78-2-170

CERN - EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Submitted to
Nuclear Physics B

CERN/EP/PHYS 77-54
5 November 1977

A CRITICAL DISCUSSION OF A PRISM PLOT ANALYSIS OF THE FOUR-BODY

REACTION $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ AT 3.93 GeV/c

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ABSTRACT

A multidimensional analysis of the reaction $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ at 3.93 GeV/c is presented. Its results are compared to those obtained with conventional methods and its limitations are discussed.

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1. INTRODUCTION

In the medium energy region, 3, 4 and 5-body final states dominate strong interactions, and the available statistics are typically of the order of 10^3 to 10^5 events per final states.

Apart from maximum likelihood techniques, which offer the possibility of using the full dimensional space but which rely on a priori assumptions about the form of the amplitudes, most analyses limit themselves to the interpretation of one or two-dimensional projections of the complete multibody phase space. The "longitudinal phase space" introduced by L. Van Hove [1-3], and which has been successfully used for the interpretation of multibody final states at high energy as long as the transverse momenta are small compared to the longitudinal ones, is not applicable at low and medium energy.

Several attempts have been made to study multiparticle processes within the full dimensional phase space. The Prism Plot Analysis (PPA), introduced by I. Pless and collaborators [4-8], represents one of the most popular ones. Recent applications of this method can be found in [9,10].

In this paper, we apply PPA to the reaction $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ with 3.93 GeV/c incident particles, and discuss the limitations of our method of analysis. We believe that our conclusions are valid for a large range of multibody final states observed at medium to high energy and should be kept in mind when quantitative results are presented on these processes. More technical details can be found in [11].

The method and definitions are given in sect. 2. In sect. 3, we apply the method to the 3.93 GeV/c $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ final state and discuss the results. We give a detailed analysis of some of the separated channels and compare their properties to those observed using classical techniques of separation. To understand better the method, we apply it to events generated by Monte-Carlo techniques.

2. THE PRISM PLOT ANALYSIS - METHOD AND DEFINITIONS

Forgetting spin variables, a n -body final state is described by $(3n-5)$ independent variables.

The aim of PPA is to separate the channels which contribute to the same final state. These channels may overlap but the use of the complete set of $3n-5$ variables should reduce these overlaps to a minimum. The standard method ignores complications due to possible interference effects.

In practice, the separation of the channels is achieved in an iterative procedure. Monte-Carlo generated events are compared with the real ones, using a complete set of $3n-5$ variables. A real event R is "hit" by a Monte-Carlo event M when, in the $3n-5$ hyperspace, R falls inside a $3n-5$ hypercube "box" built around M .

Representing by d_{MR} the "distance" between M and R and by d_{max} the maximum of this "distance", a "hit number" is defined as

$$h = \frac{d_{max}^2}{d_{MR}^2} .$$

Although the metric in this hyperspace has no physical meaning, d_{MR} will represent the euclidean distance between points M and R . For a given channel j , the hit numbers are additive in the sense that if the same event has been hit by several Monte-Carlo events i , the event R is characterized by the number

$$h_R^j = \sum_i h_i .$$

Doing the same comparison for the m channels in competition, one ends up, for a given real event R , with a vector

$$(h_R^1, h_R^2, \dots, h_R^m) .$$

From these "hit vectors" one can compute the normalized probability vectors

$$(p_R^1, p_R^2, \dots, p_R^m),$$

which gives the probability for each event to belong to a given channel.

The probability p_R^j is defined as

$$p_R^j = \frac{h_R^j \frac{M^T}{M^j} w_j}{\sum_j h_R^j \frac{M^T}{M^j} w_j}$$

$$M^T = \sum_j M^j,$$

M^j is the number of Monte-Carlo events generated to represent the channel j and w_j is the percentage contribution assumed for the channel j to the total final state.

In the first step, one generates Monte-Carlo events according to the invariant phase space modified by Breit-Wigner functions. As a result of this first step one gets production and decay angular distributions for each attempted channel. For a given channel, these distributions are used for the generation of events of the next iteration, together with the Breit-Wigner functions.

In principle, the choice of the $3n-5$ independent variables is arbitrary. It is clear that this choice does not affect the fact that two channels can populate the same region of the full dimensional space. However, a "good choice" of the variables will give a separation of two clusters related to two different channels large compared to the sizes of the boxes which are used for the separation. In practice, it is found that the Van Hove longitudinal phase space variables and the generalized Dalitz-plot variables [4] represent a good choice.

