Ref.TH.2457-CERN

NARROW STATES IN $\bar{p}p$: DUALITY EXPECTS THEM,
EXPERIMENT PRODUCES THEM, BUT ARE THEY THE SAME?

M. R. Pennington

CERN -- Geneva

ABSTRACT

The existence of four quark states coupling strongly to $\bar{B}B$ channels have been expected from duality for a long time. Recently narrow states have been produced in an Omega experiment on backward π^-p scattering, which are seen in their decay to $\bar{p}p$. We ask: are these narrow states those expected by duality? If they are, duality makes a definite statement about the cross-section for their production. This is shown to be 50 times larger than observed. If these narrow resonances are to be the $qq\bar{q}\bar{q}$ mesons of Rosner that do indeed generate $q\bar{q}$ exchange, then these can only be one of two types of such states: the others, wide and decaying to multimeson channels must be more copiously produced. These are not baryonium states, but boojums. This information can be incorporated in the colour coupling schemes of Chan and Høgaasen, and of Jaffe, but not quite naturally. Two types of four quark mesons necessarily exist in the dynamical model of Rossi and Veneziano. However, in this scheme these two types have distinctly different duality properties. These are shown to be inconsistent with phenomenology unless the ω and $\tilde{\omega}$ mix in a very special way. We find $q\bar{q}$ exchanges in virtual $\bar{B}B$ processes are built predominantly by multimeson states and only weakly, if at all, by those called baryonium. We conclude that the observed narrow states are not yet proven to be $qq\bar{q}\bar{q}$ mesons rather than, for example, $\bar{B}B$ bound states.

Ref.TH.2457-CERN
8 February 1978

1. INTRODUCTION

1.1 Preface

Ever since Harari and Rosner¹⁾ first drew diagrams depicting the flow of quark lines in order to summarize the duality properties of two-two processes, it has been recognized that the standard duality of resonance and Regge exchanges could not be realized for baryon-antibaryon ($B\bar{B}$) scattering if the only intermediate states were mesons made of just two quarks ($q\bar{q}$). It was Rosner²⁾ who emphasized that $qq\bar{q}\bar{q}$ resonance and Regge exchanges (M_4) must also exist in each $B\bar{B}$ channel, dual in turn to normal two quark Regge and resonance states, M_2 . The blatant theoretical need for such M_4 mesons was highlighted by an illustration of Lipkin's³⁾ and the experimental search began with the benefit of the theoretical advice of Refs. 3), 4) and 5). Such advice was based particularly on the imposition of the Freund-Rosner-Waltz selection rule⁴⁾. This has the effect of leaving the duality solution of Mandelstam⁶⁾ for meson-baryon scattering intact by requiring that M_4 states should, at the planar level, only couple to $B\bar{B}$ final states and not to purely mesonic decay modes.

This special relationship of M_4 states to $B\bar{B}$ channels, which has been epitomized in their recent labelling as "baryonium"⁷⁾ led to the expectation that, if they should be formed just above $\bar{p}p$ threshold and so have few available decay modes, they may be rather narrow and hence easily detected. Experiment has indeed provided us with narrow peaks formed in $\bar{p}p$ scattering close to threshold. Statistically by far the most significant is the peak at 1.936 GeV of width less than 10 MeV seen in several experiments⁸⁾. However, with just one such state definitely confirmed, it is difficult to be sure that an M_4 resonance has been observed, the T,U bumps, not being particularly narrow, may be normal $q\bar{q}$ mesons (see later).

The certain way to check that such states really belong to a family of mesons composed of four quarks is to observe them in a configuration unattainable by a pair of quarks ($q\bar{q}$), for example in a doubly charged mode^{*)}. This requires us to detect such states in a production experiment rather than by formation -- in the absence of direct $\Delta^{++}n$ scattering, for example. Since M_4 states are believed to couple strongly to $B\bar{B}$ channels, we should be able to produce them by baryon exchange in backward scattering^{4), 5)}. By using a deuteron target, experiments on $\pi^- n \rightarrow pX^-$, $\pi^- p \rightarrow pX^-$ can be performed simultaneously and hopefully M_4

*) Advice on tracking down unusual beasts is given in Ref. 9).

states in both $(\bar{p}n\bar{p}n)$ and $(\bar{p}p\bar{p}n)$ quark configurations produced. Unfortunately, two recent experiments on these channels have led to completely conflicting results¹⁰⁾: the peaks in one vanishing away^{*)}. More refined higher statistics experiments proposed for the future must be awaited¹¹⁾ before we can believe that charge two M_4 mesons have been seen.

While no boojum^{**) in a highly charged state has been indisputably observed, additional neutral candidates have been ensnared in the Omega spectrometer¹²⁾. In backward π^-p scattering at 9, 12 GeV/c two narrow states have been seen in the $\bar{p}p$ mass spectrum at 2.020 and 2.204 GeV as well as that at 1.936 GeV^{12), 8)} when a final state $p_f \pi^- \bar{p}p$ is detected (see Fig. 1). It is tempting to regard such peaks as the realization of baryonium: the successful result of an experimental search. However, such states have more than a name, a coupling to $\bar{B}B$ channels and a place in the Particle Data tables. If experiment has indeed captured the expected M_4 resonances we must be sure that they really are not only composed of four quarks, but also possess the correct duality properties, before we can call them "baryonium".}

Since their four quark nature is presently impossible to test, it is to their duality properties that this paper is addressed. We shall assume that these backward produced states^{***)} are indeed those required by duality, as the simplest available working hypothesis. Our aim is then to check whether the data are really consistent with this. Three properties only are known about these neutral states, their mass, their width and their production cross-section. We shall concern ourselves in turn with each of these within the framework of duality.

*) Perhaps not unexpectedly "if your Snark be a Boojum!" "you see". [Fit the Third and the Eighth of Ref. 9).]

**) Freund [quoted by Rosner⁵⁾] has suggested the name "gallon" for a $qq\bar{q}\bar{q}$ meson -- invoking perhaps the idea of a state of four quarts. We prefer to be more quixotic. Prompted by Ref. 9) we name an M_4 state a "boojum": defined in Ref. 13). Baryonium is a rather special breed of boojum, as will become apparent.

***) The forward produced state at 2.95 GeV¹⁴⁾ we will only be able to comment on briefly -- see Section 6.

1.2 What duality simply says

In the spirit of Harari-Rosner diagrams, M_4 states are dual to M_2 exchanges in the crossed channel for $\bar{B}B$ processes. Since we have ample evidence for $f-\omega-A_2-\rho$ Regge exchanges in $\bar{p}p$ scattering, as well as for the production of states on their grand-daughter trajectories (if not on the parent) in the direct channel^{15)†)}, we are compelled to believe in M_4 mesons being possible in all $\bar{B}B$ channels. What can we say about these beyond the Freund-Rosner-Waltz selection rule?

What about their masses for example -- on what Regge trajectory do they lie? A simple calculation allows us to estimate this. Consider the formation of a normal $q\bar{q}$ resonance, M_2 , in the s channel of each of the reactions $\bar{p}p \rightarrow M_2 \rightarrow \bar{p}p$, $\bar{p}p \rightarrow M_2 \rightarrow \pi A$, $\pi A \rightarrow M_2 \rightarrow \pi A$, where $A = \pi$ or ρ depending on the G parity of the M_2 resonance. Then let us assume that the contribution of this resonance to each of these channels saturates the appropriate finite energy sum rule^{*)} within a mass range of 1 GeV^2 around $s = M_{\text{res}}^2$, then:

i) $\bar{p}p \rightarrow M_2 \rightarrow \bar{p}p$ being dual to M_4 exchange gives at $t = 0$, for example,

$$g_{\bar{p}p M_2}^2 = \beta_{\bar{p}p M_4}^2 \left(\frac{s}{s_0} \right)^{\alpha_{M_4}(0)} \quad (1)$$

where all the angular momentum factors, isospin Clebsches etc., have been absorbed in the resonance and Regge couplings -- all these elements being factorizable.

ii) $\bar{p}p \rightarrow M_2 \rightarrow \pi A$ being dual to baryon exchange gives

$$g_{\bar{p}p M_2} g_{\pi A M_2} = \beta_{\bar{p} \pi B} \beta_{\rho A B} \left(\frac{s}{s_0} \right)^{\alpha_B(0)} \quad (2)$$

†) These states found in $\bar{p}p \rightarrow \pi^+ \pi^-$ may well be responsible for the T,U peaks in $\bar{p}p$ elastic scattering.

*) Whether f esr duality means the leading t channel trajectory is generated by resonances on the leading s channel (linear) trajectory or by resonances on a low-lying effective (square root) trajectory is irrelevant. We use here the word "dominant" s channel resonance to mean whichever is appropriate.

iii) $\pi A \rightarrow M_2 \rightarrow \pi A$ being dual to $f-\omega-A_2-\rho$ exchange gives

$$g_{\pi A M_2}^2 = \beta_{\pi A M_2}^2 \left(\frac{s}{s_0} \right)^{\alpha_{M_2}(0)} \quad (3)$$

From the factorization of resonance couplings we have on combining Eqs. (1)-(3)

$$\beta_{\bar{p} \pi B}^2 \beta_{\pi A B}^2 \left(\frac{s}{s_0} \right)^{2\alpha_B(0)} = \beta_{\bar{p} p M_4}^2 \beta_{\pi A M_2}^2 \left(\frac{s}{s_0} \right)^{\alpha_{M_2}(0) + \alpha_{M_4}(0)} \quad (4)$$

Since the residues are independent of the mass of the direct channel resonance, we see that if this relation is to hold as we pass along the dominant resonances, we must have

$$\alpha_{M_2}(0) + \alpha_{M_4}(0) = 2\alpha_B(0) \quad (5)$$

A similar relation can also be deduced at t non-zero, which implies that if $\alpha'_B = \alpha'_{M_2}$ then they also equal α'_{M_4} .

This simple argument, in principle, fixes the intercept of the M_4 trajectory. Clearly $\alpha_{M_2} = \frac{1}{2}$. However, the baryon trajectory can be either Δ or N or, as popularly taken, could be some "mean" baryon trajectory between these. As $\alpha_\Delta(0) - \alpha_N(0) = \frac{1}{2}$, the choice of α_B makes a big difference in what we expect $\alpha_{M_4}(0)$ to be and hence in the expected masses of the M_4 states.

If $\alpha_B = \alpha_\Delta = 0.15$, then $\alpha_{M_4} = -0.2$, while if $\alpha_B = \alpha_N$ then $\alpha_{M_4} = -1.2$ and the "mean" $\alpha_B = 0$ gives $\alpha_{M_4} = -0.5$. Certainly $\alpha_{M_4}(0)$ is negative but how negative is theoretically unsure.

The relation, Eq. (5), is just that obtained by a dual unitarization, multiperipheral calculation in the spirit of Huan Lee and Veneziano¹⁶⁾, in which $t_{\min}$ effects are neglected; the earliest we know of being in Ref. 17a), where baryon loops give a Pomeron of intercept $1 + \alpha_{M_2} + \alpha_{M_4} - 2\alpha_B$. Since in all dual unitarization schemes, as in multiperipheral calculations, the subenergies between clusters are not especially large, it may be argued that the nucleon is the more important baryonic exchange (see Section 2.2), in which case we have the result of Ref. 17) that

$$\alpha_{M_4}(0) = 2\alpha_N(0) - \alpha_{M_2}(0) = -1.2 \quad (6)$$

rather than up to a unit higher. With such a prejudice about the intercept and a slope of $0.85-0.9 \text{ GeV}^{-2}$ (as for the $M_{2,B}$ trajectories) the state at 1.936 GeV would sit very nicely on a linear trajectory with spin 2 and the 2.2 GeV state with spin 3 ^{*}). However, the 2.02 GeV state must lie on a different trajectory, which if it has a canonical slope would give this resonance a spin of three if $\alpha_{M_4}(0) = -0.5$: so perhaps two trajectories are explored by the Omega experiment [†]).

We shall comment later on more recent, more sophisticated calculations of "baryonium" trajectories. However, for the present the simple discussion given here will suffice, since only a future spin analysis of the final states can confirm these, or any other, trajectory assignments.

Not only does this simple argument in principle determine the M_4 trajectory composed of $q = p, n$ quarks, dual to the dominant $q\bar{q}$ vector and tensor meson trajectories, but it allows us to estimate the Regge couplings of these trajectories too. If s_0 is the same energy scale for each process ($s_0 = 1/\alpha'$ for instance) then with $A = \pi$, for example, we have from Eq. (4)

$$\beta_{N\bar{N}M_4} = \beta_{N\pi N}^2 / \beta_{\pi\pi M_2} \quad (7)$$

if the nucleon is the dominant baryon -- at $t = 0$ at least. This gives on evaluating the right-hand side of this Eq. (7) $\beta_{N\bar{N}M_4} \sim \beta_{N\bar{N}M_2}$ (see later for actual numerics), just as the simple discussion of Ref. 17) would also suggest. We will comment on the coupling of M_4 exchange to $N\bar{N}$ later, but at present say just that the simple f esr considerations given here hint at the coupling of M_4 exchange to $N\bar{N}$ being of the same order as that for M_2 exchange.

*) A unit higher intercept would, of course, fit equally well, raising these spins by one.

†) The 2.95 GeV state ¹⁴⁾ could lie on either of these trajectories with spin 6 or 7, respectively.

Very little can, in general, be said theoretically with much conviction about the width of M_4 states other than, as already put forward, we might expect that if such resonances obey the Freund-Rosner-Waltz selection rule, they should be narrow provided they have high spin and are not far above the lowest accessible $\bar{B}B$ channel. Indeed experiment confirms this intuition: the state at 1.936 GeV has a width of 10 MeV or less⁸⁾, the 2.020 GeV a width of 24 ± 12 MeV and the 2.204 GeV a width of 16^{+20}_{-16} MeV¹²⁾. No calculations yield these numbers. However, more specific schemes can, of course, be more definite: for example, $\bar{p}p$ or $(\bar{q}q)(qq)$ potential models¹⁸⁾ or different colour coupling arrangements¹⁹⁾⁻²²⁾. These we come to later. Duality alone can say very little.

