Ref.TH.2434-CERN

PION BOUND STATES IN NUCLEI

T.E.O. Ericson and F. Myhrer *)
CERN - Geneva

A B S T R A C T

Bound π nuclear states can appear under certain realistic conditions in finite nuclei. The best prospects are certain medium-weight nuclei with dense central regions and large neutron excess. Numerical illustrations are given for ^{16}O , ^{28}Si , ^{32}Na and ^{208}Pb with realistic parameters for the interaction. Further, we show that the usual low-energy π nuclear potential develops pathological features like an infinity of bound states beyond a certain critical strength. The natural remedies are indicated.

*) Present address : NORDITA, Copenhagen, Denmark.

1. The low-energy π -nuclear optical potential, as derived from the π N interaction and used in pionic atoms, results in an approximate wave equation of the Kisslinger type¹⁾

$$\nabla \cdot (1 - \alpha(\underline{r})) \nabla \phi(\underline{r}) + [-Q(\underline{r}) + k^2] \phi(\underline{r}) = 0. \quad (1)$$

The potential is thus velocity dependent (non-relativistic notation)

$$2\mu V(\underline{r}) = \nabla \cdot \alpha(\underline{r}) \nabla + Q(\underline{r}). \quad (2)$$

Here $Q(\underline{r})$ is a weakly repulsive term, while the velocity-dependent term, $\nabla \cdot \alpha(\underline{r}) \nabla$, originating from p-wave π N interaction, is attractive. The asymptotic kinetic energy is $E = k^2/2\mu$. Since pions absorb in nuclei both $\alpha(\underline{r})$ and $Q(\underline{r})$ are complex with $\text{Im } Q(\underline{r}) < 0$ and $\text{Im } \alpha(\underline{r}) > 0$.

If $\alpha(\underline{r}) < 1$, the attraction is insufficient to overcome the intrinsic repulsion of the kinetic energy in Eq. (1). However, in the region with $\alpha(\underline{r}) > 1$, the over-all "effective" kinetic energy in Eq. (1), $\nabla \cdot [1 - \alpha(\underline{r})] \nabla$ becomes attractive; bound states may then develop even in the presence of the repulsive local potential $Q(\underline{r})$. We will here demonstrate that such bound π -nuclear states are very likely for certain nuclei. This prediction is rather insensitive to the exact form of $\alpha(\underline{r})$ as long as $\alpha(\underline{r}) > 1$ in some region of the nucleus.

2. The crucial properties of Eq. (1) for the development of bound states appear clearly in the following oversimplified model. Consider the wave function to be confined to a spherical box of radius R so that $\phi(\underline{r}) = 0$ for $|\underline{r}| \geq R$. The potential parameters are constant inside the box: $\alpha(\underline{r}) = \alpha$ and $Q(\underline{r}) = Q$. For a bound state of energy $B = 2\mu\kappa^2$ ($\kappa^2 = -k^2$), the wave number K inside the box is given by Eq. (1):

$$-(1 - \alpha)K^2 - Q - \kappa^2 = 0. \quad (3)$$

The wave functions are spherical Bessel functions $j_\ell(K_n r)$ of given orbital angular momentum ℓ , for which the eigenvalues are determined by the boundary condition $j_\ell(K_n R) = 0$. Denoting the n^{th} root of $j_\ell(\pi\beta) = 0$ by $\beta_{\ell n}$, the binding energy $B_{n\ell}$ of the n^{th} state of given ℓ obeys therefore the condition

$$B_{n\ell} = -\frac{Q}{2\mu} + \frac{(\alpha - 1)\pi^2 \beta_{\ell n}^2}{2\mu R^2}, \quad (4)$$

where $\beta_{0n} = n > 0$, $\beta_{ln} \rightarrow (n + l/2)$ for large n and $l > 0$. The wave function for a given number of nodes is independent of α and Q : $j_l(K_n r) \equiv j_l[\pi\beta_{ln}(r/R)]$; it is only subject to the condition that this bound state exists, which is the positivity condition for B_{nl} in Eq. (4).