The smaller the sizes of the boxes used to compare the Monte-Carlo generated events to the real ones are, the better the separation between the different channels should be. However, if the sizes are too small, a large fraction of the full dimensional space will not be covered, and the percentage of "not hit" events will be large. As a compromise, we

have introduced, for the same iteration, several box sizes to be used successively. We start the comparison with the smallest one. For the events which have not been hit during this first attempt, we try again with the next bigger box size, etc.

3. APPLICATION OF THE PRISM PLOT ANALYSIS TO $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ AT 3.93 GeV/c

In this section, we first present the general conditions of the experiment and make the necessary choices (channels, variables, box sizes). We then discuss the general results and pay particular attention to some well known final states.

3.1. The data and the general conditions of the analysis

We have analyzed a sample of 19 442 $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ events, corresponding to 10 events/ μb , produced in the CERN 2m HBC exposed to a separated beam of 3.93 GeV/c [12].

The choice of the channels to be introduced was made by looking at the two and three-particle effective mass distributions of the data. Fig. 1 shows the presence of ρ^0 , f^0 , $\Delta^{++}(1236)$, $\Delta^0(1236)$, $N^*(1520)$, $N^*(1688)$ and A_2 , with, in addition, a shoulder in the A_1 mass region. The low mass region of the $p\pi^+\pi^-$ suggests the presence of some diffractive $p\pi^+\pi^-$ or $N^{*+}(1520)$, $N^{*+}(1688)$ production.

We therefore introduce the following channels:

- (1) $\pi^- p \rightarrow pA_1^-$, $A_1^- \rightarrow \rho^0\pi^-$, $\rho^0 \rightarrow \pi^+\pi^-$
- (2) $\pi^- p \rightarrow pA_2^-$, $A_2^- \rightarrow \rho^0\pi^-$, $\rho^0 \rightarrow \pi^+\pi^-$

- (3) $\pi^- p \rightarrow N^{*0}(1520) \rho^0, N^{*0}(1520) \rightarrow p\pi^-, \rho^0 \rightarrow \pi^+\pi^-$
- (4) $\pi^- p \rightarrow N^{*0}(1688) \rho^0, N^{*0}(1688) \rightarrow p\pi^-, \rho^0 \rightarrow \pi^+\pi^-$
- (5) $\pi^- p \rightarrow \Delta^0(1236) \rho^0, \Delta^0(1236) \rightarrow p\pi^-, \rho^0 \rightarrow \pi^+\pi^-$
- (6) $\pi^- p \rightarrow \Delta^{++}(1236) \pi^-\pi^-, \Delta^{++}(1236) \rightarrow p\pi^+$
- (7) $\pi^- p \rightarrow (DD) \pi^-, (DD) \rightarrow \Delta^{++}\pi^-, \Delta^{++} \rightarrow p\pi^+$
- (8) $\pi^- p \rightarrow (DD) \pi^-, (DD) \rightarrow p\pi^+\pi^-$
- (9) $\pi^- p \rightarrow p\pi^-\rho^0, \rho^0 \rightarrow \pi^+\pi^-$
- (10) $\pi^- p \rightarrow p\pi^+\pi^-\pi^-$ (phase space).

Seven variables describe a four-body final state. We have chosen the cartesian coordinates of the Van Hove plot and the three-dimensional energy simplex. For the seventh variable we choose the transverse momentum of the two-negative-pion system. Denoting by p , q and T , the momentum, longitudinal momentum and kinetic energy of a particle in the total c.m. system, the seven variables have the following form:

$$p_{\pi^-\pi^-}^t = \sqrt{(\vec{p}_{\pi_1^-} + \vec{p}_{\pi_2^-})^2},$$

$$x_{\text{VH}} = \frac{1}{4} [3 q_p - (q_{\pi^+} + q_{\pi_1^-} + q_{\pi_2^-})], x_{\text{ES}} = \frac{1}{4} [3 T_p - (T_{\pi^+} + T_{\pi_1^-} + T_{\pi_2^-})],$$

$$y_{\text{VH}} = \frac{1}{2\sqrt{2}} [2 q_{\pi^+} - (q_{\pi_1^-} + q_{\pi_2^-})], x_{\text{ES}} = \frac{1}{2\sqrt{2}} [2 T_{\pi^+} - (T_{\pi_1^-} + T_{\pi_2^-})],$$

$$z_{\text{VH}} = \sqrt{\frac{2}{3}} [q_{\pi_1^-} - q_{\pi_2^-}], z_{\text{ES}} = \sqrt{\frac{2}{3}} [T_{\pi_1^-} - T_{\pi_2^-}].$$

For the first step, Monte-Carlo events are generated according to phase space modified by Breit-Wigner functions and diffraction-like mass distributions characterized by the parameters given in table 1. For the subsequent steps, we also introduce the production angular distributions and the Gottfried-Jackson and Treiman-Yang decay angles of each resonance as obtained in the previous step.

