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INCLUSIVE PRODUCTION AT LARGE TRANSVERSE MOMENTUM IN QCD

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A B S T R A C T

We calculate the cross-section for the process
quark + quark $\rightarrow$ quark (with large transverse momentum) +
+ anything, up to order g^6 in the cross-section and
in the leading logarithm approximation. The result is
suggestive of a simple ansatz for the calculation of the
asymptotic behaviour of cross-sections for the production
of particles or jets with large transverse momenta.

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The parton model has proved to be a useful framework with which to study hard scattering processes, e.g., deep inelastic lepton-hadron scattering, large transverse momentum hadronic processes and massive lepton pair production¹⁾. It is hoped that many of the successful features of the parton model are also properties of Quantum Chromodynamics (QCD), the leading candidate for the theory of the strong interactions. In the case of deep inelastic lepton-hadron scattering, the use of the renormalization group and the operator product expansion, together with the property of asymptotic freedom, provide an elegant explanation of approximate Bjorken scaling, and determine the scaling violations^{2),3)}. Unfortunately, no such clean analysis exists for the predictions of QCD for the other hard scattering processes mentioned above, nevertheless some progress has recently been made in understanding these processes, as will be discussed below.

The parton model approach to hard scattering processes is to assume that the "hard scattering" occurs between constituents of the initial particles, so that one can write⁴⁾

$$\frac{d\sigma}{d^4Q} (AB \rightarrow CX) = \sum_{a,b,d} \int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{\lambda((p_a+p_b)^2, p_a^2, p_b^2)}{x_a x_b \lambda(s, p_a^2, p_b^2)} \frac{d\sigma}{d^4Q} (ab \rightarrow Cd) \quad (1)$$

$G_{b/B}(x_b)$ represents the probability of finding a constituent "b" in hadron "B" with fraction x_b of the momentum of B (in the centre-of-mass frame, for example), and "C" represents an observed particle or set of particles (e.g., a π meson at large p_T or a massive lepton pair).

We start by assuming that formula (1) is also true in QCD, where $G(x)$ is now given by the soft wave function of the hadron, and cannot be calculated using perturbation theory, whereas the hard "subprocess" $ab \rightarrow Cd$ is calculated using perturbation theory. This assumption a priori does not provide a useful prescription for calculating cross-sections, since in principle we need $d\sigma/d^4Q$ ($ab \rightarrow Cd$) to all orders in perturbation theory. This is so because, in spite of the fact that the coupling constant can be made small by an appropriate choice of

renormalization point, in each successive order of perturbation theory there appear more powers of logarithms with large arguments. Thus we obtain a power series in $\bar{g}^2(Q^2) \log Q^2$ *) [$\bar{g}(Q^2)$ is the "running coupling constant"], so that each order of perturbation theory gives a comparable contribution to $d\sigma/d^4Q$ ($ab \rightarrow Cd$). However, there are now indications that in some processes the leading logarithms can be reorganized in a particularly simple way, so that the Born term of the reorganized series reliably predicts the asymptotic behaviour of the cross-section (up to logarithmic corrections). For example, in the case where C is a lepton pair with invariant mass M , and a, b are $q\bar{q}$ ⁵⁾, qg , $q\bar{q}$ or $g\bar{g}$ ⁶⁾ (where q denotes a quark and g a gluon) low order perturbation theory calculations suggest that the leading logarithms which appear are exactly those (with the correct coefficients) which appear in the calculation of νW_2 at q^2 proportional to $-M^2$ **). Moreover, these leading logarithms multiply the point cross-section for $q\bar{q} \rightarrow$ massive photon. Thus one is led to the following formula for the process $h_1 + h_2 \rightarrow \ell^+ \ell^- X$

