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CONSEQUENCES OF A COLOUR-INDUCED VAN DER WAALS'
FORCE BETWEEN HADRONS

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A B S T R A C T

We discuss several possibly measurable consequences of an induced colour dipole-dipole long range hadronic potential $\propto r^{-6}$ mediated by the massless gluons: nuclear binding, perturbations of the energy levels of hadronic atoms, and threshold properties of hadronic scattering.

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Investigation of Quantum Chromodynamics (QCD) presents us with the possibility¹⁾ that there exist long-range forces between configurations of colour charge which are colour neutral but possess non-zero colour moments. Such forces would be mediated in perturbation theory by the exchange of pairs of gluons in a colour singlet state; since the gluons are massless the force could be long range. In the absence of macroscopic colour fields, these forces would manifest themselves as long-range Van der Waals' forces between hadrons, i.e., as an induced colour dipole-induced colour dipole interaction

It is not our purpose here to argue for the existence of such a force; indeed¹⁾ one can muster theoretical arguments against it. Rather it is to point out that, within reasonable limits on the parameters involved, the existence of such a force has not been experimentally tested and that there are several experiments which could come close to measuring it or at least place meaningful bounds on its parameters.

If one regards hadrons as colour singlet configurations of confined quarks and gluons, then the lowest order long distance potential V_0 between them, assuming the colour dipole moments to be fixed, comes from exchange of two gluons in a singlet state; this potential is $O(\alpha_s^2)$ (α_s is the running coupling constant of QCD) and is analogous to the one-photon exchange potential ($O(\alpha)$) between electrically neutral systems with fixed charge dipole moments. In other words, one finds a potential $V_0 \sim \alpha_s^2/r^3$. The Van der Waals' force is $O(V_0^2)$ and is attractive:

$$V(r) = -\alpha_s^4 x^5/r^6 \equiv C/r^6, \quad r > r_0 \quad (1)$$

The factor x^5 (measured in fm below) represents four factors of some average colour dipole moment and one factor of an inverse level splitting. For hadrons we suppose these will all be roughly $\lesssim 1$ fm and measure x accordingly. There is no firm understanding for α_s in the static region that we discuss here: but for guidance we may turn to studies²⁾ of static properties of low-lying hadrons, which find some agreement with these properties for $\alpha_s \approx 0.5$. Finally, the potential (1) must have a physical short distance cut-off r_0 fm where the multipole expansion breaks down, $r_0 \gtrsim O(x)$. We emphasize however that our understanding of such parameters is limited at best. It is unfortunately evident from Eq. (1) that our results may be rather sensitive to the choices of x , α_s , and r_0 .

We now discuss three consequences of the existence of this force. These are the best examples we have been able to find, but there may certainly be better ones.

We first estimate the total Van der Waals' energy in a nucleus of atomic number A

$$E^{VdW} = \alpha_s^4 \chi^5 \int d^3r d^3r' \frac{\rho(\vec{r}) \rho(\vec{r}')}{|\vec{r} - \vec{r}'|^6} \quad (2)$$

With a constant hadronic density $\rho_0 (= 0.16/\text{fm}^3)$, one can calculate this energy, which has a simple expression in the limit that the nuclear radius $R_0 \gg r_0$:

$$\begin{aligned} E^{VdW} &= \frac{4\pi^2}{9} \alpha_s^4 \chi^5 \rho_0^2 (R_0/r_0)^3 \\ &= \frac{1}{3} \pi \alpha_s^4 \chi^5 \rho_0^2 r_0^{-3} A \cdot 200 \text{ MeV}. \end{aligned} \quad (3)$$

This should be compared to the "ordinary" nuclear energy,

$$E^{nuc} \approx A \cdot 10 \text{ MeV}$$

or

$$R \equiv E^{VdW}/E^{nuc} \approx 3 \alpha_s^4 \chi^5 r_0^{-3} \quad (4)$$

If $(\alpha_s) = \frac{1}{2}$, $x = \frac{1}{2}$, and $r_0 = 1$, $R = 0(5\%)$. Our understanding of the binding provided by the "ordinary" nuclear force, even for large nuclei, where Eqs. (3) and (4) hold, is probably marginally too poor to make use of this estimate. We remark that qualitatively the effect is to bind large nuclei^{*)}.