While we have so far discussed only M_4 states obeying the Freund-Rosner-Waltz selection rule, it should be noted however that other $qq\bar{q}\bar{q}$ states may very well exist. Calculations in the MIT bag model¹⁹⁾ show resonant four quark configurations exist, many of which have little to do with baryonium with no particularly special relationship to $\bar{B}B$. At present, we make no distinction between all these four quark states; hence our abandonment of the evocative name baryonium in favour of boojum, suggesting an unusual and exotic creature of the snark family¹³⁾.

Since no very specific statements can be made about either the mass or width of the resonances produced in the backward $\pi^- p$ Omega experiment¹²⁾ in the absence of spin and isospin analyses, we turn to the third quantity we know about them, namely their production cross-section. It is here that the duality properties of M_4 states come into their own and will allow us to make more definite statements. It is the discussion of the magnitude of their production cross-sections that constitutes the body of this paper (Sections 2 and 3). This will lead us to the conclusion that more than one type of M_4 state exists. In a short Section 4 we consider more specific schemes, alluded to above, which have been developed during the course of this present work (or just before) in which different types of M_4 states have been proposed¹⁹⁾⁻²²⁾. In Section 5 we subject the most definite of these, namely that of Rossi and Veneziano²¹⁾ to phenomenological scrutiny. In Section 6 we discuss the results and give our conclusions.

2. HOW TO CALCULATE A PRODUCTION CROSS-SECTION

2.1 Preliminary remarks on backward processes

Two narrow states at 2.02 and 2.20 GeV have been produced in the backward direction in $\pi^- p$ scattering at 9,12 GeV/c¹²⁾. These states are seen in the $\bar{p}p$ mass spectrum provided the fast forward proton trigger has a fast π^- with it, forming a $\Delta(1232)$ or $N^*(1520)$ state. For $p\pi^-$ masses outside these resonance regions, no narrow states are seen in the $\bar{p}p$ spectrum¹²⁾. We shall study these double resonance production processes

where always, without saying, the $\Delta, N^*(1520)$ are detected by their $p\pi^-$ decay mode.

The first remark to make about such backward production processes is that the Regge exchanges in the u channel can have both isospin $\frac{1}{2}$ and $3/2$, so that both nucleon, N , and Δ exchanges are possible. It is relevant then to note that if there is no fast π^- along with the triggering proton, no peaks are seen in the $\bar{p}p\pi$, $\bar{p}p\pi\pi$ or 4π mass spectra of Boucrot et al.¹⁰⁾. Thus with some upper limits, the processes

do not produce such resonance states. Since each of these proceeds by just Δ exchange, we infer rather tentatively at this stage, that this is not the dominant mechanism for the production of the Omega states.

2.2 N or Δ exchange: which dominates?

As already indicated, two important non-strange baryon trajectories will, in general, contribute to a backward production process. These are the N_α and Δ , where the latter has the higher intercept: higher by half a unit, just like the difference between ρ and π trajectories. From this separation we expect that at high energies, the Δ contribution will duly dominate. However, by comparing the well measured Δ exchange process $\pi^- p \rightarrow p\pi^-$ with $\pi^+ p \rightarrow p\pi^+$, in which both N, Δ can be exchanged, we can conclude that at present energies with p_{lab} from 5 to 14 GeV/c, it is the nucleon exchange that is most important in describing the process $\pi^+ p \rightarrow p\pi^+$ ^{23),24)}. Moreover, the $\pi^+ p$ differential cross-section shows a dip at $u = -0.15 \text{ GeV}^2$, which has come to be regarded as a characteristic

signal for N_α exchange, as this corresponds to the position of its nonsense wrong signature zero. It is, in fact, only in this dip region, where N_α exchange vanishes, that the Δ contribution is appreciably felt. We can therefore infer that, at least at $u = 0$, the Regge couplings $\beta_{abR}(u)$ satisfy the inequality $\beta_{\pi NN}^2(0) \gg \beta_{\pi N\Delta}^2(0)$.

It appears that this type of relation, nucleon having bigger couplings than Δ , holds even when we have other than their couplings to πN . Let us consider, for example, the backward production of both $N^*(1520)$ and $N^*(1688)$ and compare for each of them $\pi^- p \rightarrow N^{*+} \pi^-$ with $\pi^- p \rightarrow N^{*0} \pi^0$, where the former process involves just Δ exchange and the latter nucleon as well. Once again, the $I_u = \frac{1}{2}$ exchange dominates²⁵⁾. Indeed, the $\pi^- p \rightarrow N^{*0} \pi^0$ differential cross-section shows for the $N^*(1688)$, at both 9 and 12 GeV/c, a sharp break in slope if not a dip, at $u = -0.15 \text{ GeV}^2$ ^{25)*)} -- indicative once again of nucleon exchange being predominant.

From this discussion, we know that not only that

$$\beta_{\pi NN}^2 \gg \beta_{\pi N\Delta}^2, \quad \text{but also} \quad \beta_{\pi N^*N}^2 \gg \beta_{\pi N^*\Delta}^2$$

As a result, we shall now be sufficiently emboldened to assume for the sake of simplicity that such inequalities continue to hold true even as the meson mass increases from that of the pion. Thus we shall simply assume at $u = 0$

$$\beta_{MNN}^2 \gg \beta_{MN\Delta}^2, \quad \beta_{MN^*N}^2 \gg \beta_{MN^*\Delta}^2 \quad (8)$$

for all mesons M on average, but not necessarily for each individual one. As we shall intimate in Section 3 when we discuss these concepts in more detail, this is in keeping with Mueller-Regge expectations.

Because of this assumption, we shall, from here on, regard all the baryon exchange processes we consider, which can have both N and Δ exchange, as being so dominated by N exchange that we can neglect the Δ contribution. This assumption, though plausible, is clearly not necessarily very exact and so we stress from the outset that we treat baryon exchange as synonymous with nucleon exchange

) It is only for the $N^(1688)$ that the statistics of the experiment of Ref. 25) are sufficient to allow this to be seen; it cannot yet be checked for the $N^*(1520)$.

(when quantum numbers allow $I_u = \frac{1}{2}$) only for simplicity and that the inclusion of Δ exchange will in almost all cases only serve to strengthen our conclusions. Its neglect is therefore of no great importance and we shall comment at suitable points on how big an effect it would in reality produce. However, its neglect makes our discussion so much simpler that its exclusion is worth while.

2.3 Duality diagrammatically

Armed with this suggestion that the dominant part of the cross-section we consider is controlled by N_α and not Δ_δ exchange, we can proceed to calculate how big we expect this dominant piece to be. Here we elucidate the notions based on duality which we will use to do this. In this we will be guided by diagrams, but with which, at present, we do not associate any specific formulae, but rather use them as a way of classifying the contributions to a given process.

The graphs we consider for the moment are of the Harari-Rosner type¹⁾ which depict the flow of quark flavours and not of colour. For mesons, these are of course the same, but for baryons care should be taken of the direction of colour. For the present, we ignore this distinction and proceed in the spirit of Harari and Rosner²⁾ in the days before it was recognized that "QCD is the underlying theory of strong interactions".

The backward production process $MN \rightarrow BX$, being a baryon exchange reaction, can be regarded basically as $\bar{B}B$ annihilation producing a state X , where the antibaryon is Reggeized. Then duality diagrams lead us to expect the production of resonance states X made both of $q\bar{q}$ and of $qq\bar{q}\bar{q}$, as well as a non-resonant background. Forming the modulus squared and hence the differential cross-section for the production of each of the two types of meson states (M_2, M_4), we have the diagrams of Fig. 2. (The square of non-resonant production is discussed separately). On summing over all missing mass states at large M^2 , Regge exchanges are generated in the crossed channel -- here called the t' channel (for which $t' = 0$). When M_2 states are produced in the missing mass channel, they generate an M_4 exchange in the crossed-channel, while M_4 production is dual to normal meson M_2 exchange (f, ω, A_2, ρ) as illustrated in the diagrams of Fig. 2. In turn, non-resonant production is assumed dual to Pomeron exchange following the hypothesis proposed by Freund and Harari²⁶⁾.

This duality implies that the ratio of M_2 to M_4 to non-resonant background production should for large M^2 behave like

$$(M^2)^{\alpha_{M_4}(0)} : (M^2)^{\alpha_{M_2}(0)} : (M^2)^{\alpha_P(0)}$$

Since we expect $\alpha_{M_4}(0) < 0$, perhaps in the range $-1.2 < \alpha_{M_4}(0) < -0.5$, as discussed in the Introduction, and $\alpha_{M_2}(0) = \frac{1}{2}$ with $\alpha_P(0) = 1$, these relative proportions are in the ratio

$$\frac{1}{M^2} \text{ or } \frac{1}{M} : M : M^2$$

Because of the semi-local nature of duality, this behaviour is not only expected to hold for asymptotic M^2 , but is realized on the average in each mass interval of typical width $\Delta M^2 \approx 1 \text{ GeV}^2$ even at much lower masses. This semi-locality has been tested in many processes, see, for example, Ref. 27), so we are naturally led to believe it applies here, with the consequence that as the missing mass increases we expect the production of M_2 states already beginning with the f , A_2 , K^{**} , g , A_3 , ω_3 , K^{***} , h , etc., to decline relative to M_4 production. Thus at large missing masses we expect M_4 production should dominate over normal meson production. How soon this actually happens depends on the relative couplings, to which we now turn briefly.

In Ref. 17) we voiced the quasi-theoretical prejudice that the couplings of M_2 and M_4 exchanges should be comparable. This was shown to be consistent with the data. Now this conclusion is reinforced by the *fesr* arguments of Section 1.2. If this is indeed roughly true then the ratio of M_4 exchange to that of M_2 exchange will already be less than 1:4 for M^2 as small as 4-5 GeV^2 . Then at such masses the bulk of the cross-section will be given by Pomeron and normal meson exchanges in the t' channel, that is by M_4 resonance and by non-resonant background production in the missing mass channel, while normal meson production is strongly suppressed. That this holds already at $M^2 \sim 4-5 \text{ GeV}^2$ is based at present solely on the prejudice of roughly comparable M_2 and M_4 couplings; however, we shall see that this has consequences borne out by the data -- so it serves as a useful guide.

2.4 Estimating a cross-section: purpose

With this in mind, our aim is to estimate the amount of M_4 meson production at 9,12 GeV/c in the processes $\pi^- p \rightarrow \Delta M_4$ and $\pi^- p \rightarrow N^*(1520) M_4$ and to compare this with the actually measured production cross-sections in which particular M_4 states decay to $\bar{p}p$. If these states are $qq\bar{q}\bar{q}$ mesons in the sense of Rosner²⁾ and obey the Freund-Rosner-Waltz rule⁴⁾ by decaying predominantly into $\bar{B}B$ channels, we may expect thereby to estimate the bulk of the measured cross-section. As just indicated in the last subsection, we believe M_4 production together with the non-resonant background to constitute most of the inclusive cross-section for

masses above 2 GeV. Sadly no data presently exist on inclusive $\pi^- p \rightarrow \Delta X$ and $\rightarrow N^* X$. If it did we could separate these into their M_2, M_4 and non-resonant production components by making use of the Mueller-Regge ideas depicted in Fig. 2. Then having determined the amount of M_4 production in some mass interval of width $\Delta M^2 \approx 1 \text{ GeV}^2$ centred on 2.02 or 2.2 GeV, we can ask how much of this production is seen in the narrow peaks decaying to $\bar{p} p$ -- we would expect most if M_2 exchange is to be built by M_4 states.

Since no inclusive data exist on these particular processes we shall use data on a reaction with very similar kinematics for which sufficient statistics exist to allow an analysis as both a function of u and M^2 , that is $K^- p \rightarrow \Lambda X$ at 14.3 GeV/c²⁸⁾). This reaction proceeds solely by nucleon exchange, because of isospin, and so will allow us to estimate the nucleon exchange contribution to $\pi N \rightarrow BX$, which we have already argued is also the dominant part.

2.5 Estimating the cross-section for M_4 production

For eventual comparison with the data on $\pi^- p \rightarrow (p\pi^-)(\bar{p}p)$ ¹²⁾ we define the cross-section integrated over $|u'| < 1 \text{ GeV}^2$ of the inclusive processes we consider by

$$\frac{d\sigma}{dM^2} = \int_{-1\text{GeV}^2}^0 du' \frac{d^2\sigma}{du dM^2} \quad (9)$$

Then from the semi-local nature of duality we expect that $d\sigma/dM^2$ averaged over some mass² interval ΔM^2 , typically 1 GeV^2 , calculated from its Regge contributions should reproduce the physical cross-section on average. Thus we define the cross-section within a bin of width ΔM^2 centred on the mass M_0 to be

$$\sigma = \int_{M_0^2 - \frac{1}{2}\Delta M^2}^{M_0^2 + \frac{1}{2}\Delta M^2} dM^2 \frac{d\sigma}{dM^2} \quad (10)$$

*) Even though, we shall, in this section, integrate over these variables, the availability of double differential cross-sections will be essential for our later discussions.

With this definition, the cross-section for $K^-p \rightarrow \Lambda X$ within a bin of width $\Delta M^2 = 1.4 \text{ GeV}^2$ and with $M_0 = 2.2 \text{ GeV}$ is from the data^{28),29)*}:

$$\sigma(K^-p \rightarrow \Lambda X) = (15.1 \pm 1.8) \mu\text{b} \quad (11)$$

For this experimental quantity we want to fix the proportion of M_4 production and so here we interject the phenomenology introduced in the earlier sections. The inclusive cross-section has contributions in the t' channel from the Pomeron, from even and odd signature $q\bar{q}$ meson exchanges -- M_2^+, M_2^- -- and from even and odd signature $qqq\bar{q}$ exchanges -- M_4^+, M_4^- . By exchange degeneracy, the two signaturred exchanges have not only degenerate trajectories, but also equal couplings (which is true within a factor of two). We can then treat these t' channel contributions as $P + 2M_2^+ + 2M_4^+$. The term M_2^+ is a sum of both $I = 0, 1$ exchanges, the f and A_2 , which are equal in turn to the ω and ρ contributions. However, since the A_2, ρ have relatively small couplings to $N\bar{N}$ (see Section 5.4) - the lower vertices in Fig. 2 - we can essentially regard M_2^+ exchange as f exchange, and this is how we shall refer to it^{**}).

