Ref.TH.2450-CERN

TRANSVERSE MOMENTUM OF MUON PAIRS PRODUCED
IN HADRONIC COLLISIONS

G. Altarelli

Istituto di Fisica dell'Università
Roma

Istituto Nazionale di Fisica Nucleare
Sezione di Roma

G. Parisi *)

Laboratoire de Physique Théorique
Ecole Normale Supérieure, Paris

and

R. Petronzio +)

CERN -- Geneva

A B S T R A C T

The observed $p_{\perp}$ distributions of large mass muon pairs produced in proton-nucleus collisions are quantitatively explained in QCD by also explicitly taking into account the intrinsic wave function $p_{\perp}$ spread of partons inside the hadrons. The experimental data indicate $\langle p_{\perp} \rangle \sim 500-600$ MeV for each parton.

*) On leave of absence from Laboratori Nazionali di Frascati.

+) On leave of absence from Istituto di Fisica dell'Università, Roma.

In quantum chromodynamics (QCD) simple scaling laws (as, for example, Bjorken scaling in deep inelastic lepton production) are logarithmically violated. These violations arise from parton interactions with exchange of large transverse momentum. Since the scaling violations are computable in perturbation theory one expects this to be the case also for the distribution of large $p_{\perp}$ partons. While in lepton production the observed $p_{\perp}$ of hadrons also depends on the fragmentation properties of partons into hadrons, in muon pair production in hadronic collisions, according to the Drell-Yan picture ¹⁾, the transverse momentum distribution of the muon pair is directly related to the $p_{\perp}$ of the annihilating partons.

In this letter we point out that the existing experimental data for muon pair production in pN collisions can be fitted at all $p_{\perp}$ by using reasonable densities for partons, provided that the intrinsic $p_{\perp}$ spread of partons inside the hadrons is also taken into account. At large $p_{\perp}$ and S , although this effect decreases as $1/S$, it turns out that it is still important for S as large as 1000 GeV^2 and $p_{\perp}$ of order a few GeV.

The cross-section for producing a muon pair of mass $Q = \sqrt{Q^2}$ at zero rapidity y , with large enough $p_{\perp}^2$ (so that the intrinsic transverse momentum of partons in the hadrons can be neglected) is given by ^{2),3)} :

$$\begin{aligned} \sigma(Q^2, \tau, x_T) \equiv \frac{d\sigma}{dQ dp_T^2 dy} \Big|_{y=0} &= \frac{8\alpha^2}{27 S^2 Q} \alpha_s(Q^2) \int dx_1 dx_2 \delta[x_1 x_2 \\ &- \frac{1}{2}(x_1 + x_2) \sqrt{x_T^2 + 4\tau} + \tau] \left\{ \sum_{i=1}^f \left[e_i^2 q_i^{h_1}(x_1, Q^2) \bar{q}_i^{h_2}(x_2, Q^2) + (1 \leftrightarrow 2) \right] \cdot \right. \\ &\cdot F_{q\bar{q}}(x_1, x_2, \tau, x_T) + \sum_{i=1}^f \left[e_i^2 \left(q_i^{h_1}(x_1, Q^2) + \bar{q}_i^{h_1}(x_1, Q^2) \right) G^{h_2}(x_2, Q^2) + (1 \leftrightarrow 2) \right] \cdot \\ &\cdot F_{qG}(x_1, x_2, \tau, x_T) \Big\} \end{aligned} \quad (1)$$

where S is the total centre-of-mass energy squared of the incoming hadrons, f is the number of quark flavours, $q_i^h, \bar{q}_i^h$ and G^h are the quark, antiquark and gluon densities in the hadron h , e_i is the charge of q_i and :

$$x_T = \frac{2 p_T}{\sqrt{S}} \quad \tau = \frac{Q^2}{S} \quad (2)$$

$$F_{q\bar{q}}(x_1, x_2, \tau, x_T) = \frac{8}{x_T^2} \left(1 + \frac{\tau^2}{x_1^2 x_2^2} - \frac{x_T^2}{2 x_1 x_2} \right) \quad (3)$$

$$F_{qG}(x_1, x_2, \tau, x_T) = \frac{3}{8 x_T^2 (x_1 x_2)^3} \left\{ [x_T^2 x_1 x_2 - (\tau - x_1 x_2)^2] \cdot \right. \\ \left. \cdot (3\tau + x_1 x_2) + 4(x_1 x_2 - \tau)^3 + (x_1 x_2 + \tau)^2 (x_1 x_2 - \tau) \right\} \quad (4)$$

