the gauge-group algebra is crucial to derive the generalized Ward identities in a gauge theory. Therefore, in the absence of closure the standard proofs of the gauge-independence and unitarity of the S-matrix will no longer be valid^{2,5)}.

Kallosh⁶⁾ has pointed out that the modification of the naive covariant quantization procedure can also be found if one compensates the effect of the non-closure by modifying the field transformations by terms quadratic in the ghost fields. She then showed that this leads to the same quartic ghost-field terms in the action as found in ref. 3. An alternative approach leading to the same results is due to Sterman, Townsend and van Nieuwenhuizen⁷⁾, who choose as a starting point the need for maintaining unitarity for the S-matrix.

Clearly in a formulation of supergravity where the algebra closes no such problems arise. It is the purpose of this note to examine the covariant quantization in such a framework, and to show that the results of refs. 3,6,7

arise naturally upon elimination of some auxiliary fields which are present in this formulation of supergravity^{*)}. Such a formulation has been given by Breitenlohner^{8,9)}. It employs in addition to the (gauge) fields $(V_\mu^a, \psi_\mu, \omega_\mu^{ab})$ of supergravity two sets of auxiliary fields, $(\chi^a, \Phi, \chi^{ab})$ and $(\mathcal{D}^a, \mathcal{D}, \mathcal{D}^{ab})$. These sets can be viewed as the partners of the gauge-field set in the same way that the fields χ and $\mathcal{D}$ are the partners of the gauge field B_μ in the globally supersymmetric vector multiplet^{10,11)}. From this analogy one is led to take χ^a, χ^{ab} and $\mathcal{D}$ as Majorana spinor fields with additional local Lorentz indices a,b, and the remaining ones as Boson fields. The field Φ is defined as a 4×4 matrix field, which can be decomposed in terms of the 16 Dirac-algebra elements:

$$\Phi = \Phi_S + i\gamma_5 \Phi_P + \gamma_a \Phi_V^a + i\gamma_a \gamma_5 \Phi_A^a + \sigma_{ab} \Phi_T^{ab} \quad (1)$$

where the field components Φ_i are real.^{**)}

In terms of these fields one can construct an invariant Lagrangian according to the procedure outlined in ref. 9. This Lagrangian can be written as follows:

$$\begin{aligned} \mathcal{L}_{INV} = & \mathcal{L}^{SG} + V \left\{ -4 \Phi_S^2 - 4 \Phi_P^2 - 8 \Phi_V^2 \right. \\ & + 3 \Phi_A^2 + \frac{11}{4} \kappa^{-2} \mathcal{D}_a \mathcal{D}^a + 5 \kappa^{-1} \mathcal{D}_a \Phi_A^a \left. \right\} \\ & - i \kappa^{-1} V \bar{\mathcal{D}} \gamma_5 \gamma \cdot \chi + V \mathcal{L}' \end{aligned} \quad (2)$$

*) This possibility is alluded to by the authors of ref. 7.

**) We use the positive Pauli flat-space metric with Hermitean γ_a ($a=1, \dots, 5$), and $\gamma_a^2 = 1$, Greek world and Latin local-Lorentz indices.

where $\mathcal{L}'$ contains the Majorana field χ^a , as an overall factor, and does not depend on the Majorana field D . $\mathcal{L}^{SG}$ denotes the standard supergravity Lagrangian^{12,13)}

$$\mathcal{L}^{SG} = -\frac{1}{2} \kappa^{-2} V R(V, \omega^{SG}) - \frac{1}{2} \epsilon^{\rho\sigma\mu\nu} \bar{\psi}_\rho \gamma_5 \gamma_\sigma D_\mu^{SG} \psi_\nu \quad (3)$$

where D_μ is the first-order covariant derivative

$$D_\mu = \partial_\mu - \frac{1}{2} \omega_\mu^{ab} \sigma_{ab} \quad (4)$$

The superscript SG indicates that ω_μ is given by

$$\omega_{\mu,ab}^{SG} = \frac{1}{2} (B_{ab,\mu} + B_{\mu b,a} + B_{a\mu,b}) \quad (5.a)$$

with

$$B_{ab,c} = V_a^\mu V_b^\nu (\partial_\mu V_{\nu c} - \partial_\nu V_{\mu c}) - \frac{1}{2} \kappa^2 \bar{\psi}_a \gamma_c \psi_b \quad (5.b)$$

From this result (2) one can see directly that if we use the equations of motion for the auxiliary fields ($\chi^a = \Phi_S = \Phi_P = \Phi_V = \Phi_A = D^a = 0$) we obtain the supergravity Lagrangian in second-order form.