20 000 events per channel are Monte-Carlo generated and four box-sizes are used at each step. These sizes correspond to 1/40, 1/30, 1/20 and 1/10 of the range of each variable.

3.2 Results of the analysis

We have formed 13 iterations with the Prism Plot Program and the rates obtained are given in table 2. All the rates become stable after the tenth iteration. We should notice that the contribution assigned to phase space is only 3%.

Fig. 2 shows typical mass distributions obtained for various channels. The $p\pi^+\pi^-$ mass distributions for the two diffraction dissociation channels $DD(\Delta^{++}\pi^-)\pi^-$ and $DD(p\pi^+\pi^-)\pi^-$ show structures which could correspond to three-body N^* resonances not introduced in PPA. The $\pi^+\pi^-$ mass distributions corresponding to the $DD(\Delta^{++}\pi^-)\pi^-$ channel (the π^- is the one associated to DD) shows an unexpected ρ^0 signal. Fig. 3 shows the Gottfried-Jackson decay angular distributions of $N^*(1688)$ and its associated ρ^0 . The decay angular distribution is not symmetric, indicating some contamination in this channel. The same remark applies to the A_2 decay.

3.3 Estimation of the contamination

In fig. 4, we display some of the mass spectra which should not show resonance production after the selection operated by PPA. One clearly observes several "wrong" signals like the Δ^{++} in the pA_1 and $\Delta^0\rho^0$ channels. Using these distributions, and similar ones, we can estimate the residual contaminations and try to find their origin, for each "separated" channel. These estimations are given in table 3. The pA_1 channel still contains a large contamination of N^* 's and Δ^{++} . This is not the case for the pA_2 channel which seems to be better separated than most of the other channels. The $\Delta^0\rho^0$ channel contains a large contamination of Δ^{++} .

The observed contaminations are obviously related to the overlap between the different channels. To obtain a quantitative estimate of the importance of the overlaps, we define the following quantity:

$$O_{ij} = \frac{\sum_{\alpha=1}^N P_i^{\alpha} P_j^{\alpha}}{\left[\sum_{\alpha=1}^N (P_i^{\alpha})^2 \sum_{\alpha=1}^N (P_j^{\alpha})^2 \right]^{1/2}}; \quad i, j = 1, \dots, 10,$$

where P_i^{α} is the probability for the event α to belong to channel i . N is the total number of events. All these quantities are given in table 4. The last row contains the total overlap of each channel with all the others. These numbers are compatible with the estimate of the contaminations deduced from the mass distributions. However, one should notice that these quantitative estimates depend on the box-size.

3.4 Partial cross sections

Table 5 gives the cross sections for the ten channels introduced in PPA. The quoted errors are only statistical. Other sources of errors will be discussed later (sect. 4). Four partial cross sections are given in ref. [13] which refer to similar π^- incident momenta and can therefore be compared to our results (table 5). Good agreement is observed for the $\pi^- p \rightarrow pA_1$, pA_2 and $\Delta^0 \rho^0$ channels. PPA gives a smaller cross section for the $\Delta^{++} \pi^- \pi^-$ channel than the "classical" analysis. As we shall discuss later, most of the difficulty comes from the large overlap between the $\Delta^{++} \pi^- \pi^-$ and the $DD(\Delta^{++} \pi^-) \pi^-$ channels to which a large partial cross section is attributed by PPA.

3.5 Spin parity analysis of the $(\pi^+ \pi^- \pi^-)$ -system in the A_2 region

We first combine the events which have been labelled pA_1 or pA_2 by PPA, and then, following the method of Abramovich et al., [14] perform a spin-parity analysis of the 3π system for those events which satisfy the condition: $1.30 \leq m(3\pi) \leq 1.34$ GeV. In order to estimate the losses of the A_2 signal into other channels, the same spin-parity analysis is applied to all of the separated channels in turn as well as to the unseparated sample of events.

Table 6 gives the number of events attributed to the usual set of spin-parity assignments, for the whole data and for each separated channel. Identifying the A_2 signal with the 2^+ contribution, we can see that 28% of the total sample contains A_2 , whereas this proportion reaches 43% for the prism plot pA selection, corresponding to 73% of the A_2 contained in the total sample. The remaining 27% of A_2 are spread over the other channels, mainly over those channels producing ρ^0 . In fact, if we antiselect the non- ρ^0 -producing channels we get a result in good agreement with the full 2^+ contribution. On the other hand, of the 552 events labelled pA₁ or pA₂ by PPA and belonging to the 3π mass region considered here, 43% have the decay properties expected for 2^+ , and 14% those expected for a 1^+ signal. Thus, more than 40% of the events must come from other channels. This fact and the decay angular distribution attributed to the A_2 selection show that PPA is unable to separate this particular channel.

