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TWO-STAGE MODEL OF HADRON STRUCTURE : PARTON DISTRIBUTIONS
AND THEIR Q^2 DEPENDENCE

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A B S T R A C T

A two-stage model - where partons belong to a constituent quark in a hadron - gives an excellent description of lepton-proton deep inelastic scattering and inclusive $\mu^+\mu^-$ production in pp collisions.

The Q^2 dependence of parton distributions is obtained by numerical integration of the Altarelli-Parisi equations. A discussion of these equations, partly based on exact analytic solutions, leads to general conclusions and suggestions on the Q^2 dependence of parton distributions in the asymptotic regions $x \approx 1$ and $x \approx 0$.

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1. INTRODUCTION

In this paper we continue the discussion on a two-stage model of parton distributions presented some time ago. In this model a hadron is pictured as containing a finite number of quarks (constituent quarks), each having a parton structure¹⁻³). We still consider this model to be of great interest, since we are convinced that the complex parton distributions observed in deep inelastic phenomena arise from a combination of short-distance and long-distance effects. While the gluon and quark-antiquark sea components are essentially a short-distance effect, certain characteristics of the valence quark distributions, such as those reflected in the large ratio of proton to neutron structure functions, are essentially a low Q^2 effect. In the model under consideration, effects of this latter type are derived from the wave function of the hadron as a system of constituent quarks, and more precisely from the deviation of such wave functions from exact $SU(6)_W$ symmetry⁴).

In the present paper we integrate the model by taking into account the Q^2 dependence of parton distributions prescribed by QCD. In fact, the availability of experimental data over a large Q^2 range now makes the scaling approximation made in Refs. 1, 2, and 3 unsatisfactory.

As we will see, the mathematical structure of the model, which we outline for completeness in Section 2, is stable under the effect of the leading QCD corrections. The Q^2 dependence of parton distributions arising from these corrections has been shown⁵⁻⁷) to be equivalent to the development of a gluon- $q\bar{q}$ pair shower; therefore, if, for a given value of Q^2 each parton can be assigned, as we assume, to an individual constituent quark, the same will be true at a larger value of Q^2 .

In Section 3 we will present the equations which describe the Q^2 dependence of parton distributions within a constituent quark, as derived from the results of Altarelli and Parisi⁷).

Section 4 is devoted to a discussion of the Q^2 dependence of parton distributions near $x = 1$ and $x = 0$. In the case of valence quarks we present exact asymptotic solutions of the evolution equations for $x \approx 0$ and $x \approx 1$, while in the case of gluons we have an asymptotic solution for $x \approx 1$ valid for $Q^2 \rightarrow \infty$. The information presented in this section is, to our knowledge, new, and can be very useful in the evaluation of different models of the parton distributions. Details on the derivation of the explicit solutions are given in the Appendix.

In Section 5 we present the results of a fit of the model to a selected sample of data on deep inelastic scattering of e ⁸) and μ ⁹) on protons and deuterons, and on μ -pair production in pp collisions¹⁰). The model gives an excellent

fit to these data and does this with a small number of adjustable parameters. The fit to the structure function of the proton allows, in this model, definite predictions on the structure functions of other hadrons, since the parton distributions inside a constituent should be the same for all hadrons. As an example, the structure functions of the pion (or K meson) will be determined by the wave function of the $q\bar{q}$ system, which requires a separate ansatz, and by the constituent structure functions, which we determine here³). Some of the features of parton distributions, such as the fraction of momentum carried by valence quarks or by the $q\bar{q}$ sea, which only depend on the parton distributions inside constituents, are expected to be the same for all hadrons.

Section 6 will be devoted to a discussion of our results and of possible extensions and alternative treatments.

2. OUTLINE OF THE MODEL

In this section we give an outline of the model in Ref. 1, to which we refer for a more detailed and explicit treatment.

A baryon of momentum p is described by a wave function

$$\Psi_{\{A^i\}}(P_T^i, x_i)$$

where A^i , p_T^i , and x_i label, respectively, the spin-isospin, transverse momentum, and longitudinal fraction of the i^{th} constituent. From the wave function we can compute the density of probability for finding (within a proton) a p or n constituent quark with a given fraction x of the proton longitudinal momentum. These functions will be denoted by $n_0(x)$, $p_0(x)$. They obey the sum rules

$$\begin{aligned} \int p_0(x) dx &= 2 \\ \int n_0(x) dx &= 1 \end{aligned} \tag{1}$$

expressing the fact that a proton contains two p and one n constituent quarks. Furthermore, we will have

$$\int x [p_0(x) + n_0(x)] dx = 1 \tag{2}$$

expressing the hypothesis that all of the proton momentum is carried by constituent quarks. This is, in fact, the basic simplifying assumption of our model. The specification of the parton distributions will require the knowledge of a set of functions $\phi_{ab}(x, Q^2)$ which express the probability distribution for finding a parton of type b in a constituent of type a . These functions depend on the fraction x of the constituent momentum carried by the parton, as well as on Q^2 , the latter dependence being a manifestation of scaling violation.