For a repulsive $Q > 0$ there can be no bound states in Eq. (4) for $\alpha < 1$, since $(\alpha - 1)$ is negative. The point $\alpha = 1$ is a singular point at which the "effective" kinetic energy term $(\alpha - 1)K^2/2\mu$ changes from repulsion to attraction. This provokes a profound change in the system. For $\alpha > 1$ an infinite number of bound states develop. It is energetically favourable to have states with more nodes, so that the level order is the inverse one of that of a normal potential: $B_{n_0, l} < B_{n_0+1, l} < \dots$, etc. This is illustrated in Fig. 1a. For an attractive $Q < 0$, the situation is slightly different. Since the normal repulsion of the kinetic energy term is weakened by the factor $(1 - \alpha)$ as α approaches unity, the attraction in Q can accommodate more and more bound states. In the limit $\alpha = 1$ an infinity of states will bind, all with the same energy $B_{nl} = -Q/2\mu$. These states have the usual level order in a potential for $\alpha < 1$, i.e. $B_{1l} > B_{2l} > B_{3l} > \dots$. At the point $\alpha = 1$, the level order is reversed (see Fig. 1b), so that the binding energies have once more the anomalous order $B_{1l} < B_{2l} < B_{3l} < \dots$. Although $\alpha = 1$ is a singular point, it is remarkable to note that any wave function with a given number of nodes $(n - 1)$ has no singularity at this point.

3. While the uniform model has many features of the general case in which $\alpha(r)$ varies from a positive, finite value near $r = 0$ to zero at $r = \infty$, there are also new features. Multiply first Eq. (1) by $\Phi^*(r)$ and make a partial integration. This gives the following relation for the binding energy B :

$$2\mu B \int |\Phi|^2 d\underline{r} = \int \{ [\alpha(\underline{r}) - 1] |\nabla \Phi|^2 - Q(\underline{r}) |\Phi|^2 \} d\underline{r}, \quad (5)$$

It is clear from Eq. (5) that in the case of a repulsive $Q(\underline{r}) > 0$, bound states can only occur if $\alpha(\underline{r}) > 1$ in some region: only then can the right-hand integral be made positive. Further, we note that for an attractive $Q(\underline{r}) < 0$, it is only possible to obtain a binding energy of more than $(-Q)_{\max}/2\mu$, if $\alpha(\underline{r}) > 1$ in some region. In both cases this is a generalization of the results of the uniform model. It is physically plausible that whenever $\alpha(r) > 1$ in some region, bound states will develop with the binding energy given by the number of nodes and the size of the region, roughly as stated in condition (4) for the uniform model. The novel feature is, however, the smooth transition from a region of $\alpha(r) > 1$ to one of $\alpha(r) < 1$. This gives rise to a singularity in the wave function as will now be discussed.

4. Bethe²⁾ pointed out that there should be a pathological behaviour of the wave equation (1) at the point $\alpha(r_0) = 1$, since the wave number becomes infinite there. He did not investigate this point further, although he proposes remedies for infinite nuclear matter^{2a)}. We will here give some important consequences of the singularity. We first transform Eq. (1) into a standard wave equation by the substitution³⁾ $\Phi(\underline{r}) = [1 - \alpha(\underline{r})]^{-\frac{1}{2}} \tilde{\Phi}(\underline{r})$

$$(\nabla^2 + k^2) \tilde{\Phi} = \frac{\{[Q(\underline{r}) - \alpha(\underline{r})k^2] - \frac{1}{2} \nabla^2 \alpha(\underline{r})\}}{[1 - \alpha(\underline{r})]} \tilde{\Phi} - \frac{1}{4} \frac{[\nabla \alpha(\underline{r})]^2}{[1 - \alpha(\underline{r})]^2} \tilde{\Phi}. \quad (6)$$

Consider the case of spherical symmetry. Since $1 - \alpha(r) \propto (r - r_0)$ near $\alpha(r) = 1$, one notes immediately that the equation has a potential term singular as $(r - r_0)^{-2}$ and $(r - r_0)^{-1}$. The $(r - r_0)^{-2}$ singularity has the following consequences:

- a) an infinity of bound states for $\alpha > 1$ as demonstrated in Sections 2 and 3;
- b) a logarithmic singularity in the wave function at the pole;
- c) an anomalous absorption layer with finite strength at $\text{Re } \alpha(r_0) = 1$ even in the limit $\text{Im } \alpha \rightarrow 0$.