3.6 Properties of the $\Delta^0 \rho^0$ channel

Another example to estimate the efficiency of the prism plot separation is to compute the joint decay correlations, in terms of the statistical tensors in the Jackson transversity frame [15], of the $\Delta^0 \rho^0$ selection and to compare these results to those obtained by selecting on mass distributions in the following way:

$$0.680 < m(\pi^+ \pi^-) < 0.860 \text{ GeV},$$

$$1.160 < m(p \pi^+) < 1.280 \text{ GeV}.$$

The tensors which must be zero by parity conservation satisfy this condition for both selections. The results obtained with the two selections for the remaining tensors are in good agreement except for T_{20}^{20} and T_{20}^{22} .

The Biaľas and Zalewski class A relationships for the transversity amplitudes [15] are shown in fig. 5. A strong violation of three of the six relations is observed. The prism plot selection shows a larger deviation from zero for the second relation which is relative to the real parts of T_{20}^{20} and T_{20}^{22} , and which is known to work well for the "clean" channel $\Delta^{++} \rho^0$ from π^+ beams [16].

In general, the two $\pi^- p$ selections lead to similar results, showing that the $\Delta^0 \rho^0$ reaction selected with PPA is not cleaner than the sample obtained by mass cuts.

4. PPA APPLIED TO SIMULATED EVENTS

To understand better the problems arising in the above analysis and to estimate the systematic errors which may affect the partial cross sections, we have applied PPA to simulated four-body final state events.

The "simulated sample" is composed of 4780 events generated according to an incoherent sum of matrix elements squared for the channels $\pi^- p \rightarrow pA_2$, $(p\pi^-)(\pi^+\pi^-)$ and $\Delta^{++}\pi^-\pi^-$. For pA_2 , the matrix elements were parametrized in terms of Breit Wigner functions, the decay angular distributions for A_2 and ρ and an exponential t dependence for the production. For the other channels the one-pion exchange formalism corresponding to the two graphs of fig. 6 were used [17]. The channel $(p\pi)^0(\pi\pi)^0$ includes Δ^0 , N_{1520}^* and N_{1688}^* , and/or ρ^0 and f^0 production. The fractions for the three channels were 16.7%, 50.0% and 33.3% respectively.

For the prism plot analysis, we introduce the same ten channels which were used for the analysis of the real events (sect. 3) although we know the "simulated sample" does not contain some of them. This will provide a further check on the reliability of the method.

At each iteration, three box sizes are successively used: $\pm 1/40$, $\pm 1/30$ and $\pm 1/20$ of the total range of each variable. All the other options (choice of variables, number of generated events per channel, and initial conditions) are identical to those used in sect. 3.

Table 7 shows the results of the three iterations. It was not found necessary to pursue the iterative procedure beyond the third iteration. The first column of the table gives the list of the channels introduced in PPA. It is followed by the number of events present in the "simulated sample". For each iteration we give the number of events attributed by PPA to the ten channels and their decomposition according to their origin.

For instance, after the first iteration, 681 events are attributed by PPA to the pA_2 channel, 319 of them coming effectively from the 800 "simulated pA_2 sample", 254 from the "simulated $(p\pi)^0 (\pi\pi)^0$ sample and 108 from the "simulated $\Delta^{++} \pi^- \pi^-$ sample". The percentage of "hit" events is rather stable from iteration to iteration and does not strongly depend on the channel (see last line of table 7). The number of events wrongly attributed to pA_1 by PPA decreases slowly from iteration 1 to 3, the main contribution (66%) coming from $(p\pi)^0 (\pi\pi)^0$. The pA_2 contribution is small and tends to decrease. Of the 800 pA_2 "simulated" events introduced in the data, 656 are hit, 343 of which being effectively attributed to pA_2 channel by PPA. An appreciable fraction of "simulated" pA_2 is attributed to the $(p\pi)^0 (\pi\pi)^0$ channels and this defect is not reduced by the iterative procedure. On the other hand, the loss of "simulated" pA_2 into the $\Delta^{++} \pi^- \pi^-$ channel is small. If we now consider the origin of the 755 events attributed by PPA to pA_2 , we find that $\Delta^{++} \pi^- \pi^-$ and $(p\pi)^0 (\pi\pi)^0$ contribute for 16 and 38% respectively. It proves that the overlap between the different channels is a serious limitation, at least at 4 GeV/c, and that the separation given by PPA for pA_2 is unsatisfactory. In these conditions, we understand better the difficulties met in the interpretation of some of the decay angular distributions. Fig. 7(a) gives the decay distribution for the "simulated" A_2 data, fig. 7(b) the same distribution for the prism-plot selection. In fig. 7(c), (d) and (e) are shown the contributions of $\Delta^{++} \pi^- \pi^-$, $(p\pi)^0 (\pi\pi)^0$ and pA_2 events, respectively to fig. 7(b). The unexpected distribution shown in fig. 3(c) is then due to a contamination coming from the $(p\pi)^0 (\pi\pi)^0$ channels. About 50% of the $(p\pi)^0 (\pi\pi)^0$ "simulated" events are correctly attributed to this channel by PPA. A large fraction of them is attributed to pA_1 and pA_2 .