We therefore have three contributions to Eq. (11), viz. $P + 2f + 2M_4$ (where we drop the signature label). Their relative sizes, at a given energy, depend on the magnitudes of their couplings and on their behaviour as a function of missing mass, which we have already discussed in Section 2.3. On quite general theoretical grounds, the couplings of the Pomeron and f are simply related³⁰⁾; such relationships arise in any scheme embodying duality. Phenomenology indicates that if the mass and energy scale is fixed by the slope α' of Regge trajectories, the f and P couplings are roughly equal. This has been confirmed in both exclusive and inclusive processes^{30),31)}. If further we assume the M_4 couplings are, at least at $u = 0$, also roughly equal to those of the f ^{***}), the relative proportions of $P : M_2 : M_4$ exchanges to the cross-section of Eq. (11) are $(38 \pm 3)\%$,

*) We take this mass interval as the data is binned in ranges of M^2/s of width 0.05. This range corresponds to $0.15 \leq M^2/s \leq 0.2$.

**) Since phenomenologically it is far easier to separate the $I = 0$ odd signature Regge exchange, viz the ω , than the non-Pomeron part of the $I = 0$ even signature exchange, the f , what we call the f in this paper is just defined by approximate exchange degeneracy from the better defined ω contribution. Some might therefore prefer to call what we refer to as f exchange ω exchange.

***) This assumption, as already indicated, is much more poorly tested than "f-dominance of the Pomeron", so we shall relax it shortly.

$(50 \pm 3)\%$, $(12 \pm 1)\%$ ^{*}), where we have taken the upper limit for $\alpha_{M_4}(0)$ of $-\frac{1}{2}$; the value of -1.2 we favour would reduce further the fraction of M_4 exchange.

With these values we see that in this mass bin the cross-section for M_4 resonance production dual to f - ω exchange is half the total of Eq. (11)

$$\sigma(K^-p \rightarrow \Lambda M_4) = 7.5 \mu b \quad (12)$$

with an uncertainty we fix below.

If the well-tested universality of P - f couplings is to remain true, it is clear that this cross-section for M_4 production cannot be larger than $9.5 \mu b$, when M_2 production would be vanishingly small. The lower limit is somewhat more problematical and clearly depends on the little known size of the M_4 exchange. However, it is seen that even if this were quadrupled so that the proportion of M_2 production was not 12% but 50% of the total, the M_4 production cross-section of Eq. (12) would be reduced by less than a factor of two. A similar reduction in the Pomeron contribution would of course also occur, because of P - f proportionality. However, such a reduction in cross-section appears to be unacceptably large. For we can check the size of the Pomeron contribution by measuring its coupling in the crossed reaction $K^+p \rightarrow \bar{\Lambda}X$ (and also in the related $pp \rightarrow \pi^+X$). This we discuss in Section 3.4 when we come to more detailed phenomenology; at present, all that need be said is that these considerations indicate that Eq. (12) is unlikely to be off by more than 30%. We therefore take

$$\sigma(K^-p \rightarrow \Lambda M_4) = (7.5 \pm 2.3) \mu b \quad (13)$$

This we then believe is the cross-section for the production of all $qq\bar{q}\bar{q}$ states within the defined mass range centred on 2.2 GeV with momentum transfer $|u'| < 1 \text{ GeV}^2$ at a lab. momentum of 14.3 GeV/c. If the mass resolution had been sufficiently good in this experiment the individual resonances contributing to this cross-section, particularly if they were narrow, would in principle have been observed. Of course, this is not the case, however the Omega experiment ¹²⁾ has seen a narrow state in just this mass bin (at $2.2 \text{ GeV} \approx M_0$) in $\pi^-p \rightarrow \Delta X_{\bar{p}p}$ at 9.12 GeV. So how big a cross-section do we expect for this, assuming the narrow state to be a four quark meson?

*) The uncertainties indicate the change brought about by replacing one choice of missing mass variable by another, see Section 3.3.

2.6 How knowing $\sigma(K^-p \rightarrow \Lambda M_4)$ tells us $\sigma(\pi^-p \rightarrow BM_4)$

We may anticipate that since these two processes are rather similar we can expect their cross-sections to be similar in the same kinematic regions, at least within a factor of two or three. The process $\pi^-p \rightarrow BX$ (for a neutral baryon B), unlike $K^-p \rightarrow \Lambda X$, involves Δ exchange as well as nucleon exchange. However, with every indication that the nucleon predominates we can compute its contribution from $K^-p \rightarrow \Lambda X$ by the use of factorization -- another concept that works well within 50%.

The nucleon exchange contribution to the cross-section for $\pi^-p \rightarrow BM_4$ is then simply related to that for $K^-p \rightarrow \Lambda M_4$. At some high energies s_1 and s_2 , not necessarily equal, but high enough that $u_{\min}$ effects can be neglected, i.e., $u_{\min} \approx 0$, we have from factorization

$$\frac{\sigma(\pi^-p \rightarrow BM_4; s_1)}{\sigma(\pi^-p \rightarrow B\pi^0; s_2)} = \frac{\sigma(K^-p \rightarrow \Lambda M_4; s_1)}{\sigma(K^-p \rightarrow \Lambda\pi^0; s_2)} \quad (14)$$

where the M_4 production cross-sections are defined in the same mass intervals for the two processes by Eqs. (9) and (10). The π^0 production cross-sections are just the appropriate differential ones integrated over $|u'| < 1 \text{ GeV}^2$. It is an important feature of factorization for a single Regge exchange that we do not require s_1, s_2 to be equal, since the energy dependence of each side of the relation cancels at a fixed missing mass. The same Omega experiment that revealed the narrow state at 2.2 GeV in $\pi^-p \rightarrow \Delta X$, upon which we presently concentrate, also measured the $\pi^-p \rightarrow \Delta\pi^0$ cross-section, as defined, at the same momenta³²⁾. Thus this simple relation, Eq. (14), allows us to compute the total M_4 production cross-section in $\pi^-p \rightarrow \Delta X$ at 9,12 GeV/c from our calculated $\sigma(K^-p \rightarrow \Lambda M_4)$ at 14.3 GeV/c provided we know the backward $\Lambda\pi^0$ cross-section at this same momentum. Unfortunately, the highest energy at which this pure nucleon exchange process has been measured is at $p_{\text{lab}} = 4.25 \text{ GeV/c}$ ³³⁾. With a model of nucleon exchange, we can of course extrapolate this from 4.25 to 14.3 GeV/c, incorporating the standard s^α behaviour. The success of simple nucleon exchange models^{23),24)} in fitting $\pi^+p \rightarrow p\pi^+$ differential cross-sections at all momenta indicates that we can do this reliably. However, lest sceptics think that such an extrapolation may be too model dependent, we can circumvent this by once again allowing ourselves only the use of experimental measurements, factorization and the idea that backward π^+p elastic scattering is controlled by nucleon exchange. Then we have

$$\frac{\sigma(K^-p \rightarrow \Lambda\pi^0; s_2)}{\sigma(K^-p \rightarrow \Lambda\pi^0; s_3)} = \frac{\sigma(\pi^+p \rightarrow p\pi^+; s_2)}{\sigma(\pi^+p \rightarrow p\pi^+; s_3)} \quad (15)$$

Thus we can use π^+p data to do our energy extrapolation in a way independent of the precise formula for nucleon exchange.

From π^+p backward scattering data at 4.25 GeV/c³⁴⁾ and at 13.75 GeV/c³⁵⁾, we extract the $K^-p \rightarrow \Lambda\pi^0$ cross-section at 13.75 GeV/c (which is almost 14.3 GeV/c) knowing its value at 4.25 GeV/c³³⁾. The experimental quantities are listed in Table 1: note the cross-sections for similar reactions are roughly the same size in the same kinematic region. We then have from Eqs. (13), (14) and (15) that at 9 GeV/c

$$\sigma(\pi^-p \rightarrow \Delta M_4) = (7.0 \pm 3.1) \mu b \quad (16)$$

and at 12 GeV/c

$$\sigma(\pi^-p \rightarrow \Delta M_4) = (3.2 \pm 1.7) \mu b \quad (17)$$

where the uncertainties in these quantities come from compounding all the experimental uncertainties listed in Table 1 as well as that of Eq. (13).

Before comparing these values with the measured cross-section for the production of the 2.2 GeV state, we need to lay to rest, at least temporarily, the fear of $u_{\min}$ effects. We have used Eq. (14) which really only holds precisely if we integrate over the same u values for each process at each energy. The cross-sections quoted¹⁶⁾⁻¹⁹⁾ are all for $|u'| < 1 \text{ GeV}^2$, which at low momenta like 9 or 12 GeV/c include sizeable values of $u_{\min}$. For example, while in $K^-p \rightarrow \Lambda M_4$ at 14.3 GeV/c the $u_{\min}$ for producing an M_4 state at 2.2 GeV is -0.19 GeV^2 , in $\pi^-p \rightarrow \Delta M_4$ at 9 GeV/c, it is -0.57 GeV^2 and for $\pi^-p \rightarrow N^*(1520)M_4$ at 9 GeV/c $u_{\min} = -1 \text{ GeV}^2$, which results in the state not being seen in this channel at such low momenta¹²⁾.

We can take approximate account of these $u_{\min}$ effects, again just using data, by only including in the $K^-p \rightarrow \Lambda X$ integration of Eq. (14), that region of u admissible in the $\pi^-p \rightarrow B M_4$ process, for which we wish to compute the cross-section.

This has the effect of reducing the 9 GeV/c result relative to that at 12 GeV/c making them somewhat more equal. While these effects reduce our calculated cross-sections [Eqs. (16) and (17)] by some 30% or even 50%, it is important to appreciate that the order of magnitude is quite unchangeable, being based solely on experimental quantities and some very general notions of duality and Regge exchanges. However, in Section 3 we shall take these detailed $u_{\min}$ effects into account.

2.7 Comparison with resonance production results

With these caveats about $u_{\min}$ effects reducing the cross-section by up to a factor of two, we believe that the cross-sections for M_4 production, within the mass range 2.035 GeV to 2.355 GeV, in conjunction with a $\Delta(1232)$ should be some 3 to 7 μb , as given by Eqs. (16) and (17). Now if just the narrow peaks seen in the $\bar{p}p$ spectrum¹²⁾ were the only M_4 states produced in this mass range then we would expect this to be their production cross-section. If moreover such M_4 states obey the Freund-Rosner-Waltz selection rules and decay predominantly to $\bar{B}B$ channels, of which $\bar{N}N$, $\bar{N}N\pi$, $\bar{N}N\pi\pi$, $\Delta\bar{N}$, $N\bar{\Delta}$ are the only ones available to a 2.2 GeV state, we should expect the cross-section to the specific $\bar{p}p$ channel to be an appreciable fraction of this total 3-7 μb , say 25%. The experimentally determined cross-sections within the resonance peak at 2.2 GeV are¹²⁾

$$9 \text{ GeV/c} : \quad \sigma(\pi^- p \rightarrow \Delta \bar{p} p) = (17 \pm 5) \text{ nb} \quad (18)$$

$$12 \text{ GeV/c} : \quad \sigma(\pi^- p \rightarrow \Delta \bar{p} p) = (21 \pm 5) \text{ nb} \quad (19)$$

and if we include the contribution from the part of the 2.02 GeV peak in this mass range, these both become $25 \pm 5 \text{ nb}$. It is seen that these are a hundred times smaller than the calculated total M_4 production cross-section, in this region! The ratio of the experimental cross-section to this total M_4 cross-section is just the fraction of $q\bar{q}$ exchanges ($f-\omega-A_2-\rho$) built by these narrow resonances in their $\bar{p}p$ decay mode. We see that this is only one per cent! Where is the remaining 99%?

Clearly it cannot be in the undetected decay modes of the narrow states, otherwise they would be much broader: nevertheless these undetected modes may

increase the 1% in the $\bar{p}p$ channel by a factor of five or so, but still most of the cross-section is unexplained.

This requires that the narrow states sit not just on a non-resonant background, but also on a background of broad M_4 states, which then make up the bulk of the cross-section and so realize the M_2 - M_4 duality. Now if these wide M_4 states obey the Freund-Rosner-Waltz rule too, as we expect simple M_4 states to do, then this bulk of the calculated M_4 cross-section should be seen in $\bar{B}B$ decay channels. So we ask can our 3-7 μb cross-section in this mass range be predominantly $\bar{B}B$? This can, in fact, be seen to be impossible in two ways.

a) Firstly by looking at our reference process $K^-p \rightarrow \Lambda X$. Here only the total $K^-p \rightarrow \Lambda p p$ and $K^-p \rightarrow \Lambda p X$ cross-sections have been measured³⁶⁾. However, the kinematic region we study $\{0.15 \leq M^2/s \leq 0.2, |u'| \leq 1 \text{ GeV}^2\}$ contains less than 1% of the total inclusive $K^-p \rightarrow \Lambda X$ cross-section (15 μb out of 1.9 mb), and we note that the measured fraction of $\bar{p}p$'s is less than 1%, and pX is 2.5% of the total inclusive cross-section. We see that if the M_4 states produced in this mass bin are to decay overwhelmingly to $\bar{B}B$, we require a fortuitous coincidence in which in just the kinematic region we consider, which has only 1% of the total inclusive cross-section, should be seen almost 100% of the $\bar{p}p$ mode and 40% of the pX mode. If the distribution of $\bar{p}p$'s was like this, what would happen in the higher mass bins, $0.2 \leq M^2/s \leq 0.25$ for example, in which we would have few $\bar{p}p$'s left to go there, even though they would be just as needed? This suggests that such a distribution of $\bar{p}p$'s into this one mass bin considered here is highly unlikely, to say the least.

b) We can check this same conclusion by considering the results of the Omega experiment, which detects the narrow peaks in $\pi^-p \rightarrow \Delta(\bar{p}p)$. In Fig. 1 we show the experimental $\bar{p}p$ mass spectrum averaged over the incident momentum of 9, 12 GeV/c with a cut in the Jackson angle to ensure the $(\bar{p}p)$ pair is produced in the backward hemisphere. It is seen that the narrow peaks sit not on an enormous background as would be required if most of the M_4 production were to go into $\bar{B}B$ channels. Indeed the total $\bar{p}p$ background in this bin is no more than a factor of four greater than the peak cross-sections and even half of this may be due to non-resonant production dual to the Pomeron and not to the f . Thus the branching ratio of all M_4 states, in this mass range whether narrow or wide, into $\bar{p}p$ can be no more than a few per cent.