To exhibit the last two features consider the special case of a diffuse plane surface, i.e. $\alpha(\underline{r}) = \alpha(x)$ and $Q(\underline{r}) = Q(x)$. We take $\alpha(x)$ to vary linearly through the critical region, while all other potential terms are constant there: $\alpha(x) - 1 = \alpha'(x - x_0)$ and $Q(x) = Q$. The most general solution to Eq. (1) is then in this case

$$\Phi(x) = f(x) + \ln(x - x_0) g(x) \equiv A J_0 \left[2 \left\{ \frac{k^2 - Q}{\alpha'} (x_0 - x) \right\}^{\frac{1}{2}} \right] + B N_0 \left[2 \left\{ \frac{k^2 - Q}{\alpha'} (x_0 - x) \right\}^{\frac{1}{2}} \right]. \quad (7)$$

Here $f(x)$ and $g(x)$ are regular functions, $J_0(x)$ and $N_0(x)$ are the regular and irregular cylindrical Bessel functions, while A and B are arbitrary constants. The result (7) exhibits point (b) above.

Consider once more the plane surface discussed above with the wave function given by Eq. (7) near the singular region. The derivative will then take the form

$$\Phi'(x) \xrightarrow{x \rightarrow x_0} \frac{g(x_0)}{(x - x_0)},$$

i.e. it becomes very large there.

The absorption generally reflects itself by an imaginary part of the potential. In the limit of a real potential the pole position x_0 moves to the real axis. There will still be absorption in this limit. To see this, consider the

imaginary part of Eq. (5). The first term in the r.h.s. integrand is then

$$\begin{aligned} \text{Im} \left\{ [\alpha(x) - 1] |\Phi'(x)|^2 \right\} &\xrightarrow{x \rightarrow x_0} \text{Im} \left\{ [\alpha(x) - 1] \frac{|g(x_0)|^2}{|x - x_0|^2} \right\} = \\ &= \frac{\alpha' \text{Im}(x_0) |g(x_0)|^2}{[x - \text{Re}(x_0)]^2 + [\text{Im}(x_0)]^2} \xrightarrow{\text{Im}(x_0) \rightarrow 0} \pi \alpha' |g(x_0)|^2 \delta[x - \text{Re}(x_0)]. \end{aligned}$$

The consequence is thus that there will be a non-vanishing absorption from this term even in the limit of purely real potential. The origin of the anomaly is an absorptive dipole layer in the region $x = \text{Re}(x_0)$.

5. The singularities of the wave equation (1) reflect two simplifying approximations in the original π -nuclear multiple scattering equations: i) the use of a vanishing range in the p-wave πN interaction, and ii) the neglect of the range of the NN pair correlation function. These approximations are not necessary¹⁾. In the actual description an averaging factor is introduced, so that there is no singularity in the critical region. The mechanism for this is easily seen as follows. Replace $\alpha(\underline{x}) \nabla \Phi(\underline{x})$ by the averaged Hermitian form

$$\alpha(\underline{x}) \nabla \Phi(\underline{x}) \longrightarrow \int \alpha(\underline{x} + \frac{\underline{\zeta}}{2}) F(\underline{\zeta}) \nabla \Phi(\underline{x} + \frac{\underline{\zeta}}{2}) d\underline{\zeta} \quad (8)$$

Here $F(\underline{\zeta})$ is an averaging function which we take spherically symmetrical with a range $\langle \zeta^2 \rangle = \int F(\underline{\zeta}) \zeta^2 d\underline{\zeta}$. To leading order in the range parameter $\langle \zeta^2 \rangle$, Eq. (8) leads to a fourth-order non-singular differential equation.