An interesting remark, drawn from table 7, is that if the $DD(\Delta^{++} \pi^- \pi^-)$ channel is present, it is impossible to disentangle it from $\Delta^{++} \pi^- \pi^-$, the number of "simulated" $\Delta^{++} \pi^- \pi^-$ events attributed to $DD(\Delta^{++} \pi^- \pi^-) \pi^-$ by PPA being larger than the one attributed to $\Delta^{++} \pi^- \pi^-$ itself. Moreover, this feature gets worse as the number of iterations increases. This trend was already observed for real events (sect. 3, table 2). On the contrary, the $DD(p\pi^+ \pi^-) \pi^-$ channel tends to disappear, from iteration to iteration, as it should.

To appreciate the effect of the size of the boxes on the results, we have repeated the analysis with smaller boxes limiting ourselves to two iterations. The new sizes correspond to 1/50 to 1/35 of the total range of each variable. The percentage of "hit" events is uniformly reduced from 73% for the previous box sizes to 50% for the new ones, but all the other features remain the same. This may indicate that the large overlaps observed between the channels have a physical origin. In the overlap regions, where two channels are present with different intensities, the probability to attribute wrongly and uniquely an event to the weakly populated channel is quite large. This is corroborated by a more unsatisfactory decay angular distribution for the A_2 , for instance, when using smaller box sizes.

5. CONCLUSIONS

The use of PPA for the separation of the constituent channels of a multibody final state, at least as seen from 4 GeV/c π^-p interactions, suffers from several limitations:

- (i) The systematic errors on the partial cross sections can reach 20% and are channel dependent [18].
- (ii) The basic difficulty is related to the presence of large overlap regions, important for four-body final states at these medium high energies. This difficulty becomes very serious when the densities of events corresponding to two channels are different in the overlap.
- (iii) Attempts made with different box sizes have shown that these overlaps have a physical origin and are therefore irreducible.
- (iv) The contamination between channels can reach 40%. For example, in the case of the pA_2 channel we detect a contamination larger than 40% and the analysis of the polarization of the $\Delta^0 \rho^0$ channel shows that the results of PPA are not better than those obtained with a simple mass cut.

These conclusions should remain valid for any method which does not include amplitudes in the description of the channels. Nevertheless, one should point out that these methods have the good property of detecting low frequency processes [18]. The methods that can deal with the unavoidable complexities due to the overlaps and interference effects must include analytical functions for the description of the channel involved in the final state [19].

We wish to thank Professor L. Van Hove for his continuous interest in this work and his valuable advice as well as Professor I. Pless and Dr. R. Eisner for useful discussions.

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TABLE CAPTIONS

- Table 1 Mass and width used for resonances and mass shape for diffraction dissociation DD in the $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ data.
- Table 2 Input channel percentages along the iterations for the $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ data.
- Table 3 Numbers of events attributed to each channel after 13 iterations of the Prism Plot Analysis (PPA) on the $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ data and quantitative estimation of the residual contaminations between the channels.
- Table 4 Estimation of the overlaps between the channels in the $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ data.
- Table 5 Cross sections for $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ data obtained with PPA and with classical methods.
- Table 6 Spin-parity analysis of the 3π system in the region $1.30 < m(3\pi) < 1.34$ GeV.
- Table 7 Quantitative results for three iterations of prism-plot analysis of $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ simulated data.