The parton distributions in a proton, $q(x, Q^2)$, will then be obtained by a convolution of the ϕ_{ab} distributions and the n_0 and p_0 distributions^{*}):

$$q(x, Q^2) = \int_x^1 \frac{dz}{z} \left[p_0(z) \phi_{pq}\left(\frac{x}{z}, Q^2\right) + n_0(z) \phi_{nq}\left(\frac{x}{z}, Q^2\right) \right] \quad (3)$$

The parton q can be a quark ($p, n, \lambda, c, \dots$) or an antiquark, or a gluon (g).

Our model will be specified by assigning an ansatz for the wave function ψ and for the parton distributions ϕ_{ab} . The ansatz for ϕ_{ab} is based on the idea that a constituent contains a single quark-parton carrying its flavour quantum numbers (the valence quark), and a sea of gluons and quark antiquark pairs. We will thus have^{**})

$$\phi_{aa}(\xi, Q^2) = \phi^v(\xi, Q^2) + \phi^s(\xi, Q^2) \quad (4)$$

$$\phi_{aq}(\xi, Q^2) = \phi^s(\xi, Q^2) \quad (5)$$

$$\phi_{ag}(\xi, Q^2) = \phi^g(\xi, Q^2) \quad (6)$$

where q is a non-charmed quark (different from a) or antiquark, and g denotes a gluon. ϕ^v , ϕ^s and ϕ^g describe respectively the valence, non-charmed $q\bar{q}$ sea, and gluon component. We will also denote by $\phi^c(\xi, Q^2)$ the distribution of c and $\bar{c}$ quarks, assumed to be different from that of other sea q and $\bar{q}$. We will assume the following form for ϕ 's at some fixed value of Q^2 , Q_0^2 :

$$\phi^v(\xi, Q_0^2) = \frac{\Gamma(A + 1/2)}{\Gamma(1/2) \Gamma(A)} \frac{(1-\xi)^{A-1}}{\sqrt{\xi}} \quad (7)$$

$$\phi^s(\xi, Q_0^2) = \frac{C}{\xi} (1-\xi)^{D-1} \quad (8)$$

$$\phi^g(\xi, Q_0^2) = \frac{G}{\xi} (1-\xi)^{E-1} \quad (9)$$

$$\phi^c(\xi, Q_0^2) = 0 \quad (10)$$

A few comments on this choice are in order:

- i) The power behaviour near $x = 0$ has been fixed on the basis of Regge-pole arguments¹⁾.

^{*}) Equation (3) has been discussed for the first time in the framework of QCD by Parisi and one of us¹¹⁾ with a different ansatz for the probability distributions $\phi_{ab}(\xi, Q^2)$.

^{**}) We depart here from the notation of Ref. 1 where ϕ^v and ϕ^s were denoted by $\phi^{(a)}$ and $\phi^{(c)}$.

- ii) The behaviour near $x = 1$ has been assumed to be a simple power behaviour determined in each case by a parameter, A, D, E, to be fixed by fitting experimental data.
- iii) The numerical coefficient of ϕ^v is determined by the sum rule

$$\int \phi^v(\xi, Q_0^2) d\xi = 1 \quad (11)$$

expressing the presence of one valence quark in each constituent.

- iv) One of the remaining parameters can be eliminated through the momentum sum rule:

$$\int \xi (\phi^v + 6\phi^s + \phi^g) d\xi = 1 \quad (12)$$

From this we obtain

$$G = E \left(1 - 6 \frac{C}{D} - \frac{1}{2A+1} \right) \quad (13)$$

- v) The assumption $\phi^c(\xi, Q_0^2) = 0$ represents a rough attempt at introducing the threshold for $c\bar{c}$ production. This is clearly not very satisfactory, since charm can be produced at any value of Q^2 , at a small enough x . Some sign of this is probably seen in the narrow spike shown by proton structure functions near $x = 0$, at moderate Q^2 values. We will discuss this point in Section 5.

We will maintain in the present paper the ansatz for the proton wave function used in Ref. 1. It contains two adjustable parameters: one, β , is associated with the width of the x distributions $n_0(x)$, $p_0(x)$; the other, a , is a mixing parameter associated with a breaking of $SU(6)_W$ ⁴⁾ and a difference between $p_0(x)$ and $2n_0(x)$. We recall that this mixing of $SU(6)_W$ representations was introduced in Ref. 1 in order to explain the large values of F_2^{ep}/F_2^{en} near $x = 1$.