The nature of this equation is apparent in the previously discussed case of a plane surface with a linear variation $\alpha(x) = \alpha_0 + \alpha_1 x$,

$$\frac{\langle \zeta^2 \rangle}{6} \alpha(x) \frac{d^4 \Phi(x)}{dx^4} + \frac{\alpha_1 \langle \zeta^2 \rangle}{3} \frac{d^3 \Phi(x)}{dx^3} + \frac{d}{dx} [\alpha(x) - 1] \frac{d \Phi(x)}{dx} + [k^2 - Q(x)] \Phi(x) = 0 \quad (9)$$

It is essential to note that the limit $\langle \zeta^2 \rangle \rightarrow 0$ is not a smooth one as $\alpha(x) = 1$.

For a uniform medium with $\alpha(x) = \alpha > 1$ and $Q(x) = Q$, a crucial parameter is $\beta = \frac{2}{3}(\kappa^2 + Q) \langle \zeta^2 \rangle \alpha(\alpha - 1)^{-2}$, which determines the magnitude of the fourth-order term as compared to each of the other terms. The eigen-frequencies λ_1 and λ_2 are in this case

$$\left. \begin{aligned} \lambda_1^2 &= \frac{2(\kappa^2 + Q)}{(\alpha - 1)(1 + \sqrt{1 - \beta})} \approx \frac{\kappa^2 + Q}{(\alpha - 1)} \\ \lambda_2^2 &= \frac{3(\alpha - 1)}{\alpha \langle J^2 \rangle} [1 + \sqrt{1 - \beta}] \approx \frac{6(\alpha - 1)}{\alpha \langle J^2 \rangle} \end{aligned} \right\} \beta < 1. \quad (10)$$

Therefore $\lambda_1^2/\lambda_2^2 \approx \frac{1}{4}\beta \ll 1$.

The normal "long wave length" solution λ_1 , has a small fourth-order term as required as long as β is small. The high-frequency solution λ_2 has a large such term in this case and must be expected to be highly model-dependent.

The number of bound states is now finite. For a real uniform potential of radius R this number is approximately obtained from the two conditions: $\kappa^2 > 0$ and $\lambda_1 R = n\pi$ (s-waves only). Neglecting the weak local potential Q , we find for large n :

$$n^2 \lesssim \frac{3(\alpha - 1)R^2}{\pi^2 \langle J^2 \rangle \alpha} \quad (11)$$

In addition, the range parameter $\langle \zeta^2 \rangle$ limits the binding energy of the lowest state in Eq. (9). Although we have discussed non-relativistic binding, Eq. (1) is in fact relativistic with a total π -energy $\omega = \sqrt{k^2 + \mu^2}$. The binding energy therefore equals the π rest mass for $\omega = 0$ or $\kappa = \mu$. This is a necessary condition for a π condensate⁴⁾. If we neglect the weak local potential Q , we have⁵⁾ for the uniform model (which has no response of the nuclear degrees of freedom) by this condition and by the reality condition $\beta \leq 1$ (undamped waves)

$$\alpha > 1 + \left(\frac{2}{3}\right)^{1/2} \frac{\mu \langle J^2 \rangle^{1/2}}{\alpha^{1/2}} \quad (12)$$

This demonstrates incidentally that the condition for bound states as discussed here is not the same as the condition for a π -condensate.

6. In actual nuclei bound π -nuclear states, as discussed above, will occur as a general phenomenon provided the decisive condition $\text{Re}(\alpha) > 1$ is valid in a sufficiently large region. In its gross features the shape of $\alpha(r)$ is that of the nuclear matter distribution, although its detailed behaviour is a different one. In fact, microscopic theories for $\alpha(r)$ lead to a non-linear dependence on the nuclear density^{1,3,6,7)}. For the present considerations these non-linear effects are largely irrelevant, while it is crucial that the effective strength of $\alpha(r)$ of the nucleus is consistent with the large amount of information from pionic atoms. As a consequence we have used $\alpha(r)$ and $Q(r)$ with experimentally determined parameters; these do not vary much from one analysis to another^{1,8,9)}.