TABLE 1

Resonance	M(MeV)	Γ (MeV)
ρ	765	135
Δ^0	1236	120
N_{1520}^{*0}	1520	120
N_{1688}^{*0}	1688	120
A_1	1100	200
A_2	1310	100
f^0	1250	140
DD [8]

TABLE 2

Iteration Channel	pA_1	pA_2	N_{1520}^{ρ}	N_{1688}^{ρ}	Δ_{1236}^{ρ}	$\Delta_{1236}^{++\pi^-}$	$DD(\Delta^{++}\pi^-)$	$DD(p\pi^+\pi^-)$	$p\pi^- f^0$	ph. sp.
0	11.0	11.3	9.7	9.6	9.6	11.7	11.7	9.5	7.0	9.0
1	11.4	12.9	10.1	9.5	10.3	12.7	12.5	9.0	5.0	6.6
2	11.9	12.6	9.9	9.8	10.9	13.2	13.1	8.2	4.4	5.9
3	12.1	12.6	10.2	10.0	10.5	12.7	14.1	8.4	4.3	5.2
4	12.4	12.5	10.0	9.3	10.2	12.8	15.2	8.4	4.5	4.7
5	13.0	12.5	9.8	9.5	10.0	12.5	16.0	8.4	4.3	4.1
6	13.1	12.2	9.6	9.8	9.9	12.2	17.2	8.2	4.1	3.6
7	12.5	11.5	9.2	10.0	10.6	11.5	17.6	8.9	4.2	3.9
8	12.8	11.2	9.5	10.6	10.8	10.6	17.8	9.3	3.9	3.6
9	12.8	11.6	9.7	10.9	11.1	9.8	17.6	9.5	3.7	3.3
10	13.4	11.9	8.3	10.9	11.4	9.7	17.2	10.2	3.7	3.2
11	13.2	11.6	8.0	10.4	12.2	9.3	17.4	10.9	3.9	3.1
12	13.7	11.9	7.7	10.3	12.5	8.9	17.8	10.5	3.7	3.1
13	13.1	11.7	8.5	10.7	11.9	9.2	17.5	10.5	3.7	3.1

TABLE 3

	Rates (%)	Number of events	Contamination of the channels (in nb. of events). The origin of the contamination is indicated in the first column.										Total losses (lower limit)
			pA ₁	pA ₂	N ₁₅₂₀ ^{ρ⁰} *	N ₁₆₈₈ ^{ρ⁰} *	Δ ^{ρ⁰}	Δ ^{ρ⁰}	Δ ^{ρ⁰}	Δ ^{ρ⁰}	Δ ^{ρ⁰}	Δ ^{ρ⁰}	
pA ₁	13.1	2538			0	15	20	0	0	0	0	0	35
pA ₂	11.7	2267			15	0	20	68	0	0	0	18	161
N ₁₅₂₀ ^{ρ⁰} *	8.5	1654	75	46		20	0	20	0	0	0	0	161
N ₁₆₈₈ ^{ρ⁰} *	10.7	2077	100	25	0		0	0	0	0	0	0	165
Δ ^{ρ⁰}	11.9	2313	50	70	0	66		?	?	?	?	0	236
Δ ^{ρ⁰}	9.2	1793	250	46	104	130	300		?	?	?	30	902
DD(Δ ^{ρ⁰} π ⁻)π ⁻	17.5	3409											?
DD(pπ ⁺ π ⁻)π ⁻	10.5	2050											?
pπ ⁻ f ⁰	3.7	728	?	?	20	20	46	0	0	0	0	0	86
ph. sp.	3.1	614											
Total contamination (lower limit)			475	187	139	251	386	150	68	0	42	48	

TABLE 4

	pA_1	pA_2	$N_{1520}^* \rho^0$	$N_{1688}^* \rho^0$	$\Delta^0 \rho^0$	$\Delta^{++} \pi^- \pi^-$	$DD(\Delta^{++} \pi^-) \pi^-$	$DD(p \pi^+ \pi^-) \pi^-$	$p \pi^- f^0$	ph. sp.
pA_1	1.00	.07	.04	.05	.07	.03	.06	.03	.01	.02
pA_2		1.00	.06	.07	.08	.04	.04	.03	.02	.04
$N_{1520}^* \rho^0$			1.00	.13	.05	.05	.03	.02	.08	.09
$N_{1688}^* \rho^0$				1.00	.03	.07	.05	.02	.08	.10
$\Delta^0 \rho^0$					1.00	.02	.03	.05	.04	.03
$\Delta^{++} \pi^- \pi^-$						1.00	.12	.07	.05	.05
$DD(\Delta^{++} \pi^-) \pi^-$							1.00	.10	.02	.02
$DD(p \pi^+ \pi^-) \pi^-$								1.00	.03	.02
$p \pi^- f^0$									1.00	.10
ph. sp.										1.00
Total overlap	.38	.45	.59	.60	.40	.50	.47	.37	.43	.47