We therefore conclude at this stage that:

- i) the narrow peaks seen in the Omega experiment in the $\bar{p}p$ decay mode are only a small fraction of the total M_4 production required by duality.
- ii) The branching ratio of all M_4 states, whether narrow or broad, produced within this mass interval $\Delta M^2 \approx 1.4 \text{ GeV}^2$ decaying to the specific $\bar{p}p$ channel is at most $(2 \pm 1)\%$ (modulo $u_{\min}$ effects, i.e., factors of two).

These results are quite out of keeping with our expectation from the Freund-Rosner-Waltz selection rule that M_4 states should have $\bar{B}B$ channels as their dominant decay mode and places at risk our belief that the narrow states in the Omega experiment have anything to do with the four quark states expected from duality. However, before analyzing these conclusions, in detail, it seems wise to show that the results we have obtained here based solely on experimental numbers and a few simple, but quite general, theoretical notions are still borne out when we add the flesh of specific model forms to the bare bones of these theoretical precepts. This we do in the next Section.

3. MODEL OF NUCLEON EXCHANGE: EXCLUSIVE AND INCLUSIVE PHENOMENOLOGY

3.1 Aims

The purpose of this section is to use a simple model for nucleon exchange to show that the results just obtained in Section 2 using only experimental quantities with no assumption about phenomenological forms remain true under closer examination.

The use of a model form will allow us to use the $K^-p \rightarrow \Lambda X$ data at 14.3 GeV/c²⁸⁾

- i) to analyze separately the 2.02 and 2.2 GeV states in both Δ and $N^*(1520)$ production;
- ii) to take account in detail of energy dependences and $u_{\min}$ effects, while
- iii) at the same time justifying the claim that M_2 production cannot contribute appreciably in the mass region of interest as indicated in Section 2.5.

The basic aim of fitting the experimental results on $K^-p \rightarrow \Lambda X$, for example, is to allow us to interpolate the data and so obtain quantities of interest in kinematic ranges differing from those in which they were originally binned.

In this our purpose is not to fit the data to a high degree of precision, but rather to use a simple model with just one or two parameters that works to within a factor of two or better and provides an easily understandable form associated with each of the duality diagrams considered in the last section and shown in Fig. 2.

3.2 Model: exclusive processes

The model, we introduce here, for nucleon exchange is a modified form of that discussed previously in Ref. 24). This is based on the large subenergy limit of a B_{2N} amplitude for $N \rightarrow N$ processes, simply extended to fermions by the ad hoc replacement of α by $\alpha - \frac{1}{2}$. The problem of parity doublets is circumvented by restricting our attention to a finite range of the scattering region. As discussed in Ref. 24), it can be shown that in considering such a region, for which $\sqrt{u}$ is purely imaginary, the presence or not of particle poles for $\sqrt{u}$ purely real is a quite separable problem and can be ignored. (This is quite unlike the meson sector, where the analogous extrapolation is from u real and negative to u real and positive.) Thus unlike the earlier similar simple models of Barger and Cline, and of Berger and Fox²³⁾, we ignore the issue of correct extrapolation to the nucleon pole and its non-existent parity partner by staying within the region of $u < 0$.

Then the differential cross-section for the nucleon exchange contribution to the process $ab \rightarrow cd$ is given by

$$\frac{d\sigma}{du} = \frac{1}{16\pi} \frac{s^2}{\lambda(s, m_a^2, m_b^2)} \frac{1}{(2S_a+1)} \frac{1}{(2S_b+1)} \times$$

$$\beta_{acN}^2(u) \beta_{bdN}^2(u) |\xi(u)|^2 |f(u)|^2 (\alpha' s)^{2\alpha_N(u)-2} \quad (20)$$

where the isospin Clebsches have been absorbed in the real residues β (see below); where S_a, S_b are the spins of the initial particles. $\xi(u)$ is the signature factor, which for nucleon exchange is $[i - \cot \frac{1}{2} \pi(\alpha - \frac{1}{2})]$. The function $f(\sqrt{u})$ is that part of the residue that provides the correct extrapolation to the particle poles which lie on the real $\sqrt{u}$ axis: it is a complex function normalized to one at $u = 0$. As already explained, we can set $|f(\sqrt{u})| = 1$ if we consider only the finite part of the $u < 0$ region we wish to fit.

In Eq. (20) we could equally choose the energy variable to be $\alpha(s)$ rather than $\alpha's$. For high energies, this makes little difference and so in keeping with common usage we quote results for $\alpha's$. However, when we come to 3-3 amplitudes, in which M^2 is not so large the choice of variable is more important.

The residues, β , are normalized by the optical theorem such that the contribution of Reggeon R to the total cross-section for ab scattering is given by

$$\sigma_{ab}^{tot}(s) = \frac{s}{\lambda^{1/2}(s, m_a^2, m_b^2)} \frac{\beta_{aar}(0)}{(2S_a+1)} \frac{\beta_{bbr}(0)}{(2S_b+1)} (\alpha's)^{\alpha_r(0)-1} \quad (21)$$

The residues, β , have dimensions of GeV^{-1} .

The important feature of nucleon exchange is that data show that it has a pronounced nonsense wrong signature zero at $\alpha_N = -\frac{1}{2}$. This is introduced into u dependence of the nucleon residues in B_u fashion by

$$\beta_{acN}(u) \beta_{bdN}(u) = \beta_{acN}(0) \beta_{bdN}(0) \left[\frac{\Gamma(\alpha_N(0) + \frac{1}{2})}{\Gamma(\alpha_N(u) + \frac{1}{2})} \right] \quad (22)$$

The nucleon trajectory is taken to be

$$\alpha_N(u) = -0.35 + 0.85 u \quad (23)$$

which is roughly $\frac{1}{2} + \alpha'(u - m_N^2)$. Lastly, we define the isospin invariant residues as follows. If particles a,c are charge states of particle type A,C, respectively, and r is the Reggeon of charge $\bar{ac}$ of type R, then the residues in Eqs. (20) and (22) are related to β_{ACR} by the isospin Clebsch-Gordan $\beta_{acr} = \beta_{ACR} \langle ac | r \rangle$. Results are quoted for $\beta_{ACR}(0)$, etc.

With such a form for the nucleon exchange contribution, Eqs. (20) and (22), the only free parameter for each exclusive 2-2 process is the value of the residue at $u = 0$. We determine these by fitting the reaction cross-sections [defined by Eq. (9)] for $\pi^- p \rightarrow \Delta\pi^0$ and $\pi^- p \rightarrow N^*\pi^0$ (25), (32), knowing that the form of Eqs. (20) and (22) describe the differential data for $\pi^+ p \rightarrow p\pi^+$ from 5.9 to 13.7 GeV/c and for $K^- p \rightarrow \Lambda\pi^0$ at 3.3, 4.2 GeV/c (34), (35), (24). These latter reactions give

$$\beta_{\pi NN}^2(0) = (61.5 \pm 3) \text{ GeV}^{-2} \quad (24)$$

$$\beta_{KAN}^2(0) = (32 \pm 3) \text{ GeV}^{-2} \quad (25)$$

while the 9, 12 GeV/c reaction cross-sections*) for $\pi^- p \rightarrow \Delta\pi^0$ and $N^*(1520)\pi^0$ give

$$\beta_{KAN}^2(0) = (45 \pm 11) \text{ GeV}^{-2} \quad (26)$$

$$\beta_{KN^*N}^2(0) = (18 \pm 5) \text{ GeV}^{-2} \quad (27)$$

where these residues include the branching ratio $\Delta \rightarrow p\pi^-$, $N^* \rightarrow p\pi^-$, respectively -- the only mode in which they are detected in the Omega experiment (12), (32).

3.3 Model: inclusive reactions

In the same spirit as in Section 3.2, we take the large s , large M^2 limit of a B_6 amplitude, to obtain the form for the forward 3-3 amplitude describing the invariant inclusive cross-section for $a \xrightarrow{b} c$, where the Reggeon $a\bar{c}$ is a nucleon, then**)

*) Integrated over u from $u' = 0$ to -1 GeV^2 as in Eq. (9).

**) The Lorentz invariant cross-section $E_c d^3\sigma/dp_c$ is

$$\frac{1}{\pi} \lambda^{1/2}(s, m_a^2, m_b^2) d^2\sigma/dudM^2 ;$$

we shall have occasion to plot this cross-section.

$$\begin{aligned}
 \frac{d^2 \sigma}{du dM^2/s} &= \frac{1}{16\pi} \frac{1}{(2S_a+1)} \frac{1}{(2S_b+1)} \sum_R \beta_{acN}^2(u) \\
 &\times g_{NNR}(u) \beta_{bbr}(0) |\xi(u)|^2 |f(u)|^2 \\
 &\times (\alpha's)^{2\alpha_N(u)-1} (\alpha(M^2))^{\alpha_r(0)-2\alpha_N(u)}
 \end{aligned} \tag{28}$$

where the Reggeons R are P, M_2^+, M_4^+ discussed in Section 2.3, 2.5. Note that Eq. (28) is defined with $\alpha(M^2)$ as the missing mass variable. Other choices, like $\alpha'M^2$ and $v = M^2 - u - m_b^2$, in fact, make rather little difference in the mass range of 2-2.2 GeV of the narrow peaks. Indeed, this small difference is indicated in the uncertainties in the percentage contributions of the different types of Regge exchanges quoted in Section 2.5. However, for small M^2 , the variable $\alpha(M^2)$ naturally suggested by the B_6 model, has the virtue that the factor in Eq. (28), which for large M^2 is $(s/M^2)^{2\alpha_N(u)-1}$, has a sensible limit for small M^2 : $(\alpha's/\alpha(M_{th}^2))$. Typically as $M^2 \rightarrow M_{th}^2$, $\alpha(M^2) \rightarrow \frac{1}{2}$, so the factor becomes $(2\alpha's)^{2\alpha_N(u)-1}$. We shall quote results for this missing mass variable, even though different choices produce only a small effect in the mass range of the narrow peaks, since this has a more sensible low mass continuation.

We thus use for $K^- p \rightarrow \Lambda X$

$$\alpha(M^2) \equiv \alpha_{\Sigma}(M^2) = \frac{1}{2} + \alpha'(M^2 - m_{\Sigma}^2) \tag{29}$$

for $\pi^- p \rightarrow \Delta, N^* X$

$$\alpha(M^2) \equiv \alpha_{\Delta}(M^2) = \frac{1}{2} + \alpha'(M^2 - m_{\Delta}^2) \tag{30}$$

When no obvious single trajectory arises in the missing mass channel, we take guided by Eqs. (29) and (30)

$$\alpha(M^2) = \frac{1}{2} + \alpha'(M^2 - M_u^2) \tag{31}$$

where M_{th} is the lowest two-body threshold in the missing mass channel [this is very close to Eqs. (29) and (30) for the appropriate processes and for many others too: it serves as a useful mnemonic]. Examples of channels with no single trajectory are $K^+p \rightarrow \bar{\Lambda}X$, $pp \rightarrow \pi^+X$, for which we take^{*)}

$$\alpha(M^2) = \frac{1}{2} + \alpha' (M^2 - 4m_\pi^2) \quad (32)$$

While viewing the triple-Regge form of Eq. (28) it is useful to recall that we shall, for simplicity, only include the nucleon exchange contribution to these processes and not the $I_u = 3/2 \Delta$ component. However it is seen from Eq. (28) that the assumption of nucleon dominance, Eq. (8), is expected to hold increasingly well for larger missing mass (at a fixed energy), since the N_α trajectory, having the lower intercept by half a unit, contributes a factor of $\alpha(M^2)$ more than the Δ -- thereby partially justifying its neglect: at least to within a factor of two.

The use of a B_6 model provides us with more than the triple-Regge form of Eq. (28); it determines the u dependence of both the Regge residues β , Eq. (22), and the triple Regge coupling $g_{NNR}(u)$. Once again replacing α by $\alpha - \frac{1}{2}$ in the meson amplitude to take some account of the fermionic nature of the $\bar{ac}$ Reggeon we have

$$g_{NNR}(u) = g_{NNR}(0) \frac{\Gamma(\alpha_N(0) + \frac{1}{2})}{\Gamma(\alpha_N(u) + \frac{1}{2})} \frac{\Gamma(2 + \alpha_R(0) - 2\alpha_N(0))}{\Gamma(2 + \alpha_R(0) - 2\alpha_N(u))} \quad (33)$$

This leaves just the product of Regge couplings at $u = 0$ as the only free parameters in describing each inclusive process for $u < 0$. Results for these are quoted with all isospin Clebsches removed as in Section 3.2.