In conclusion, we use six adjustable parameters in the description of the parton distributions for a fixed value of Q^2 : two, β and a , enter into the construction of the wave function, the other four, A, C, D, E, determine the parton distribution within a constituent.

3. Q^2 DEPENDENCE

The Q^2 dependence implied by leading log QCD corrections can be equivalently expressed either as a set of equations for the moments¹²⁾ of parton distributions or as a system of integral differential equations for the distributions themselves^{6,7)}. We will adopt here the second formulation. The Q^2 dependence of the functions ϕ^v , ϕ^s , ϕ^g , and ϕ^c is determined by

$$\frac{d\phi^v(x, Q^2)}{d[\ln Q^2/Q_0^2]} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \phi^v(y, Q^2) P_{qq}(x/y) \quad (14)$$

$$\frac{d\phi^{s,c}(x, Q^2)}{d[\ln Q^2/Q_0^2]} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} [\phi^{s,c}(y, Q^2) P_{qq}(x/y) + \phi^g(y, Q^2) P_{qg}(x/y)] \quad (15)$$

$$\frac{d\phi^g(x, Q^2)}{d[\ln Q^2/Q_0^2]} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left\{ [\phi^v(y, Q^2) + 6\phi^s(y, Q^2) + 2\phi^c(y, Q^2)] P_{gq}(x/y) + \phi^g(y, Q^2) P_{gg}(x/y) \right\} \quad (16)$$

where the kernels P_{qq} , P_{qg} , P_{gq} , and P_{gg} have been explicitly written in Ref. 7, and $\alpha(Q^2)$ is the Q^2 dependent effective strong coupling constant

$$\alpha(Q^2) = \frac{12\pi}{33-2n_F} \left[\ln(Q^2/\Lambda^2) \right]^{-1} \equiv \frac{1}{b \ln(Q^2/\Lambda^2)} \quad (17)$$

It is easily seen by substitution that, if the $\phi^{v,s,g}(\xi, Q^2)$ obey the equations written above, the parton distribution given in Eq. (3) will obey the equations given in Ref. 7, and have therefore the correct Q^2 dependence.

In order to integrate Eqs. (14)-(16) numerically, it is convenient to adopt a variable T :

$$T(Q^2) = \ln \left[\frac{\alpha(Q_0^2)}{\alpha(Q^2)} \right] = -\ln[\alpha(Q^2)] + \text{const.}$$

In terms of this variable and using results from Ref. 7, we write explicitly Eqs. (14)-(16) in the form:

$$\frac{d\phi^v}{dT} = \mathcal{A} \phi^v \quad (18)$$

$$\frac{d\phi^{s,c}}{dT} = \mathcal{A} \phi^{s,c} + \mathcal{B} \phi^g \quad (19)$$

$$\frac{d\phi^g}{dT} = \mathcal{C} \phi^g + \mathcal{D} (\phi^v + 6\phi^s + 2\phi^c) \quad (20)$$

where $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ are integral operators defined as

$$\begin{aligned}
 (\mathcal{A}q)_x &= \frac{2}{3\pi b} \left\{ \int_x^1 dy \frac{(1 + (\frac{x}{y})^2)}{y-x} (q(y) - \frac{x}{y} q(x)) + \right. \\
 &\quad \left. + [2 \ln(1-x) + x + \frac{x^2}{2}] q(x) \right\} \\
 (\mathcal{B}q)_x &= \frac{1}{4\pi b} \int_x^1 \frac{dy}{y} [2(\frac{x}{y})^2 - 2\frac{x}{y} + 1] q(y) \\
 (\mathcal{C}q)_x &= \frac{3}{\pi b} \left\{ \int_x^1 \frac{dy}{y-x} [q(y) - \frac{x}{y} q(x)] + \right. \\
 &\quad \left. + \int_x^1 \frac{dy}{y} (-2 + \frac{y}{x} + \frac{x}{y} - \frac{x^2}{y^2}) q(y) + (\frac{25}{36} + \ln(1-x)) q(x) \right\} \\
 (\mathcal{D}q)_x &= \frac{2}{3\pi b} \frac{1}{x} \int_x^1 dy (2 - 2\frac{x}{y} + \frac{x^2}{y^2}) q(y)
 \end{aligned} \tag{21}$$

These equations, together with the ansatz for $\phi^i(\xi, Q_0^2)$, determine the parton distribution for any value of ξ and Q^2 . For $Q^2 \leq Q_0^2$ we will assume that $\phi^C(x, Q^2) = 0$, while for $Q^2 > Q_0^2$ we will determine ϕ^C through Eq. (15).