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3nM^4} \sum_a G_a^2 \int_0^1 dx_1 dx_2 x_1 x_2 \left[G_{q\bar{q}/h_1}(x_1, M^2) \right. \\ \left. G_{q\bar{q}/h_2}(x_2, M^2) + G_{q\bar{q}/h_1}(x_1, M^2) G_{q\bar{q}/h_2}(x_2, M^2) \right] \delta(x_1 x_2 - \frac{M^2}{s}) \quad (2)$$

where the G 's are extracted from deep inelastic data at $q^2 \sim -M^2$, and n is the number of colours. This, of course, is just the Drell-Yan ⁸⁾ formula with scale dependent distribution functions replacing the scaling ones used in the parton model. There will be logarithmic corrections to formula (2). The object of this letter is to present an example of inclusive production at large transverse momentum, a genuine strong interaction process, where a similar phenomenon occurs.

*) Although individual diagrams may give two logarithms per loop, these double logarithms cancel when all relevant diagrams are considered.

**) For other examples where a similar feature occurs, see Ref. 7).

An extension of (1) to the case of $A + B \rightarrow C + X$, where C is a particle with large transverse momentum, is given by the formula⁴⁾

$$E_C \frac{d\sigma}{d^3p_C} (AB \rightarrow CX) = \sum_{ab,cd} \int_0^1 dx_a \int_0^1 dx_b \int_0^1 \frac{dx_c}{x_c^2} G_{A/A}(x_a) G_{B/B}(x_b) \tilde{G}_{C/C}(x_c) \delta(s'+t'+u') \frac{s'}{\pi} \frac{d\sigma}{dt'} (ab \rightarrow cd) \quad (3)$$

where

$$s' = x_a x_b s, \quad t' = \frac{x_a}{x_c} t, \quad u' = \frac{x_b}{x_c} u$$

and the fragmentation function $\tilde{G}_{C/C}(x_c)$ is defined to be the probability, normalized to the multiplicity, for particle C to have a fraction x_c of the momentum along the direction of constituent c . s, t, u are defined by $s \equiv (p_A + p_B)^2$, $t \equiv (p_A - p_C)^2$ and $u \equiv (p_B - p_C)^2$. We shall work in the kinematic region where $s \sim t \sim u$, and let Q^2 be a typical invariant mass squared of order s, t, u . The results in this paper are for massless quarks, the conclusions however are unchanged if the quarks are given a mass. In this letter we study a sample subprocess in lowest non-trivial order of perturbation theory ($qq \rightarrow qX$, where the initial quarks have different flavours), and from the results conjecture how within our framework formula (3) should be modified.

The lowest order inelastic diagrams which contribute to the subprocess $qq \rightarrow qX$ are shown in Fig. 1. We start by calculating the cross-section for $x < 1$, where $x = E_C/E_{C,\max}$ in the centre-of-mass frame, in this case only the inelastic diagrams contribute. The $\log Q^2/p_1^2$ factors then arise from regions of integration where k is parallel to either p_A, p_B or p_C and we now consider these three regions separately. It is convenient to perform the intermediate steps of the calculation in the rest frame of the missing mass system, i.e., in the rest frame of $p_A + p_B - p_C$. As an example, consider the region where k is parallel to p_C in this frame, so that we can write

$$k = \beta p_C + \hat{k} \quad (4)$$

where $\hat{k}$ does not contain any components as large as $O(\sqrt{s})$. β is straightforward to evaluate and is $(1-x)/x$. We denote by $\sum_{ij}$ the contribution to $E_C(d\sigma/d^4p_C)$ from the convolution of the amplitude corresponding to Fig. 1(i) with that of Fig. 1(j) in the Feynman gauge. Then by setting $k = \beta p_C$ in all dot-products which give large invariants, one readily finds

$$\sum_{ad} + \sum_{da} = \frac{2}{\beta} \left(-\frac{2a^2 C_F}{n^2} \right) \cdot I \cdot X \quad (5a)$$

$$\sum_{bd} + \sum_{db} = -\frac{2}{\beta} \left(\frac{a C_F^2}{n} - \frac{a^2 C_F}{n^2} \right) \cdot I \cdot X \quad (5b)$$