Our aim is now to "fit" the form of Eqs. (28), (22) and (33) to the process $K^-p \rightarrow \Lambda X$ at 14.3 GeV/c so as to obtain the coupling $g_{NNf}(0) = g_{NN\omega}(0)$. This in turn determines the $\pi^-p \rightarrow \Delta, N^* M_4$ production cross-sections by way of Eqs. (9), (10), (26), (27) and (28). However, in determining this g_{NNf} coupling we have to recall that the Reggeons R of Eq. (28) are the Pomeron, the $M_2^\pm$ ($= f + A_2 + \omega + \rho$) and $M_4^\pm$ of Fig. 2 as discussed in Section 2.3, 2.5. Their

*) This is rather close to the variable considered in Ref. 37).

relative couplings are fixed as described in Section 2.5. Namely, we assume the M_4 exchange couplings at $u = 0$ are roughly equal to those of the M_2 exchange, viz. the f -- an assumption discussed in Sections 1.2 and 2.5, and to be checked in Section 3.4. The Pomeron couplings are fixed by f dominance of the Pomeron³⁰⁾, which gives for inclusive processes

$$g_{RRf}(0) \beta_{bbf}(0) = r_{fP} g_{RRP}(0) \beta_{bbP}(0) \quad (34)$$

where r_{fP} should be independent of the reaction for non-strange particles b . This has been tested to be roughly so by Inami and Roberts³¹⁾ with r_{fP} typically 1.5 [see Eq. (45) for generalization to strange particles b].

With all the parameters fixed by the one f - ω coupling we can determine this by use of the data on $K^-p \rightarrow \Lambda X$ at 14.3 GeV/c as a function of u and M^2/s ^{28),29)}. No attempt has been made to fit these data in the sense of χ^2 minimization, but rather the nearest round number for the coupling is taken to give an eyeball fit^{*)}. This is

$$\beta_{KAN}^2(0) g_{NNf}(0) \beta_{NNf}(0) = 3000 \text{ mb GeV}^{-2} \quad (35)$$

This choice gives the one parameter curves shown in Figs. 3 and 4. With typical uncertainties being given by the errors on the final data set, we have using Eq. (25)

$$g_{NNf}(0) \beta_{NNf}(0) = (230 \pm 80) \text{ GeV}^{-2} \quad (36)$$

Having determined this crucial coupling we can then multiply it by $\beta_{\pi\Delta N}$ and $\beta_{\pi N^*N}$ using Eqs. (26) and (27) and calculate the reaction cross-section for $\pi p \rightarrow \Delta, N^* M_4$ in any mass and u range we wish and at any energy -- M_4 production being dual to M_2 exchange. In doing this we can specifically integrate the

*) This is not the same as in Ref. 24) on two counts. Firstly, a different form, Eq. (28), is used here and secondly in Ref. 24) only the preliminary sample of the data then available was used. I am very grateful to Alain Borg for providing me with tables of the final results of this experiment²⁹⁾.

the form of Eqs. (22), (28) and (33) from $u' = 0$ to $u' = -1 \text{ GeV}^2$ [Eqs. (9)] and calculate the cross-sections for M_4 production at 9,12 GeV/c. This we do in two roughly equal mass bins, Eq. (10), centred on $M = 2.02$ and 2.2 GeV , i.e., $1.88 - 2.16 \text{ GeV}$ and $2.08 - 2.32 \text{ GeV}$. These bins are shown shaded in Fig. 1. Using the results of Eqs. (26), (27) and (36) we then find the cross-sections for M_4 production given in Table 2, where they are listed together with the corresponding peak cross-sections. Note that both the 1.936 GeV peak and that at 2.02 GeV occur in the first mass bin as seen in Fig. 1. It is seen that the correct treatment of $u_{\min}$ effects and the reduction of ΔM^2 from 1.4 to 1 GeV^2 reduces the cross-sections of Eqs. (16) and (17) by no more than a factor of two. The ratio of the experimental peak cross-sections to the calculated M_4 production cross-sections expected by duality, within each of these mass bins of width 1 GeV^2 can be summarized as

$$\frac{\sigma(\pi^- p \rightarrow B M_4 [\text{peak} \rightarrow \bar{p}p])}{\sigma(\pi^- p \rightarrow B M_4 [\text{all}])} = (1 \pm 0.5)\% \quad (37)$$

whether $B = \Delta$ or N^* or whether at 9 or 12 GeV/c. Indeed the fraction of all $\bar{p}p$ production compared to the calculated M_4 cross-section is only $(4 \pm 2)\%$ and even then part of this must be background (i.e., Pomeron exchange) production of $\bar{p}p$ pairs. We have calculated only the nucleon exchange contribution to these processes (Table 2). The inclusion of Δ exchange may increase these cross-sections by a quarter or even a third, and hence further reduce the right-hand side of Eq. (37).

The ratio of Eq. (37) gives the fraction of $f-\omega-A_2-\rho$ exchange built by these narrow resonances in their $\bar{p}p$ decay mode. Moreover, at most $(4 \pm 2)\%$ of $f-\omega-A_2-\rho$ exchange can be built by $\bar{p}p$ intermediate states, quite at variance with our previously espoused notion of duality for $\bar{B}B$ processes. This ratio should not be confused with the actual branching ratio to $\bar{p}p$ of these narrow states [Eq. (37) would only be such a branching ratio, if the finite mass sum rule for $f-\omega-A_2-\rho$ exchange were saturated by these narrow resonances]. Indeed, we can see that the multimeson background in each mass bin (Fig. 1) would within just the widths of the peaks be only a small fraction of the total production cross-sections, given in column 4 of Table 2. Thus, for example, within just the width

of the 2.2 GeV peak strict local duality would give the ratio of the experimental $\bar{p}p$ decay mode to the cross-section for all channels to be $(34 \pm 14)\%$ ^{*} when an $N^*(1520)$ is also produced, with similar results for $\Delta(1232)$ production.

Equation (37) confirms completely the conclusions of the previous section, in which we shied away from such sophisticated forms as Eqs. (20), (22), (28) and (33). However, we have one last check to make.

3.4 Indirect check of the M_4 exchange contribution

Here we want to check that the values of the couplings and cross-sections given above cannot be grossly wrong because of an inaccurate estimate of the size of M_2 production, which is rather poorly known being dual to M_4 exchange. In Section 2.5 we mentioned that the cross-sections of Eq. (11), and hence those of Table 2, would only be decreased by a factor of two, if our estimate of the M_4 coupling was even a factor of four too small. The purpose of this section is to show that even this degree of uncertainty is not consistent with the data. We can in fact independently check the Pomeron couplings (which are in turn related to those of the $f^{(30)}$) in the following ways:

a) the process $K^+p \rightarrow \bar{\Lambda}X$ in the beam fragmentation region is just the crossed channel of $K^-p \rightarrow \Lambda X$ and so is described by the same Regge exchanges. However, the $K^+p\Lambda$ channel being exotic leads to approximate exchange degeneracy and so this crossed process involves only Pomeron exchange but with just the same coupling as in $K^-p \rightarrow \Lambda X$ [†]). Because of comparatively low statistics in the fragmentation regions, the data on $K^+p \rightarrow \bar{\Lambda}X$ exist only as a function of a single variable, in particular of x where p_T is integrated over. With $x = p_L^*/p_{\max}^*$ we have the CERN 8.2, 16 GeV/c data ³⁷⁾ and the very recently finalized Serpukhov data at 32 GeV/c ³⁸⁾ both of Chliapnikov et al. The one parameter curves through these data are shown in Fig. 5, where these correspond to

$$\beta_{KAN}^2(0) g_{NNR}(0) \beta_{NNR}(0) = 4.4 \cdot 10^3 \text{ GeV}^{-4} \quad (38)$$

^{*}) Note that this is quoted merely as an indication; unlike Eq. (37) this is based on strictly local duality which is not such a realistic hypothesis. Note added: After writing this paper I received a private communication from Dr J. Six, to whom I am most grateful, confirming that the local indications quoted here are well within the experimental limits for the channels other than $\bar{p}p$ detected in his Omega experiment¹²⁾.

[†]) If we had data on the four processes $K^\pm p(K^\pm n) \rightarrow \Lambda X$ as a function of u, M^2 , we could separate the M_2 exchanges ω, A_2, ρ , cleanly from the f without assuming approximate exchange degeneracy, which is what we do here in the absence of such data.

Within 30% the agreement is quite reasonable considering the low statistics of the data. We see that comparing this coupling with that of Eq. (36) from $K^-p \rightarrow \Lambda X$ we have

$$\frac{g_{NNf}(0) \beta_{NNf}(0)}{g_{NNP}(0) \beta_{NNP}(0)} = 1.7 \pm 0.8 \quad (39)$$

in agreement with the P-f universality ratio of $(1.5 \pm 0.3)^{31)}$, thus confirming the estimates previously made.

b) We can moreover test this coupling further by the use of more precise data on $pp \rightarrow \pi^+ X$, which has been taken at the CERN PS, FNAL and at the ISR. This process can be expected to be controlled by space-like nucleon exchange too for momentum transfers in the regions of $-u \in (0, 0.1), (0.5, 1) \text{ GeV}^2$, for example. The $pp\pi^-$ channel being exotic, this process is also controlled by Pomeron exchange in the t' channel. By factorization its coupling is just as for $K^+p \rightarrow \bar{\Lambda}X$, but with $\beta_{KAN}^2(0)$ in Eq. (38) replaced by $\beta_{\pi NN}^2(0)$.

In Fig. 6 we show as an illustration the curve for the invariant cross-section at the ISR energy of $\sqrt{s} = 44.6 \text{ GeV}$ with $p_T = 0.8 \text{ GeV}/c$ ³⁹⁾ and mean coupling

$$\beta_{\pi NN}^2(0) g_{NNP}(0) \beta_{NNP}(0) = (7.5 \pm 1.2) 10^3 \text{ GeV}^{-4} \quad (40)$$

Using Eq. (24) this gives

$$g_{NNP}(0) \beta_{NNP}(0) = (123 \pm 20) \text{ GeV}^{-2} \quad (41)$$

while from $K^+p \rightarrow \bar{\Lambda}X$ data, Eq. (38), we have a value of $(137 \pm 48) \text{ GeV}^{-2}$ and using $r_{rP} = (1.5 \pm 0.3)$ the $K^-p \rightarrow \Lambda X$ data, Eq. (36), gives $(153 \pm 60) \text{ GeV}^{-2}$; all in very good agreement. Moreover, this same coupling agrees with the new FNAL 100, 200, 400 GeV/c data at $p_T = 0.75 \text{ GeV}/c$ ⁴⁰⁾ and with the 24 GeV/c data of Allaby et al.⁴¹⁾ at similar transverse momenta for $pp \rightarrow \pi^+ X$. Only the ISR curve is shown in Fig. 6 to illustrate how remarkably this simple model works at M^2

of 1000 GeV^2 as well as at 4 GeV^2 ^{*)}. This confirms that the Pomeron and hence the related $f-\omega$ coupling are indeed of the size anticipated in Sections 2.3 and 3.3. This in turn implies that the M_4 exchange contribution cannot be much bigger either and has therefore been correctly taken into account in the mass range of the narrow states.

One loose end to tie up is to note that in both the data on $K^- p \rightarrow \Lambda X$ ²⁸⁾ and on $K^+ p \rightarrow \bar{\Lambda} X$ ^{37), 38)} the $\Lambda, \bar{\Lambda}$ cannot be distinguished from a $\Sigma^0, \bar{\Sigma}^0$. So what we have called the inclusive Λ cross-section really includes Σ^0 production too, which is found to be 25% of the data ^{36), 37)}. Thus in Eqs. (35) and (38) the coupling β_{KAN} includes a Σ^0 component. However, this is not the case for the residue quoted in Eq. (40) for which data with Λ, Σ^0 separated was used ³³⁾. This might suggest that the triple Regge couplings quoted in Eqs. (36) and (39) should perhaps have somewhat larger uncertainties. That this is really unnecessary is seen in the fact that the Pomeron coupling of Eq. (41) from $pp \rightarrow \pi^+ X$ data, which has no such ambiguity, agrees very well with those from $\Lambda, \bar{\Lambda}$ production. These effects are anyway rather small and confirm that our calculations should be accurate to within a factor of two.

4. A COLLECTION OF MODELS

4.1 Prerequisites

Our analysis, sketched in Section 2 and detailed in Section 3, forces us to conclude that any model for four quark states must, if it is to be consistent with duality and compatible with experimental data, allow two distinct types of boojum states: A and B, which must in turn satisfy the following conditions.

- A) If the narrow peaks seen in the Omega experiment ¹²⁾ are to be interpreted as $qq\bar{q}\bar{q}$ states they, being narrow, do obey the Freund-Waltz-Rosner selection rule and are closest to those named baryonium ⁷⁾. However, they constitute only a small fraction of all M_4 mesons required to be produced according to duality.
- B) The bulk of M_4 states produced know nought of the Freund et al. rule and of any special relationship to baryons, decay predominantly to mesonic channels and have only a small (few per cent) branching ratio to the $\bar{p}p$ channel.

These are boojums A and B: we wish to find homes for them within the available collection of models.

*) Note that in extrapolating the results of Fig. 6 from ISR energies down to PS momenta the use of the variable $\alpha(s)$ rather than α' s in Eq. (28) works a little better.

4.2 The Rossi-Veneziano description of baryon dynamics

In our use of duality we have only been concerned with the flow of quark flavour and not of colour. For mesons, the colour indices follow the flavour lines. However, for baryons the direction taken by the sink and sources of colour are, of course, distinct. In the string picture of hadrons, baryons take on a Y shaped configuration. Rossi and Veneziano²¹⁾ have studied the dynamics of such baryons in a dual approximation to QCD and found the path followed by the junction of the baryonic string to play an important rôle. Moreover, this has fundamental consequences for the spectroscopy of mesons of both the two and four quark type. Each arrangement of quarks has two possible forms: either without junctions or with two junctions, which we denote in an obvious way by M_2^{0J} , M_2^{2J} , M_4^{0J} , M_4^{2J} (Fig. 7). In this scheme²¹⁾ those states with two junctions obey a new Zweig-like rule, which forbids them decaying to other than $\bar{B}B$ channels, since the junction lines must in lowest order flow on, just as with quark flavour lines. In this way a dynamical explanation of the Freund-Waltz-Rosner selection rule⁴⁾ for some M_4 states, the M_4^{2J} , is provided. Normal mesons π , η , ρ , etc., are then of the M_2^{0J} type and like M_4^{0J} mesons can be formed in $\bar{B}B$ channels^{*)} by the annihilation of the baryon's junction lines.