4. ASYMPTOTIC BEHAVIOUR OF DISTRIBUTIONS NEAR $x = 1$ and $x = 0$

We have obtained asymptotic solutions for $x \approx 1$ and $x \approx 0$ of the evolution equation for the valence distribution, Eq. (18), which match our ansatz for $Q^2 = Q_0^2$, and an asymptotic solution for the gluon distribution near $x = 1$ at large Q^2 . The asymptotic behaviour of the valence distributions has already been presented for $x \approx 1$ by Gribov and Lipatov¹³⁾ and for $x \approx 0$ by Parisi¹⁴⁾; the use of the evolution equations allows, however, a more transparent derivation, which we present in the Appendix. The asymptotic solution for the gluon distribution is, to our knowledge, new. Some suggestion on the Q^2 dependence of the gluon distribution near $x = 0$ and of the sea quarks near $x = 0$ and $x = 1$ can be obtained from Eqs. (19) and (20). We will present here some results of this analysis.

i) Asymptotic behaviour of ϕ^V for $x \approx 1$

The solution of Eq. (A2), normalized as in Eq. (11), is

$$\phi^V(x, Q^2) \approx \frac{\Gamma(A+\frac{1}{2})}{\Gamma(\frac{1}{2}) \Gamma(A(\tau))} \exp(K\tau) (1-x)^{A(\tau)-1} + O[(1-x)^{A(\tau)}] \quad (22)$$

where

$$A(\tau) = A + \frac{4}{3\pi b} \tau \quad (23)$$

and

$$K = \frac{4}{3\pi b} (0.75 - C)$$

b has been defined in Eq. (17), and C is the Euler constant: $C \approx 0.57$. We note that the effective exponent $A(\tau)$ is an increasing function of Q^2 , while the coefficient decreases with Q^2 .

ii) Behaviour of ϕ^V for $x \approx 0$

From Eq. (A9), normalizing to Eq. (11), we find

$$\phi^V(x, Q^2) \approx \frac{\Gamma(A+\frac{1}{2})}{\Gamma(\frac{1}{2}) \Gamma(A)} \exp(K'\tau) x^{-\frac{1}{2}} + O(x^0) \quad (23)$$

where

$$K' = \frac{2}{3\pi b} (4 \ln 2 - 7/6)$$

iii) Asymptotic behaviour of ϕ^g for $x \approx 1$

Substituting Eqs. (7)-(9) in Eq. (20), we can derive the following asymptotic expression for ϕ^g , valid for $x \approx 1$ and large Q^2 (see Appendix)

$$\phi^g(x, Q^2) \xrightarrow[Q^2 \rightarrow \infty]{} \frac{2}{5A(\tau)} \frac{1-x}{|\ln(1-x)|} \phi^V(x, Q^2) \quad (24)$$

iv) Behaviour of ϕ^g near $x = 0$

If we rewrite Eq. (9) as

$$\phi^g(x, Q^2) = \frac{G}{x} + \tilde{\phi}^g(x, Q_0^2)$$

where $\tilde{\phi}^g(x, Q_0^2)$ is integrable at $x = 0$, we obtain from Eq. (20):

$$\frac{d\phi^g(x, Q^2)}{d\tau} \Big|_{Q^2=Q_0^2} = \frac{3}{\pi b x} \left\{ G \left(\ln \frac{1}{x} - \frac{41}{36} \right) + \int_0^1 [\tilde{\phi}^g(y, Q_0^2) + \frac{4}{9} \phi^V(y, Q_0^2)] dy \right\} + O(x^0) \quad (25)$$

We thus see that the $1/x$ behaviour, associated with the Pomeron singularity, reproduces itself, but is complicated by slightly more divergent $(1/x) \ln(1/x)$ terms $[(1/x) \ln^n(1/x)]$ at higher iterations of Eq. (20).

v) Behaviour of the sea distribution at $x \approx 1$ and $x \approx 0$

The gluon term in Eq. (19) gives a contribution near $x = 1$ of the form

$$\left. \frac{d\phi^S}{dT} \right|_{Q^2=Q_0^2} \propto (1-x)^E$$

This suggests that the exponent of the sea distribution will roughly be related to that for the gluon: $D \approx E + 1$.

At the same time, the asymptotic behaviour of ϕ^g in Eq. (24) suggests

$$E \approx A + 1$$

This suggestion is based on the idea that the ϕ 's at $Q^2 = Q_0^2$, which we take as initial data for integrating the evolution equations, can also be seen as the result of the evolution from smaller Q^2 values. One should, however, keep in mind that the equations lose their validity at small Q^2 , where the $\alpha(Q^2)$ becomes large. In Section 5 we will see that the fitted values for D , E , and A are only in rough agreement with these suggestions.