$$\sum_{cd} + \sum_{dc} = -\frac{2}{1+\beta} \left(-a \frac{C_F C_A}{2n} \right) \cdot I \cdot X \quad (5c)$$

$$\sum_{ed} + \sum_{de} = -\frac{2}{\beta(1+\beta)} \left(-\frac{a^2 C_F}{n^2} \right) \cdot I \cdot X \quad (5d)$$

$$\sum_{dd} = -\frac{\beta}{(1+\beta)^2} \frac{a C_F^2}{n} \cdot I \cdot X \quad (5e)$$

where

$$I \equiv \int d^4k \frac{\delta^{(4)}(k^2) \delta^{(4)}((p_A + p_B - p_C - k)^2)}{(p_C + k)^2} \approx \frac{\pi}{2} \frac{1}{xs} \log \frac{Q^2}{p_C^2} \quad (6)$$

and

$$X = -\frac{1}{2s} \frac{1}{(2\pi)^3} \frac{1}{(2p_A \cdot p_C)^2} \frac{\text{Tr}[(\not{p}_A - \not{p}_B - \frac{\not{k}}{x}) \gamma^\mu \not{p}_B \gamma^\nu]}{\text{Tr}[\not{p}_C \gamma^\mu \not{p}_A \gamma^\nu]}$$

If T_i are the generators of the gauge group [assumed to be $SU(n)$] then a is

defined by $\text{Tr}[T_i T_j] = a \delta_{ij}$, C_F is the eigenvalue of the quadratic Casimir operator for the fundamental representation [$C_F = a(n^2-1/n)$ for $SU(n)$] and C_A is that for the adjoint representation [$C_A = 2an$ for $SU(n)$]. Adding up the right-hand sides of Equation (5) one finds that the total has a very simple group theory factor $\frac{1}{n} a C_F^2$, which is precisely C_F times the group theory factor of the Born term in elastic qq scattering. Thus we find that the contribution to the inclusive cross-section $E_C (d\sigma/d^3p_C)$ from the region where k is parallel to p_C can be written in the form

$$\int \frac{dx_c}{x_c^2} \left\{ \frac{2g^2}{(4\pi)^2} C_F \left(1 - x_c + \frac{2x_c}{(1-x_c)} \right) \log \frac{Q^2}{p_c^2} \right\} \left\{ \frac{s'}{\pi} \delta(s' + t' + u') \frac{d\sigma^{(0)}}{dt'} \right\} \quad (7)$$

where $s' = s$, $t' = \frac{t}{x_c}$

and

$$u' = \frac{u}{x_c}$$

$d\sigma^{(0)}/dt'$ denotes the Born term contribution to the elastic cross-section for qq scattering where the initial quarks have momenta p_A and p_B , and the final quarks have momenta p_C/x_c and $p_A + p_B - p_C/x_c$.

$$\frac{2g^2}{(4\pi)^2} C_F \left(1 - x_c + \frac{2x_c}{1-x_c} \right) \log \frac{Q^2}{p_c^2}$$

is exactly the lowest order contribution to the fragmentation function $G_{C/c}(x_c, Q^2)$ for $x_c < 1$, which is calculable in $e^+e^- \rightarrow qX$. It is related to the deep inelastic distribution functions by the Gribov-Lipatov reciprocity relations⁹⁾. The contributions from the regions where k is parallel to p_A and p_B can be calculated similarly, both of these contributions can be written in a form similar to (7), with $1/x_c^2$ times the fragmentation function replaced by the appropriate distribution functions.