On the other hand, the Drell-Yan total cross-section at $y=0$ is given by :

$$\sigma_{DY}(Q^2, \tau) \equiv \frac{d\sigma}{dQ dy} \Big|_{y=0} = \frac{8\pi d^2 \tau}{9 Q^3} \sum_{i=1}^f e_i^2 \left[q_i^{h_1}(\sqrt{\tau}, Q^2) \bar{q}_i^{h_2}(\sqrt{\tau}, Q^2) + (1 \leftrightarrow 2) \right] \quad (5)$$

In Eqs. (1) and (5) we replaced $\alpha_s \equiv g_s^2/4\pi$ (g_s being the quark gluon coupling constant) with the running coupling constant and inserted the exact Q^2 dependent densities for quarks and gluons as discussed in Refs. 2)-6). This procedure has been supported by computations in perturbation theory in the leading logarithmic approximation ^{6),7)}. Equations (1) and (5) are presumably valid up to terms of higher order in α_s (without logarithmic factors of Q^2).

Note that σ_{DY} in Eq. (5) cannot be directly derived from Eq. (1) because the p_T^2 integration would diverge logarithmically at $x_T=0$. In fact, as already mentioned, $\sigma(Q^2, \tau, x_T)$ only approaches the correct p_T^2 distribution at sufficiently large p_T^2 and S . As a consequence a comparison of $\sigma(Q^2, \tau, x_T)$ with the available data which only extend up to moderate values of p_T^2 , as was attempted in Ref. 3), shows

that the theoretical curve is above the data at low $p_{\perp}^2$ and below the experimental points at the maximum measured values of $p_{\perp}^2$. This reflects in the rather low values with respect to experiment of the average $p_{\perp}^2$ as a function of τ , which have been derived in Ref. 2). It is our purpose to show that, while these difficulties cannot be removed by reasonable changes of the parton densities and of $\alpha_s(Q^2)$, they are immediately disposed of by introducing a simple model for the intrinsic transverse momentum of partons inside the hadrons which has been neglected up to now. To this aim we observe that a distribution with regular behaviour at $x_T = 0$ that approaches $\sigma(Q^2, \tau, x_T)$ at large $p_{\perp}^2$ and S can be constructed as follows :

$$\sigma^{\text{reg}}(Q^2, \tau, p_{\perp}^2) = \sigma_{DY}(Q^2, \tau) f(p_{\perp}^2) + \frac{1}{\pi} \int d^2 \vec{k}_{\perp} \sigma(Q^2, \tau, \frac{2k_{\perp}}{\sqrt{S}}) \cdot [f((p_{\perp} - k_{\perp})^2) - f(p_{\perp}^2)] \quad (6)$$

where $f(p_{\perp}^2)$ is a distribution which at large $p_{\perp}^2$ decreases much faster than $\sigma(Q^2, \tau, x_T)$ and is normalized according to :

$$\int_0^{\infty} d p_{\perp}^2 f(p_{\perp}^2) = 1 \quad (7)$$

$f(p_{\perp}^2)$ is meant to reproduce the spread in $p_{\perp}$ of partons inside the hadrons in the absence of scaling violations and is therefore independent of Q^2 . For simplicity we neglected a possible dependence on τ . As a consequence of Eqs. (6), (7), we have :

$$\int d p_{\perp}^2 \sigma^{\text{reg}}(Q^2, \tau, p_{\perp}^2) = \sigma_{DY}(Q^2, \tau) \quad (8)$$