For our second test, we discuss the perturbation of energy levels in hadronic atoms. Then it is simplest to measure everything in atomic units, $C \rightarrow C/(1/mZe^2)^6$, where m is the reduced mass for the atom and Z is the nuclear charge. We assume that the colour polarizability of a nucleus grows as the linear dimension, i.e., we take as a rough measure $x \rightarrow xA^{1/3}$. This may be too strong an assumption. Linear A dependence seems ruled out by the test described below. The first order perturbation on states with principal quantum number n and angular momentum ℓ is, for states with $\ell \geq 2$,

$$\Delta E_{n\ell} = -C \frac{32 (2\ell-4)! (n-\ell-1)!}{n^7 (n+\ell+2)(n+\ell+1)} \left\{ \frac{1}{(n-\ell-1)!(2\ell+1)!} + \frac{5!}{3! (n-\ell-2)!(2\ell+2)!} + \frac{6!}{(2!)^3 (n-\ell-3)!(2\ell+3)!} + \frac{7!}{(3!)^2 (n-\ell-4)!(2\ell+4)!} + \frac{8!}{(4!)^2 (n-\ell-5)!(2\ell+5)!} \right\} \quad (5)$$

In practice it is simplest to measure levels for which ℓ is near n ; then Eq. (5) simplifies. For example,

$$\Delta E_{n,n-1} = -C \frac{16 (2n-6)!}{n^8 (2n-1)! (2n+1)} \quad , \quad (6)$$

which for large n falls as n^{-14} .

*) It is amusing that the same calculation applied to a neutron star of radius 10^4 m gives $E_{VdW}/E_{grav.} \approx 10^{-1}(\alpha_s)^4 r_0^{-3} x^5$, not an astronomically small number.

It is clear that to maximize the shift one should pull the orbit in as far as one can consonant with a tolerable annihilation width. This can be done by increasing m and Z , since $C \sim m^6 Z^6$. Thus we concentrate on large $\bar{p}$ atoms henceforth. For such atoms, the lowest observed states³⁾ are those for $n = 8$, $\ell = 7$. We find using (6)

$$\Delta E_{8,7} \approx 1.5 \times 10^{-13} x^5 Z^6 \alpha_s^{-1} A^{5/3} \text{ eV.}$$

If we make $x = 0.5 = \alpha_s$ as before and for illustration choose $Z = 100$ and $A = 216$, then $\Delta E_{8,7} = 0(2 \text{ eV})$. These are numbers which will be within experimental reach shortly^{*)}.

Finally we turn to low energy hadronic scattering. In such a process we can, by isolating large partial waves, eliminate the central interaction region and get an r_0 independent result. (At high energies this is not possible. Moreover there seems to be no prominent signature for the Van der Waals' interaction at high energies such as we shall find at low energies.) Assuming the target is a proton, we should take as projectile some neutral particle to avoid Coulomb interference. Nevertheless, we will in general have an electric monopole-magnetic dipole Van der Waals' potential V' , proportional to α^2/r^4 , to deal with; this is of course completely calculable.

We consider the first Born approximation to the amplitude M ,

$$M(\vec{q}) = \int d^3r e^{i\vec{q} \cdot \vec{r}} V(\vec{r})$$

where q is the momentum transfer. The ℓ^{th} partial wave expansion of this amplitude can be calculated. If k is the magnitude of the centre-of-mass three-momentum we find for small k

*) There is at present a theoretical uncertainty due to electron screening in such levels of $0(5 \text{ eV})$. Work is in progress on this problem.

$$M_{\ell=0} = -\frac{8\pi}{3n_0^3} \alpha_s^4 \chi^5 + O(k^2 n_0^{-1})$$

$$M_{\ell=1} = -\frac{8\pi}{9n_0} k^2 \alpha_s^4 \chi^5 + O(k^4 n_0) \quad (7)$$

$$M_{\ell \geq 2} \approx \frac{3\pi^2}{4} \frac{(\ell - \frac{1}{2})!}{(\ell + \frac{5}{2})!} k^3 \alpha_s^4 \chi^5$$

d waves and higher are independent of r_0 and moreover have threshold behaviour $\propto k^3$ rather than the $k^{2\ell}$ characteristic of a short-range potential. This is easily understood from a scaling argument. (V' gives for p waves and above linear threshold dependence in k , but the calculable coefficient is very small - 0(1%) - for the range of QCD parameters we have been discussing.) While difficult, an experiment to search for k^6 rather than k^8 threshold behaviour in d waves of $d\sigma/d\Omega$ would seem to be worth serious consideration.

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