Near $x = 0$ the evolution equation for the sea distribution ϕ^S behaves in much the same way as that for ϕ^g : the $1/x$ singularity reproduces itself, with the addition of more singular $[(1/x) \ln(1/x), (1/x) \ln^2(1/x), \text{etc.}]$ terms.

5. A FIT TO EXPERIMENTAL DATA

We have used the model specified in the previous sections to fit data from lepton-nucleon deep inelastic scattering and from muon-pair production in proton-nucleus collisions. Data on these processes are very abundant, and come from different sources. We have selected a sample of such data which includes

- D₁) Values of $F_2^p(x, Q^2)$ and $F_2^d(x, Q^2)$ at $(8 \text{ GeV}^2) \leq Q^2 \leq (10 \text{ GeV}^2)$. In this range data from ep⁸⁾ and from μp ⁹⁾ cover the range from $x = 0.05$ to $x = 0.75$. In our fit we have considered all data in this range as corresponding to $Q^2 = 9 (\text{GeV})^2$, taking for each x the average of points from the same source, when more than one point existed.
- D₂) For selected values of x -- $1/3$, $1/2$, $2/3$ -- we have considered the Q^2 dependence of F_2^p as given in Ref. 8.
- D₃) We have considered the recent data of $pp \rightarrow \mu^+\mu^- + X$ of Ref. 10. These data test -- given the Drell-Yan mechanism¹⁵⁾ -- the sea and valence distributions

in the x range from $x \approx 0.2$ to $x \approx 0.5$ and, correspondingly, $Q^2 \approx M_{\mu\mu}^2$ from 30 GeV^2 to $\sim 200 \text{ GeV}^2$. Fitting these data gives thus a stringent test of the correct treatment of sea quarks, and of the scaling violations. Rather than the data themselves, which have only been shown in graphical form, we have compared the model with a numerical fit given in Ref. 10:

$$\frac{d^2\sigma}{dM dy} \Big|_{y=0} = (1.26 \pm 0.02) \exp[-(0.953 \pm 0.01) M^{n^*n^*}] \times 10^{-33} \text{ cm}^2 / \text{GeV} \cdot \text{nucleon} \quad (26)$$

The model, as specified in Sections 2 and 3, has been fitted to these data with $Q_0^2 = 9 \text{ GeV}^2$. We first considered the data D_1 . These do not require, given our choice of Q_0^2 , the use of the evolution equations, and are only sensitive to the five parameters β , a , A , C , and D defined in Section 2. The fitting procedure follows the same lines discussed in Ref. 1. In this way, we obtain a starting value for these parameters. As for the $q\bar{q}$ sea parameters C and D , this first step gives only an indication, since data on deep inelastic scattering are not very sensitive to the sea distributions.

In order to treat data, and give predictions, at $Q^2 \neq Q_0^2$, we set up a program for the stepwise integration of the evolution equations. This is quite easy, since these equations give an explicit expression for the derivative of the distribution functions, and the expressions on the right-hand side involve convergent integrals. As a test of our procedure, we show in Fig. 1 $\phi^V(x)$ at $Q^2 = 200 \text{ GeV}^2$, compared with the two exact asymptotic solutions (see Section 4) for $x \approx 0$ and $x \approx 1$. In this figure we use the parameters determined by our fit as presented in this section.

To determine the value of Λ^2 , which defines the value of $\alpha_s(Q^2)$, we examine the Q^2 dependence of F_2^{ep} at $x = 0.5$. At this value of x the sea contribution is negligible, while the mass corrections are expected to be small^{16,17)}. A possibly non-negligible correction we have not taken into account is that arising from next to leading corrections to the evolution equations, corresponding to higher order corrections to the Callan-Symanzig functions β and γ . These corrections could amount to 10-20% effects, as indicated by the explicit computations given in Buras et al.¹⁸⁾, but their omission will be partly compensated -- *over a limited Q^2 range* -- by a readjustment of the value of Λ^2 .

Finally, data D_3 are used to fix the parameters C , D , and E , which determine the sea and gluon contributions. The fitting procedure leads to a tight determination of the parameters in the model, except for the gluon exponent E . In fact we have obtained good fits for E varying between 2 and 3. The parameters in Table 1 are used in the fit presented in the figures 2 to 5.