The following two conditions favour bound states. Firstly, a high nuclear density near the central region favours bound states of π^- , π^0 and π^+ . Secondly, since the p-wave π^-n interaction at low energy is about 10 times the π^-p interaction, it is very favourable for π^- -nuclear bound states to have a large neutron excess in this region, while this is unfavourable for π^+ binding, and indifferent for π^0 binding.

For a numerical investigation of favourable cases we have first considered the $T = 0$ nuclei ^{16}O , ^{28}Si and ^{40}Ca with equal neutron and proton densities; using the experimental charge density for the proton density^{10,11)}, and parametrizing experiments as in Krell and Ericson¹⁾, the maximal value for $\alpha(r)$ of 0.80, 0.88 and 0.83, respectively, is obtained. These nuclei which have higher densities than normal in their central regions, therefore all fail to meet the criterion for bound states; this conclusion *a fortiori* also extends to the vast majority of other nuclei. We have also considered the doubly magic nucleus ^{208}Pb , for which we optimistically took the neutron and proton distributions to have equal shape¹¹⁾. In this case $\alpha(r) \approx 1$ from the centre to nearly 4 fm. Here the possibility of bound states is a more realistic one, since small changes in the parameters or in the detailed shape of $\alpha(r)$ would be sufficient to fulfil the conditions. It is therefore difficult to make a firm prediction, since this is a transition case. As the neutron distribution is more concentrated in the outer region of this nucleus than we have assumed, we feel, however, that chances are that ^{208}Pb just fails to meet the necessary conditions.

These results are a consequence of experimentally constrained values for the "effective" $\alpha(r)$. In terms of a theoretical description based on the πN interaction there are rather important higher order corrections to $\alpha(r)$ [Lorentz-Lorentz effect^{1,6)}]. If we ignore such effects and simply construct the leading order π -nuclear optical potential linear in the density, nearly all nuclei would have $\alpha(r) > 1$ in the central region and π -nuclear bound states would be the rule and not the exception; this is not a realistic description, however.

Returning to the empirical treatment, we have deliberately tried to find a nucleus with particularly high central neutron density. Conditions are particularly favourable in neutron-rich medium-weight nuclei, for which the 2s neutron shell is filled while the corresponding proton shell is still empty. An example is ^{32}Na [lifetime ≈ 10 msec¹³⁾], for which theoretical Hartree-Fock neutron and proton densities are available¹⁴⁾. In this case the maximal value of $\alpha(r)$ is comfortably larger than unity, making this nucleus a very serious candidate for the effect we discuss.

We have numerically integrated Eq. (1) with the potential parameters as above¹⁾. The spurious singularity at $\alpha(r) = 1$ has been avoided by linearly

interpolating the wave function in this region (other interpolation procedures were attempted with only minor changes in eigenvalues). Results for binding energies and widths for various assumptions about parameters are given in Table 1. As a whole these results confirm qualitatively the results of the uniform model. In particular, the remarkably steep slope of the binding energy with increasing strength of $\alpha(r)$ is confirmed (see Fig. 1).

The surprisingly large widths of the bound states are noteworthy. The explanation is readily seen in the uniform model with binding energies proportional to $(\alpha - 1)$. While $\text{Im } \alpha \ll \text{Re } \alpha$, $\text{Im } \alpha$ is of the same order as $\text{Re } (\alpha - 1)$.

The typical shapes of the 1s and 2s wave functions for $\pi^{-32}\text{Na}$ are shown in Fig. 2. Both of these wave functions vary over regions larger than internucleon spacing, which is a necessary condition for our discussion.

The closeness of $\alpha(r)$ to unity in most nuclei has an important practical consequence for the discussion and analysis of experimental data on π -nuclear scattering, pionic atoms, etc. An inadvertent description of such data with $\alpha(r) > 1$ could result simply from a minor variation in parameters. If improperly handled this may lead to unrealistic and unphysical conclusions.