We will now briefly describe the procedure which leads to eq. (2). According to arguments given in ref. 9 an invariant Lagrangian can be obtained by performing two consecutive supersymmetry transformations with parameter

$$\epsilon_L = \frac{1}{2} (1 + \gamma_5) \epsilon \quad \text{on the quantity}$$

$$V (\Phi_S - i \Phi_P + \frac{1}{8} \bar{\chi}^a (1 - \gamma_5) \chi^a) \quad (6)$$

The form of the supersymmetry transformation⁸ can be found from ref. 8. This yields an expression proportional to $\bar{\epsilon} (1 + \gamma_5) \epsilon$. The real part of this

expression is an invariant Lagrangian $\mathcal{L}_B$. One then observes that the fields form a reducible representation of the supersymmetry algebra, namely the 126 field components reduce to the sum of a 54- and a 72-dimensional representation. Furthermore, it turns out that the previously constructed invariant Lagrangian $\mathcal{L}_B$ does not depend on the 72 field components. As was shown in ref. 9 this can be deduced from the fact that the Lagrangian $\mathcal{L}_B$ is invariant under a larger local gauge group than that given by the supersymmetry algebra; the corresponding extra 72 gauge degrees of freedom can be removed by choosing a set of 72 gauge conditions, which transform among themselves under supersymmetry transformations. (These gauge conditions are somewhat analogous to the Wess-Zumino gauge conditions¹⁰⁾ for the vector-superfield Lagrangian, except that here they may lead to a local contribution to the integration measure of the functional integral.)

The gauge conditions can be used to express the fields χ^{ab} and D^{ab} in terms of the other fields. Furthermore they yield the relations

$$\chi^a = \frac{1}{4} \gamma^a \gamma \cdot \chi \quad ,$$

$$\Phi_T^{ab} = 0 \quad , \quad (7)$$

$$\omega_{\mu,ab} = \omega_{\mu,ab}^{SG} + \kappa \bar{\omega}_{\mu,ab}$$

with

$$\bar{\omega}_{c,ab} = \frac{1}{2} (\bar{B}_{c,ab} + \bar{B}_{a,cb} + \bar{B}_{b,ac}) \quad ,$$

$$\begin{aligned} \bar{B}_{c,ab} = & -\frac{1}{8} \bar{\gamma}_b \gamma_a \gamma_c \gamma \cdot \chi + \frac{1}{8} \bar{\gamma}_a \gamma_b \gamma_c \gamma \cdot \chi \\ & + 2 \delta_{ac} \Phi_{Vb} - 2 \delta_{bc} \Phi_{Va} \end{aligned}$$

$$+ i \epsilon^{abcd} (\kappa^{-1} D_d + 2 \Phi_{Ad} - \frac{1}{16} i \kappa^{-1} \chi \cdot \gamma \gamma_d \gamma_5 \gamma \cdot \chi). \quad (8)$$

Substituting these relations into $\mathcal{L}_B$ leads then directly to the Lagrangian (2). We wish to emphasize that since the imposed gauge conditions transform among themselves under supersymmetry transformations, the Lagrangian (2) is still invariant under local supersymmetry transformations, and the algebra still closes on the remaining fields. Therefore eq.(2) is a good starting point for the covariant quantization of supergravity.

Before discussing the actual quantization procedure we give the supersymmetry transformations for the relevant fields (compare ref. 8.):

$$\delta V_\mu^a = \kappa \bar{\epsilon} \gamma^a \not{\psi}_\mu - \bar{\epsilon} \gamma_\mu \chi^a, \quad (9)$$

$$\begin{aligned} \delta \not{\psi}_\mu &= 2 \kappa^{-1} \bar{\epsilon} \not{D}_\mu - \bar{\epsilon} \gamma_\mu \Phi \\ &= 2 \kappa^{-1} \bar{\epsilon} \not{D}_\mu^{SG} + \bar{\epsilon} (\sigma_{ab} \bar{\omega}_\mu^{ab} - \gamma_\mu \Phi). \end{aligned} \quad (10)$$

To quantize the Lagrangian (2) we must eliminate the degrees of freedom related to local supersymmetry, local Lorentz transformations, and general coordinate transformations. The elimination of the latter two proceeds as usual and gives nothing new. Therefore we concentrate immediately on the gauge choice for the supersymmetry components. For purpose of illustration we choose the standard gauge-fixing term¹⁴⁾ $F = \gamma \cdot \not{\psi}$, although our procedure can directly be used for any choice of the gauge.