The fit to F_2^p and F_2^d at $Q^2 = Q_0^2 = 9 \text{ GeV}^2$ is shown in Fig. 2 and the Q^2 dependence of F_2^p at three values of x , in Fig. 3. Figure 4 gives a comparison of our results with data at $Q^2 = 3.5 \text{ GeV}^2$. The fit to the Lederman data is represented in Fig. 5 as a ratio of our result to the central value of the fit in Eq. (26). This figure does not do justice to the quality of our fit. If we plot our results on the figure presented in Ref. 10, the fit is indistinguishable from that of Eq. (26) and follows the data for about five decades. We tried to compare our results with the data of Antreasyan et al.¹⁹⁾. We find a good agreement at those points which are not influenced by the Y resonance, corresponding to $M_{\mu^+\mu^-}$ less than 7-8 GeV.

The over-all quality of the fit is very good, with the exception of a few very low x points for F_2^p at $Q^2 \approx 3.5 \text{ GeV}^2$. The failure to fit these points probably reflects our rough treatment of the charm threshold. In fact, we assume $\phi^c = 0$ for $Q^2 \leq Q_0^2$, while $c\bar{c}$ pairs can be produced, even at small Q^2 , at small enough x . The effect of this on F_2 can be gauged by the relation:

$$\Delta^{charm} F_2^{ep} = \frac{Q^2}{4\pi^2\alpha} \sigma_\tau^{charm} \approx \frac{Q^2}{4\pi^2\alpha} \sigma_\gamma^{charm}(\nu) \quad (27)$$

A cross-section for charm photoproduction $\sigma_\gamma^{charm} \approx 1-2 \text{ } \mu\text{b}$ corresponds at $Q^2 \approx 3.5 \text{ GeV}^2$ to a change in F_2^{ep} of 0.03-0.06, which is of the correct magnitude to explain the low x discrepancies in Fig. 4. We propose to return to this problem in the future.

6. CONCLUDING REMARKS

The basic ingredients of our method of evaluation of parton distributions are:

- a) an exact treatment of the evolution equations presented in Section 2;
- b) the use of the model of non-pointlike constituents presented in Section 1.

The second of these ingredients has an important heuristic value, since parton distributions in constituent quarks are more likely to have a simple structure than parton distributions in a hadron, so that simple ansätze, such as those in Eqs. (7)-(10), are more justified than the similar ones usually given for the proton structure functions²⁰⁾.

An important feature of the model is that the ϕ 's are expected to be the same for all hadrons, so that detailed predictions can be made on processes such as the lepton pair production in πN or KN collisions. This point was already discussed in Ref. 3, where the scaling distributions of Ref. 1 were used to

predict lepton pair production in πN and KN collisions. A better prediction can now be made with the present ϕ 's, which incorporate the most important source of Q^2 dependence, and give an excellent fit to present data on $pp \rightarrow e^+ + e^- + \dots$.

Two applications of the proton wave function were presented in Ref. 1 [a correction to the SU(6) prediction for G_A/G_V] and Ref. 21 [a calculation of D/F for SU(3) breaking in the baryon octet]. The results presented there are modified by the new fit leading to:

$$G_A/G_V = 1.33 \quad (= 1.52 \text{ in Ref. 1})$$

$$D/F = -0.45 \quad (= -0.3 \text{ in Ref. 21})$$

We have a clear improvement in the prediction for G_A/G_V , while the excellent agreement found in Ref. 21 with the experimental value, $D/F = -0.31$, is substantially spoiled. It will be interesting at some point to review these predictions, and to follow the evolution of the fit parameters which will certainly be required by the future availability of more accurate deep inelastic data.

Another important application of the model is to deep inelastic neutrino data. This was discussed in Ref. 2 on the basis of the scaling distributions of Ref. 1. We will now be able to give more detailed predictions on the basis of our Q^2 dependence distributions. However, a detailed description of charm production in neutrino processes will require two refinements in respect to the model presented here: the first is to abandon the hypothesis of an SU(3) symmetric $q\bar{q}$ sea: $\lambda\bar{\lambda}$ pairs are likely to be less abundant than $p\bar{p}$ or $n\bar{n}$ pairs*). Another needed improvement of particular importance at intermediate neutrino energies (30-70 GeV) is a more accurate treatment of the charm threshold, which, as noted in the closing of Section 5, is already required by present electroproduction data.

The other ingredient in our treatment is the exact treatment of the evolution equations. Our explicit asymptotic solutions for ϕ^V , as well as the indications on the behaviour of ϕ^S and ϕ^8 presented in Section 4, show that forms of the kind $x^\beta(1-x)^\gamma$ cannot give an adequate description of parton distributions *at all* Q^2 . The forcing of such a fit is bound to give difficulties, especially at $x \approx 0$ **), and might lead to unrealistic distributions for the $q\bar{q}$ sea.