7. Our treatment has implicitly assumed that the nucleus is an inert source of the potential. In fact, the nucleus will respond to the presence of the pion. In particular, the binding energy of the pion is an extremely rapid function of the density and neutron excess in the critical region. As a consequence it is in principle advantageous for the nucleus to adjust to the presence of the pion. This can be done in two ways particularly. The nucleus may be compressed in volume to a higher density ($T = 0$ monopole compression). It may also respond to a π^- by compressing the neutron distribution and dilating the proton distribution ($T = 1$ monopole compression). In the absence of absorption these effects would be important, and bound states would result under more favourable conditions than described here. However, the adjustment time is linked to the monopole frequencies, which are of the same magnitude as the typical absorption width. Consequently, the nuclear response time is at best similar to the absorption time. An important exception to this situation is a π^- in a finite piece of neutron matter, in which case there is no absorption. Even in the absence of an adjustment of the density, the π^- will bind at a density of 0.11 fm^{-3} , i.e. at less than the nuclear density.

8. We have demonstrated here that our present understanding of low-energy π -nuclear interactions makes it necessary to consider a novel physical effect. It follows conclusively from our investigation that when a critical value for the matter density and neutron density is exceeded, the "effective" kinetic energy

of the pion in the nucleus becomes attractive. Single-pion bound states in the Schrödinger sense will then rapidly develop with increasing density. The level order of such states are highly characteristic with an inverse ordering of states of a given number of nodes as compared with that in an ordinary potential. The phenomenon we discuss is a single-pion effect; it is thus quite distinct from the development of π condensates in nuclear matter⁴⁾, at least as long as the binding of the pion is small compared with its rest mass. Using experimental values for densities and interaction strength we find that actual nuclei are close to fulfilling the necessary conditions for bound states. Minor variations in parameters are decisive. Particularly favourable cases have not been systematically searched for among stable elements, but the unstable nucleus ^{32}Na is expected to fulfil the conditions, and ^{208}Pb may possibly do so.

During this work we had to face the consequences of the singular behaviour of the π -nuclear interaction in the region of a vanishing effective kinetic energy, where the pion wave-length goes to zero. At this point the wave function develops a logarithmic singularity. A consequence of this is that an absorptive layer develops at the critical distance. It will give finite absorption even in the limit, when the potential parameters become real. A further consequence of the singularity is an infinity of bound states. All of these features are associated with approximations to the microscopic description of the π -nuclear interaction. They are removed in the more complete versions of the microscopic description equivalent to an averaging procedure.

Bound π -nucleon states would manifest themselves in various contexts associated with virtual pions near the threshold of physical pion production. Examples are large-energy transfer μ -capture¹⁶⁾, photonuclear cross-sections near π production threshold^{*}), direct radiative transitions from an atomic state to a π -nuclear bound state, etc. In all of these cases the (broad) bound states will lead to enhancements. One should also note that the same mechanism which produces the bound states will also produce π -nuclear resonant states in low energy π -nuclear scattering¹⁷⁾.

We have enjoyed discussions on the topic of the present article with a number of colleagues and are in particular indebted to Drs. J. Bernabéu, G. Brown, M. Ericson, M. Johnson, R.J. Lombard, A.B. Migdal, C. Pethick and T.T. Wu.

*) Suggested to us by M. Ericson.