At intermediate values of S the difference between σ^{reg} and σ (written as functions of x_T) is approximately given by

$$\sigma^{\text{reg}}(x_T) \simeq \sigma(x_T) + \left[\frac{1}{x_T} \sigma'(x_T) + \sigma''(x_T) \right] \frac{\langle p_{\perp}^2 \rangle f}{S} + \sigma \left(\frac{\langle p_{\perp}^4 \rangle f}{S^2} \right) \quad (9)$$

where $\langle p_{\perp}^2 \rangle_f$ is the average value of $p_{\perp}^2$ with the distribution $f(p_{\perp}^2)$. Since $\sigma(x_T)$ is a very steep function the derivatives are rather large and the correction turns out to be important in the experimental range of $p_{\perp}^2$ as we shall now discuss.

For a numerical evaluation of σ^{reg} we take a Gaussian form for $f(p_{\perp}^2)$:

$$f(p_{\perp}^2) = A \exp(-A p_{\perp}^2) \quad (10)$$

The averages of $p_{\perp}$ and $p_{\perp}^2$ implied by this distribution are given by

$$\langle p_{\perp} \rangle_f = \sqrt{\frac{\pi}{4A}} \quad \langle p_{\perp}^2 \rangle_f = \frac{1}{A} \quad (11)$$

Assuming that $f(p_{\perp}^2)$ arises as a convolution of two universal distributions for each of the annihilating partons, it follows that the average transverse momentum of each parton is given by

$$\langle p_{\perp} \rangle_f^{\text{parton}} = \sqrt{\frac{\pi}{8A}} \quad (12)$$

In Fig. 1, $\sigma_{\text{DY}}^{\text{reg}}(Q^2, \tau, p_{\perp}^2)$ is compared with the available data ⁸⁾. The absolute normalization of the data has been fixed from the quoted values ⁸⁾ for $\sigma_{\text{DY}}(Q^2)$ at $Q = 7.5$ GeV and $\sqrt{S} = 27$ GeV. The parton densities were taken as approximately Q^2 independent and given by ^{*}) :

$$P_v(x) = 1.79 (1-x)^3 (1+2.3x)/\sqrt{x}$$

$$h_v(x) = 1.07 (1-x)^{3.1}/\sqrt{x}$$

$$S(x) = .34 (1-x)^9/x$$

$$G(x) = A_n (1-x)^n/x$$

with $n=5$, A_n being fixed by momentum conservation.

^{*}) We used the parton densities of Wang ⁹⁾ for the valence quarks ; the sea density has been modified.

The quark densities given above lead to a fair fit to the data for the total cross-section. We also set $\alpha_s(1 \text{ GeV}^2) = 1$ and $A = 1.3 \text{ GeV}^{-2}$. The data points at $Q = 9-10 \text{ GeV}$ are in excess with respect to the solid curve owing to the presence of the $T(9.5)$ structure^{8),10)}. The transverse momentum distribution is not necessarily the same as the continuum and in particular an increase of the average $p_\perp$ may derive from the contribution of gluon-gluon annihilation mechanism for producing the T .

In Fig. 2 a comparison of $\sigma^{\text{reg}}(Q^2, \tau, p_\perp^2)$ with $\sigma(Q^2, \tau, x_\perp)$ and $\sigma_{\text{dy}}(Q^2, \tau) f(p_\perp^2)$ for $Q = 7.5 \text{ GeV}$ and $\sqrt{S} = 27 \text{ GeV}$ is shown. The difference between σ^{reg} and σ at large $p_\perp$ is important to bring the predicted and observed rates into agreement. In Fig. 2b, the effect on σ^{reg} of altering the gluon x dependence is also displayed. At larger τ values the sensitivity to the gluon density is increasing²⁾. Note that even in absence of logarithmic scaling violations, while $s^{\frac{5}{2}}\sigma_{\text{DY}}$ in Eq. (1) would be s independent at fixed τ and $x_\perp$, this is not the case for $s^{\frac{5}{2}}\sigma_{\text{DY}}^{\text{reg}}$ in Eq. (6), because of the scale of intrinsic transverse momentum introduced by A in Eq. (10). This produces scale breaking terms of order $1/AS$, whose effect is displayed in Fig. 3 and extends up to rather large values of s [because of the large derivatives appearing in Eq. (9)].