This scheme, moreover, has two types of non-resonant production in $\bar{B}B$ channels, not just the M_6^{2J} states (which are $\bar{B}B$ states) which generate the usual Pomeron, but also M_6^{0J} states (which are $q\bar{q}q\bar{q}q\bar{q}$ mesons) which generate a Reggeon with junction, but not quark, exchange (Fig. 8). This is called M_0^{2J} exchange, while in this notation M_0^{0J} would be the Pomeron. This is quite different from the simple Rosner approach discussed in Sections 1-3.

This scheme rather naturally provides us with two types of $q\bar{q}q\bar{q}$ states: those with two junctions, M_4^{2J} , decaying only to $\bar{B}B$ channels and those without junctions, M_4^{0J} , decaying predominantly to multimeson modes. These two types appear to overlap rather well with the two phenomenological categories A,B we have found necessary to explain the data. It is therefore appealing to identify type A mesons as M_4^{2J} resonances, while type B include both M_4^{0J} and M_6^{0J} states. This description of Rossi and Veneziano having developed from a specific attempt to incorporate baryons into topological theories (dual and gauge) is more definite than the other, more recent, schemes for M_4 states. Because of this, it has, when combined with our result of Eq. (37), consequences which can be tested; Section 5 is devoted to just that.

*) Where, of course, either B or $\bar{B}$ must be off-shell to produce a low mass meson.

4.3 Bags and baryonium

During the course of this present work, two, rather similar, schemes have been proposed for four quark mesons coupling to $\bar{B}B$ channels^{20),22)}; similar in that the properties of these states are wholly dictated by the particular colour coupling of the diquark to antidiquark which form the colour singlet $qq\bar{q}\bar{q}$ meson. These schemes not being tied to a detailed field theory, like that of Rossi and Veneziano²¹⁾, are more general. They give rise to three types of boojum states, two of which have been studied in detail by Chan and Høgaasen²²⁾. These are the mock baryonium states, which must be narrow: being $6-\bar{6}$ of colour, they have difficulty in decaying to any pair of colour singlets whether mesons or baryons. These we must identify with the narrow peaks observed in the Omega experiment¹²⁾, i.e., our type A. Then there are the true baryonium states with 100 MeV widths and large couplings to $\bar{B}B$ channels and small to meson decay modes. Phenomenology, Section 4.1, dictates that these are part of our category B. However, they can only be a very small part in the 1.88-2.32 GeV region, since, as already discussed in Section 2.7, the $\bar{p}p$ production cross-section is only a small part of the total M_4 production required by duality. These true baryonium states could be identified with the T,U mesons⁴²⁾ observed in $\bar{p}p$ scattering, which we may alternatively regard as $q\bar{q}$ states of the normal type¹⁵⁾. Nonetheless, the so-called true baryonium states can, like mock baryonium, only be a small fraction of the total boojum production -- the bulk being made up of $qq\bar{q}\bar{q}$ states with no special relationship to baryons, but which may be regarded rather as two meson states. These, not studied in any detail by Chan and Høgaasen²²⁾, have been classified by Jaffe²⁰⁾ along with all M_4 mesons. However, while these two meson boojums do exist, as well as true baryonic ones, in bag and string models, only their couplings to on-shell $\bar{B}B$ initial states have been calculated²⁰⁾ and nothing can presently be predicted about their relative couplings in production experiments. Instead, the phenomenology presented here fixes these and dictates that these models accommodate this information.

Lastly, there exist calculations of $\bar{B}B$ bound states in potential models^{*)}, these do provide a spectrum of expected mesons, but which, embodying no aspect of duality, stand outside the framework in which our discussion takes place.

*) For an early example of such calculations, see Ref. 18), while Ref. 43) shows the present state of this subject.

5. THE ROSSI-VENEZIANO SCHEME FOR $\bar{q}q\bar{q}q$ STATES

5.1 Introduction

The aim of this section is to subject the model of Rossi and Veneziano for M_4 states²¹⁾ to detailed scrutiny by testing whether the identification, required by the junction Zweig rule, of our type A and B states of Section 4.1 with M_4^{2J} and M_4^{0J}, M_6^{0J} mesons provides consistent results. In this scheme these types of multiquark mesons are dual to M_2^{0J}, M_2^{2J} and M_0^{2J} exchanges, respectively (see Figs. 7c, 7d and 8). Recall that we calculated the total M_4 production, in Sections 2 and 3, as that component dual to M_2 exchange, in particular dual to the normal M_2^{0J} Reggeons. Thus we may believe we have computed, in Eqs. (13), (16) and (17) and Table 2, the production of just M_4^{2J} states, which, because of the junction Zweig rule must have a large branching ratio to $\bar{p}p$. Since this is not fulfilled, Eq. (37), we must admit the possibility that the part of the $Mp \rightarrow BX$ cross-section attributed to M_2 exchange includes not just $f-\omega-A_2-\rho$ exchange, but also M_2^{2J}, M_0^{2J} exchanges. Then Eq. (37) would be interpreted, if we even include all produced $\bar{p}p$'s as

$$\frac{\sigma(\pi^- p \rightarrow B M_4^{2J})}{\sigma(\pi^- p \rightarrow B(M_4^{0J} + M_6^{0J})) + \sigma(\pi^- p \rightarrow B M_4^{2J})} \lesssim (4 \pm 2)\% \quad (42)$$

If the junction Zweig rule, so crucial to the scheme of Rossi and Veneziano, is then to be valid, we must find that M_4^{0J}, M_6^{0J} production in the denominator of Eq. (42) must dominate over M_4^{2J} production (i.e., B boojums must be more plentiful than A's, Section 4.1). Indeed, Rossi and Veneziano argue²¹⁾ that M_6^{0J} production should dominate over both M_4^{0J}, M_4^{2J} , in order to ensure the suppression of junction Zweig rule violating terms. While M_6^{0J} production must dominate for internal consistency of this scheme, which would make the phenomenological result of Eq. (42) just what we would expect, the aim of this section is to test whether experiment confirms this result of the Rossi and Veneziano scheme.

5.2 "Up the junction" *)

In order to study this in greater detail than sketched above, we begin by being more specific about all the exchanges that contribute to the inclusive process $Mp \rightarrow BX$, when we take account of junction lines. All the contributions, in

*) As common in Didcot or Clapham see Ref. 13), Vol. V, 629.

which quarks are exchanged in the t' channel, are shown in Figs. 7a, 7b, 7c and 7d, which display their corresponding duality properties. The exchanges of Fig. 2 have been doubled by the inclusion of the junction lines. Similarly, we have two types of background contributions: the Pomeron (M_0^{0J}) and M_0^{2J} (cf. Fig. 8). What of the trajectories of these exchanges?

Our previous result of Eq. (5) for the trajectory intercepts becomes subdivided too:

$$\alpha_{M_2^{0J}} + \alpha_{M_4^{2J}} = 2\alpha_B = \alpha_{M_2^{2J}} + \alpha_{M_4^{0J}} \quad (43)$$

and the discussion of Section 1.2 of the intercept of the $qq\bar{q}\bar{q}$ exchange now applies to just the left-hand half of this Eq. (43), viz. to M_4^{2J} exchange, Eq. (6). It is the right half of this Eq. (43) that is new. Since M_4^{0J} exchange generates a Regge-Regge cut with intercept $2\alpha_B - 1 = 0$, Eq. (43) gives the intercept of the M_2^{2J} exchange to be

$$\alpha_{M_2^{2J}}(0) = 2\alpha_B(0) - 2\alpha_{M_2^{0J}}(0) + 1 \approx 2\alpha_B(0) \quad (44)$$

Moreover, we have a new non-resonant component in this scheme. This M_0^{2J} exchange has intercept of

$$2\alpha_B(0) + 2 - 3\alpha_{M_2^{0J}}(0) = 2\alpha_B(0) + \frac{1}{2} \approx \frac{1}{2},$$

and is dual to $qq\bar{q}\bar{q}\bar{q}\bar{q}$ production, i.e., three meson states. What about the contributions of these exchanges

$$M_0^{0J}, M_0^{2J}, M_2^{0J}, M_2^{2J}, M_4^{0J}, M_4^{2J}$$

with intercepts of

$$1, (2\alpha_B + \frac{1}{2}), \frac{1}{2}, 2\alpha_B, 0, (2\alpha_B - \frac{1}{2})$$

respectively?

As discussed before, M_4^{2J} exchange only gives a small contribution in the mass range we consider -- a component the size of which we have already taken account. However, we have not considered M_4^{0J} exchange, viz. the Regge-Regge cut. Previous investigations by Harari⁴⁴⁾, for instance, suggest that with a scale of 1 GeV^2 the coupling of this component should be roughly equal to that of $M_2^{0J}(f)$ Regge exchange. Thus, if we separate the cut contribution, it will in this mass range, around 2 to 2.2 GeV, at most reduce what we previously considered to be f exchange by 30%. So the right-hand side of Eq. (42), for example, may be correspondingly increased to $(1.4 \pm 0.7)\%$ -- a change well within the present uncertainties anyway. We see that if the Rossi-Veneziano scheme is to remain viable, it really must be M_4^{0J}, M_6^{0J} production that dominates over M_4^{2J} production, since M_2^{0J} and M_2^{2J} production are too small to make much difference. Though the M_2^{2J} exchange component will decrease, relative to both the normal M_2^{0J} and the M_0^{2J} exchanges, with increasing missing mass, the relevant couplings are a priori quite unknown. Indeed, Rossi and Veneziano²¹⁾ argue that M_0^{2J} should dominate over the other two to ensure the consistency of their junction Zweig rule. In this scheme the production of M_4^{2J} states, which must decay mainly to $\bar{B}B$ channels because of the new Zweig rule, should build $f(M_2^{0J})$ exchange. However, the total $\bar{p}p$ production in the mass range we consider is only a few per cent of what we previously thought was dual to this M_2^{0J} exchange (Sections 2 and 3). Thus for the scheme to make sense and $\bar{p}p$ intermediate states to predominate in the building of $f\omega A_2 \rho$ exchange, we must find that M_4^{0J}, M_6^{0J} production must, in this mass range, be at least five, if not ten, times larger than M_4^{2J} production. This is what we now test.

As illustrated in Figs. 7c, 7d and 8 the size of multiquark production in $Mp \rightarrow BX$ (i.e., in $K^-p \rightarrow \Lambda X$, $\pi^-p \rightarrow \Delta X$, $\pi^-p \rightarrow N^* X$, etc.) is given by the sum of M_2^{0J} , M_2^{2J} and M_0^{2J} exchange contributions. Since these three components cannot be easily separated by their missing mass dependences, we must describe these processes by an effective M_2 exchange just as in Section 3.3, Eq. (36). However, this effective coupling should, if the Rossi-Veneziano scheme is correct, be much larger than the pure M_2^{0J} coupling. How can we tell how big this is?

While $Mp \rightarrow BX$, being essentially $\bar{B}(\equiv \bar{M}B)p$ scattering in the missing mass channel, has both M_2 exchanges, the process $M_1 p \rightarrow M_f X$, being essentially $M_1 B(\equiv p \bar{M}_f)$ scattering, can only have M_2^{0J} exchange, as seen in Fig. 9. Thus such a process measures just the coupling we need to compare with Eq. (36) provided it is dominated by N_α exchange in the u channel.

5.3 Choice of data

It is crucial for a meaningful determination of triple Regge couplings to choose inclusive data as a function of two variables, e.g., u and M^2 . This is because fits to integrated cross-sections as a function of x can easily yield couplings varying by an order of magnitude depending on the p_T or t dependence assumed. The use of double differential cross-sections avoids these ambiguities and our "fits" with specific triple Regge forms will provide couplings meaningful at least within the present context.

Thus for the test described above we want to choose a process $p \xrightarrow{M_i} M_f$ for which data exist in two variable forms. Moreover, we want nucleon exchange in the u channel to dominate, so we take $M_f = \pi^+$. However, the initial (spectator) meson M_i can be either a pion or kaon. It is useful to note that the maximum value of the mass squared of the baryon exchange, u , becomes increasingly negative for increasing missing mass for $K^-p \rightarrow \Delta X$ as for $\pi^-p \rightarrow \Delta, N^* X$, while for $K^-p \rightarrow \pi^+ X$, the process considered first, $u_{\min}$ becomes more positive, indeed coming close to the nucleon pole itself. For this reaction we have two sets of available data at 10 and 16 GeV/c of Baker et al.⁴⁵⁾ and at 14.3 GeV/c of Borg et al.⁴⁶⁾. While the latter is available in all accessible kinematic regions, the former has only been published in the proton fragmentation region integrated over the momentum transfer u , when this is positive. This integration is in fact over only a small kinematic region for small missing mass and so provides us with an almost double differential cross-section there. However, because u is positive, these data explore the domain of $\sqrt{u}$ real, which for baryon exchanges N_α, Δ_δ fixes the function $f(\sqrt{u})$ of Eqs. (20) and (28) rather than the $u = 0$ couplings we need. Fortunately, the data at 14.3 GeV/c⁴⁶⁾ have been taken in the negative u region too, where we have to a good approximation $|f(\sqrt{u})| \approx 1$ (as in Sections 3.2 and 3.3), and this can then determine $g_{NNf}(0)$. Nonetheless, we can compare the data at 10 and 16 GeV/c with that at 14.3 GeV/c in the region of $u > 0$, where they both have been published. This comparison is shown in Fig. 10. A serious discrepancy between these sets of data is seen, which does not appear to be due to some strange energy dependence between 10 and 16 GeV/c through 14.3 GeV/c, but rather arises in the analyses of these experiments. Since this conflict has not been previously pointed out, it seems necessary to do so here to warn an unsuspecting user of this fact. Though the reason for this discrepancy is unresolved^{*)}, we have faith that the 14.3 GeV/c K^-p data used here is not far

*) I am very grateful to Alain Borg and Douglas Morrison for discussions about their respective experiments.

wrong^{*)}. The only two variable data on $\pi^\pm p \rightarrow \pi^\pm X$ ⁴⁷⁾ come from the same K^-p group as in Ref. 45) analyzed in just the same way. We therefore restrict attention to the reaction $Mp \rightarrow \pi^\pm X$ with $M = K^-$.