*) Strange quarks play a minor role in the electromagnetic phenomena considered here, so the simplifying assumption of SU(3) symmetry was well justified.

**) We are thinking here of the method of Buras et al.²²⁾, where the exponents β and γ were obtained by fitting the Q^2 dependence of moments of parton distribution. This procedure favours a good fit at large x values.

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1. VALENCE DISTRIBUTION FOR $x \approx 1$

Let us make the following ansatz:

$$\phi^v(x, Q^2) = B(Q^2) (1-x)^{A(Q^2)-1} + O[(1-x)^{A(Q^2)}] \quad (A1)$$

Substituting this in Eq. (18), we find, by direct integration²³, that

$$\frac{d\phi^v}{dT} = \frac{4B}{3\pi b} \left\{ -\frac{d}{dA} \ln \Gamma(A) - C + \frac{3}{4} + \ln(1-x) \right\} (1-x)^{A-1} + O[(1-x)^A] \quad (A2)$$

where C is the Euler constant, $C = 0.5772$. We can compare this with the explicit derivative of (A1):

$$\frac{d\phi^v}{dT} = [\dot{B} + B \dot{A} \ln(1-x)] (1-x)^{A-1} + O[(1-x)^A] \quad (A3)$$

and obtain

$$\dot{A} = \frac{4}{3\pi b} \quad (A4)$$

and [here we re-express $4/3 \pi b$ as $\dot{A}$, according to Eq. (A4)]

$$\frac{\dot{B}}{B} = -\dot{A} \left[\frac{d}{dA} \ln \Gamma(A) + \frac{3}{4} - C \right] \quad (A5)$$

Equations (A4) and (A5) are then integrated to give

$$A(Q^2) = A(Q_0^2) + \frac{4}{3\pi b} T \quad (A6)$$

$$B(Q^2) = B(Q_0^2) \frac{\Gamma[A(Q_0^2)]}{\Gamma[A(Q^2)]} \exp \left[\left(\frac{3}{4} - C \right) \frac{4T}{3\pi b} \right] \quad (A7)$$

Putting

$$A(Q_0^2) = A$$

$$B(Q_0^2) = \Gamma(A + 1/2) / [\Gamma(A) \Gamma(1/2)]$$

and substituting (A6) and (A7) in (A1) we have the solution in Eq. (22) of Section 4.

2. VALENCE DISTRIBUTION FOR $x \approx 0$

If we substitute into the right-hand side of Eq. (18) a ϕ^v of the form:

$$\phi^v = \frac{B(Q^2)}{x^\alpha} + o(x^0) \quad (A8)$$

with $0 < \alpha < 1$, we obtain, by explicit integration,

$$\frac{d\phi^v}{dT} = \frac{2}{3\pi b} I(\alpha) \frac{B(Q^2)}{x^\alpha} + f(x) \quad (A9)$$

where

$$I(\alpha) = \int_0^1 \frac{d\xi}{\xi} (1+\xi^2) \frac{\xi^\alpha - \xi}{1-\xi} \quad (A10)$$

and $f(x)$ has a $\log x$ singularity at $x = 0$. We thus obtain asymptotic solutions of the form

$$\phi^v(x, Q^2) = B(Q_0^2) \exp \left[\frac{2}{3\pi b} I(\alpha) T \right] x^{-\alpha} + R(x) \quad (A11)$$

where $R(x)$ can be shown to be less singular, at $x = 0$, than $x^{-\epsilon}$ with ϵ arbitrarily small. For $\alpha = 1/2$ we find

$$I(1/2) = 4 \ln 2 - 7/6$$

from which we get our Eq. (23) in Section 4.

3. GLUON DISTRIBUTION NEAR $x = 1$

Equation (20) for the gluon distribution is inhomogeneous. To solve it for $x \approx 1$ we first look at the homogeneous equation, obtained by putting $\phi^v = 0$. The solution of this can be worked out by the same method adopted in (A1)-(A9) for the solutions of the valence equations. In this way we find that the homogeneous equation:

$$\frac{d\phi^g}{dT} = C \phi^g \quad (A12)$$

has a family of asymptotic solutions of the form

$$g(T, T_0, E_0) = \frac{\Gamma(E_0)}{\Gamma[E_0 + \frac{3}{\pi b}(T-T_0)]} e^{\tilde{K}(T-T_0)} (1-x)^{E_0 + \frac{3}{\pi b}(T-T_0)-1} [1 + o(1-x)] \quad (A13)$$