To determine the ghost-field Lagrangian is straightforward. We have commuting spinor ghost fields c and c^* , and their Lagrangian follows from the transformation rule of $\not{\psi}_\mu$. We find

$$\mathcal{L}(c, c^*) = 2 \bar{c}^* \not{D}^{SG} c - \kappa \bar{c}^* (\gamma^c \sigma_{ab} \bar{\omega}_c^{ab} - \gamma_a \Phi^c \gamma^a) c \quad (11)$$

with $\Phi^c \equiv \gamma_4 \Phi^\dagger \gamma_4$.

As we have mentioned before there is nothing new for the ghost-field contributions related to Lorentz and general coordinate transformations. If we now add $\mathcal{L}(c, c^*)$ to the Lagrangian (2) the equations of motion give

$$\gamma \cdot \chi = 0 ,$$

$$\Phi_S = \frac{1}{2} \kappa \bar{c}^* c ,$$

$$\Phi_P = -\frac{1}{2} i \kappa \bar{c}^* \gamma_5 c ,$$

$$\Phi_V^a = -\frac{1}{4} \kappa \bar{c}^* \gamma^a c , \quad (12)$$

$$\Phi_A^a = -\frac{1}{4} i \kappa \bar{c}^* \gamma^a \gamma_5 c ,$$

$$D^a = \frac{1}{2} i \kappa^2 \bar{c}^* \gamma^a \gamma_5 c .$$

Using these equations to eliminate the auxiliary fields from the full Lagrangian we find after Fierz reordering

$$\mathcal{L}(c, c^*) = 2 \bar{c}^* \not{D}^{SG} c - \frac{5}{8} \kappa^2 (\bar{c}^* \gamma^a c^*) (\bar{c} \gamma_a c) \quad (13)$$

This result is in agreement with that of refs. 3 and 7.

The elimination of the auxiliary fields also introduces a ghost-dependent term in the transformation properties of the fields. For instance, we find that eq. (10) for $\delta \bar{\psi}_\mu$ becomes:

$$\begin{aligned} \delta \bar{\psi}_\mu &= 2 \kappa^{-1} \bar{\epsilon} \overleftarrow{D}_\mu^{SG} \\ &- \frac{1}{4} \kappa \bar{\epsilon} \left\{ 2 \gamma_\mu (\bar{c}^* c) + 2 \gamma_\mu \gamma_5 (\bar{c}^* \gamma_5 c) \right. \\ &\quad \left. - \gamma_a \gamma_\mu (\bar{c}^* \gamma^a c) + \gamma_\mu \gamma_a \gamma_5 (\bar{c}^* \gamma^a \gamma_5 c) \right\}. \end{aligned} \quad (14)$$

This leads to a modification in the BRS transformation⁵⁾, which is obtained from (14) by choosing $\bar{\epsilon} = \kappa \lambda \bar{c}$, where λ is an anticommuting parameter. Fierz reordering leads directly to

$$\begin{aligned} \delta_{BRS} \bar{\psi}_\mu &= 2 \lambda \bar{c} \overleftarrow{D}_\mu^{SG} \\ &+ \lambda \kappa^2 \bar{c}^* \left\{ \frac{3}{8} \gamma_\mu \gamma_a (\bar{c} \gamma^a c) - \frac{1}{2} (\bar{c} \gamma_\mu c) \right. \\ &\quad \left. + \frac{1}{4} \gamma_\mu \sigma^{ab} (\bar{c} \sigma_{ab} c) \right\}. \end{aligned} \quad (15)$$

which can be compared to the result of ref. 7.

We have thus shown that conventional gauge-field quantization procedures can be applied to supergravity theory provided it is formulated in a manner which respects the group property of the local symmetries. Breitenlohner's prescription gives such a formulation, and seems to be an adequate way of understanding supergravity at the quantum level. We have not addressed ourselves here to the question of the integration measure of the functional integral,

which is known to be $(\det(V_{a\mu}))^{-4}$ after the auxiliary fields have been removed.³⁾ We wish to stress that although we have given the results for a particular gauge choice it is straightforward to apply this quantization prescription in any gauge. Also, since we use standard covariant quantization methods, it is obvious that the resulting quantum theory leads to a unitary and gauge-independent S-matrix.

Details and further results will be published elsewhere.

Acknowledgment

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12. The following table shows the number of people who attended the concert on each day.

Day	Number of people
Monday	120
Tuesday	150
Wednesday	180
Thursday	200
Friday	220
Saturday	250
Sunday	280

13. The following table shows the number of people who attended the concert on each day.

Day	Number of people
Monday	120
Tuesday	150
Wednesday	180
Thursday	200
Friday	220
Saturday	250
Sunday	280

14. The following table shows the number of people who attended the concert on each day.

Day	Number of people
Monday	120
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Sunday	280

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Friday	220
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Day	Number of people
Monday	120
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