5.4 Test of duality with junction lines

Before we can analyze these data on $K^-p \rightarrow \pi^\pm X$ in the manner of Section 3.3 we must take note of a couple of minor complications:

a) since the Pomeron can couple to $K\bar{K}$ through the f' as well as the f , the relation of Pomeron to f couplings of Eq. (34) becomes³⁰⁾

$$\left[1 + \frac{1 - \alpha_f(0)}{1 - \alpha_{f'}(0)} \right] \frac{g_{RRf}(0) \beta_{KKf}(0)}{g_{RRP}(0) \beta_{KKP}(0)} = \tau_{fP} \quad (45)$$

b) in $\bar{N}N$ scattering, which is basically what we consider in $K^-p \rightarrow \Lambda X$, the M_2^{0J} Reggeons are $f + \omega + A_2 + \rho$, where exchange degeneracy makes the contributions of $f \approx \omega$, $A_2 \approx \rho$. Moreover, from data on $\bar{p}p$, pp , $\bar{p}n$, pn total cross-sections⁴⁸⁾ we see that

$$\frac{\beta_{NN\omega}^2(0)}{\beta_{NN\rho}^2(0)} = (11.2 \pm 5.7) \quad (46)$$

which means that, if we assume a similar result holds for $g_{NN\omega}/g_{NN\rho}$, we can neglect the contribution of A_2, ρ compared to f, ω (Sections 2.5 and 3.3). In $K^-p \rightarrow \pi^\pm X$, we consider basically K^-n scattering (assuming nucleon dominance once again). This has contributions from $f + \omega - A_2 - \rho$, which equal $2(f - A_2)$ by exchange degeneracy. Then, again assuming Eq. (46) holds true within the errors for the triple couplings too, we have

$$\frac{\beta_{KKA_2}(0) g_{NNA_2}(0)}{\beta_{KKf}(0) g_{NNf}(0)} = (0.3 \pm 0.1) \quad (47)$$

*) For instance, in this same kinematic region of $u > 0$ the data on the process $K^-p \rightarrow \pi^- X$ at 14.3 GeV/c⁴⁶⁾ are in complete agreement (taking account of the different Regge exchanges) with the $K^+p \rightarrow \pi^- X$ data at neighbouring energies of the Goldschmidt-Clermont group also published in Ref. 45).

so that the sum over non-Pomeron couplings in Eq. (28) becomes

$$\sum_{R \neq P} \beta_{KKR}(0) g_{NNR}(0) = (1.4 \pm 0.2) \beta_{KKf}(0) g_{NNf}(0) \quad (48)$$

Can this f residue, which is the true M_2^{0J} coupling, really be a factor of five or 10 smaller than the effective M_2 coupling of Eq. (36), which in this scheme is now a sum of even (or equally, of odd) signature couplings of M_2^{0J} , M_2^{2J} , M_0^{2J} exchanges, viz.

$$\sum_{\substack{M = M_2^{0J}, M_2^{2J}, M_0^{2J} \\ \text{of } + \text{ or } - \text{ signature}}} g_{NNM}(0) \beta_{NNM}(0) = (230 \pm 80) \text{ GeV}^{-2} \quad (49)$$

As an illustration let us take in this sum, contrary to what the Rossi-Veneziano scheme requires, the dominant contribution to be just $M = f = \omega$ [exactly as stated in the Harari-Rosner analysis leading to Eq. (36)]. This means let

$$\beta_{\pi NN}^2(0) g_{NNf}(0) \beta_{KKf}(0) = (3.8 \pm 1.3) 10^2 \text{ mb. GeV}^{-2} \quad (50)$$

We then calculate $d^2\sigma/dudM^2$ from the forms of Section 3^{*}) and see if this overshoots the data by an order of magnitude. The result is shown in Figs. 11 and 12. Remarkably the band given by Eq. (49) passes through the data, rather than overshooting. If anything it is slightly below, as we would expect since we have omitted the Δ_3 contribution which may be as much as 30%. Quite unexpectedly the sum of M_2^{0J} , M_2^{2J} , M_0^{2J} couplings of Eq. (36) is almost exactly just the M_2^{0J} coupling alone.

This is analogous to the result in the single Regge limit of ω universality⁴⁹⁾, that is the ω contribution in NN scattering, viz.

$$\frac{1}{4} (\sigma_{\bar{p}p} - \sigma_{pp} + \sigma_{\bar{p}n} - \sigma_{pn}),$$

is simply related to the ω contribution in KN scattering at $t = 0$ ⁴⁸⁾. In

^{*}) For $K^-p \rightarrow \pi^+X$, $\alpha(M^2)$ of Eq. (28) is the Σ trajectory.

the Rossi-Veneziano scheme²¹⁾, the $I = 0$ odd signature exchange in KN scattering is M_2^{0J} , the ω , as discussed above for an off-shell nucleon. However, in NN scattering it is both M_2^{0J} and M_0^{2J} , i.e., $\omega + \tilde{\omega}$. That these should be simply related, viz. equal, requires, in this scheme, a very special mixing of the bare $\omega, \tilde{\omega}$ (equally $f, \tilde{f}$) couplings at $t = 0$ to give the physically observed ω universality⁴⁹⁾. Unless such "magic" *) mixing occurs, the M_4^{0J}, M_6^{0J} production of Eq. (42) not only fails to dominate M_4^{2J} production, but if anything is much smaller^{**)}. We see then that the fraction of $f\text{-}\omega\text{-}A_2\text{-}\rho$ exchange built by $M_4^{2J} \rightarrow \bar{p}p$ states, wide or narrow, can only be as large as $(6 \pm 4)\%$ ^{†)}, in complete contradiction with the duality of the Rossi-Veneziano scheme. For, unlike the Rosner duality²⁾, in which $f\text{-}\omega\text{-}A_2\text{-}\rho$ exchange is built by all $qq\bar{q}\bar{q}$ intermediate states, the Rossi-Veneziano scheme requires this duality to be fulfilled by a subset of all multiquark intermediate states, viz. M_4^{2J} , which couple strongly to $\bar{B}B$ channels. This is shown to be incorrect. The duality of Rosner, in which colour is not considered, has less trouble with the data, provided, however, that we abandon the Freund-Waltz-Rosner selection rule⁴⁾ and allow the bulk of $qq\bar{q}\bar{q}$ states to be multimeson boojums and not baryonium states.

In contrast, the more restrictive duality scheme of Rossi and Veneziano makes predictions inconsistent with the data, unless "magic" mixing occurs between the M_2^{0J}, M_0^{2J} exchanges at $t' = 0$.

6. CONCLUSIONS

Rosner²⁾ has shown that the jigsaw puzzle, picturing the duality of $q\bar{q}$ and $qq\bar{q}\bar{q}$ states, can only be completed if $qq\bar{q}\bar{q}$ mesons also exist. Ever since then, these missing pieces have been sought in both the s and t channels -- in formation, production and exchange.

*) In the sense of being presently unexplained and ill-understood.

**) Within the errors it could almost be equal in magnitude.

†) This includes $\bar{p}p$ intermediate states that are probably dual to M_0^{0J}, M_0^{2J} exchanges and so is an overestimate.

A phenomenological analysis of Hoyer, Roberts and Roy⁵⁰⁾ was the first to isolate a low-lying trajectory, which duality could identify as M_4 exchange^{*)}. More recently, Nicolescu⁵³⁾ has assembled in great detail the evidence for such $qq\bar{q}\bar{q}$ exchanges in intermediate and high energy reactions, in which trajectories with intercepts from -1.2 up to -0.13 have been found^{**)}. Moreover, he deduces that their couplings to the baryon octet are similar in size to those of $q\bar{q}$ exchange, as expected from Sections 1.2, 2.3 and 3.4. However, he notes that their couplings to the baryon decuplet are unexpectedly (and inexplicably) enhanced. Nonetheless, the suggestion of M_4 exchanges, much as duality diagrams require, is clear.

The situation in both the direct and missing mass channels is somewhat more confusing. In the wake of Chew's discussion of baryonium⁷⁾, many papers have been written on the rich spectrum of states expected coupling strongly to $\bar{B}B$ channels^{54), 20)-22)***)}. These expectations have been highlighted by the discovery in the last year of several narrow states coupling to such channels^{8), 12), 14), 42)}. However, in the absence of spin and isospin analyses of these resonances, their inclusion in the Chew-Frautschi plot on the appropriate trajectory remains a matter of speculation. Nor, can we be sure from the present experimental situation that we really are studying $qq\bar{q}\bar{q}$ mesons. We still await the discovery of a truly exotic resonance, which the present round of contradictory experiments, may or may not have seen¹⁰⁾. Future experiments, now at the proposal stage¹¹⁾, should provide the answer.

The narrowness of these states brings the need for some dynamical suppression. For those close to $\bar{p}p$ or $\bar{p}\Delta$ thresholds the old Freund-Waltz-Rosner selection rule⁴⁾ (or its more recent variations²¹⁾) might be sufficient, but the 2.95 GeV state¹⁴⁾ produced forwards in π^-p scattering, decaying to $(\bar{p}p\pi^-)$, poses a major

*) However, their much quoted result for this trajectory's intercept (-0.5 ± 0.3) is somewhat suspect and should really have a much larger uncertainty. This result was obtained by evaluating a finite mass sum rule for the backward production of ordinary mesons π, ρ, A_1, A_2, g in $\pi^-p \rightarrow pX$ at one value of u (viz. -0.2 GeV^2) for this Δ exchange process. Unfortunately, the u dependence of these resonance cross-sections from the data used⁵¹⁾ is not in agreement with Mueller-Regge expectations (too low mass?), so a different result is obtained at other values of u ⁵²⁾. Nonetheless a low-lying trajectory results. Indeed, Hoyer et al. look at other processes in wider u bins and find intercepts of -0.8 and -1.4 with undetermined errors.

***) These intercepts depend on the isospin of the exchange.

***) These use either more sophisticated versions of the calculations of Section 1.2, or dual bootstrap conditions in the j plane.

challenge. With the present demise of the Rossi-Veneziano scheme²¹⁾ (Section 5), the colour scheme of Chan and Høgaasen²²⁾ provides the best explanation⁵⁵⁾. Moreover, the cascade decay of the 2.95 GeV state to the 2.2 GeV meson, we have considered here, will, if confirmed, provide a powerful handle on the relationship of these states¹⁴⁾.

The third experimental quantity that we know about these resonances is their production cross-section. It is about this that duality is most definite, for it allows these cross-sections to be calculated once, of course, we understand their production mechanism. It is here that the backward produced states¹²⁾ at 1.936, 2.020 and 2.204 GeV stand apart for they are most decidedly created by baryon exchange. If these are really to be $qq\bar{q}\bar{q}$ mesons, which according to the duality properties implied by quark diagrams^{1),2)} build up $f-\omega-A_2-\rho$ exchange, we can calculate the cross-sections for their production. This we have done in this paper, performing many consistency checks to be sure we cannot be far wrong. We find that the observed narrow states contribute only 1/50 or 1/100 of the M_4 production needed to build these $q\bar{q}$ exchanges. We deduce from this that duality requires the existence of two distinct types of four quark states, which we call generically boojums^{9),13)} (Section 4.1). Those of type A give the narrow peaks seen in the Omega experiment¹²⁾ and may be labelled baryonium. However, they constitute only a few per cent of all M_4 production. Type B, the remaining 90%+, consists of broad $qq\bar{q}\bar{q}$ states with no special relationship to $B\bar{B}$ channels, which decay to multimeson modes.

As discussed in Sections 4 and 5, both such categories of states naturally arise in the QCD description of baryon dynamics of Rossi and Veneziano²¹⁾. However, in this scheme, it is only those coupling strongly to $B\bar{B}$ channels that generate $f-\omega-A_2-\rho$ exchange. We have shown that this is inconsistent with the data, casting doubt on the whole duality scheme incorporating junctions, unless the $\omega, \tilde{\omega}$ mix in a "magic" way.

This leaves us with the colouring schemes of Chan and Høgaasen²²⁾ and of Jaffe²⁰⁾ as the most likely home for our boojum states A and B. However, even here they rest uneasily requiring both baryoniums, mock and true, to be only a very small fraction of all boojums produced in backward scattering. For this, there is as yet no obvious theoretical explanation within these schemes.

While we believe the $qq\bar{q}\bar{q}$ interpretation of these backward produced peaks¹²⁾ is still, despite these complications, the most appealing, we mention in passing two other possibilities. Brayshaw⁵⁶⁾ has proposed a way to generate narrow states

from a subenergy singularity in three body processes, which impressively reproduces, with no free parameters, the masses of many experimental peaks. $\bar{N}\bar{N}$ bound states also seem very likely to exist, as potential model calculations suggest^{18),43),57)}.