TABLE 5

Final state	Prism plot cross section at 3.93 GeV(c) (mb)	Cross section (mb) and beam momentum of the nearest quoted experiment	
pA_1	0.254 ± 0.005	0.282 ± 0.032	3.89 GeV/c
pA_2	0.227 ± 0.005	0.215 ± 0.032	3.89 GeV/c
$N_{1520}^* \rho^0$	0.165 ± 0.004	-	-
$N_{1688}^* \rho^0$	0.208 ± 0.005	-	-
$\Delta_{\rho^0}^0$	0.231 ± 0.005	0.203 ± 0.029	3.9 GeV/c
$\Delta^{++} \pi^- \pi^-$	0.179 ± 0.004	0.303 ± 0.033	3.89 GeV/c
$DD(\Delta^{++} \pi^-) \pi^-$	0.341 ± 0.006	-	-
$DD(p \pi^+ \pi^-) \pi^-$	0.205 ± 0.005	-	-
$p \pi^- f^0$	0.073 ± 0.003	-	-
ph. sp.	0.061 ± 0.003	0.634	3.89 GeV/c

TABLE 6

Channel	Number of events	2^+	$1^+/s$	$1^+/d$	$0^-/00$	% $\rho\pi$	% backgr.
All channels, all data	1139	321 ± 43	233 ± 42	13 ± 13	386 ± 47	49 ± 3	51 ± 3
$pA_1 + pA_2$	552	235 ± 22	64 ± 21	28 ± 17	53 ± 46	57 ± 4	43 ± 4
$N_{1520}^* \rho^0$	59	16 ± 6	10 ± 6	3 ± 5	33 ± 5	47 ± 9	53 ± 9
$N_{1688}^* \rho^0$	77	21 ± 8	21 ± 8	-	36 ± 6	55 ± 8	45 ± 8
$\Delta_{1236}^0 \rho^0$	112	7 ± 12	51 ± 14	9 ± 9	45 ± 10	58 ± 6	42 ± 6
$\Delta^{++} \pi^- \pi^-$	78	9 ± 4	5 ± 5	6 ± 6	27 ± 7	25 ± 8	75 ± 8
$DD(\Delta^{++} \pi^-) \pi^-$	139	26 ± 9	13 ± 9	11 ± 8	45 ± 10	27 ± 5	73 ± 5
$DD(p \pi^+ \pi^-) \pi^-$	88	-	-	6 ± 3	60 ± 10	8 ± 6	92 ± 6
$p \pi^- f^0$	4	-	-	-	-	-	-
ph. sp.	30	-	-	-	-	-	-
Only pro- ducing channels	844	295 ± 33	198 ± 33	25 ± 22	35 ± 35	60 ± 4	40 ± 4

TABLE 7

Channel	Generated	1st iteration				2nd iteration				3rd iteration			
		Hits	$\Delta^{++}\pi\pi^-$	$(p\pi)^0(\pi\pi)^0$	pA_2	Hits	$\Delta^{++}\pi\pi^-$	$(p\pi)^0(\pi\pi)^0$	pA_2	Hits	$\Delta^{++}\pi\pi^-$	$(p\pi)^0(\pi\pi)^0$	pA_2
pA_1	0	428	97	269	62	12.3	96	251	55	11.5	372	81	243
pA_2	800	681	108	254	319	19.6	135	283	322	21.2	755	129	283
$N_{1520}^{*\rho^0}$	2	294	78	170	46	8.5	64	185	51	8.6	293	64	180
$N_{1688}^{*\rho^0}$	3	293	75	145	73	8.4	82	131	62	7.8	278	68	140
Δ^{ρ^0}	8	506	64	371	71	14.5	73	474	73	17.7	701	82	541
$p\pi f^0$	0	58	28	28	2	1.7	36	20	0	1.0	9	2	5
$\Delta^{++}\pi\pi^-$	1600	422	277	116	29	12.1	403	272	34	11.5	387	258	108
$DD(\Delta^{++}\pi^-)\pi^-$	0	488	309	160	19	14.0	523	338	23	15.0	547	373	145
$DD(p\pi^+\pi^-)\pi^-$	0	266	112	140	14	7.6	188	93	11	5.4	139	61	72
ph. sp.	0	47	17	21	9	1.3	8	6	0	0.2	2	0	2
Total	4780	3483	1165	1674	644		3495	1166	631		3483	1118	1719
Percent of hits		73	73	70	80		74	73	79		74	70	81

FIGURE CAPTIONS

Fig. 1 Mass spectra for the total data $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$:

- (a) $\pi^+ \pi^-$ effective mass;
- (b) $p \pi^+$ effective mass;
- (c) $p \pi^-$ effective mass;
- (d) $\pi^+ \pi^- \pi^-$ effective mass;
- (e) $p \pi^+ \pi^-$ effective mass.