We note that the effective exponent increases, for such solutions, *faster* than in those of Eq. (22). The inhomogeneous terms in Eq. (20) can be computed explicitly by substituting for ϕ^v the asymptotic solution in Eq. (22):

$$\mathcal{D}\phi^V = \frac{2}{3\pi b} \frac{\Gamma(A+1/2) \exp[KT]}{\Gamma(1/2) \Gamma[A(\tau)+1]} (1-x)^{A(\tau)} [1 + \sigma(1-x)] \quad (A14)$$

It is then easy to see that a general asymptotic solution for the inhomogeneous equation (20) is given by

$$\phi^g(x, Q^2) = \frac{2}{3\pi b} \frac{\Gamma(A+1/2)}{\Gamma(1/2)} \int_0^\tau d\tau' \frac{\exp K\tau'}{\Gamma[A(\tau')+1]} g(\tau, \tau', A(\tau')+1) + \tilde{\phi}^g(x, Q^2) \quad (A15)$$

where $\tilde{\phi}^c$ is a solution of the homogeneous equation (A12). By introducing the explicit expression (A13), and changing the integration variable, we have

$$\phi^g(x, Q^2) = \frac{\Gamma(A+1/2)}{\Gamma(1/2)} \frac{2}{5} e^{KT} (1-x)^{A(\tau)} \int_0^{\frac{5\tau}{3\pi b}} d\xi \frac{\exp[\xi(U + \ln(1-x))]}{\Gamma(A(\tau)+1+\xi)} + \tilde{\phi}^g(x, Q^2) \quad (A16)$$

where

$$U = 0.65 - G \simeq 0.073$$

Apart from logarithmic corrections, the first term on the right-hand side of Eq. (A16) has an effective exponent which is one unit larger than that of the valence distribution at the same Q^2 . At large enough Q^2 this exponent will be smaller than that of $\tilde{\phi}^c$, so that $\tilde{\phi}^c$ will become negligible for large Q^2 . For large Q^2 the integral in the first term on the right-hand side becomes²⁴⁾

$$\int_0^\infty \frac{e^{-a\xi}}{\Gamma(A(\tau)+1+\xi)} d\xi = e^{A(\tau)a} \gamma(e^{-a}, A(\tau)) \approx \frac{1}{\Gamma(A(\tau)+1)} \frac{1}{a} + \mathcal{O}\left(\frac{1}{a^2}\right) \quad (A17)$$

where we put $a = \ln[1/(1-x)] - U$. This leads to the result in Eq. (24).

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Table 1

Fitted values of different parameters in the model, compared to those of Ref. 1. E and Λ were not required in Ref. 1.

	Present fit	Fit in Ref. 1
β	0.310	0.35
a^2	0.80	0.91
A	0.847	0.80
D	3.29	1.4
C	0.082	0.03
E	2.4	
Λ	0.48 GeV	

Figure captions

- Fig. 1 : The function $\phi^V(x, Q^2 = 200)$ as computed by numerical integration of Eq. (18) is compared with the exact solutions for $x \approx 0$, Eq. (23), and $x \approx 1$, Eq. (22).
- Fig. 2 : Our fit for $\nu W_2^P(x)$ and $\nu W_2^d(x)$ at $Q^2 = 9 \text{ GeV}^2$ is compared with the experimental points, averaged over $8 \leq Q^2 \leq 10 \text{ GeV}^2$, taken from Ref. 8 ($\bullet$) and Ref. 9 ($\blacklozenge$). The point ($\blacklozenge$) is obtained averaging only over $8 \leq Q^2 \leq 9 \text{ GeV}^2$.
- Fig. 3 : The prediction of the model for the Q^2 dependence of νW_2^P at fixed x is compared to the data of Ref. 9 for three different values of x : $1/3$, $1/2$, and $2/3$.
- Fig. 4 : The prediction for $\nu W_2^P(x)$ and $\nu W_2^d(x)$ at $Q^2 = 3.5 \text{ GeV}^2$ is compared with experimental points taken from Ref. 8 ($\bullet$) averaged over $3 \leq Q^2 \leq 4 \text{ GeV}^2$ and Ref. 9 ($\blacklozenge$) averaged over $2.75 \leq Q^2 \leq 4 \text{ GeV}^2$. The point ($\blacklozenge$) is obtained averaging only on $3 \leq Q^2 \leq 4 \text{ GeV}^2$ and the points ($\bullet$) on $3.0 \leq Q^2 \leq 3.5 \text{ GeV}^2$.
- Fig. 5 : The ratio of our result for the cross-section $d\sigma^{\mu^+\mu^-}/dMdy$ to the central value of the experimental fit in Eq. (26) quoted in Ref. 10. The dashed area represents the experimental errors on the fit parameters.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5