We have studied how in virtual antibaryon-baryon scattering the duality between normal meson exchanges $f-\omega-A_2-\rho$, and M_4 resonance states is satisfied. We had expected that the narrow peaks recently observed in the $\bar{p}p$ mass spectrum of backward π^+p scattering would make an appreciable contribution, as suggested by simple notions of duality²⁾ and baryonium⁷⁾ and by the models of Ref. 21). We find that this is not the case. Rather this duality must be fulfilled by the existence of $qq\bar{q}\bar{q}$ states that form a meson continuum -- for these, baryonium is a complete misnomer. The narrow peaks observed in the $\bar{p}p$ spectrum may be $qq\bar{q}\bar{q}$ states, but if so, they make only a small contribution to generating $q\bar{q}$ exchanges. Thus the observed states play a minor role in building duality and not the central one subsumed in the baryonium schemes of Rosner, of Chew and of Rossi and Veneziano. They are thus not the states expected to complete our picture of duality for $\bar{B}B$ processes.

Note added: For completeness we mention that after this paper was typed, it was recognized that the highest momentum, at which the process $K^-p \rightarrow \Lambda\pi^0$ has been measured, is not 4.25 GeV/c³³⁾ as used in Sections 2 and 3 but 5 GeV/c⁵⁸⁾. The inclusion of these latter data reduces the β_{KAN}^2 coupling of Eq. (25) and consequently increases g_{NNf} in Eq. (36) by some 30%. This in turn increases the predicted cross-sections of Table 2 towards the top end of their error bars and hence would further strengthen our conclusions.

Acknowledgements

It is a pleasure to thank my many colleagues, both theorists and experimentalists, for helpful discussions. In particular I am grateful to Ed Berger, Alain Borg, Chan Hong-Mo, Haim Harari, Hallstein Høgaasen, Paul Hoyer, Basarab Nicolescu, Arthur Rogers, Giancarlo Rossi, Fridger Schrempp and Gabriele Veneziano. I am especially indebted to Daniel Treille and other members of the Omega group.

Table 1

The cross-sections for the various backward scattering processes, integrated over $0 \leq -u' \leq 1 \text{ GeV}^2$, used in the calculations of Section 2

process	p_{lab} GeV/c	reaction cross-section μb	Ref.
$K^- p \rightarrow \Lambda \pi^0$	4.25	5.3 ± 0.5	17
$\pi^+ p \rightarrow p \pi^+$	4.25	13.9 ± 0.5	18
$\pi^+ p \rightarrow p \pi^+$	13.75	0.31 ± 0.04	19
$\pi^- p \rightarrow \Delta \pi^0$	9	0.11 ± 0.03	16
$\pi^- p \rightarrow \Delta \pi^0$	12	0.05 ± 0.02	16

Table 2

The calculated cross-sections are given for the processes

in the two final state mass bins, shown shaded in Fig. 1 - that labelled 1 is $1.88 - 2.16 \text{ GeV}$ and 2 is $2.08 - 2.32 \text{ GeV}$. The last column gives the experimentally measured cross-sections for the peaks seen in these mass bins for

process $\pi^- p \rightarrow$	mass bins	p_{lab} GeV/c	calculated cross-sections μb	peak cross-sections nb
ΔM_4	1	9	3.5 ± 1.5	18 ± 5
ΔM_4	1	12	1.4 ± 0.6	19 ± 9
ΔM_4	2	9	3.8 ± 1.6	17 ± 5
ΔM_4	2	12	1.8 ± 0.8	21 ± 5
$N^* M_4$	1	9	2.8 ± 1.3	30 ± 12
$N^* M_4$	1	12	1.8 ± 0.8	33 ± 13

REFERENCES

- 1) H. Harari, Phys. Rev. Letters 22 (1969) 562.
J.L. Rosner, Phys. Rev. Letters 22 (1969) 689.
- 2) J.L. Rosner, Phys. Rev. Letters 21 (1968) 950.
- 3) H.J. Lipkin, Nuclear Phys. B9 (1969) 349; Phys. Letters 32B (1970) 301.
- 4) P.G.O. Freund, R. Waltz and J.L. Rosner, Nuclear Phys. B13 (1969) 237.
- 5) M. Jacob and J. Weyers, Nuovo Cimento 69 (1970) 521.
H.J. Lipkin, Phys. Rev. D7 (1973) 2262.
J.L. Rosner, Phys. Reports 11C (1974) 189.
- 6) S. Mandelstam, Phys. Rev. D1 (1970) 1734.
- 7) G.F. Chew, "States of Baryonium within the Hadron Family", Invited talk at the 3rd European Symposium on Antinucleon-Nucleon Interaction, Stockholm, July 1976, LBL-5391 preprint (1976).
- 8) A.S. Carroll et al., Phys. Rev. Letters 32 (1974) 247.
T.E. Kalogeropoulos and G.S. Tzanakos, Phys. Rev. Letters 34 (1975) 1047.
V. Chaloupka et al., Phys. Letters 61B (1976) 487.
W. Brückner et al., Phys. Letters 67B (1977) 222.
- 9) C.L. Dodgson (Lewis Carroll), "The Hunting of the Snark", Macmillan (1876).
- 10) J. Boucrot et al., Nuclear Phys. B121 (1977) 251.
A. Rogers, report on preliminary results of the Indiana-Purdue-SLAC-Vanderbilt Collaboration, given at the 5th Int. Conf. on Experimental Meson Spectroscopy, Boston (April 1977).
- 11) V. Chaloupka et al., SLAC-Tufts proposal at SLAC (July 1977).
J. Six and D. Treille, private communication (CERN).
- 12) P. Benkheiri et al., Phys. Letters 68B (1977) 483.
- 13) Oxford English Dictionary Supp. Vol. I, 323, Oxford (1972).
- 14) C. Evangelista, Phys. Letters 72B (1977) 139.
- 15) H. Nicholson et al., Phys. Rev. D7 (1973) 2572.
B. Hyams et al., Nuclear Phys. B73 (1974) 202.
A.A. Carter et al., Phys. Letters 67B (1977) 117.
- 16) Huan Lee, Phys. Rev. Letters 30 (1973) 719.
G. Veneziano, Phys. Letters 43B (1973) 413.
- 17) M.R. Pennington and A. Guña, Rutherford Lab. preprint RL-75-024, T.106 (February 1975) (unpublished); Nuclear Phys. B96 (1975) 535.
- 18) See, for example, L.N. Bogdanova, O.D. Dalkarov and I.S. Shapiro, Ann. Phys. 84 (1974) 261.

- 19) K. Johnson and C.B. Thorn, Phys. Rev. D13 (1976) 1934.
- 20) R.L. Jaffe, MIT preprint CTP 657 (August 1977).
- 21) G.C. Rossi and G. Veneziano, Nuclear Phys. B123 (1977) 507.
M. Imachi, S. Otsuki and F. Toyoda (Kyushu University preprint 76-HE-4, 1976).
- 22) Chan Hong-Mo and H. Høgaasen, Phys. Letters 72B (1977) 121.
- 23) V. Barger and D. Cline, Phys. Rev. Letters 21 (1968) 392 (erratum 1132).
A.D. Martin and C. Michael, Phys. Letters 32B (1970) 297.
E.L. Berger and G.C. Fox, Nuclear Phys. B26 (1971) 1.
J.K. Storrow and G.A. Winbow, Nuclear Phys. B53 (1973) 62; B54 (1973) 560.
- 24) A. Guila and M.R. Pennington, Nuclear Phys. B97 (1975) 461.
- 25) A. Rougé et al., Phys. Letters 69B (1977) 115.
- 26) P.G.O. Freund, Phys. Rev. Letters 20 (1968) 235.
H. Harari, Phys. Rev. Letters 20 (1968) 1395.
- 27) P. Hoyer, R.G. Roberts and D.P. Roy, Nuclear Phys. B56 (1973) 173.
- 28) A. Borg et al., Nuovo Cimento 22A (1974) 559.
K. Paler et al., Phys. Letters 43B (1973) 437.
- 29) A. Borg, Thesis, Université de Paris VI (May 1977).
- 30) R. Carlitz, M.B. Green and A. Zee, Phys. Rev. Letters 26 (1971) 1515;
Phys. Rev. D4 (1971) 3439, 3501.
- 31) T. Inami and R.G. Roberts, Nuclear Phys. B93 (1975) 497.
- 32) P. Benkheiri et al., contribution to the European Conf. on Particle
Physics, Budapest (July 1977); Ecole Polytechnique preprint LPNHE/X/77.
- 33) F. Marzano et al., Nuclear Phys. B123 (1977) 203.
- 34) R.A. Sidwell et al., Phys. Rev. D3 (1971) 1523.
- 35) D.P. Owen et al., Phys. Rev. 181 (1969) 1794.
- 36) C. Louedec et al., Nuovo Cimento 41A (1977) 166.

R. Barloutaud, invited talk at XVIII Int. Conf. on High Energy Physics,
Tbilisi, July (1976); Saclay preprint DPhPE 76-17.
- 37) P.V. Chliapnikov et al., Nuclear Phys. B112 (1976) 1.
- 38) P.V. Chliapnikov et al., Nuclear Phys. B131 (1977) 93.
- 39) M.G. Albrow et al., Contribution to the XVI Int. Conf. on High Energy
Physics, Batavia, September (1972).

- 40) J.R. Johnson et al., Phys. Rev. Letters 39 (1977) 1173.
- 41) J.V. Allaby et al., Contribution to the IVth Int. Conf. on High Energy Collisions, Oxford (1972).
A.N. Diddens and K. Schlüpmann, Landolt-Börnstein 6 (1972) 53.
- 42) L. Montanet, Contribution to the Vth Int. Conf. on Experimental Meson Spectroscopy, Boston, April (1977); CERN/EP/PHYS 77-22 (June 1977).
- 43) W.W. Buck, C.B. Dover and J.M. Richard, BNL preprint-23315 (September 1977).
- 44) H. Harari, Phys. Rev. Letters 26 (1971) 1079.
- 45) P.A. Baker et al., Nuclear Phys. B89 (1975) 189.
- 46) A. Borg et al., Nuclear Phys. B106 (1976) 430.
- 47) P. Bosetti et al., Nuclear Phys. B54 (1973) 141.
- 48) P. Kroll, Fort. der Physik 24 (1976) 565.
- 49) C.A. Levinson, N.S. Wall and H.J. Lipkin, Phys. Rev. Letters 17 (1966) 1122.
- 50) P. Hoyer, R.G. Roberts and D.P. Roy, Phys. Letters 44B (1973) 258.
- 51) E.W. Anderson et al., Phys. Rev. Letters 22 (1969) 1390.
- 52) Correspondence between A. Rougé and M.R. Pennington (unpublished).
- 53) B. Nicolescu, LBL preprint 6701 (July 1977).
- 54) C. Rosenzweig, Phys. Rev. Letters 36 (1976) 697.
K. Konishi, Nuclear Phys. 131 (1977) 143.
F. Toyoda and M. Uehara, Kyushu-77-HE-4 preprint (1977).
L.A.P. Balazs and B. Nicolescu, Phys. Letters 72B (1977) 240.
- 55) Chan Hong-Mo and H. Høgaasen, Rutherford Lab. preprint RL-77-121/A (October 1977).
- 56) D.D. Brayshaw, Phys. Rev. Letters 39 (1977) 371.
- 57) B.R. Karlsson and B. Kerbikov, CERN preprint TH.2436 (January 1978).
- 58) J. Gallivan et al., Nuclear Phys. B117 (1976) 269.

Figure captions

- Fig. 1 : Plot of $\bar{p}p$ mass spectrum from $\pi^- p \rightarrow \Delta(\bar{p}p)$ in which the 9 and 12 GeV/c data¹²⁾ have been combined and a cut on the Jackson angle made to ensure the $\bar{p}p$ pair are produced in the backward hemisphere. The shaded regions show the two mass bins in which the complete $M_{\bar{p}p}$ production cross-sections are calculated in Section 3 and listed in Table 2.
- Fig. 2 : The quark content of the Mueller-Regge diagrams contributing to the resonant cross-section for $MN \rightarrow BX$ in the beam fragmentation region is shown, particularly displaying their duality properties.
- Fig. 3 : Plot of the invariant cross-section for $K^- p \rightarrow \Lambda X$ at 14.3 GeV/c²⁸⁾ compared with curves calculated as described in Section 3 with coupling of Eq. (35): here as a function of M^2/s at fixed values of u . The curves are plotted for the central value of u in the range of the data.
- Fig. 4 : As for Fig. 3, but shown as a function of u at fixed values of M^2/s .
- Fig. 5 : Integrated inclusive cross-section for $K^+ p \rightarrow \bar{\Lambda} X$ in the beam fragmentation region at fixed x ^{37),38)}. The curves are calculated with the coupling of Eq. (38).
- Fig. 6 : Invariant inclusive cross-section for $pp \rightarrow \pi^+ X$ as a function of x at fixed $p_T = 0.8$ GeV/c from the ISR³⁹⁾. The curve is calculated with the coupling of Eq. (40).
- Fig. 7 : The quark and junction content of the Mueller-Regge diagrams contributing to the resonant cross-section for $MN \rightarrow BX$ in the beam fragmentation region, to be compared with Fig. 2.
- Fig. 8 : The quark and junction content of the Mueller-Regge diagram contributing in addition to the Pomeron to the non-resonant background for $MN \rightarrow BX$ in the beam fragmentation region.

- Fig. 9 : The quark and junction content of the Mueller-Regge diagram contributing to the resonant cross-section for $M_i B \rightarrow M_f X$ in the target fragmentation region.
- Fig. 10 : Plot of invariant cross-section for $K^- p \rightarrow \pi^+ X$ in the target fragmentation region integrated over the kinematically allowed domain of positive u as a function of M^2/s comparing the data of Refs. 45) and 46).
- Fig. 11 : Plot of invariant cross-section for $K^- p \rightarrow \pi^+ X$ at 14.3 GeV/c⁴⁶⁾ compared with the shaded bands calculated with the coupling of Eq. (50): here as a function of M^2/s at fixed values of u ; the curves are plotted for the central value of u in the range of the data.
- Fig. 12 : As for Fig. 11 but shown as a function of u at fixed values of M^2/s .

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

(a)

(b)

(c)

(d)

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

60
~~59~~