Fig. 2 Mass spectra for some of the channels separated by the prism-plot analysis after 13 iterations:

- (a) $\pi^+ \pi^- \pi^-$ effective mass for the pA_1 channel;
- (b) $\pi^+ \pi^- \pi^-$ effective mass for the pA_2 channel;
- (c) $\pi^+ \pi_1^-$ effective mass for the pA_2 channel, $A_2 \rightarrow \rho \pi_2^-$, $\rho \rightarrow \pi^+ \pi_1^-$;
- (d) $p \pi_1^-$ effective mass for the $\Delta^0 \rho^0$ channel, $\Delta^0 \rightarrow p \pi_1^-$, $\rho^0 \rightarrow \pi^+ \pi_2^-$;
- (e) $\pi^+ \pi_2^-$ effective mass for the $\Delta^0 \rho^0$ channel, $\Delta^0 \rightarrow p \pi_1^-$, $\rho^0 \rightarrow \pi^+ \pi_2^-$;
- (f) $p \pi^+$ effective mass for the $\Delta^{++} \pi^- \pi^-$ channel;
- (g) $\pi^- \pi^-$ effective mass for the $\Delta^{++} \pi^- \pi^-$ channel;
- (h) $p \pi^+ \pi_1^-$ effective mass for the $(DD)_{\Delta} \pi^-$ channel, $(DD)_{\Delta} \rightarrow \Delta^{++} \pi_1^-$, $\Delta^{++} \rightarrow p \pi^+$;
- (i) $p \pi^+ \pi_1^-$ effective mass for the $(DD) \pi^-$ channel, $(DD) \rightarrow p \pi^+ \pi_1^-$;
- (j) $\pi^+ \pi_1^-$ effective mass for the $(DD)_{\Delta} \pi^-$ channel, $(DD)_{\Delta} \rightarrow \Delta^{++} \pi_1^-$;
- (k) $\pi^+ \pi_1^-$ effective mass for the $(DD) \pi^-$ channel, $(DD) \rightarrow p \pi^+ \pi_1^-$.

Fig. 3 Gottfried-Jackson decay angular distributions for the $N^*(1688) \rho^0$ channel and the pA_2 channel:

- (a) decay of $N^*(1688)$;
- (b) decay of ρ^0 ;
- (c) decay of A_2 ;
- (d) decay of ρ^0 .

FIGURE CAPTIONS (Cont'd)

Fig. 4 Effective masses after separation by the PPA of the $\pi^- p \rightarrow \pi^- p \pi^+ \pi^-$ data:

- (a) $p\pi^+$ effective mass for the pA_1 channel;
- (b) $p\pi^-$ effective mass for the pA_1 channel;
- (c) $p\pi^+$ effective mass for the pA_2 channel;
- (d) $p\pi^+$ effective mass for the $\Delta^0 \rho^0$ channel.

Fig. 5 Białas-Zalewski class A relationships as observed for the $\Delta^0 \rho^0$ reaction. PPA (dots) and mass cut selection (circles):

- A.1: $T_{00}^{20} - \sqrt{2} T_{00}^{02} = 0;$
- A.2: $\text{Re } T_{20}^{22} - \frac{1}{2} \text{Re } T_{20}^{20} = 0;$
- A.3: $\text{Re } T_{02}^{22} - \frac{1}{2} \text{Re } T_{02}^{02} = 0;$
- A.4: $T_{00}^{22} - \left(\frac{1}{2\sqrt{6}} - \frac{1}{\sqrt{2}} T_{00}^{02} \right) = 0;$
- A.5: $\text{Im } T_{20}^{22} - \frac{1}{2} \text{Im } T_{20}^{20} = 0;$
- A.6: $\text{Im } T_{02}^{22} - \frac{1}{\sqrt{2}} \text{Im } T_{02}^{02} = 0.$

Fig. 6 Two OPE diagrams used to generate the $\Delta^{++} \pi^- \pi^-$ and $(p\pi)^0 (\pi\pi)^0$ events.

Fig. 7 Gottfried-Jackson decay angular distribution of $A_2 \rightarrow \rho\pi$:

- (a) for the "simulated" $\pi^- p \rightarrow pA_2$ sample;
- (b) for the $\pi^- p \rightarrow pA_2$ prism-plot selection;
- (c) contribution to (b) coming from the "simulated" $\Delta^{++} \pi^- \pi^-$ events;
- (d) contribution to (b) coming from the "simulated" $(p\pi)^0 (\pi\pi)^0$ events;
- (e) contribution to (b) coming from the "simulated" pA_2 events.

Fig. 1 (a-e)

Fig. 2 (a-e)

Fig. 2 (f-k)

Fig. 3 (a-d)

Fig. 4 (a-d)

Fig. 5

Fig. 6

Fig. 7 (a-e)