REFERENCES

- 1) M. Ericson and T.E.O. Ericson, Ann. Phys. (NY) 36, 323 (1966).
M. Krell and T.E.O. Ericson, Nuclear Phys. B11, 521 (1969).
- 2) H.A. Bethe, Phys. Rev. Letters 30, 105 (1973).
- 2a) M. Johnson and H. Bethe, Nuclear Phys., to be published.
- 3) T.E.O. Ericson, The investigation of nuclear structure by scattering processes at high energy (ed. H. Schopper) (North Holland, Amsterdam, 1975), p. 165.
- 4) A.B. Migdal, Zh. Eksper. Teor. Fiz. 61, 2210 (1971) [Transl.: Soviet Phys. JETP 34, 1184 (1972)].
G.E. Brown and W. Weise, Phys. Reports 27C, 1 (1976).
- 5) An analogous condition based on a different consideration has been obtained independently by M. Ericson, private communication.
- 6) G. Baym and G.E. Brown, Nuclear Phys. A247, 395 (1975).
- 7) J.M. Eisenberg, J. Hufner and E.J. Moniz, Phys. Letters 47B, 381 (1973).
- 8) L. Tauscher, Proc. 6th Internat. Conf. on High-Energy Physics and Nuclear Structure, Santa Fe, USA, June 1975 (eds. D.E. Nagle et al.) (American Institute of Physics, New York, 1975), AIP Conf. Proc. No. 26, p. 541.
- 9) C.J. Batty, S.F. Biagi, E. Friedman, S.D. Hoath, J.D. Davies, G.J. Pyle, G.T.A. Squier and D.M. Asbury, Rutherford Lab. Preprint RL 77-101A (1977).
- 10) R. Engfer, H. Schneuwly, J.L. Vuilleumier, H.K. Walter and A. Zehnder, Atomic Data and Nuclear Data Tables 14, 509 (1974).
- 11) I. Sick, Proc. 6th Internat. Conf. on High-Energy Physics and Nuclear Structure, Santa Fe, USA, June 1975 (ed. D. Nagle et al.) (American Institute of Physics, New York, 1975), AIP Conf. Proc. No. 26, p. 388.
- 12) J.W. Negele, Invited talk at Internat. Conf. on High-Energy Physics and Nuclear Structure, Zürich, August-September 1977.
- 13) R. Klapisch et al., private communication.
- 14) M. Beiner, R.J. Lombard and D. Mas, Nuclear Phys. A249, 1 (1973).
X. Campi, H. Flocard, A.K. Kerman and S. Koonin, Nuclear Phys. A251, 193 (1975).
- 15) J.P. Blaizot, D. Gogny and B. Grammatikos, Nuclear Phys. A265, 315 (1976).
- 16) J. Bernabéu, T.E.O. Ericson and C. Jarlskog, Phys. Letters 69B, 161 (1977).
- 17) M. Ericson and M. Krell, Phys. Letters 38B, 359 (1972).

Table 1

The binding energies B_{1s} in MeV for ^{32}Na as a function of the different terms in the optical potential (see Ref. 1). The Fermi parameters in the proton and neutron density distribution are $c_p = 3.1$ fm and $t_p = 2.3$ fm, and $c_n = 3.2$ fm and $t_n = 3.0$ fm, respectively. Parameters are given in the standard notation of Krell and Ericson^{1,8,9}.

b_0 (m_π^{-1})	b_1 (m_π^{-1})	$\text{Im } B_0$ (m_π^{-4})	c_0 (m_π^{-3})	c_1 (m_π^{-3})	$\text{Im } C_0$ (m_π^{-6})	ξ	B_{ns} (MeV)	State (ns)
-0.030	0	0	0.26	0.18	0.005	0.5	$1.0 + i 0.5$	1s
-0.030	0	0	0.26	0.18	0.140	0.5	$2.5 + i 12.5$	1s
-0.030	-0.080	0.040	0.26	0.18	0.140	0.5	$0.3 + i 15.0$	1s
-0.030	-0.080	0.040	0.24	0.18	0.140	1.0	unbound	1s
-0.030	-0.080	0.040	0.26	0.18	0.140	0.9	$20.2 + i 83$	2s

Figure captions

Fig. 1 : Binding energies of ns states in the uniform model according to Eq. (4). Radius $R = 2$ fm. a) repulsive local potential $Q/2\mu = 10$ MeV; b) attractive local potential $Q/2\mu = -10$ MeV.

Fig. 2 : Typical shapes of the real part of the wave function, $u(r) = r\phi(r)$, as a function of r for the 1s and 2s pion states in ^{32}Na . The optical potential parameters are $b_0 = -0.030 m_\pi^{-1}$, $\text{Im } B_0 = 0$, $c_0 = 0.26 m_\pi^{-3}$, $c_1 = 0.18 m_\pi^{-3}$, $\text{Im } C_0 = 0.005 m_\pi^{-6}$ and $\xi = 0.6$. The binding energies are $B_{1s} = (1.4 + i0.5)$ MeV and $B_{2s} = (34 + i2)$ MeV. The singularity $\alpha(r_0) = 1$ occurs for $r_0 \approx 2.36$ fm. The interpolated region is shown dotted.

Fig. 1

Fig. 2