At $x = 1$ in addition to the contributions from the diagrams of Fig. 1, one must also include the diagrams which contribute to the elastic qq scattering amplitude. It is well known that $\log p^2$ terms can now arise from either of two sets

of integration regions: (i) where one of the gluons is soft (e.g., in the centre-of-mass frame) and (ii) where one of the gluons is parallel to one of the external particles whose momentum is measured. Singularities of type (i) are cancelled by the Bloch-Nordsieck mechanism^{10),11),12)}, the cancellation occurring in the usual way between diagrams with soft real gluons and those with soft virtual ones. The singularities of type (ii) can be treated in an analogous manner to those for $x < 1$. The total result (for all values of x) for the term containing $\log p_C^2$ is $\frac{1}{n} a C_F^2$ times the integral over the region where k is parallel to p_C in the QED diagrams shown in Fig. 2. This of course is precisely the $\log p_C^2$ term in the fragmentation function $\tilde{G}_{C/c}(x_c, Q^2)$. In a similar fashion it is found that the $\log p_A^2$ and $\log p_B^2$ terms are exactly those in the quark distribution functions.

In addition to the $\log(Q^2/p^2)$ terms studied above, the diagrams which contribute to the elastic qq scattering amplitude, after renormalization will include terms which have $g^6(\mu^2) \log(Q^2/\mu^2)$ as a factor, where μ is the renormalization point. These terms are exactly those which, when combined with the $g^4(\mu^2)$ terms from the Born term, give the first two terms in the expansion of $g^4(Q^2)$, the running coupling constant.

We conclude that the cross-section for $qq \rightarrow qX$ to order g^6 in the cross-section and in the leading logarithm approximation can be written in the form

$$E_c \frac{d\sigma}{d^3p_c} (A+B \rightarrow CX) = \int_0^1 dx_a dx_b \frac{dx_c}{x_c} G_{a/A}(x_a, Q^2) G_{b/B}(x_b, Q^2) \tilde{G}_{C/c}(x_c, Q^2) S(s'+t'+u') \frac{s'}{\pi} \frac{d\sigma^{(B)}}{dt'} (a+b \rightarrow cq) \Big|_{s'=x_a x_b s, t'=\frac{x_a}{x_c} t, u'=\frac{x_b}{x_c} u} \quad (8)$$

where $d\sigma^{(B)}/dt'$ is the Born term contribution to the elastic cross-section for qq scattering, calculated using the running coupling constant.

We have studied here only one subprocess, and only the lowest non-trivial order of perturbation theory (a more comprehensive study is now being carried out), nevertheless we feel that the result is non-trivial and suggestive of an ansatz for calculating cross-sections for inclusive production processes at large transverse

momenta. The ansatz is to take formula (3) and replace the scaling distribution and fragmentation functions by the appropriate non-scaling ones, and to keep only the Born term for the hard subprocesses, using the running coupling constant. There will of course be corrections, which are smaller than the leading behaviour by powers of logarithms and which are not included in the ansatz. Such an ansatz has already been used by several authors^{13),14)15)}. This ansatz is very reminiscent of that for lepton pair production [formula (2)] except that now we have a phenomenological difficulty in that whereas the non-scaling quark and antiquark distributions are measured in deep inelastic lepton-hadron scattering, there is no such measurement of the gluon distributions which are needed for these large transverse momentum processes.

The above results also suggest that the cross-sections for "jet" production at large transverse momenta can be reliably calculated within the present framework; (up to order g^6 in qq scattering this is a simple consequence of the calculation presented above). All the $\log p_A^2$ and $\log p_B^2$ terms can be absorbed into the initial distribution functions as above, leaving us with a series in $\bar{g}(Q^2)$ whose coefficients are calculable and depend on the kinematic parameters of the jet.

In this letter we have presented an example of a hard scattering process in a theory of strong interactions, where the radiative corrections have a particularly simple form. If this simplicity is a general feature of the theory, and if our assumptions about the soft nature of the hadronic wave functions are correct, then we have a reliable prescription for the calculation of the asymptotic behaviour of hard scattering cross-sections.

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FIGURE CAPTIONS

- Figure 1 Lowest order diagrams contributing to the inelastic cross-section in the reaction $qq \rightarrow qX$.
- Figure 2 Auxiliary QED diagrams used in the calculation of the inclusive cross-section for the reaction $qq \rightarrow qX$.

FIG.1

FIG. 2

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