In Fig. 4, we finally give the predicted results in the case of $p\bar{p}$ scattering with the same $f(p_\perp^2)$ at different Q values and $\sqrt{S} = 27 \text{ GeV}$.

In conclusion the value of A that gives the best fit to the existing data suggests, from Eq. (12), an average intrinsic transverse momentum for each parton inside the hadrons of about 500-600 MeV. This is in agreement with the qualitative conclusions of Ref. 2). The effect of this spread of the transverse momentum distribution of partons together with the perturbative calculation of the hard tail of the $p_\perp$ spectrum is important at current values of $p_\perp$ and S for a quantitative description of the transverse momentum distribution of muon pairs.

We acknowledge useful discussions with D. Amati, H. Fritzsch and G. Veneziano.

REFERENCES

- 1) S.D. Drell and T.M. Yan - Phys.Rev.Letters 25, 316 (1970). For a recent comparison of the data, see D. Antreasyan et al. - Phys.Rev.Letters 39, 906 (1977) and references therein.
- 2) G. Altarelli, G. Parisi and R. Petronzio - "Transverse Momentum in Drell-Yan Processes", CERN Preprint TH. 2413 (1977).
- 3) K. Kajantie and R. Raitio - "Gluon Effects in Muon Pair Production", Helsinki University Preprint HU-TFT-77-21 (1977).
- 4) J.B. Kogut - Phys.Letters 65B, 377 (1976) ;
D. Soper - Phys.Rev.Letters 38, 461 (1977).
- 5) A.V. Radyushkin - Phys.Letters 69B, 245 (1977).
- 6) H.D. Politzer - "Gluon Corrections to Drell-Yan Processes", Harvard Preprint (May 1977).
- 7) K.H. Craig and C.H. Llewellyn Smith - "QCD Calculation of Three Jet Processes in Leptoproduction and Hadronic Mu-Pair Production", Oxford University Preprint 67/77 (1977).
- 8) S.W. Herb et al. - Presented by L. Lederman at the 1977 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg (August 1977).
- 9) L.L. Wang - "Anticipating the Intermediate Boson", BNL Preprint 22661 (April 1977).
- 10) S.W. Herb et al. - Phys.Rev.Letters 39, 1240 (1977).

FIGURE CAPTIONS

- Figure 1 The theoretical prediction is compared to the experimental data of Ref. 8) for P nucleus collisions normalized to the total cross-section.
- Figure 2 a) The cross-section $\sigma^{\text{reg}}(Q^2, \tau, p_{\perp}^2)$ (full line) is compared to $\sigma(Q^2, \tau, x_T)$ (dashed line) and to $\sigma_{\text{DY}}(Q^2, \tau) \cdot f(p_{\perp}^2)$ (dotted line) at $\sqrt{S} = 27$ GeV.
b) The dependence of $\sigma^{\text{reg}}(Q^2, \tau, p_{\perp}^2)$ on the power n of the gluon density is shown for $n = 4, 7$ at $\sqrt{S} = 27$ GeV.
- Figure 3 The dimensionless cross-sections $s^{\frac{1}{2}}\sigma^{\text{reg}}(Q^2, \tau, p_{\perp}^2)$ (full line) and $s^{\frac{1}{2}}\sigma_{\text{DY}}(Q^2, \tau) \cdot f(p_{\perp}^2)$ (dashed line) are plotted as functions of S for different values of x_T .
- Figure 4 The theoretical prediction for $\sigma^{\text{reg}}(Q^2, \tau, p_{\perp}^2)$ in the case of $p\bar{p}$ annihilation is shown for different values of Q and $p_{\perp}$ at $\sqrt{S} = 27$ GeV.

FIG.1

FIG. 2a

FIG. 2 b

FIG.3

FIG.4